Outline

Probability

Probability rules (cont)

- A company gathers data on the gender of its employees and their volleyball membership status

```
volley1 <- matrix(c(30, 20, 60, 30, 20, 20), ncol = 2, byrow = TRUE)
colnames(volley1) <- c("Male", "Female")
rownames(volley1) <- c("Current", "Former", "Never")
volley1 <- as.table(volley1)
100 * prop.table(volley1)
```

```
##          Male Female
## Current 16.67  11.11
## Former  33.33  16.67
## Never   11.11  11.11
```

The above information represents probabilities. e.g., 16.7 is the joint probability of male and current membership, i.e., $P(male \cap current) = 0.16$, which means that 16.7% of the employees are male as well as current members of the club.

Exercise

Question: 1a

- **What is the probability that an employee is a male or current member of the volleyball club, i.e., find $P(male \cup current)$?**
- **Solution:**
  To find $P(male \cup current)$, we use the formula;

  $$P(male \cup current) = P(male) + P(current) - P(male \cap current)$$

  First we calculate the probabilites that we need in the above formula:
  - Probability of being a male: $P(male) = 0.16 + 0.33 + 0.11 = 0.611$
  - Probability of being a current member $P(current) = 0.16 + 0.11 = 0.27$
  - Probability of being a male and current member $P(male \cap current) = 0.16$
- Now plugin the values in the formula:

  $$P(male \cup current) = 0.61 + 0.27 - 0.16 = 0.72$$

Exercise

Question: 1a

- **What is the probability that an employee is a female or a former member of the volleyball club, i.e., find $P(female \cup never)$?**
- **Solution:**
  To find $P(female \cup never)$, we use the formula;

  $$P(female \cup never) = P(female) + P(never) - P(female \cap never)$$

  First we calculate the probabilites that we need in the above formula:
  - Probability of being a female: $P(female) = 1 - 0.61 = 0.39$
  - Probability of never being a member $P(never) = 0.22$
  - Probability of being a female and never a member $P(female \cap never) = 0.11$
- Now plugin the values in the formula:

  $$P(female \cup never) = 0.39 + 0.22 - 0.11 = 0.50$$

Exercise

Question: 1b

- **What is the probability that an employee who is a male is also a former club member?**
- **Solution:**
  Here we have a case of conditional probability. The condition is that given we have a male member, what is the probability that he is a former club member. The formula to compute P(former|male) is:

$$P(former|male) = \frac{P(former \cap male)}{P(male)} = \frac{0.33}{0.61} = 0.54$$

- **What is the probability that an employee who is a female is also a current club member?**
- **Solution:**
  Here we have a case of conditional probability. The condition is that given we have a female member, what is the probability that she is a current club member. The formula to compute P(current|female) is:

$$P(current|female) = \frac{P(current \cap female)}{P(female)} = \frac{0.11}{0.39} = 0.28$$

Exercise

Question: 2

- Given $P(A_1) = 0.4$, $P(B_1|A_1) = 0.6$, and $P(B_1|A_2) = 0.7$, what is the probability of $P(A_1|B_1)$?

- Given $P(A_1) = 0.4$, $P(B_1|A_1) = 0.6$, and $P(B_1|A_2) = 0.7$, what is the probability of $P(A_2|B_2)$?

Exercise

Question: 2

- Given $P(A_1) = 0.4$, $P(B_1|A_1) = 0.6$, and $P(B_1|A_2) = 0.7$, what is the probability of $P(A_1|B_1)$?
- Solution:
  From the given information we can only find $P(A_1|B_1)$ using Bayes' theorem. You will notice that you cannot directly the formula as we did in the simple conditional probability case.
  We use Bayes'theorem:
  $$P(A_1|B_1) = \frac{P(B_1|A_1).P(A_1)}{P(B_1)}$$
  But you can see the $P(B_1)$ is also not given. Therefore we have to apply the other version of Bayes' theorem:
  $$P(A_1|B_1) = \frac{P(B_1|A_1).P(A_1)}{P(B_1|A_1).P(A_1) + P(B_1|A_2)P(A_2)}$$
- For the above formula we have all the necessary information except $P(A_2)$, which is the complement of $P(A_1)$, which can very easily be calculated.

