Eksamensopgave 1

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Opgave 1

1. Explain crucial differences between the Solow model and the Ramsey, Cass and Koopman model. Does it affect the overall conclusions?

Solow antager konstant opsparingsrate (lille s), så kapitalakkumuleringen kan skrives som: $\dot{K}(t) = sY(t) - \delta K(t)$, da $I = S$

Den største forskel mellem Solow-modellen og Ramsey-Kass-Koopman modellen er, at sidstnævnte implementerer opsparing som en endogen faktor i modellen.

Solow-modellen holder opsparingsraten konstant, og således eksogent bestemt i modellen.

I Ramsey-modellen vælger husholdningerne den optimale opsparingsrate og denne er således endogeniseret. Her introduceres en utålmodighedsrate, ρ, og en rate der "smoother" forbruget, *theta*. Således vil opsparingsraten ikke længere være konstant udenfor steady state.

Derudover ligger der en forskel i de 2 modellers "balanced growth paths": Det er nemlig ikke muligt for Ramsey-Cass-Koopman at have en balanced growth path med kapitalniveau over golden rule niveauet (golden rule kapitalniveauet er det højeste niveau af forbrug som kan vedligeholdes. Det er givet ved $f'(k) = n + g$). Dette skyldes, at opsparingsraten er udledt af husholdningerne ud fra nyttemaksimering.

Udover ovenstående faktorer, er Ramsey- og Solow-modellen ens og der er ikke forskel på de overordnede konklusioner.

2. Piketty (2014) argues that a fall in the growth rate of the economy is likely to an increase in the difference between the real interest rate and the growth rate. This problem asks you to investigate this issue in the context of the Ramsey Cass Koopmans model. Specially, consider a Ramsey Cass Koopmans economy that is on its balanced growth path, and suppose there is a permanent fall in g.

2.1 How, if at all, does this affect the $\dot{k} = 0$ curve?

Det vides fra opgaven, at der sker et permanent fald i vækstraten g Her skal vi kigge på følgende ligning:

$$\dot{k}(t) = f(k(t)) - c(t) - (n + g)k(t)$$

.

Effekten skal udledes fra sidste led af ligningen:

$$-(n + g)k(t)$$

.

Da g falder bliver tallet større (mindre negativt). Vi ser ved denne omskrivning (hvor $\dot{k} = 0$),

$$c = f(k) - (n+g)k$$

at faldet i g medfører en stigning i $c(t)$ for at opretholde niveauet i $\dot{k}(t)$. Denne stigning i c skubber $\dot{k}$ kurven op og gør den dermed højere.

Derudover ved vi, at $f'(k) = n + g$ (ved at sætte $\dot{k} = 0$ og isolere f(k) og differentiere). Her ser vi, at et fald i g, får marginalproduktet af kapital ($f'(k)$) til at falde, hvilket får k til at stige, hvilket ligeledes forskyder kurven opad og udad.

(Inada betingelsen)

2.2 How, if at all, does this affect the $\dot{c} = 0$ curve?

Vi starter med Eulerligningen:

$$\frac{\dot{c}}{c} = \frac{r(t) - \rho - \theta g}{\theta}$$

Her ved vi, at $r(t)$ (realrenten) er lig med $f'(k(t))$ (marginalproduktet af kapital), da der ingen depreciering antages, hvorved vi også kan få:

$$\frac{\dot{c}}{c} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

Her sætter vi $\dot{c} = 0$ og isolerer for $f'(k(t))$:

$$f'(k(t)) = \rho + \theta g$$

Her ser vi, at et fald i g må betyde et tilsvarende fald i $f'(k(t))$. Dermed må det betyde at det k som er nødvendigt for at opretholde $\dot{c} = 0$ stiger, og således bliver kurven skubbet til højre, som det ses på billedet.

