# Modeling Volatility Dynamics in Commodity ETFs: A Comparative Study using VAR-RV and HAR Models

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**Abstract**

In this research, we investigate the volatility dynamics in the commodity exchange-traded fund (ETF) market, a rapidly expanding sector. We utilize the Bayesian Vector Autoregression (BVAR) and Heterogeneous Autoregressive (HAR) models to gain insights into market behavior. Our analysis confirms the effectiveness of both models in capturing volatility dynamics. Particularly, we emphasize the influence of commodity prices on the volatility of commodity ETFs, underlining their interlinked nature. In the BVAR model, we observe a predominant spillover effect from the indicative net asset value (iNAV) to the ETF. This suggests that the iNAV, more than just reflecting market movements, actively influences the ETF's volatility. The HAR model, with its time-varying betas, provides a detailed view of the changing risk exposures in this market, complementing our understanding of these dynamics. Our study contributes to existing literature by developing an intraday series for the ETF's NAV, or iNAV. We document a bidirectional spillover between the ETF and its iNAV for ETFs backed by energy futures contracts, while a more pronounced spillover from iNAV to ETF is observed for those backed by metal futures contracts.

**Keywords:** commodities, energy, futures, information diffusion, financialization, high-frequency, speculation, sustainable, commercial, institutional, volatility, macro, announcements, surprise, events.

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# 1 Introduction

In recent years, the commodity exchange-traded fund (ETF) market has witnessed significant growth and has become an increasingly important component of investors' portfolios. As a result, understanding the dynamics of volatility in this market has become crucial for market participants and researchers alike. This paper aims to provide a comprehensive analysis of volatility dynamics within the commodity ETF market by employing two widely used models: the Bayesian VAR-RV model and the HAR model.

The first key finding of our analysis is the strong performance of both the Bayesian VAR-RV and HAR models in capturing the volatility dynamics within the commodity ETF market. This suggests that these models offer valuable insights into the market's behavior and provide reliable measures of volatility.

Furthermore, our investigation reveals that the volatility observed in commodity ETFs is significantly influenced by the underlying commodity prices. This finding highlights the interconnectedness between commodity prices and the volatility experienced by investors in commodity ETFs.

When examining the Bayesian VAR-RV model, we uncover compelling evidence of changing relationships between the ETF and its net asset value (NAV) within the volatility process. This finding suggests that the NAV of commodity ETFs is not only impacted by changes in commodity prices but also by other factors that influence the relationship between the ETF and its underlying assets.

In addition, the HAR model employed in our analysis allows for the incorporation of time-varying betas, enabling a more flexible representation of the changing risk exposures within the commodity ETF market. Moreover, our research provides evidence of long memory in the volatility process, indicating that past volatility levels have a persistent influence on future volatility, thereby supporting the existence of long-term patterns in the commodity ETF market. The results of the HAR model also show that for metals ETFs, volatility propagation seems to be unidirectional (from NAV to ETF), whereas in the case of energy, propagation is rather bidirectional.

By conducting this comprehensive analysis of volatility dynamics in the commodity ETF market using the Bayesian VAR-RV and HAR models, our research aims to deepen our understanding of the market's behavior and provide valuable insights for investors, market participants, and policymakers.

# 2 Literatture Review

## 2.1 Arbitrage mechanism behind ETFs

ETFs are investment entities that issue securities that trade continuously on public exchanges. Unlike mutual funds, which only allow investors to purchase or redeem shares at the end of the trading day, ETFs allow investors to trade their shares continuously throughout the trading day. ETFs combine the features of both open-end and closed-end funds. The very fact that ETFs are traded continuously significantly reduces arbitrage opportunities.(**?**) The market price of ETF shares often diverges from the NAV of the underlying basket because of asynchronous trading of the ETF and the underlying assets. This fact can generate an opportunity for arbitrage between the ETF shares and the underlying basket of securities when the discrepancy exceeds the transaction costs. Two types of market participants are poised to benefit from such differences in prices: APs and secondary market arbitrageurs. APs are a small group of institutions that are allowed to trade directly with the ETF sponsor in the primary market. These transactions typically take place in kind, with securities being exchanged for ETF shares. If the price of ETF shares is below the NAV, arbitrageurs will buy ETF shares and sell the underlying securities. If the price of ETF shares is above the NAV, arbitrageurs will buy the underlying securities and exchange them for newly issued ETF shares. This creates downward pressure on the high priced asset and upward pressure on the low priced asset, preventing discrepancies from widening.

## 2.2 Impact of ETFs on underlying asset prices

### 2.2.1 ETFs improve price discovery

Some of the literature clearly shows the benefits of ETFs on the individual stock market. In other words, it shows that the presence of ETFs improves the efficiency of individual stock prices. **?** and **?** argue that ETFs are cost efficient and provide an easy tool for investors to make directional long bets, and the market benefits from this. As long as there is no friction in the capital markets for arbitrage, ETFs do not propagate shocks into securities, but rather expedite price discovery. The price discovery at the ETF level leads to price discovery at the underlying security level.

Several studies confirm empirically that ETFs enhance price discovery. **?** compare the comovement of S&P 500 futures, the main ETF on this index (SPY), and the underlying portfolio. They conclude that prices deviate little between the futures contract and ETFs but that there are larger deviations from the underlying portfolio. **?** find that stocks incorporate information more quickly once they are in ETF portfolios. They argue that some of the increased comovement of stocks with indices that has been documented by other researchers (see below) can be explained by better incorporation of systematic information into stock prices.

### 2.2.2 ETFs degrade price and information discovery

Subsequently, another portion of the literature argues that ETFs are harmful to individual stock markets. In other words, ETFs would degrade the informational efficiency of the securities in their baskets. **?** document increased comovement in returns in the stocks that are part of an index. They argue that when investors trade on news related to the index, they trade the ETF more actively. **?** show that stocks owned by ETFs have higher trading costs, have higher comovement with the index, exhibit lower informational efficiency and receive less analyst coverage. **?** argue that private firms are reluctant to list on stock exchanges because passive investors, primarily ETFs, slow down price discovery. **?** document that the degree and direction of mispricing between ETFs and their underlying securities comove across ETFs. Both groups conclude that ETFs attract short-horizon noise traders with correlated demand across investment styles. **?** provides further evidence that ETFs attract sentiment-driven noise traders..

### 2.2.3 ETF increase underlying stock liquidity

**?** document patterns that illustrate the activity of arbitrageurs. They find that the liquidity of ETFs is correlated with the liquidity of the underlying stocks. The more liquid the underlying stocks are, the greater the ability of arbitrageurs to engage in arbitrage trades, making the ETF liquid as well. **?** document that the liquidity of ETFs comoves with the liquidity of the assets in the ETF baskets. The authors show that higher ETF ownership is associated with higher comovement of liquidity12 among large and small stocks alike. They further document that this comovement of liquidity has increased in recent years and that it is greater during crisis versus non-crisis periods. **?** show that ownership by mutual funds increases the liquidity of the underlying bonds due to flows to and from the mutual funds, which induce trading.

### 2.2.4 ETF decrease underlying stock liquidity

**?** finds a significant deviation of ETF prices from those of the underlying assets, especially for illiquid assets. **?** documents that in some ETFs, the deviation from the value of the underlying assets is permanent, which he argues may be the result of market segmentation. Investors may be willing to pay a premium for access to assets with greater liquidity.

## 2.3 Behavior of ETFs during a market panic

During several episodes in recent years, ETFs have displayed a high level of illiquidity during times of market turbulence. The most famous example is the flash crash that took place on May 6, 2010. The reason for the higher frequency of flash crashes in the ETF market is that ETFs become illiquid during a financial panic. Borkovec, **?** report that the liquidity of ETFs declined dramatically during the crash: Spreads widened significantly, and the limit order book dried up. They interpret this finding as evidence that market participants exited the market once signs of extreme volatility and illiquidity appeared. As investors provide liquidity to sell their positions, this will affect the price discovery process between ETFs and their underlying assets. Subsequently, **?** argues that the departure of ETF prices from those of the underlying securities was rooted in the fragmentation of markets. Madhavan claims that stocks are more sensitive to liquidity shocks when markets are fragmented.

### 2.4 Commodity Price Volatility and ETF Volatility

The relationship between commodity price volatility and ETF volatility is a complex and multifaceted issue. Empirical evidence suggests that commodity price volatility significantly impacts the volatility of commodity ETFs (**?**). This impact is particularly pronounced during periods of uncertainty and shocks, such as those caused by global demand and oil supply shocks (**?**). Primary commodity prices are inherently more volatile than those of manufactured goods and services (**?**). The demand for ETFs has been identified as a driver of high fluctuations in the underlying assets and futures markets of commodities, further emphasizing the interplay between commodity prices and ETF volatility (**?**).

Additionally, the impact of ETFs on the prices of underlying assets has been well-documented, with evidence suggesting that ETFs may lead to increased price volatility and affect market efficiency (**?**). The exponential growth in exchange-traded fund (ETF) trading has made ETFs a significant factor in the volatility-generating process of their largest component stocks (**?**). This suggests that the relationship between underlying commodity prices and commodity ETF volatility is influenced by the broader dynamics of ETF trading and market interactions.

Moreover, the tracking performance and tracking error of ETFs are determined, in part, by the volatility of the underlying benchmark, indicating a direct link between underlying asset volatility and ETF behavior (**?**). Furthermore, the pricing deviation of ETFs from their underlying value can be influenced by ETF volatility, supply and demand imbalances, and other market dynamics (**??**).

Underlying commodity prices play a crucial role in the volatility of commodity ETFs. The relationship between commodity price volatility and ETF volatility is influenced by various factors, including uncertainty shocks, demand for ETFs, market efficiency, and tracking performance. Understanding and managing this relationship is essential for investors and market participants in the commodity ETF space.

## 2.5 Realized Variance

Realized variance (RV) is a commonly used measure of asset price variation constructed from high-frequency data. It provides an estimate of the volatility of an asset's returns over a specific time period. Several studies have examined the accuracy and performance of different estimators of realized variance and compared them to the benchmark 5-minute RV. **?** conducted a comprehensive analysis of almost 400 different estimators of asset price variation, including realized measures, applied to 31 different financial assets spanning five asset classes. They found little evidence that any of the other measures outperformed the 5-minute RV when it was taken as the benchmark realized measure. However, when using inference methods that do not require specifying a benchmark, they found some evidence that more sophisticated realized measures significantly outperformed the 5-minute RV. In forecasting applications, they found that a low-frequency "truncated" RV performed better than most other realized measures. **?** also examined the forecasting performance of different models using realized variance decomposed into its positive and negative parts (semivariances) and its continuous and discontinuous parts (jumps). They found that while these decompositions were important in predictive regressions, considering these components did not significantly improve the forecasting accuracy compared to a simple autoregressive specification using past realized variances as predictors. In the context of volatility spillovers, **?** used realized volatility (RV) to quantify stock market risk and developed a spillover model that divided RV into positive realized semi-variance (RS+) and negative realized semi-variance (RS-) based on positive or negative returns. This allowed them to analyze the asymmetric volatility spillovers between economic policy uncertainty and stock markets in China. Furthermore, **?** compared implied volatility (represented by VIX and VXO) with realized volatility and studied the distribution of their ratio. They found that the ratio was best fitted by heavy-tailed log-normal and fat-tailed power-law distributions, depending on the time period of realized variance used. Overall, while the benchmark 5-minute RV has shown to be difficult to significantly beat, more sophisticated realized measures and models that consider different components of realized variance have shown some potential for improved accuracy and forecasting performance in certain contexts.

## 2.6 Bayesian VAR-RV and HAR models in analyzing volatility

The Bayesian VAR-RV and HAR models have significantly contributed to understanding the behavior and volatility measures of the commodity ETF market. These models have been instrumental in capturing the dynamics of realized volatility (RV) and forecasting volatility in various financial markets, including commodity markets (**???**). Additionally, the HAR-type volatility forecasting models, which incorporate continuous and discontinuous (jump) components of volatility, have been widely used in studies of stock and commodity markets, further emphasizing their relevance in understanding commodity ETF market behavior (**?**). Moreover, these models have also been employed to analyze the transmission of volatility from different markets to the commodity ETF market.

The effectiveness of the Bayesian VAR-RV and HAR models in analyzing volatility has been extensively studied in the literature. These models have been shown to offer significant improvements over traditional parametric multivariate ARCH or stochastic volatility models, particularly at intraday frequencies (**?**). The HAR-RV model, in particular, has demonstrated superior performance in forecasting return volatility, outperforming other models such as GARCH-RV and ARFIMA-RV, especially in the context of high-frequency data (**?**). Additionally, the HAR-RV model has been found to be more accurate and stable in predicting long-term volatility, making it a valuable tool for analyzing and forecasting volatility in various financial markets, including the Chinese stock market and futures markets (**??**). Furthermore, the inclusion of jump components in the HAR-RV model has been shown to enhance its forecasting capabilities, as evidenced by its higher R2 values compared to other models (**?**).

