Videregående makroøkonomi E2022 - Opgavesæt 1

Exercise 1: - Solowmodel

Consider an economy with technical progress but without population growth that is on its balanced growth path. Now suppose there is a one-time jump in the number of workers.

1. At the time of the jump, does output per unit of effective labor rise, fall, or stay the same? Why?
2. After the initial change (if any) in output per unit of effective labor when the new workers appear, is there any further change in output per unit of effective labor ? If so, does it rise or fall ? Why?
3. Once the economy has again reached a balance growth path, is output per unit of effective labor higher, lower, or at the same as it was before the new workers appeared? Why?

Exercise 2: - Ramsey, Cass and Koopman

Piketty (2014) argues that a fall in the growth rate of the economy is likely to an increase in the difference between the real interest rate and the growth rate. This problem asks you to investigate this issue in the context of the Ramsey-Cass-Koopmans model. Specially, consider a Ramsey-Cass-Koopmans economy that is on its balanced growth path, and suppose there is a permanent fall in g.

1. How, if at all, does this affect the $\dot{k} = 0$-curve?
2. How, if at all, does this affect the $\dot{c} = 0$-curve?
3. At the time of the change, does c rise, fall, or stay the same, or is it not possible to tell?
4. At the time of the change, does $r - g$ rise, fall, or stay the same, or is it not possible to tell?
5. In the long run, does $r - g$ rise, fall, or stay the same, or is it not possible to tell?

Exercise 3: Diamond model

Consider the following overlapping generations framework. Welfare is equal to:

$$U_t = u(C_{1t}) + \beta u(C_{2t+1})$$

with $\beta = \frac{1}{1+\rho} \leq 1$ the discount factor.
Assume the utility function is logrithmic:

$$u(C_{jt}) = lnC_{jt}$$

.

We will assume that the government implements a pension scheme, more specifically, a pay-as-you-go social security scheme, where the government taxes each young individual by an amount T , and uses that amount to pay benefits to old individuals. Hence, the budget constraints is given by:

$$C_{1t} + S_t = w - T$$

whereas period 2 consumption is given by:

$$C_{2t+1} = S_t(1+r) + (1+n)T$$

. Note that n represents population growth. Take the wage w and interest rate r as exogenous and solve the following:

- Set up the intertemporal maximization problem and derive the Euler equation
- Derive S_t as a function of r, w and T. How does an increase in T affect savings (show mathematically)? Discuss the result.

Exercise 4: Endogenous growth theory:

b. Assume production function of final goods sector is given by:

$$Y(t) = [(1-a_K)K(t)^{\alpha}][A(t)(1-a_L)L(t)]^{1-\alpha}$$

where $1 - a_L$ and $1 - a_K$ is the proportion of labour and capital allocated for final production

Capital stock is given by:

$$\dot{K}(t) = sY(t)$$

Production function of R&D sector is given by:

$$\dot{A}(t) = B[a_K K(t)]^{\beta}[a_L L(t)]^{\gamma} A(t)^{\theta}$$

where $B > 0$ shows efficiency of research, $\gamma \in (0, 1)$ is the output elasticity of labour allocated in R&D, and $\theta \leq 1$ is a parameter describing the elasticity of existing knowledge (A) for the production of increases in the stock of knowledge.

Population growth is exogenous:

$$\frac{\dot{L}(t)}{L(t)} = n \text{ .... OR we can write } \dot{L}(t) = nL(t)$$

- Derive an expression for the growth rate of capital $g_K(t)$ and growth rate of technology $g_A(t)$
- Using the fact that $g_A(t)$ is constant overtime (i.e., $\frac{\partial g_A(t)}{\partial t} = 0$), derive an expression for the growth rate of technology g_A^* along a balanced growth path (steady state), for the case when $\theta + \beta < 1$? Intuitively, explain the equation