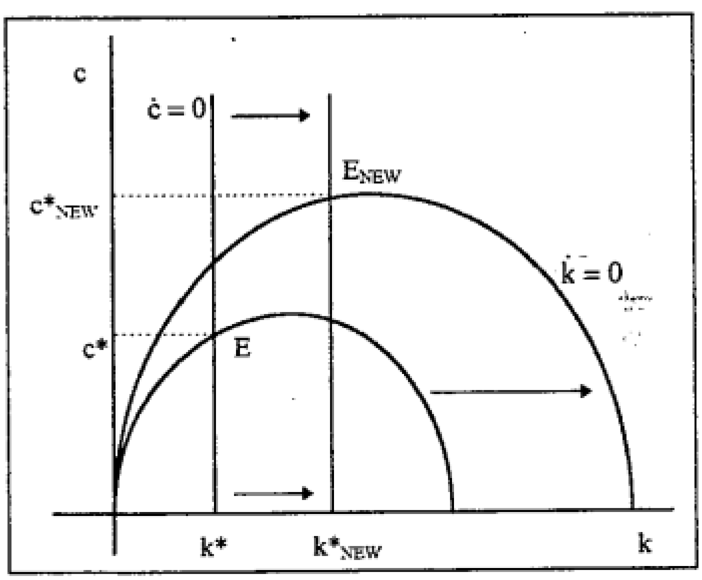

Figure 1: dotkc

2.3 At the time of the change, does c rise, fall, or stay the same, or is it not possible to tell?

At the time of change in g, the value of k, the stock of capital per unit of effective labor, is given by the history of the economy, and it cannot change discontinuosly. It remains equal to the k^* on the old balanced growth path.

In contrast, c, the rate at which households are consuming in units of effective labor, can jump at the time of the shock. In order for the economy to reach the new balanced growth path, c must jump at the instant of the change so that the economy is on the new saddle path.

However, we cannot tell whether the new saddle path passes above or below the original point E. Thus we cannot tell whether c jumps up or down and in fact, if the new saddle path passes right through point E, c might even remain the same at the instant the g falls. Thereafter, c and k rise gradually to their new balanced-growth-path values; these are higher than their values on the original balanced growth path.

2.4 At the time of the change, does r-g rise, fall, or stay the same, or is it not possible to tell?

If we assume that the shock is unexpected then the amount of capital is historically build up and will not imidially change when the shock is made to g. Therefor there will be no changes in $r(t)$ because there will be no changes in k therefor no changes in $f(k)$ and last no change in $f'(k)$ and we know $f'(k) = r(t)$. Therefor the distance will be larger.

2.5 In the long run, does r-g rise, fall, or stay the same, or is it not possible to tell?

Vi starter ud fra følgende ligning

$$\frac{\dot{c}}{c} = \frac{f'(k) - \rho - \theta g}{\theta}$$

Man sætter $\dot{c} = 0$ og isolerer $f(k)$ og differencierer og får:

$$f'(k) = \rho + \theta g$$

Vi ved også at $f'(k) = r$, så på lang sigt gælder det, at

$$r = \rho + \theta g$$

Her trækkes g fra på begge sider, så vi får

$$r - g = \rho + \theta g - g$$

Der faktoreres for g på højresiden

$$r - g = \rho + g(\theta - 1)$$

Til sidst differentierer vi for g og får:

$$\frac{\partial r - g}{\partial g} = \theta - 1$$

Dermed kan følgende konkluderes om θ (villighed til at ændre forbrug på tværs af perioder):

Hvis $\theta > 1$ vil

2.6 Find an expression for the impact of a marginal change in g on the fraction of output that is saved on the balanced growth path. Can one tell whether this expression is positive or negative?

2.7 For the case where the production function is Cobb-Douglas, $f(x) = k^{\alpha}$, rewrite your answer to part (2.6) in terms of ρ, n, g, θ and α (Hint: Use the fact that $f'(k^*) = \rho + \theta g$

3. Analyze the effect of a public procurement, including a thorough presentation of the dynamics in Figures 2.8 and 2.9

$T = G$ $\dot{k} = f(k) - C - T - (n+g)k$

$\dot{c} = 0$ kurven (den som er lodret) og 2 halvcirkler med $\dot{k} = 0$

Opgave 2 Diamond

Consider the following overlapping generations framework. Welfare is equal to:

$$U_t = u(C_{1t}) + \beta u(C_{2t+1})$$

with $\beta = \frac{1}{1+\rho} \leq$ the discount factor.

Assume the utility function is logarithmic:

$$u(C_{jt}) = lnC_{jt}$$

We will assume that the government implements a pension scheme, more specifically, a pay-as-you-go social security scheme, where the government taxes each young individual by an amount T , and uses that amount to pay benefits to old individuals. Hence, the budget constraints is given by:

$$C_{1t} + S_t = w - T$$

whereas period 2 consumption is givien by:

$$C_{2t+1} = S_t(1+r) + (1+n)T$$

Note that n - $L_t = (1+n)L_{t-1}$ represents population growth. Take the wage w and interest rate r as exogenous and solve the following:

Set up the intertemporal maximazation problem and derive the Euler equation:

sæt op lagrangian ((ln af?) nyttefunktionerne og lambda ganget med budgetrestriktionen) og for at finde euler ligningen differentiere man i forhold til den ene forbrug og sætter ind i den anden forbrug.

facit bliver: $\frac{1+r}{1+\rho} = \frac{c_2}{c_1}$

Derive S_t as a function of r, w and T. How does an increase in T affect savings (show mathematically)? Discuss the result.

Brug facit ovenfra: $\frac{1+r}{1+\rho} = \frac{c_2}{c_1}$