While the HAR-RV model has demonstrated strong predictive power, it is essential to note that its performance may vary across different markets and time periods. For instance, comparative analysis has revealed that the HAR-CJN model outperformed the HAR-RV model in forecasting future volatility in the Chinese stock market (**?**). Moreover, the effectiveness of the HAR-RV model has been demonstrated in forecasting volatility in diverse financial instruments, including the SP 500 index, gold futures, and copper futures (**???**).

# 3 Data

In this comprehensive study, we delve into the behaviors of various Exchange-Traded Funds (ETFs), particularly focusing on commodity ETFs. The selected data spans from January 1, 2010, to January 1, 2023, offering a robust 13-year period for analysis. This time frame covers various market conditions, including periods of economic growth, downturns, and market volatilities, providing a rich context for evaluating ETF performances.

## 3.1 ETFs Analyzed:

**GLD (Commodity ETF - Gold):**

- Nature: GLD tracks the price of gold bullion, offering an investment vehicle that mirrors the movements in gold prices.
- Relevance: It's a popular choice for investors looking to hedge against inflation or seeking a safe-haven asset during economic uncertainties.
- Utility: GLD provides an accessible way to invest in gold without the need for physical ownership, making it a vital tool for portfolio diversification.

**SLV (Commodity ETF - Silver):**

- Nature: SLV aims to reflect the performance of silver, allowing investors to participate in the price fluctuations of this precious metal.
- Investment Focus: It's a strategic option for investors interested in commodities with both industrial applications and investment appeal.
- Utility: SLV caters to those who wish to invest in silver for its dual roles in industrial usage and as a store of value.

**USO (Commodity ETF - Oil):**

- Nature: USO tracks the performance of West Texas Intermediate (WTI) crude oil futures, providing exposure to oil price dynamics.
- Significance: As a crucial energy commodity, crude oil's price movements have broad implications for global economic conditions.

- Utility: USO offers a direct path for investors to gain exposure to the energy sector, reflecting the intricacies of oil supply, demand, and geopolitical factors.

**UNG (Commodity ETF - Natural Gas):**

- Nature: UNG is designed to mirror the price movements of natural gas, a key energy commodity.
- Role in Energy Markets: It's particularly relevant for understanding energy market dynamics, especially in heating and electricity generation.
- Investment Potential: UNG provides investors with a means to engage with natural gas market trends, without the complexities of direct commodity trading.

The ETF and NAV price data were obtained from Bloomberg, ensuring high-quality and reliable market information. For each ETF, we have extracted three different time series of price data:

- 1-minute series: This high-frequency data provides insights into the immediate market reactions and short-term volatility, offering a granular view of market dynamics.
- 5-minute series: Serving as a middle ground, this time series allows for analysis of slightly longer trends and patterns while still capturing the nuances of intraday movements.
- 30-minute series: This lower-frequency data helps in understanding longer-term trends and market sentiments, smoothing out the noise present in higher-frequency data.

# 4 Econometric framework and methods

The importance of volatility in asset returns is a central theme in asset pricing within financial literature. As such, financial market volatility plays a key role in various areas ranging from investment decisions to derivatives pricing and financial market regulation, according to **?**. The level and nature of volatility, which is a measure of market risk, generates anxiety for market participants. However, volatility is an unobservable variable, and its effects on financial markets are difficult to predict, making asset return volatilities of utmost importance in empirical finance. This is because they are essential in assessing asset or portfolio risk and play a critical role in asset pricing models that heavily rely on underlying asset return dynamics. As such, asset management and asset pricing models require proper volatility modeling of financial assets.

Various tools have been suggested for modeling time-varying volatility in the existing literature. For instance, **?** showed that ARCH models produce accurate inter-daily forecasts for the latent volatility in most financial applications. **?** found evidence that realized volatilities and correlations move together in a manner consistent with a latent factor structure. Realized volatility, a quadratic variation-like measure of activity in financial markets, has been used to investigate the stochastic properties of returns by **?**. They further suggested that realized bi-power variation estimates integrated variance in stochastic volatility models, providing a model-free and consistent alternative to realized variance in special cases.

**?** utilized a high-frequency futures dataset to examine the effects of news on financial market volatility and concluded that news produces conditional mean jumps. **?** used realized kernels to investigate efficient feasible inference on the ex post variation of underlying equity prices in the presence of simple models of market frictions. **?** proposed an estimator of the quadratic variation, which removes the effects of market microstructure noise, based on the theory of Markov chains and consistent with a Gaussian limit distribution.

In order to exploit the high frequency data on ETFs and to obtain a measure of variability quantifying the intraday variance, the use of the realized variance is necessary. Realized variance is a statistical measure of the volatility of financial returns, which is based on the observed or "realized" returns over a certain period of time. The realized variance is calculated by summing the squared returns over a certain time period and then scaling the sum by the number of returns.

## 4.1 Indicative Net Asset Value

In order to obtain a value for the intraday NAV, we will construct a series approximating the iNAV (Indicative Net Asset Value) for the 5 ETFs in question in this paper. The standard iNAV formula is:

$$iNAV = \frac{\sum_{i=1}^{n} x_{i,t} \times p_{i,t} + Cash_t}{CU_t} \tag{1}$$

Where

- $x_{i,t}$: = number of shares of component $i$ at datetime $t$
- $p_{i,t}$: price of component $i$ at datetime $t$
- $CU_t$: creation units at datetime $t$

## 4.2 Realized variance

The theoretical foundation of Realized Variance (RV) in a market model is rooted in classical diffusion models that incorporate jumps.The presentation of this theory is particularly pertinent as (Realized Variance) RV, along with Bipower Variation (BV), forms the core of our analysis in the Bayesian Vector Autoregression (VAR) and Heterogeneous Autoregressive (HAR) models. There is already plenty of literature reviewing the theoretical and practical foundation of the classical diffusion model without jumps (see, (**??????**)).

Let $P_t$ be the price of an asset and $S_t = \log(P_t)$ its log-transformation. We suppose that the following stochastic differential equation characterizes the log-price dynamics of this asset:

$$dS_t = \mu_t dt + \sigma_t dW_t + dJ_t \tag{2}$$

where $W_t$ is a Brownian motion, $J_t$ is a jump process. $\mu_t$ is the drift and $\sigma_t$ the instantaneous stochastic volatility, strictly positive and square integrable.

The quadratic variation of a stochastic process as shown in **?** is the following:

$$[X]_T = QV_T = \lim_{n\to\infty} \sum_{i=1}^{n} (X_{t_i} - X_{t_{i-1}})^2 \tag{3}$$

In the context of the market model with jumps, the quadratic variation is equivalent to the following formulation for the whole duration of the process:

$$[S]_T = QV_T = \int_0^T \sigma_t^2 ds + \sum_{i=1}^{N_T} Y_i^2 \tag{4}$$

where $Y_i$ corresponds to the size of jump $i$, and $N_T$ to the total number of jumps on the whole trajectory.

Let an intraday return between $t$ and $t + \tau$ be:

$$r_t = S_{t+\tau} - S_{t-(i-1)\tau}, \quad i \in [1, n] \tag{5}$$

and the realized variance, which is a good proxy for the quadratic variation:

$$RV_{t+\tau} = \sum_{i=1}^{n} r_{t+i\tau}^2 \tag{6}$$

It follows that the daily quadratic variance from $t$ to $t+T$ satisfies:

$$
\begin{aligned}
QV_{t,t+\tau} &= [S]_{t+\tau} - [S]_t && (7)\\
&= \lim_{n\to\infty} \sum_{i=1}^{n} (S_{t+i\frac{\tau}{n}} - S_{t+(i-1)\frac{\tau}{n}})^2 && (8)\\
&= \lim_{n\to\infty} \sum_{i=1}^{n} r_{t+i\frac{\tau}{n}} && (9)\\
&= RV_{t,t+\tau} && (10)
\end{aligned}
$$

Therefore the daily quadratic variation for the jump process is:

$$QV_{t,t+\tau} = \int_t^{t+\tau} \sigma_s^2 ds + \sum_{i=N_i}^{N_{t+\tau}} Y_i^2 \tag{11}$$

**?** show that the quadratic variation attributable to the continuous part of the process can be estimated by the bipower variation as follows:

$$BV_{t,t+\tau} = \frac{\pi}{2} \sum_{i=2}^{n} \mid r_{t+i\frac{\tau}{n}} r_{t+(i-2)\frac{\tau}{n}} \mid \tag{12}$$

The advantage of this measure is that the realized variance is based on actual returns rather than assumptions about the distribution of returns or market expectations.

## 4.3 HAR-RV Model

Despite its simplicity, the HAR model proposed by Corsi (2009) can produce rich dynamics for the variance that closely resemble the empirical data. The HAR is based on the heterogeneous market hypothesis and the asymmetric propagation of variance between long and short-time horizons. To be more precise, the HAR contains three components of variance. The short-time component represents one-day variance, the medium time component corresponding to the average variance over a week, and finally, the long-term component for the monthly average variance. The heterogeneous market hypothesis claims that the three components represent the behavior of different types of market participants with different investment time horizons. The basic specification in Corsi (2009) is as follows.

$$log(RV_t) = \beta_1 log(RV_{t-1}) + \beta_2 log(\overline{RV}_{t-5}) + \beta_3 log(\overline{RV}_{t-22}) \tag{13}$$

where $RV_t$ is the realized variance for day t, $RV_{t-1}$ is the realized variance from the previous trading day, $\overline{RV}_{t-5}$ is the average realized variance from the previous five trading days, $\overline{RV}_{t-22}$ is the average realized variance from the previous 22 trading days. The average realized variance over the past n days is defined as follows:

$$\overline{RV}_{t-n} = \frac{1}{n} \sum_{i=1}^{n} RV_{t-i} \tag{14}$$

In addition, With the growing literature on jumps in asset prices, **?** and **?** decompose an asset's price process into two components: the continuous component C and the jump component J. In their study, **?** rely on the bipower variation proposed by **??**. The jump component on day t, $J_t$, is estimated as the difference between the realized variance and the realized bipower variation. In theory, the realized variance should always be greater than the bipower variation, but since it is not always the case in practice, $J_t$ should be limited to positive numbers:

$$J_t = max(RV_t - BV_t, 0) \tag{15}$$

The continuous part of the process is the difference between the quadratic variation, estimated by the realized variance, and the jump component:

$$C_t = RV_t - J_t \tag{16}$$

$$\overline{BV}_{t-n} = \frac{1}{n}\sum_{i=1}^{n} BV_{t-i} \tag{17}$$

$$\bar{J}_{t-n} = \frac{1}{n}\sum_{i=1}^{n} J_{t-i} \tag{18}$$

Descriptive statistics concerning realized variance, quadractic bipower variation and jumps are presented in tables 1,2 and 3 with frequencies of 5 minutes, 1 minute and 30 minutes respectively.

Here are the HAR models we'll be using to model the propagation of volatility from ETF to NAV and from NAV to ETF. We'll also look at how the presence of jumps can lead to an increase in realizer variance in the following period.

#### 4.3.1 Model 1: HAR-X Model (impact on NAV)

$$\begin{aligned}\log(RV_{t,NAV}) = \beta_1 \log(RV_{t-1,NAV}) + \beta_2 \log(\overline{RV}_{t-5,NAV}) + \beta_3 \log(\overline{RV}_{t-22,NAV}) \\ + \alpha_1 \log(RV_{t-1,ETF}) + \epsilon_t\end{aligned}$$

- $log(RV_{t,NAV})$: Logarithm of Realized Volatility of NAV returns at time t.
- $\beta_1$, $\beta_2$, $\beta_3$: Coefficients for lagged NAV realized volatilities.
- $\alpha_1$: Coefficient for lagged ETF realized volatility.
- $\epsilon_t$: Error term.

In this model, the $\alpha_1$ coefficient helps us determine if there is a spillover effect from the ETF returns to the NAV returns.

#### 4.3.2 Model 1: HAR-X Model (impact on ETF)

$$\begin{aligned}\log(RV_{t,ETF}) = \beta_1 \log(RV_{t-1,ETF}) + \beta_2 \log(\overline{RV}_{t-5,ETF}) + \beta_3 \log(\overline{RV}_{t-22,ETF}) \\ + \alpha_1 \log(RV_{t-1,NAV}) + \epsilon_t\end{aligned}$$

- $log(RV_{t,ETF})$: Logarithm of Realized Volatility of ETF returns at time t.
- $\beta_1$, $\beta_2$, $\beta_3$: Coefficients for lagged ETF realized volatilities.
- $\alpha_1$: Coefficient for lagged NAV realized volatility.
- $\epsilon_t$: Error term.