Exercise

Question: 2

- **Remember:** Before applying Bayes theorem, always follow the following steps:
  - Define the probabilities and conditional probabilities for the events defined in the problem.
  - Compute the complements of the probabilities.
  - Formally state and apply Bayes' theorem to compute the solution probability.
- In our question, we just need $P(A_2)$, which is: $1 - P(A_1) = 0.6$
- We now plug in the values in the formula:

$$P(A_1|B_1) = \frac{P(B_1|A_1).P(A_1)}{P(B_1|A_1).P(A_1) + P(B_1|A_2)P(A_2)} \tag{1}$$

$$= \frac{(0.6)(0.4)}{(0.6)(0.4) + (0.7)(0.6)} = \frac{0.24}{0.66} = 0.36$$

Exercise

Question: 2

- Given $P(A_1) = 0.4$, $P(B_1|A_1) = 0.6$, and $P(B_1|A_2) = 0.7$, what is the probability of $P(A_2|B_2)$?
- **Solution:**
  We use Bayes'theorem:
  $$P(A_2|B_2) = \frac{P(B_2|A_2)P(A_2)}{P(B_2)}$$
  Again, we do not know the $P(B_2)$. Therefore we have to apply the other version of Bayes' theorem:
  $$P(A_2|B_2) = \frac{P(B_2|A_2)P(A_2)}{P(B_2|A_2)P(A_2) + P(B_2|A_1)P(A_1)}$$
- For the above formula are missing the terms highlighted in red, but these can be easily calculated, given the information in the question:
  - $P(B_2|A_2) = 1 - P(B_1|A_2) = 0.3$
  - $P(B_2|A_1) = 1 - P(B_1|A_1) = 0.4$
  - $P(A_2) = 1 - P(A_1) = 0.6$
- $$P(A_2|B_2) = \frac{(0.3)(0.6)}{(0.3)(0.6) + (0.4)(0.4)} = \frac{0.18}{0.34} = 0.52$$

Exercise

Question: 3

A publisher sends advertising materials for an accounting text to 80% of all professors teaching the appropriate accounting course. 30% of the professors who received this material adopted the book, as did 10% of the professors who did not receive the material. What is the probability that a professor who adopts the book has received the advertising material?

- **solution:** Let us define the probabilities and conditional probabilities for the events defined in the problem
- R_1 denotes prof. who receives the book (then $R_2 = 1 - R_1$ is someone who does not receive the book)
- A_1 denotes the prof. who adopts that book (then $A_2 = 1 - A_1$ is someone who does not adopt the book)

$$P(R_1) = 0.8, \qquad P(R_2) = 0.2$$

$$P(A_1|R_1) = 0.3, \qquad P(A_2|R_1) = 0.7$$
$$P(A_1|R_2) = 0.1, \qquad P(A_2|R_2) = 0.9$$

Exercise

Question: 3

- **solution (cont):** We can now apply Bayes' theorem to find $P(R_1|A_1)$:

$$P(R_1|A_1) = \frac{P(A_1|R_1).P(R_1)}{P(A_1)}$$

Since we do not have $P(A_1)$, we apply the other version of Bayes' theorem:

$$P(R_1|A_1) = \frac{P(A_1|R_1).P(R_1)}{P(A_1|R_1).P(R_1) + P(A_1|R_2).P(R_2)}$$

$$P(R_1|A_1) = \frac{(0.3)(0.8)}{(0.3)(0.8) + (0.1)(0.2)} = \frac{0.24}{0.26} = 0.92$$

- So, the probability that a professor who adopts the book has received the advertising book is 92 percent.