In this model, the $\alpha_1$ coefficient helps us determine if there is a spillover effect from the NAV returns to the ETF returns.

#### 4.3.3 Model 2: HAR-CJ-X Model (impact on NAV)

$$\begin{aligned}\log(RV_{t,NAV}) = \beta_1 \log(QPV_{t-1,NAV}) + \beta_2 \log(\overline{QPV}_{t-5,NAV}) + \beta_3 \log(\overline{QPV}_{t-22,NAV}) \\ + \beta_4 \log(J_{t-1,NAV}) + \beta_5 \log(\bar{J}_{t-5,NAV}) + \beta_6 \log(\bar{J}_{t-22,NAV}) \\ + \alpha_1 \log(QPV_{t-1,ETF}) + \alpha_2 \log(J_{t-1,ETF}) + \epsilon_t\end{aligned}$$

- $log(RV_{t,NAV})$: Logarithm of Realized Volatility of NAV returns at time t.
- $\beta_1$, $\beta_2$, $\beta_3$: Coefficients for lagged NAV realized volatilities.
- $\beta_4$, $\beta_5$, $\beta_6$: Coefficients for lagged NAV-related variables (QPV and J).
- $\alpha_1$, $\alpha_2$: Coefficients for lagged ETF-related variables (QPV and J).

- $\epsilon_t$: Error term.

In this model, the $\alpha_1$ and $\alpha_2$ coefficients collectively help us examine if there is a spillover effect from the ETF returns to the NAV returns.

#### 4.3.4 Model 2: HAR-CJ-X Model (impact on ETF)

$$
\begin{aligned}
\log(RV_{t,ETF}) = {} & \beta_1 \log(QPV_{t-1,ETF}) + \beta_2 \log(\overline{QPV}_{t-5,ETF}) + \beta_3 \log(\overline{QPV}_{t-22,ETF}) \\
& + \beta_4 \log(J_{t-1,ETF}) + \beta_5 \log(\bar{J}_{t-5,ETF}) + \beta_6 \log(\bar{J}_{t-22,ETF}) \\
& + \alpha_1 \log(QPV_{t-1,NAV}) + \alpha_2 \log(J_{t-1,NAV}) + \epsilon_t
\end{aligned}
$$

- $log(RV_{t,ETF})$: Logarithm of Realized Volatility of ETF returns at time t.
- $\beta_1$, $\beta_2$, $\beta_3$: Coefficients for lagged ETF realized volatilities.
- $\beta_4$, $\beta_5$, $\beta_6$: Coefficients for lagged ETF-related variables (QPV and J).
- $\alpha_1$, $\alpha_2$: Coefficients for lagged NAV-related variables (QPV and J).
- $\epsilon_t$: Error term.

In this model, the $\alpha_1$ and $\alpha_2$ coefficients collectively help us examine if there is a spillover effect from the NAV returns to the ETF returns.

### 4.4 Vector Autoregressive Modelling

The use of Vector Autoregressive (VAR) models is essential for understanding the complex interactions between variables in time series data. These models are also incredibly useful for forecasting purposes, as they can take into account the relationships between variables when making predictions.

Researchers continue to build on the seminal work of **?** to further develop the VAR framework. This has led to the development of statistical tests to identify dependencies and the nature of dynamics between variables. Additionally, recent innovations in the field include the use of structural decompositions, sign restrictions, time-varying parameters, structural breaks, and stochastic volatility.

IThe Bayesian Vector Autoregressive (BVAR) approach has gained prominence due to its ability to incorporate prior information into the model, allowing for a more nuanced and informed analysis **?**. This incorporation of prior beliefs with observed data enables BVAR models to yield more robust and credible inferences about the dynamic relationships among variables **?**. BVAR methodology introduces flexibility in modeling time series data, particularly in stabilizing estimates in the presence of multicollinearity or model overfitting **?**. This is crucial in macroeconomic forecasting, where the number of potential predictors often exceeds the number of observations (Koop, 2011). Additionally, BVAR models are user-friendly and provide an accessible reference implementation, making them valuable tools for researchers and practitioners **?**. Furthermore, the Bayesian framework facilitates the incorporation of structural changes and non-linearities in the data, which are often encountered in real-world scenarios **?**. The BVAR approach has been found to outperform standard models in forecasting household credit, demonstrating its superiority in predictive performance **?**. In summary, the BVAR approach offers several advantages, including the ability to handle small sample sizes, model complex structures, stabilize estimates in the presence of multicollinearity, and incorporate structural changes and non-linearities in the data. These advantages make BVAR models a valuable tool for analyzing and forecasting macroeconomic and financial time series data.

Another notable advantage of the BVAR approach is its ability to quantify the uncertainty of forecasts and model estimates. Through the use of posterior distributions for the model parameters, researchers can obtain not only point estimates but also credible intervals, which provide a range of plausible values for the forecasts and model coefficients. This aspect is particularly valuable in policy analysis and risk assessment, where understanding the range of possible outcomes is as important as the outcomes themselves.

The VAR model is represented as follows:

$$\begin{bmatrix} RV_{t,ETF} \\ RV_{t,NAV} \end{bmatrix} = \Phi_0 + \Phi_1 \begin{bmatrix} RV_{t-1,ETF} \\ RV_{t-1,NAV} \end{bmatrix} + \Phi_2 \begin{bmatrix} RV_{t-2,ETF} \\ RV_{t-2,NAV} \end{bmatrix} + \epsilon_t \tag{19}$$

Here, $\Phi_0$ is a $2 \times 1$ vector of intercepts, $\Phi_1$ and $\Phi_2$ are $2 \times 2$ coefficient matrices for the lags, and $\epsilon_t$ is a $2 \times 1$ vector of error terms assumed to follow a multivariate normal distribution.

To estimate the parameters $\Phi_0$, $\Phi_1$, and $\Phi_2$ in a Bayesian framework, we need to specify prior distributions for these parameters and update them based on observed data.

The Bayesian VAR estimation involves the following steps:

#### 4.4.1 Prior Distributions

We specify prior distributions for the parameters $\Phi_0$, $\Phi_1$, and $\Phi_2$. These priors can be chosen based on prior knowledge or beliefs about the parameters. For simplicity, we can assume normal priors:

$$\begin{aligned} \Phi_0 &\sim \mathcal{N}(\mu_0, \Sigma_0) \\ \Phi_1 &\sim \mathcal{N}(\mu_1, \Sigma_1) \\ \Phi_2 &\sim \mathcal{N}(\mu_2, \Sigma_2) \end{aligned} \tag{20}$$

#### 4.4.2 Likelihood

We model the likelihood of the data as a multivariate normal distribution:

$$\begin{bmatrix} RV_{t,ETF} \\ RV_{t,NAV} \end{bmatrix} \sim \mathcal{N}\left( \Phi_0 + \Phi_1 \begin{bmatrix} RV_{t-1,ETF} \\ RV_{t-1,NAV} \end{bmatrix} + \Phi_2 \begin{bmatrix} RV_{t-2,ETF} \\ RV_{t-2,NAV} \end{bmatrix}, \Sigma \right) \tag{21}$$

#### 4.4.3 Posterior Distribution

Using Bayes' theorem, we update our prior beliefs with observed data to obtain the posterior distribution for the parameters:

$$P(\Phi_0, \Phi_1, \Phi_2 | \text{Data}) \propto P(\text{Data} | \Phi_0, \Phi_1, \Phi_2) \cdot P(\Phi_0) \cdot P(\Phi_1) \cdot P(\Phi_2) \tag{22}$$

The posterior distribution reflects our updated knowledge about the parameters given the data.

#### 4.4.4 Gibbs Sampling in Bayesian VAR Estimation

We can draw samples from the posterior distribution using Markov Chain Monte Carlo (MCMC) methods, such as Gibbs sampling or Metropolis-Hastings, to estimate the parameters $\Phi_0$, $\Phi_1$, and $\Phi_2$. In our case, we'll use Gibbs sampling. Gibbs sampling is a Markov Chain Monte Carlo (MCMC) method used in Bayesian VAR estimation to draw samples from the joint posterior distribution of the parameters $\Phi_0$, $\Phi_1$, and $\Phi_2$ given the data. In this context, we are using Gibbs sampling to estimate the parameters of a VAR model with two variables, $RV_{t,ETF}$ (Realized Volatility of ETF returns) and $RV_{t,NAV}$ (Realized Volatility of Net Asset Value returns), each with two lags.

The Gibbs sampling algorithm involves iteratively sampling each parameter from its conditional posterior distribution while keeping the other parameters fixed. Here's how it works:

**Step 1: Initialize Parameters**

Start with initial values for the parameters $\Phi_0$, $\Phi_1$, and $\Phi_2$.

**Step 2: Sample $\Phi_0$**

To sample $\Phi_0$ from its conditional posterior distribution, you need to compute the following conditional distribution:

$$P(\Phi_0|\Phi_1, \Phi_2, \text{Data}) \propto P(\text{Data}|\Phi_0, \Phi_1, \Phi_2) \cdot P(\Phi_0) \tag{23}$$

In this step, you treat $\Phi_1$ and $\Phi_2$ as constants and sample $\Phi_0$ from its posterior distribution. You can use standard statistical software or algorithms to perform this sampling.

**Step 3: Sample $\Phi_1$ and $\Phi_2$**

Similarly, to sample $\Phi_1$ and $\Phi_2$, you need to compute their conditional posterior distributions:

$$\begin{aligned} P(\Phi_1|\Phi_0, \Phi_2, \text{Data}) &\propto P(\text{Data}|\Phi_0, \Phi_1, \Phi_2) \cdot P(\Phi_1) \\ P(\Phi_2|\Phi_0, \Phi_1, \text{Data}) &\propto P(\text{Data}|\Phi_0, \Phi_1, \Phi_2) \cdot P(\Phi_2) \end{aligned} \tag{24}$$

**Step 4: Repeat**

Repeat steps 2 and 3 for a large number of iterations. The samples drawn from each step will converge to samples from the joint posterior distribution of $\Phi_0$, $\Phi_1$, and $\Phi_2$.

**Step 5: Posterior Analysis**

After collecting a sufficient number of samples, you can perform posterior analysis on the samples to estimate the parameters and assess their uncertainty.

Gibbs sampling allows you to incorporate prior information, capture parameter dependencies, and estimate the uncertainty associated with the parameter estimates in Bayesian VAR modeling.

## 5 Result

Tables 4, 5, and 6 provide detailed summaries of the HAR-X model results for the 5-minute, 1-minute, and 30-minute realized variance, respectively. It is important to note that the results we are about to detail are valid for all three frequencies used

- **Crude Oil:** There exists a significant positive relationship between past NAV volatility, both short-term and long-term, and current NAV and ETF volatility. Coefficients for $RV_{t-1,NAV}$, $\overline{RV}_{t-5,NAV}$, and $\overline{RV}_{t-22,NAV}$ are positive and statistically significant, indicating strong predictive power of past NAV volatility on current NAV and ETF volatility. The influence of past ETF volatility ($RV_{t-1,ETF}$) on current volatility is weaker but still significant compared to NAV's effect.

- **Gold:** Similar to Crude Oil, a strong and significant positive relationship is observed between past NAV volatility and current NAV and ETF volatility. Short-term NAV volatility ($RV_{t-1,NAV}$) has a particularly strong impact on ETF volatility. However, the influence of past ETF volatility ($RV_{t-1,ETF}$) on NAV volatility is mixed, with a negative and significant effect.

- **Silver:** This commodity exhibits a significant positive relationship between past NAV volatility and current NAV and ETF volatility, although the effects are generally weaker compared to Crude Oil and Gold. Past ETF volatility doesn't significantly affect current volatility, suggesting a weaker or non-existent spillover from ETF to NAV in the Silver market.

- **Natural Gas:** The results indicate a strong relationship between past NAV volatility and current NAV and ETF volatility. Past ETF volatility has a more pronounced influence on current volatility in Natural Gas compared to Silver, with significant positive coefficients for both short-term and long-term ETF volatility.

Overall, these results indicate a notable spillover effect of volatility from NAV to ETF across all commodities, with varying degrees of influence. The spillover from ETF to NAV is more mixed and commodity-dependent. The results suggest that market participants in ETFs are likely responding to the volatility observed in the NAV, and to a lesser extent, the reverse is also true, especially in commodities like Natural Gas and Crude Oil.

## 5.1 HAR-CJ-X Model (Table 7)

Tables 7, 8, and 9 provide detailed summaries of the HAR-CJ-X model results for the 5-minute, 1-minute, and 30-minute realized variance, respectively. It is important to note that the results we are about to detail are valid for all three frequencies used

- **Crude Oil:**
  - QPV: The coefficients for $QPV_{t-1,NAV}$ and $QPV_{t-1,ETF}$ suggest a minimal influence of quadratic bipower variation on current volatility. The negative coefficient for $\overline{QPV}_{t-22,NAV}$ indicates a possible inverse relationship over a longer-term.
  - Jumps: Significant positive coefficients for $J_{t-1,NAV}$, $\overline{J}_{t-5,NAV}$, and $\overline{J}_{t-22,NAV}$ suggest that jumps in NAV greatly influence current volatility in both NAV and ETF. The effect of jumps in ETFs is also significant but less pronounced.
- **Gold:**
  - QPV: Mixed coefficients, suggesting a complex or non-linear relationship between quadratic bipower variation and current volatility.
  - Jumps: Strong positive relationship between jumps in NAV and current volatility, indicating that sudden large movements in NAV are critical in predicting current market volatility in both NAV and ETF. Jumps in ETFs also show some impact but are less consistent.
- **Silver:**
  - QPV: Mixed coefficients, generally indicating a weaker relationship between quadratic bipower variation and current volatility.
  - Jumps: Jumps in NAV have a significant positive impact on current volatility, underscoring the importance of large, sudden movements in NAV. The impact of ETF jumps is less clear.

- **Natural Gas:**
  - QPV: Mostly positive coefficients, suggesting some level of influence, particularly over the longer term.
  - Jumps: Unlike other commodities, jumps in NAV do not show a consistent significant impact on current volatility. However, the impact of jumps in ETFs appears to be more significant.

## 5.2 Bayesian Vector Autoregression (VAR)-USO (Table 10)

**NAV Volatility Analysis ($log(RV_{t,NAV})$):** The model reveals a strong positive correlation between the NAV's previous day's volatility and its current day volatility, indicated by a mean of 0.5704. This suggests that the NAV's immediate past volatility is a strong predictor of its current volatility. A significant, albeit smaller, positive effect is seen from the NAV's volatility two days ago on its current volatility, with a mean of 0.2606. This indicates that the influence of past volatility, while still significant, diminishes with time. The positive mean of 0.0735 suggests a smaller yet significant impact of the ETF's previous day volatility on the current NAV volatility. This implies that the ETF's immediate past volatility modestly influences the NAV's current volatility. With a modest positive mean (0.0156), this indicates a small but present influence of the ETF's volatility from two days ago on the current NAV volatility.

**ETF Volatility Analysis ($log(RV_{t,ETF})$):** A moderately strong positive correlation is observed, with a mean of 0.2861, suggesting that the NAV's volatility from the previous day significantly influences the current day's ETF volatility. This indicates a notable positive influence (mean of 0.1934) of the NAV's volatility from two days ago on the current day's ETF volatility, though slightly less impactful than the immediate past volatility. A significant positive mean of 0.2941 indicates that the ETF's own volatility from the previous day plays a substantial role in determining its current volatility. The positive mean (0.1057) suggests a smaller, yet existing, influence of the ETF's volatility from two days ago on its current volatility.

The Bayesian VAR model results for USO ETF and NAV highlight the interconnectedness of their volatilities. Both the immediate and two-day past volatilities of NAV and ETF significantly influence their current volatilities. The most pronounced effects are observed for the immediate past volatilities (Lag 1), indicating stronger short-term effects. The impacts from two days prior (Lag 2), though still significant, are relatively less influential, pointing to a diminishing effect of past volatilities over time.

## 5.3 Bayesian Vector Autoregression (VAR)-GLD (Table 11)

**NAV Volatility Analysis ($log(RV_{t,NAV})$):** The model reveals a strong positive correlation between the NAV's previous day's volatility and its current day volatility, indicated by a mean of 0.5506. This suggests that the NAV's immediate past volatility is a strong predictor of its current volatility. A significant, albeit smaller, positive effect is seen from the NAV's volatility two days ago on its current volatility, with a mean of 0.2480. This indicates that the influence of past volatility, while still significant, diminishes with time. The negative mean (-0.0101) suggests a negligible or slightly inverse relationship between the ETF's previous day volatility and the current NAV volatility. This implies that the ETF's immediate past volatility does not significantly influence the NAV's current volatility. With a modest positive mean (0.0383), this indicates a small but present influence of the ETF's volatility from two days ago on the current NAV volatility.

**ETF Volatility Analysis ($log(RV_{t,ETF})$):** A moderately strong positive correlation is observed, with a mean of 0.3487, suggesting that the NAV's volatility from the previous day significantly influences the current day's ETF volatility. This indicates a notable positive influence (mean of 0.2758) of the NAV's volatility from two days ago on the current day's ETF volatility, though slightly less impactful than the immediate past volatility. A significant positive mean of 0.1549 indicates that the ETF's own volatility from the previous day plays a substantial role in determining its current volatility. The positive mean (0.0480) suggests a smaller, yet existing, influence of the ETF's volatility from two days ago on its current volatility.

The Bayesian VAR model results underscore a notable interdependence between the volatilities of GLD's NAV and ETF. The immediate past (one-day lag) volatilities of both NAV and ETF exhibit the strongest predictive power for their current volatilities. This highlights a significant short-term memory effect in both markets. The two-day lag effects, while still influential, show a decreasing impact, suggesting a diminishing influence of past volatility over time.

### 5.4 Bayesian Vector Autoregression (VAR)-SLV (Table 12)

**NAV Volatility Analysis ($log(RV_{t,NAV})$):** The model reveals a very strong positive correlation between the NAV's previous day's volatility and its current day volatility, indicated by a mean of 0.6192. This suggests that the NAV's immediate past volatility is a highly influential predictor of its current volatility. A significant, albeit smaller, positive effect is observed from the NAV's volatility two days ago on its current volatility, with a mean of 0.2144. This indicates that while the influence of past NAV volatility is still significant, it diminishes over time. The negative mean (-0.0446) suggests a small inverse relationship or negligible impact of the ETF's previous day volatility on the current NAV volatility. This implies that the ETF's immediate past volatility might not be a strong factor influencing the NAV's current volatility. A modest positive mean (0.0584) indicates a small but present influence of the ETF's volatility from two days ago on the current NAV volatility.

**ETF Volatility Analysis ($log(RV_{t,ETF})$):** A moderately strong positive correlation is observed, with a mean of 0.3760, suggesting that the NAV's volatility from the previous day significantly influences the current day's ETF volatility. This indicates a notable positive influence (mean of 0.2016) of the NAV's volatility from two days ago on the current day's ETF volatility, though slightly less impactful than the immediate past volatility. A significant positive mean of 0.1790 indicates that the ETF's own volatility from the previous day is an important factor in determining its current volatility. The positive mean (0.0777) suggests a smaller yet existing influence of the ETF's volatility from two days ago on its current volatility.

The Bayesian VAR model results for SLV ETF and NAV underscore the interconnectedness of their volatilities. Immediate past volatilities (Lag 1) of both NAV and ETF are shown to have the most significant predictive power for their current volatilities. The effects from two days prior (Lag 2), while still influential, are less pronounced, indicating a diminishing effect of past volatilities over time.

## 5.5 Bayesian Vector Autoregression (VAR)-UNG (Table 13)

**NAV Volatility Analysis ($log(RV_{t,NAV})$):** The model reveals a strong positive correlation between the NAV's previous day's volatility and its current day volatility, indicated by a mean of 0.3919. This suggests that the NAV's immediate past volatility is a significant predictor of its current volatility. A notable positive effect is seen from the NAV's volatility two days ago on its current volatility, with a mean of 0.2313. This indicates that while the influence of past NAV volatility is still significant, it is less pronounced compared to the immediate past. The positive mean of 0.0799 suggests a modest yet noticeable impact of the ETF's previous day volatility on the current NAV volatility. This implies that the ETF's immediate past volatility has a perceptible influence on the NAV's current volatility. With a positive mean of 0.1567, this indicates a significant influence of the ETF's volatility from two days ago on the current NAV volatility.

**ETF Volatility Analysis ($log(RV_{t,ETF})$):** A positive correlation is observed, with a mean of 0.1297, suggesting that the NAV's volatility from the previous day has a moderate influence on the current day's ETF volatility. This indicates a substantial positive influence (mean of 0.1653) of the NAV's volatility from two days ago on the current day's ETF volatility, highlighting the lasting impact of NAV volatility on the ETF. A significantly high positive mean of 0.3388 indicates that the ETF's own volatility from the previous day is a major factor in determining its current volatility. The positive mean (0.2396) suggests a notable influence of the ETF's volatility from two days ago on its current volatility.

The Bayesian VAR model results for UNG ETF and NAV demonstrate a complex interplay between their volatilities. Both the immediate and two-day past volatilities of NAV and ETF significantly influence their current volatilities. The most pronounced effects are observed for the immediate past volatilities (Lag 1), indicating stronger short-term effects. The impacts from two days prior (Lag 2) are also significant but to a lesser extent, pointing to a diminishing but still relevant effect of past volatilities over time.

## 5.6 Interpretation of results

Across all ETFs analyzed (GLD, SLV, USO, UNG), a consistent and striking observation is the significant impact of the immediate past volatility (Lag 1) on the current day's volatility. This finding underscores the importance of short-term market movements and investor reactions in shaping the current market dynamics. The high dependency on the recent past suggests that traders and market participants respond quickly to changes and that these markets exhibit a strong memory effect. While the two-day past volatilities (Lag 2) also influence current volatilities, their impact is generally less pronounced compared to the immediate past. This trend indicates that as the market moves forward, the relevance of past volatility data decreases, highlighting a diminishing memory effect in these markets. This observation aligns with the common market perception that more recent information is often more valuable for predicting short-term market behavior. The degree to which the ETF's past volatilities impact the current NAV volatility varies across different ETFs. This variation could be attributed to the specific market conditions, investor sentiment, and unique characteristics of each commodity market. For instance, in markets where ETFs have a substantial impact, it may reflect a higher integration or interdependence between the ETF and its underlying assets. These results also shed light on the diverse market dynamics and investor behaviors across different commodity ETFs. Factors such as market liquidity, investor risk appetite, and external economic events can significantly influence how past volatilities impact current market conditions. Markets with higher liquidity and investor participation might exhibit a stronger short-term memory effect due to quicker price adjustments.

## 6 Conclusion

The paper provides a comprehensive analysis of the volatility dynamics in the commodity ETF market using the VAR-RV and HAR models. We find that both models perform well in capturing the volatility dynamics in the market and that the volatility in commodity ETFs is driven by the underlying commodity prices. The VAR-RV model shows evidence of changing relationships between the ETF and its NAV in the volatility process. The HAR model allows for time-varying betas and evidence of long memory in the volatility process. The findings have important implications for investors and policymakers who need to understand the risk dynamics of commodity ETFs. The results suggest that investors in commodity ETFs should be aware of the volatility in the underlying commodity prices and the potential impact on the ETF's performance. Policymakers may have to consider

**Table 1:** Descriptive statistics for RV, QPV and J variables constructed with price data every 5 minutes

| | Variable | Obs | Mean | Std.dev. | Min | Max |
|---|---|---|---|---|---|---|
| **Crude Oil** | $RV_{t,NAV}$ | 3935 | 0,081% | 0,717% | 0,002% | 42,082% |
| | $RV_{t,ETF}$ | 3935 | 0,066% | 0,204% | 0,001% | 10,335% |
| **Gold** | $RV_{t,NAV}$ | 3935 | 0,012% | 0,017% | 0,001% | 0,327% |
| | $RV_{t,ETF}$ | 3935 | 0,034% | 1,259% | 0,001% | 78,988% |
| **Silver** | $RV_{t,NAV}$ | 3935 | 0,043% | 0,063% | 0,002% | 1,120% |
| | $RV_{t,ETF}$ | 3935 | 0,042% | 0,067% | 0,004% | 1,286% |
| **Natural Gas** | $RV_{t,NAV}$ | 3935 | 0,104% | 0,173% | 0,010% | 6,635% |
| | $RV_{t,ETF}$ | 3935 | 0,103% | 0,116% | 0,004% | 1,651% |
| **Crude Oil** | $QPV_{t,NAV}$ | 3935 | 0,110% | 6,797% | 0,000% | 426,363% |
| | $QPV_{t,ETF}$ | 3935 | 0,003% | 0,121% | 0,000% | 7,295% |
| **Gold** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,000% | 0,000% | 0,007% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,000% | 0,000% | 0,027% |
| **Silver** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,003% | 0,000% | 0,139% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,043% |
| **Natural Gas** | $QPV_{t,NAV}$ | 3935 | 0,003% | 0,152% | 0,000% | 9,555% |
| | $QPV_{t,ETF}$ | 3935 | 0,001% | 0,030% | 0,000% | 1,881% |
| **Crude Oil** | $J_{t,NAV}$ | 3935 | -0,028% | 6,132% | 0,000% | 8,941% |
| | $J_{t,ETF}$ | 3935 | 0,063% | 0,124% | 0,000% | 3,040% |
| **Gold** | $J_{t,NAV}$ | 3935 | 0,012% | 0,017% | 0,001% | 0,324% |
| | $J_{t,ETF}$ | 3935 | 0,034% | 1,259% | 0,001% | 78,981% |
| **Silver** | $J_{t,NAV}$ | 3935 | 0,042% | 0,061% | 0,002% | 1,094% |
| | $J_{t,ETF}$ | 3935 | 0,042% | 0,066% | 0,004% | 1,243% |
| **Natural Gas** | $J_{t,NAV}$ | 3935 | 0,101% | 0,209% | 0,000% | 6,634% |
| | $J_{t,ETF}$ | 3935 | 0,102% | 0,113% | 0,000% | 1,651% |

the unique risk dynamics of commodity ETFs when regulating these markets. Overall, our paper contributes to a better understanding of the volatility dynamics in the commodity ETF market and provides useful insights for investors and policymakers

# 7 Tables

**Table 2:** Descriptive statistics for RV, QPV and J variables constructed with price data every 1 minutes

| | Variable | Obs | Mean | Std.dev. | Min | Max |
|---|---|---|---|---|---|---|
| **Crude Oil** | $RV_{t,NAV}$ | 3935 | 0,083% | 0,959% | 0,001% | 58,167% |
| | $RV_{t,ETF}$ | 3935 | 0,058% | 0,250% | 0,001% | 14,051% |
| **Gold** | $RV_{t,NAV}$ | 3935 | 0,012% | 0,018% | 0,001% | 0,302% |
| | $RV_{t,ETF}$ | 3935 | 0,031% | 1,259% | 0,001% | 78,986% |
| **Silver** | $RV_{t,NAV}$ | 3935 | 0,040% | 0,064% | 0,001% | 1,347% |
| | $RV_{t,ETF}$ | 3935 | 0,036% | 0,061% | 0,002% | 1,367% |
| **Natural Gas** | $RV_{t,NAV}$ | 3935 | 0,099% | 0,174% | 0,007% | 6,699% |
| | $RV_{t,ETF}$ | 3935 | 0,086% | 0,096% | 0,004% | 1,634% |
| **Crude Oil** | $QPV_{t,NAV}$ | 3935 | 0,019% | 1,152% | 0,000% | 72,288% |
| | $QPV_{t,ETF}$ | 3935 | 0,004% | 0,191% | 0,000% | 11,936% |
| **Gold** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,000% | 0,000% | 0,004% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,000% | 0,000% | 0,016% |
| **Silver** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,060% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,025% |
| **Natural Gas** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,004% | 0,000% | 0,232% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,039% |
| **Crude Oil** | $J_{t,NAV}$ | 3935 | 0,064% | 0,330% | 0,000% | 8,933% |
| | $J_{t,ETF}$ | 3935 | 0,055% | 0,112% | 0,001% | 3,172% |
| **Gold** | $J_{t,NAV}$ | 3935 | 0,012% | 0,018% | 0,001% | 0,300% |
| | $J_{t,ETF}$ | 3935 | 0,031% | 1,259% | 0,001% | 78,969% |
| **Silver** | $J_{t,NAV}$ | 3935 | 0,040% | 0,063% | 0,001% | 1,328% |
| | $J_{t,ETF}$ | 3935 | 0,036% | 0,060% | 0,002% | 1,344% |
| **Natural Gas** | $J_{t,NAV}$ | 3935 | 0,098% | 0,173% | 0,007% | 6,697% |
| | $J_{t,ETF}$ | 3935 | 0,086% | 0,095% | 0,004% | 1,634% |

**Table 3:** Descriptive statistics for RV, QPV and J variables constructed with price data every 30 minutes

| | Variable | Obs | Mean | Std.dev. | Min | Max |
|---|---|---|---|---|---|---|
| **Crude Oil** | $RV_{t,NAV}$ | 3935 | 0,073% | 0,459% | 0,001% | 23,817% |
| | $RV_{t,ETF}$ | 3935 | 0,052% | 0,176% | 0,000% | 8,995% |
| **Gold** | $RV_{t,NAV}$ | 3935 | 0,011% | 0,019% | 0,001% | 0,407% |
| | $RV_{t,ETF}$ | 3935 | 0,030% | 1,257% | 0,000% | 78,854% |
| **Silver** | $RV_{t,NAV}$ | 3935 | 0,039% | 0,069% | 0,001% | 1,701% |
| | $RV_{t,ETF}$ | 3935 | 0,033% | 0,060% | 0,001% | 1,618% |
| **Natural Gas** | $RV_{t,NAV}$ | 3935 | 0,093% | 0,176% | 0,004% | 6,829% |
| | $RV_{t,ETF}$ | 3935 | 0,077% | 0,093% | 0,002% | 1,668% |
| **Crude Oil** | $QPV_{t,NAV}$ | 3935 | 0,002% | 0,067% | 0,000% | 4,036% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,013% | 0,000% | 0,831% |
| **Gold** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,000% | 0,000% | 0,002% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,000% | 0,000% | 0,002% |
| **Silver** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,050% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,037% |
| **Natural Gas** | $QPV_{t,NAV}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,044% |
| | $QPV_{t,ETF}$ | 3935 | 0,000% | 0,001% | 0,000% | 0,035% |
| **Crude Oil** | $J_{t,NAV}$ | 3935 | 0,072% | 0,401% | 0,001% | 19,781% |
| | $J_{t,ETF}$ | 3935 | 0,051% | 0,165% | 0,000% | 8,164% |
| **Gold** | $J_{t,NAV}$ | 3935 | 0,011% | 0,018% | 0,001% | 0,404% |
| | $J_{t,ETF}$ | 3935 | 0,030% | 1,257% | 0,000% | 78,852% |
| **Silver** | $J_{t,NAV}$ | 3935 | 0,038% | 0,068% | 0,001% | 1,665% |
| | $J_{t,ETF}$ | 3935 | 0,033% | 0,059% | 0,001% | 1,583% |
| **Natural Gas** | $J_{t,NAV}$ | 3935 | 0,093% | 0,175% | 0,004% | 6,823% |
| | $J_{t,ETF}$ | 3935 | 0,077% | 0,093% | 0,002% | 1,633% |

**Table 4:** HAR-X Model with 5 minutes realized variance

| | Crude Oil | | Gold | | Silver | | Natural Gas | |
|---|---|---|---|---|---|---|---|---|
| | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ |
| $RV_{t-1,NAV}$ | 0.2877623*** | 0.3105033*** | 0.2529546*** | 0.5148665*** | 0.3446605*** | 0.3826652*** | 0.0924232** | 0.1050916*** |
| $\overline{RV}_{t-5,NAV}$ | 0.407598*** | | 0.3382115*** | | 0.3184849*** | | 0.3930579*** | |
| $\overline{RV}_{t-22,NAV}$ | 0.1444736*** | | 0.3214955*** | | 0.2706576*** | | 0.3125695*** | |
| $RV_{t-1,ETF}$ | 0.1112259*** | 0.0984888*** | 0.0208678 | -0.0842967* | -0.0086121 | -0.0148356 | 0.1264126*** | 0.0844965*** |
| $\overline{RV}_{t-5,ETF}$ | | 0.2965396*** | | 0.2699655*** | | 0.2092887*** | | 0.4306773*** |
| $\overline{RV}_{t-22,ETF}$ | | 0.2269969*** | | 0.163896*** | | 0.3419885*** | | 0.3319939*** |



**Table 5:** HAR-X Model with 1 minutes realized variance

| | Crude Oil | | Gold | | Silver | | Natural Gas | |
|---|---|---|---|---|---|---|---|---|
| | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ |
| $RV_{t-1,NAV}$ | 0.3714679*** | 0.3782994*** | 0.3309549*** | 0.6319796*** | 0.4041135*** | 0.4183781*** | 0.1206816*** | 0.079018*** |
| $\overline{RV}_{t-5,NAV}$ | 0.3866316*** | | 0.329186*** | | 0.2973761*** | | 0.3861102*** | |
| $\overline{RV}_{t-22,NAV}$ | 0.109249*** | | 0.2797703*** | | 0.2258195*** | | 0.2845063*** | |
| $RV_{t-1,ETF}$ | 0.08946*** | 0.0971226*** | -0.0007677 | -0.0591496* | 0.0041008 | 0.0148553 | 0.1271672*** | 0.123824*** |
| $\overline{RV}_{t-5,ETF}$ | | 0.2719414*** | | 0.1957926*** | | 0.2059932*** | | 0.4378609*** |
| $\overline{RV}_{t-22,ETF}$ | | 0.200038*** | | 0.1134628*** | | 0.2761932*** | | 0.3182265*** |

**Table 6:** HAR-X Model with 30 minutes realized variance

| | **Crude Oil** | | **Gold** | | **Silver** | | **Natural Gas** | |
|---|---|---|---|---|---|---|---|---|
| | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ |
| $RV_{t-1,NAV}$ | 0.1334384*** | 0.1995633*** | 0.1499283*** | 0.3865535*** | 0.2260959*** | 0.2765517*** | 0.0429689 | 0.0810632*** |
| $\overline{RV}_{t-5,NAV}$ | 0.4607951*** | | 0.3354556*** | | 0.3190018*** | | 0.3986872*** | |
| $\overline{RV}_{t-22,NAV}$ | 0.2431567*** | | 0.4453711*** | | 0.393724*** | | 0.3929249*** | |
| $RV_{t-1,ETF}$ | 0.1025262*** | 0.0491635 | -0.0146057 | -0.1283879** | -0.0339618 | -0.0749709** | 0.073678** | 0.0117463 |
| $\overline{RV}_{t-5,ETF}$ | | 0.362827*** | | 0.3323706*** | | 0.2384086*** | | 0.4054139*** |
| $\overline{RV}_{t-22,ETF}$ | | 0.3193907*** | | 0.2119943*** | | 0.4595315*** | | 0.4474487*** |

**Table 7:** HAR-CJ-X Model with 5 minutes realized variance

| | **Crude Oil** | | **Gold** | | **Silver** | | **Natural Gas** | |
|---|---|---|---|---|---|---|---|---|
| | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ |
| $QPV_{t-1,NAV}$ | 0.0490583** | 0.0079393 | 0.0257434 | -0.0009149 | -0.0152919 | -0.0256667 | 0.0645067*** | 0.0164956 |
| $\overline{QPV}_{t-5,NAV}$ | 0.0324494 | | -0.0267742 | | -0.0302175 | | 0.1097163*** | |
| $\overline{QPV}_{t-22,NAV}$ | -0.04553*** | | -0.0208293 | | -0.0053534 | | 0.0443421** | |
| $QPV_{t-1,ETF}$ | 0.0006008 | 0.0434199** | -0.0146375 | -0.0304232 | 0.0064549 | 0.0100746 | 0.0112614 | 0.0275242* |
| $\overline{QPV}_{t-5,ETF}$ | | 0.0166125 | | -0.0079885 | | -0.0034221 | | 0.0621459*** |
| $\overline{QPV}_{t-22,ETF}$ | | -0.0399056** | | 0.1327477*** | | 0.0063268 | | 0.0375879 |
| $J_{t-1,NAV}$ | 0.2071681*** | 0.279848*** | 0.2118451*** | 0.4906024*** | 0.3747311*** | 0.4314626*** | -0.0026933 | 0.0565611* |
| $\overline{J}_{t-5,NAV}$ | 0.3383152*** | | 0.4067403*** | | 0.3886982*** | | 0.2200274*** | |
| $\overline{J}_{t-22,NAV}$ | 0.2668151*** | | 0.3535649*** | | 0.2719239*** | | 0.2132049*** | |
| $J_{t-1,ETF}$ | 0.0943863*** | 0.0189719 | 0.0407096 | -0.0225409 | -0.0161098 | -0.0299429 | 0.0443812 | 0.0389113 |
| $\overline{J}_{t-5,ETF}$ | | 0.2637849*** | | 0.2463867*** | | 0.2144421*** | | 0.306782*** |
| $\overline{J}_{t-22,ETF}$ | | 0.3285126*** | | -0.0142732 | | 0.3270304*** | | 0.2500081*** |

**Table 8:** HAR-CJ-X Model with 1 minutes realized variance

| | **Crude Oil** | | **Gold** | | **Silver** | | **Natural Gas** | |
|---|---|---|---|---|---|---|---|---|
| | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ |
| $QPV_{t-1,NAV}$ | 0.0363062 | -0.0191013 | -0.0330827* | -0.12317*** | -0.0406337** | -0.0653139*** | 0.0082145 | -0.0024863 |
| $\overline{QPV}_{t-5,NAV}$ | -0.0208693 | | -0.0762007*** | | -0.0675044*** | | 0.1163383*** | |
| $\overline{QPV}_{t-22,NAV}$ | -0.0306007 | | -0.000817 | | 0.0087315 | | -0.0401051** | |
| $QPV_{t-1,ETF}$ | -0.0109489 | 0.009254 | -0.0202785* | -0.0320848* | -0.011154 | -0.0139627 | -0.0113016 | -0.0209799* |
| $\overline{QPV}_{t-5,ETF}$ | | -0.014021 | | 0.0164616 | | 0.0034283 | | 0.0162852 |
| $\overline{QPV}_{t-22,ETF}$ | | -0.0235749** | | 0.0598899*** | | -0.0166442 | | -0.0138206 |
| $J_{t-1,NAV}$ | 0.3060633*** | 0.4315109*** | 0.4255088*** | 0.8922364*** | 0.5098133*** | 0.5775538*** | 0.0923634* | 0.0904133*** |
| $\bar{J}_{t-5,NAV}$ | 0.4351271*** | | 0.5091064*** | | 0.4678481*** | | 0.2189002*** | |
| $\bar{J}_{t-22,NAV}$ | 0.1896227*** | | 0.2341039*** | | 0.1609894*** | | 0.3461739*** | |
| $J_{t-1,ETF}$ | 0.0955601*** | 0.0773423** | 0.0311733 | 0.0112357 | 0.0282037 | 0.0510185 | 0.1283538*** | 0.1662645*** |
| $\bar{J}_{t-5,ETF}$ | | 0.3002446*** | | 0.1320609** | | 0.1838292*** | | 0.4014837*** |
| $\bar{J}_{t-22,ETF}$ | | 0.2495922*** | | 0.0277424 | | 0.2948618*** | | 0.3495071*** |

**Table 9:** HAR-CJ-X Model with 30 minutes realized variance

| | **Crude Oil** | | **Gold** | | **Silver** | | **Natural Gas** | |
|---|---|---|---|---|---|---|---|---|
| | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ | $RV_{t,NAV}$ | $RV_{t,ETF}$ |
| $QPV_{t-1,NAV}$ | 0.0191586 | -0.025803 | 0.0578991*** | 0.0672191*** | 0.0130017 | 0.0332086 | 0.0571801*** | 0.0270091 |
| $\overline{QPV}_{t-5,NAV}$ | 0.0286454 | | -0.0508322** | | -0.0084194 | | 0.1136484*** | |
| $\overline{QPV}_{t-22,NAV}$ | -0.0009409 | | 0.0006133 | | -0.0453743* | | 0.1558169*** | |
| $QPV_{t-1,ETF}$ | -0.0102057 | 0.0153828 | -0.0182844 | -0.0265051 | -0.0063637 | 0.0005231 | -0.0141659 | 0.0009177 |
| $\overline{QPV}_{t-5,ETF}$ | | 0.0552223** | | -0.0170282 | | -0.0010585 | | 0.0787447*** |
| $\overline{QPV}_{t-22,ETF}$ | | 0.0160596 | | 0.2336415*** | | -0.035093 | | 0.1684398*** |
| $J_{t-1,NAV}$ | 0.1101896** | 0.2209544*** | 0.0536858 | 0.205545*** | 0.2006159*** | 0.2159805*** | -0.0057401 | 0.0266475 |
| $\overline{J}_{t-5,NAV}$ | 0.4030334*** | | 0.4357502*** | | 0.3394677*** | | 0.2043785*** | |
| $\overline{J}_{t-22,NAV}$ | 0.2477565*** | | 0.439587*** | | 0.4885628*** | | 0.0893825 | |
| $J_{t-1,ETF}$ | 0.1101033*** | 0.0425918 | 0.0086643 | -0.0592772 | -0.0234756 | -0.0803946* | 0.0157848 | 0.0066244 |
| $\overline{J}_{t-5,ETF}$ | | 0.2591833*** | | 0.2736371*** | | 0.2406158*** | | 0.2433137*** |
| $\overline{J}_{t-22,ETF}$ | | 0.2831252*** | | -0.0815792* | | 0.5321428*** | | 0.0877828 |

| | | | $log(RV_{t,NAV})$ | | | |
|---|---|---|---|---|---|---|
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.5704063 | 0.0236163 | 0.000236 | 0.5704446 | 0.5236736 | 0.6170733 |
| $log(RV_{t-2,NAV})$ | 0.2605721 | 0.0206081 | 0.000206 | 0.2607468 | 0.2205824 | 0.3004614 |
| $log(RV_{t-1,ETF})$ | 0.073531 | 0.0201458 | 0.000201 | 0.0735165 | 0.0342794 | 0.1134928 |
| $log(RV_{t-2,ETF})$ | 0.0155775 | 0.0178843 | 0.000179 | 0.0158028 | -0.0194301 | 0.0511915 |
| | | | $log(RV_{t,ETF})$ | | | |
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.2861492 | 0.0283011 | 0.000283 | 0.2860354 | 0.2314617 | 0.3432742 |
| $log(RV_{t-2,NAV})$ | 0.1934378 | 0.0246438 | 0.000243 | 0.1932421 | 0.1443466 | 0.2409736 |
| $log(RV_{t-1,ETF})$ | 0.2940656 | 0.0243559 | 0.000244 | 0.2940536 | 0.245977 | 0.3418339 |
| $log(RV_{t-2,ETF})$ | 0.1056721 | 0.0215096 | 0.000212 | 0.1059451 | 0.0642683 | 0.1475865 |

**Table 10:** results of the Bayesian Vector Autoregression (VAR) derived from modeling the USO ETF and its Net Asset Value (NAV)

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | $log(RV_{t,NAV})$ | | | |
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.5506335 | 0.0265677 | 0.000268 | 0.5505196 | 0.4984977 | 0.6027287 |
| $log(RV_{t-2,NAV})$ | 0.2480039 | 0.0226964 | 0.000227 | 0.24802 | 0.2033675 | 0.2930322 |
| $log(RV_{t-1,ETF})$ | -0.0101261 | 0.0225007 | 0.000225 | -0.009853 | -0.0547467 | 0.0335829 |
| $log(RV_{t-2,ETF})$ | 0.0382607 | 0.0193646 | 0.000194 | 0.0382007 | 0.0001143 | 0.0759385 |
| | | | $log(RV_{t,ETF})$ | | | |
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.3487317 | 0.032542 | 0.000332 | 0.3481733 | 0.2847775 | 0.4126296 |
| $log(RV_{t-2,NAV})$ | 0.2757957 | 0.0276061 | 0.000276 | 0.2756654 | 0.2221232 | 0.3306929 |
| $log(RV_{t-1,ETF})$ | 0.1549426 | 0.0275764 | 0.000276 | 0.1552797 | 0.1004078 | 0.2083055 |
| $log(RV_{t-2,ETF})$ | 0.048005 | 0.0233456 | 0.000233 | 0.048072 | 0.0025562 | 0.0945181 |

**Table 11:** results of the Bayesian Vector Autoregression (VAR) derived from modeling the GLD ETF and its Net Asset Value (NAV)

| | $log(RV_{t,NAV})$ | | | | | |
|---|---|---|---|---|---|---|
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.6191766 | 0.0282475 | 0.000279 | 0.619402 | 0.5631442 | 0.6737706 |
| $log(RV_{t-2,NAV})$ | 0.214354 | 0.0237759 | 0.000238 | 0.2143959 | 0.1677967 | 0.2606003 |
| $log(RV_{t-1,ETF})$ | -0.0446257 | 0.0248539 | 0.000249 | -0.0445203 | -0.0932139 | 0.0051081 |
| $log(RV_{t-2,ETF})$ | 0.0583869 | 0.0208764 | 0.000209 | 0.0581792 | 0.0176497 | 0.0996259 |
| | $log(RV_{t,ETF})$ | | | | | |
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.3759512 | 0.0329039 | 0.00032 | 0.3765365 | 0.3102376 | 0.4404484 |
| $log(RV_{t-2,NAV})$ | 0.2015704 | 0.0277495 | 0.000277 | 0.201502 | 0.1471034 | 0.2562645 |
| $log(RV_{t-1,ETF})$ | 0.1790262 | 0.0290228 | 0.000287 | 0.1787673 | 0.1225737 | 0.2361899 |
| $log(RV_{t-2,ETF})$ | 0.0777421 | 0.0244172 | 0.000244 | 0.0778238 | 0.0305811 | 0.126285 |

**Table 12:** results of the Bayesian Vector Autoregression (VAR) derived from modeling the SLV ETF and its Net Asset Value (NAV)

| | $log(RV_{t,NAV})$ | | | | | |
|---|---|---|---|---|---|---|
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.3918631 | 0.0224541 | 0.000225 | 0.3917972 | 0.348754 | 0.4366317 |
| $log(RV_{t-2,NAV})$ | 0.2313034 | 0.0203958 | 0.000204 | 0.2311102 | 0.1915471 | 0.2715316 |
| $log(RV_{t-1,ETF})$ | 0.0798518 | 0.021189 | 0.000212 | 0.0799138 | 0.0381142 | 0.1214105 |
| $log(RV_{t-2,ETF})$ | 0.1567487 | 0.0192459 | 0.000192 | 0.1571281 | 0.1188634 | 0.193788 |
| | $log(RV_{t,ETF})$ | | | | | |
| | **Mean** | **Std. dev.** | **MCSE** | **Median** | **95% cred. interval** | |
| $log(RV_{t-1,NAV})$ | 0.1296578 | 0.023824 | 0.000238 | 0.129506 | 0.083075 | 0.1772056 |
| $log(RV_{t-2,NAV})$ | 0.1653116 | 0.0215382 | 0.000215 | 0.165259 | 0.1236417 | 0.2081028 |
| $log(RV_{t-1,ETF})$ | 0.3387547 | 0.0225042 | 0.000222 | 0.3387 | 0.2942176 | 0.3829999 |
| $log(RV_{t-2,ETF})$ | 0.2396132 | 0.020431 | 0.000204 | 0.2395922 | 0.1998239 | 0.279715 |

**Table 13:** results of the Bayesian Vector Autoregression (VAR) derived from modeling the UNG ETF and its Net Asset Value (NAV)

## 8 Figures

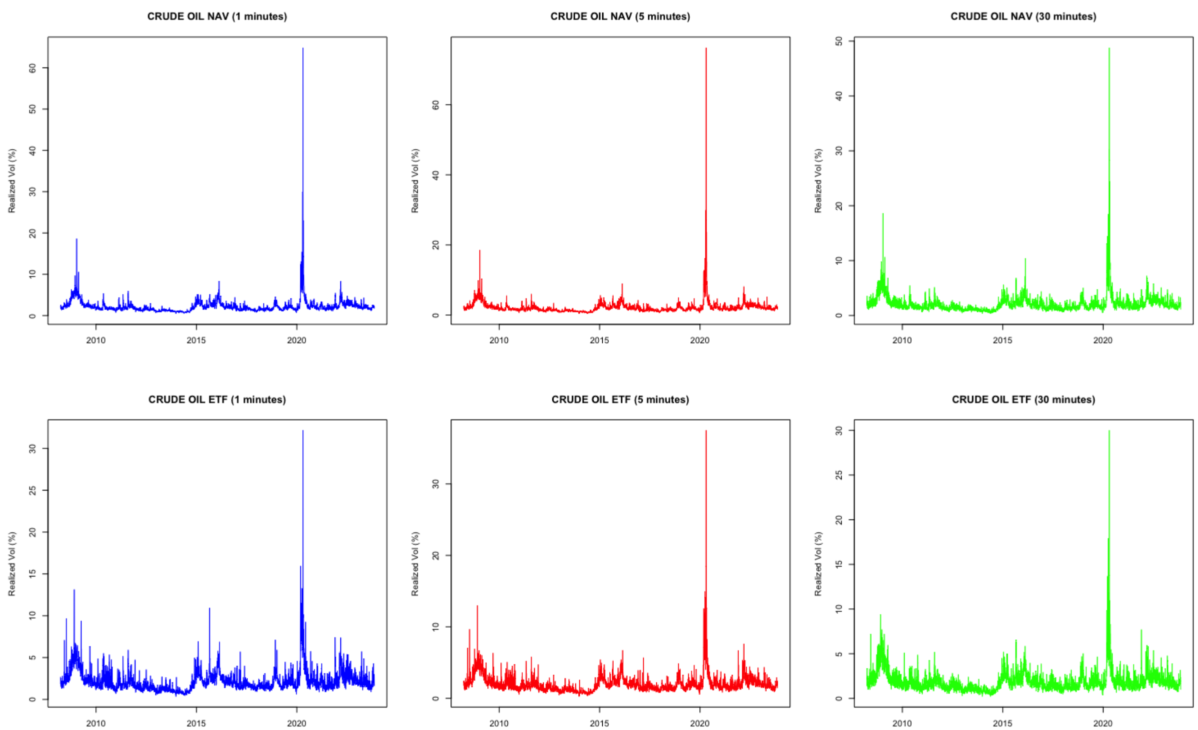


**Figure 1:** Realized Volatility for Crude Oil (ETF and NAV)

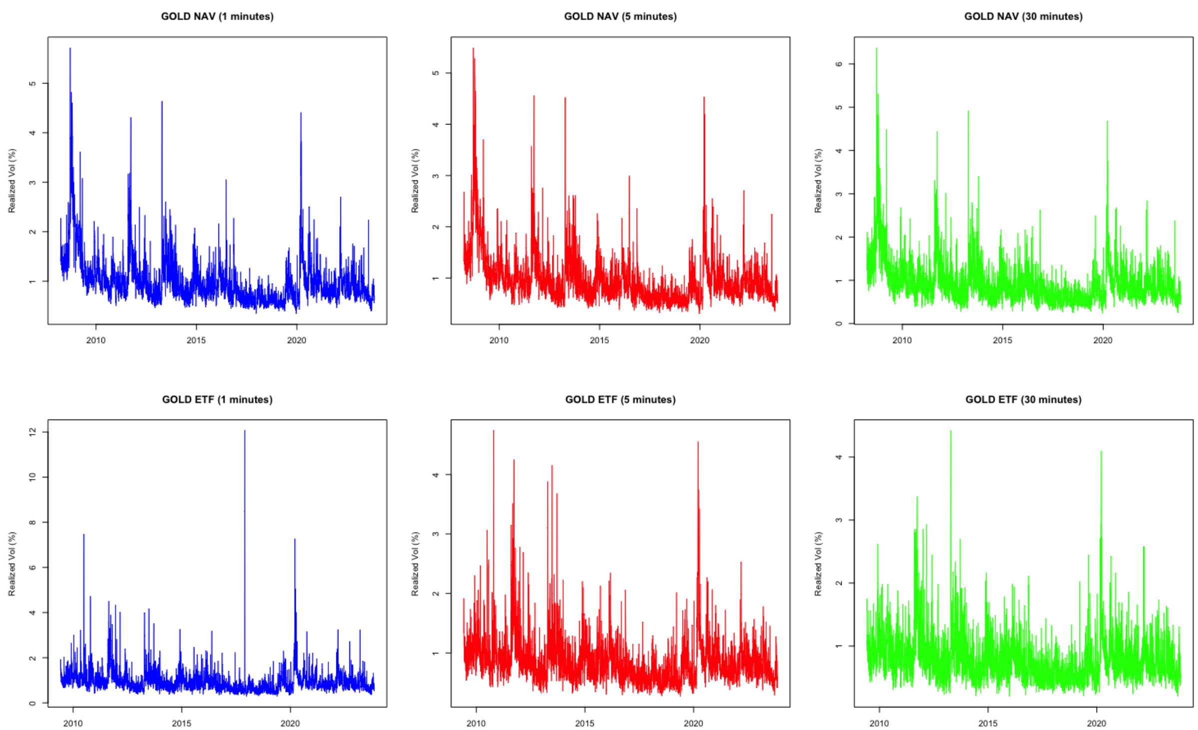


**Figure 2:** Realized Volatility for Gold (ETF and NAV)

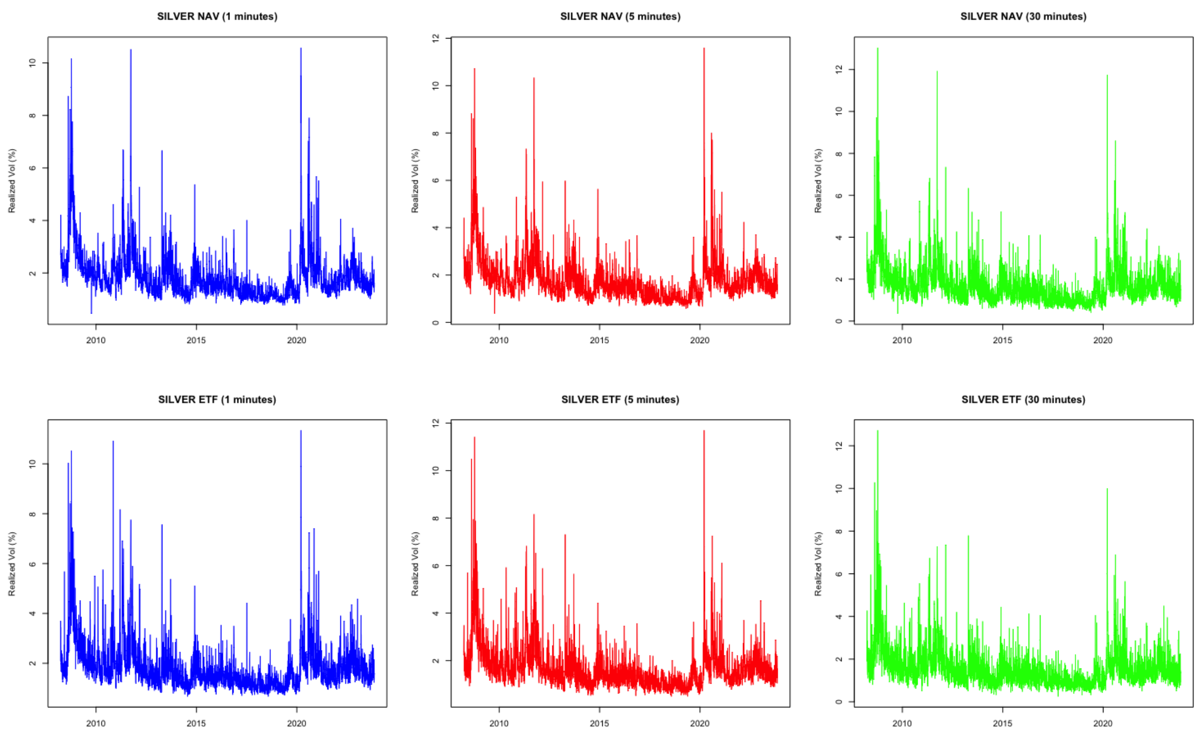


**Figure 3:** Realized Volatility for Silver (ETF and NAV)

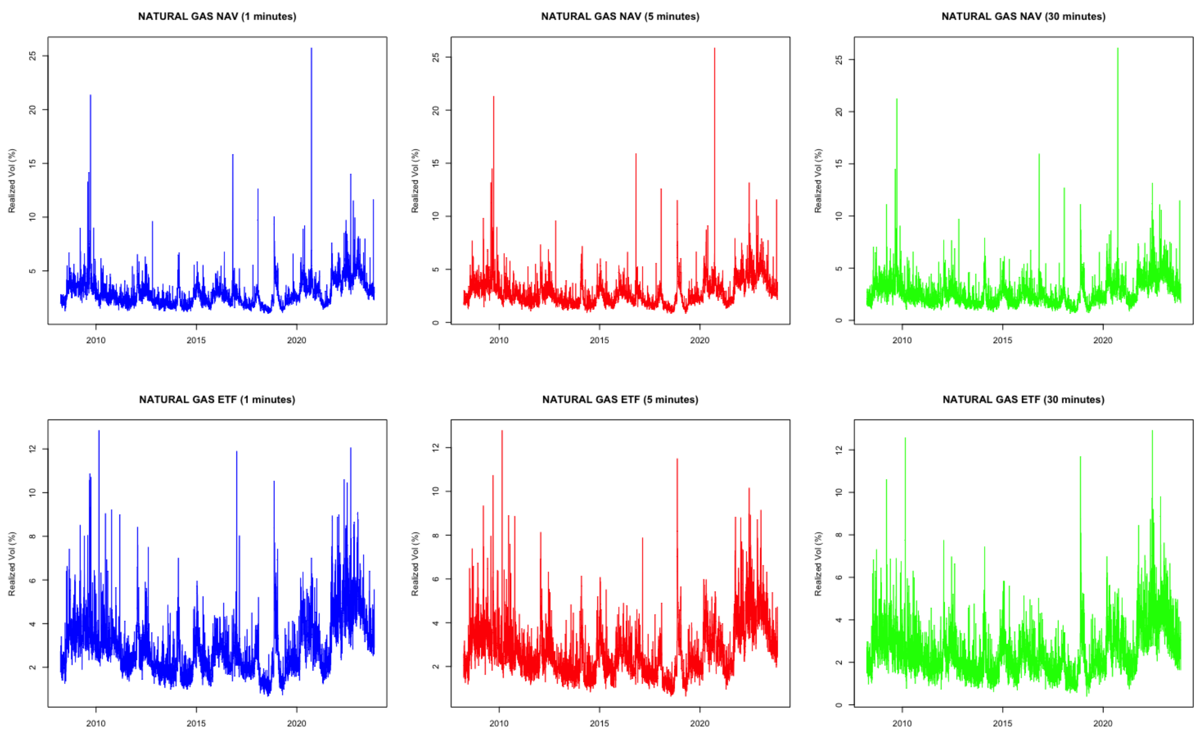


**Figure 4:** Realized Volatility for Natural Gas (ETF and NAV)

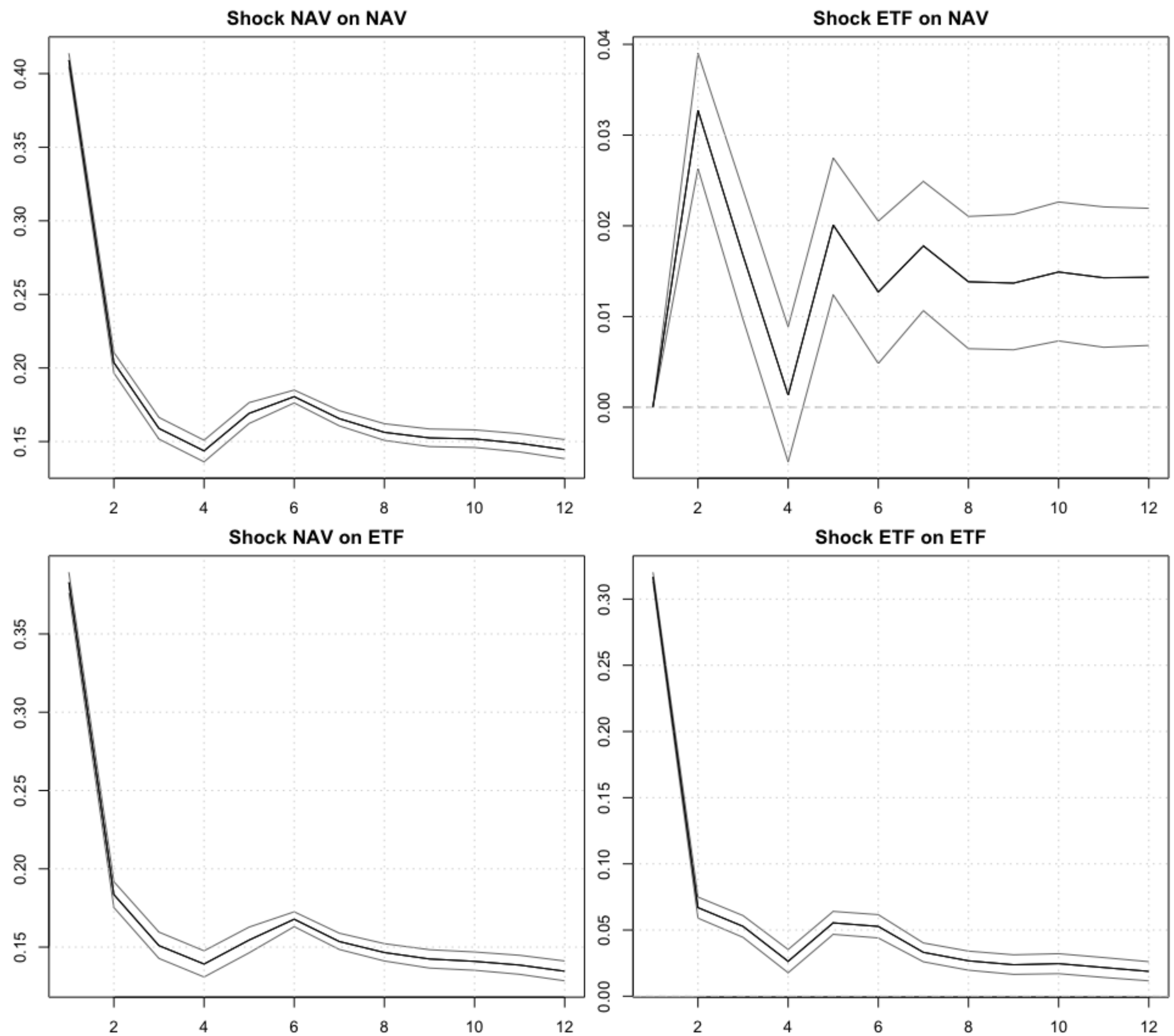


**Figure 5:** Impulse Response Function (IRF) resulting from the modeling of the USO ETF and its Net Asset Value (NAV)

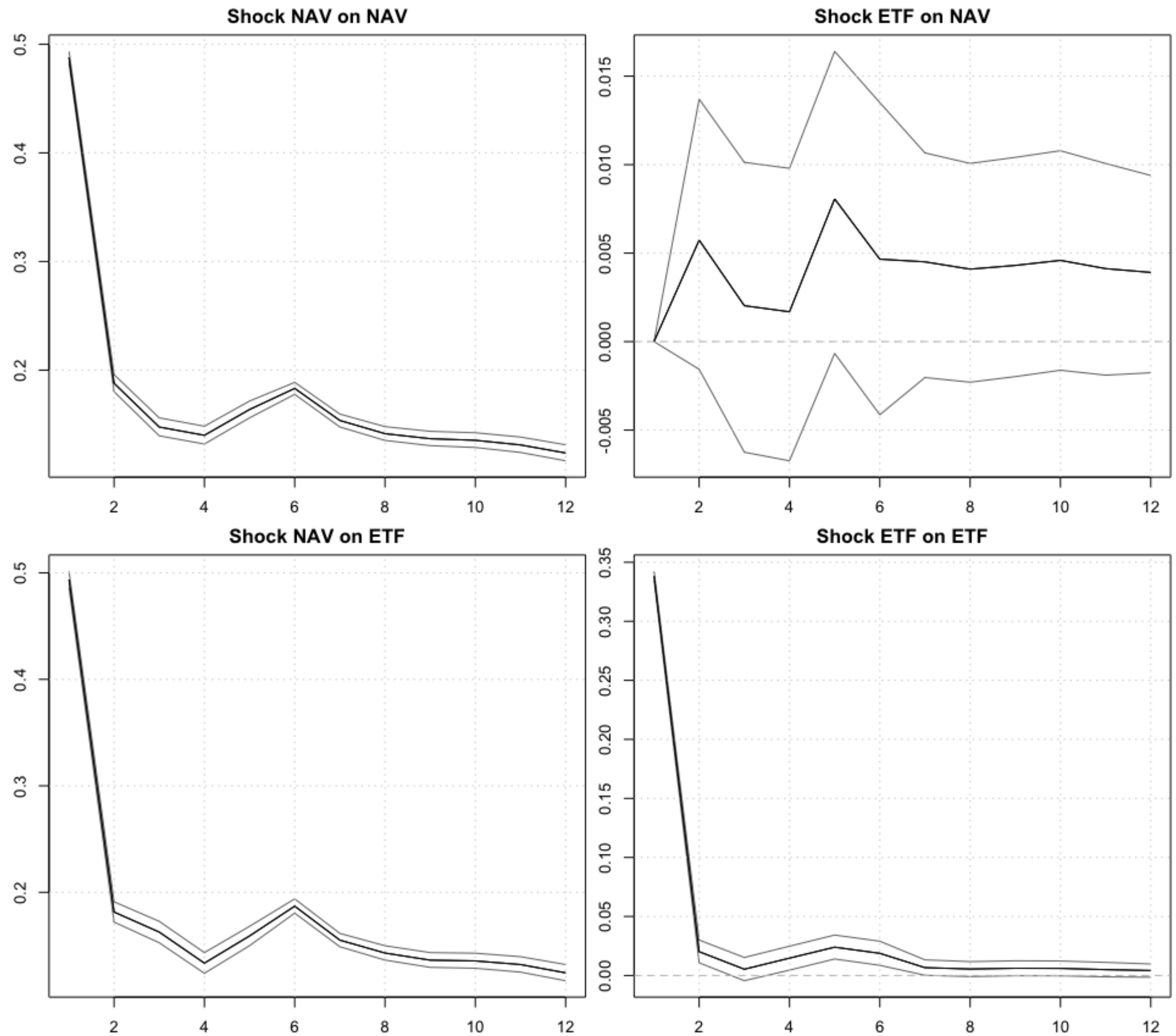


**Figure 6:** Impulse Response Function (IRF) resulting from the modeling of the GLD ETF and its Net Asset Value (NAV)

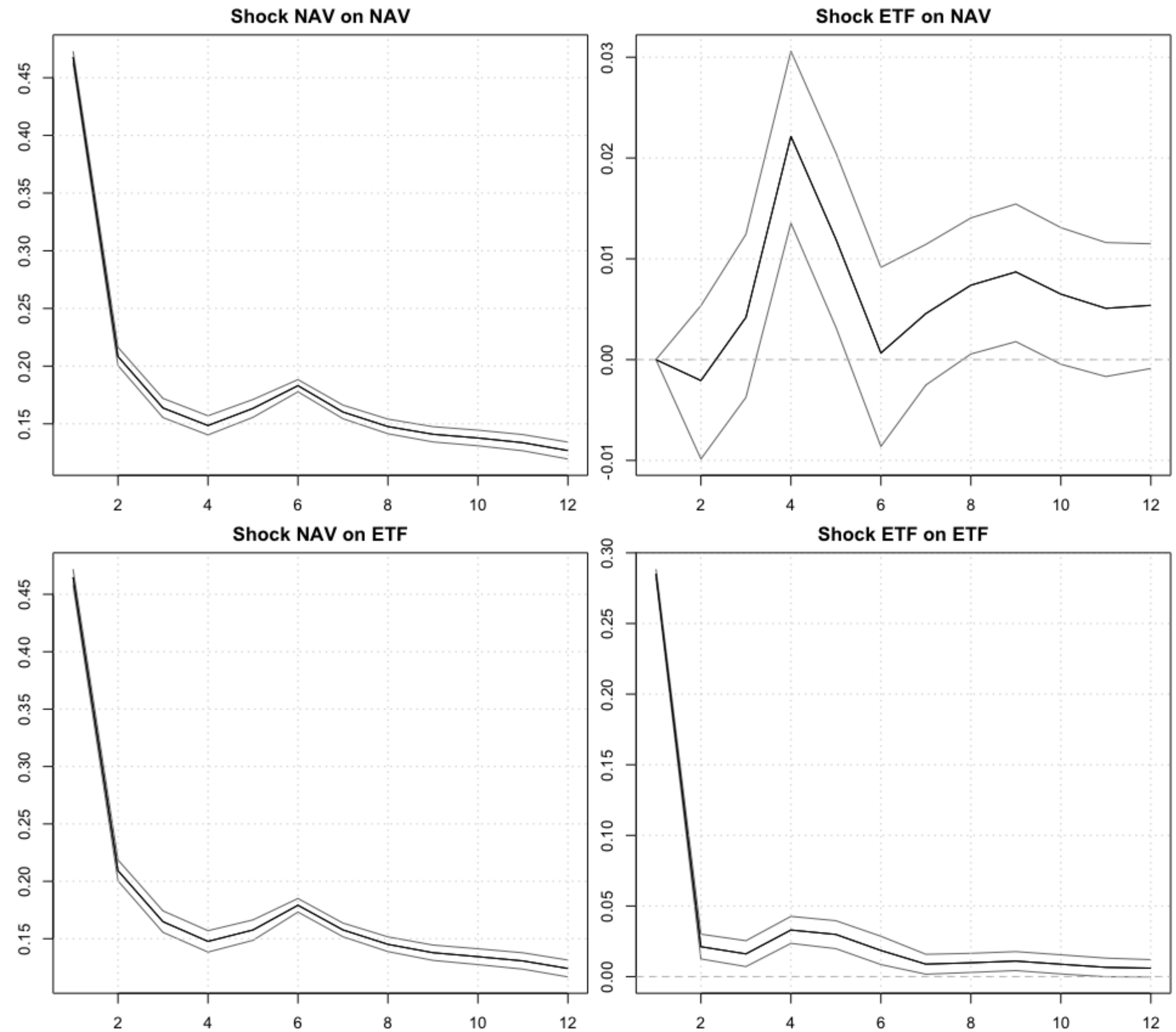


**Figure 7:** Impulse Response Function (IRF) resulting from the modeling of the SLV ETF and its Net Asset Value (NAV)

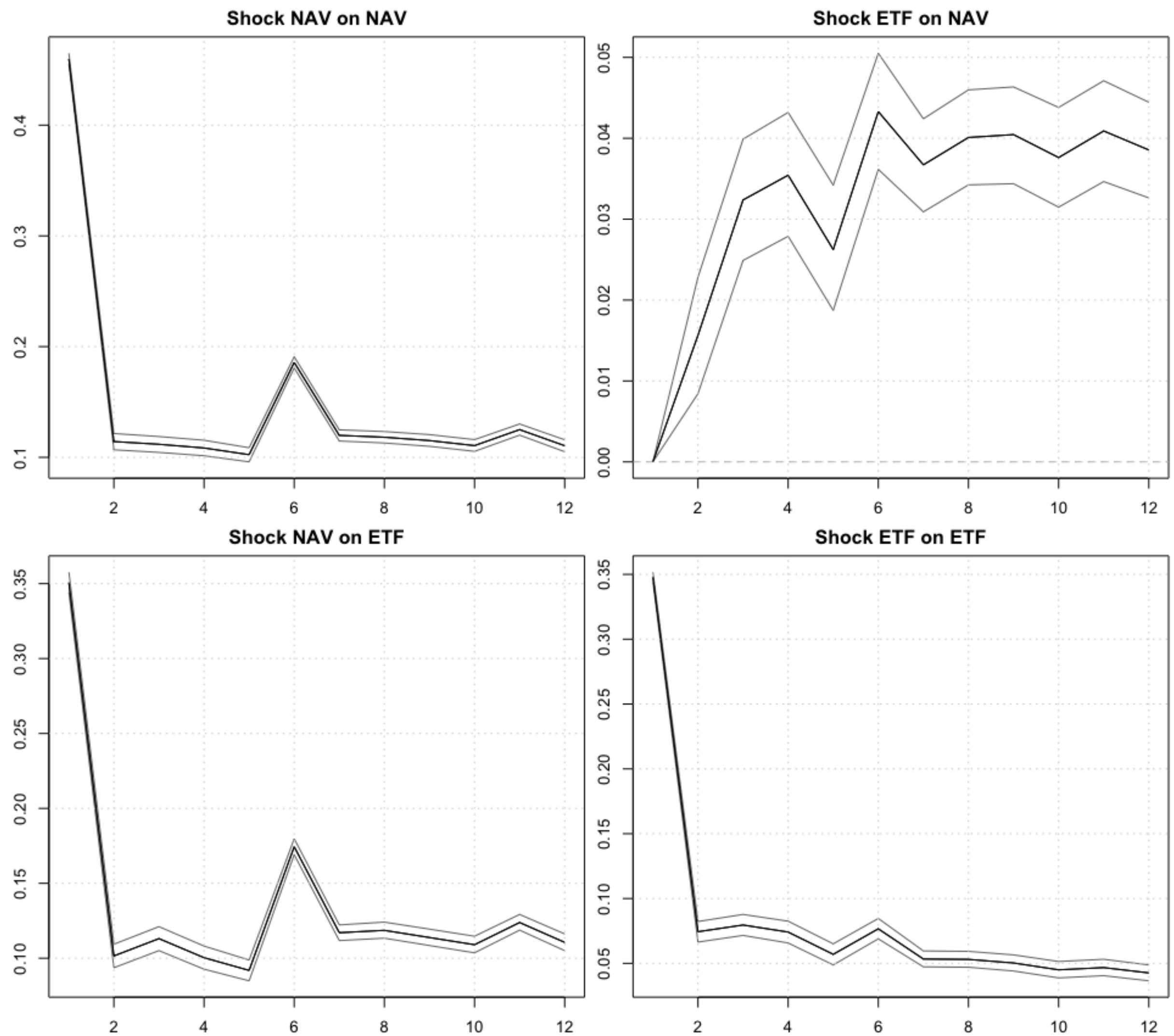


**Figure 8:** Impulse Response Function (IRF) resulting from the modeling of the UNG ETF and its Net Asset Value (NAV)