11.4.17

Collision Tensor (in ridgelet basis)

$$r_{i_1}(x) \leftrightarrow \hat{\psi}_{i_1}(\hat{x},\hat{y}) \in \mathbb{R}^{n_1 \times m_1}$$

$$r_{i_2}(x) \leftrightarrow \hat{\psi}_{i_2}(\hat{x},\hat{y}) \in \mathbb{R}^{n_2 \times m_2}$$

$$r_{i_3}(x) \leftrightarrow \hat{\psi}_{i_3}(\hat{x},\hat{y}) \in \mathbb{R}^{n_3 \times m_3}$$

$$r_{i_1}(x)\, r_{i_2}(x) = \sum_{\hat{x}_1,\hat{y}_1} \sum_{\hat{x}_2,\hat{y}_2} e^{2\pi i (\hat{x}_1+\hat{x}_2)\frac{x}{L_x}} e^{2\pi i (\hat{y}_1+\hat{y}_2)\frac{y}{L_y}} e^{-2\pi i \vec{t}_1 \cdot \begin{bmatrix}\hat{x}_1\\ \hat{y}_1\end{bmatrix}} e^{-2\pi i \vec{t}_2 \cdot \begin{bmatrix}\hat{x}_2\\ \hat{y}_2\end{bmatrix}} \hat{\psi}_{i_1}(\hat{x}_1,\hat{y}_1)\, \hat{\psi}_{i_2}(\hat{x}_2,\hat{y}_2)$$

$$\hat{x} = \hat{x}_1 + \hat{x}_2$$
$$\hat{y} = \hat{y}_1 + \hat{y}_2$$

$$= \sum_{\hat{x},\hat{y}} e^{2\pi i \left(\hat{x}\frac{x}{L_x} + \hat{y}\frac{y}{L_y}\right)} \sum_{\hat{x}',\hat{y}'} e^{-2\pi i \vec{t}_1 \begin{bmatrix}\hat{x}'\\ \hat{y}'\end{bmatrix}} e^{-2\pi i \vec{t}_2 \cdot \begin{bmatrix}\hat{x}-\hat{x}'\\ \hat{y}-\hat{y}'\end{bmatrix}} \hat{\psi}_{i_1}(\hat{x}',\hat{y}')\, \hat{\psi}_{i_2}(\hat{x}-\hat{x}',\hat{y}-\hat{y}')$$

$$= \sum_{\hat{x},\hat{y}} e^{2\pi i \left(\hat{x}\frac{x}{L_x} + \hat{y}\frac{y}{L_y}\right)} \sum_{\hat{x}',\hat{y}'} \underbrace{e^{-2\pi i (\vec{t}_1-\vec{t}_2)\cdot \begin{bmatrix}\hat{x}'\\ \hat{y}'\end{bmatrix}} e^{-2\pi i \vec{t}_2 \cdot \begin{bmatrix}\hat{x}\\ \hat{y}\end{bmatrix}} \hat{\psi}_{i_1}(\hat{x}',\hat{y}')\, \hat{\psi}_{i_2}(\hat{x}-\hat{x}',\hat{y}-\hat{y}')}_{(*)}$$

$$\int_D r_{i_1}(x)\, r_{i_2}(x)\, r_{i_3}(x)\, dx = \sum_{\hat{x}_3,\hat{y}_3} \sum_{\hat{x},\hat{y}} e^{2\pi i \underbrace{(\hat{x}+\hat{x}_3)}\frac{x}{L_x}} e^{2\pi i (\hat{y}+\hat{y}_3)\frac{y}{L_y}} \hat{\psi}_{i_3}(\hat{x}_3,\hat{y}_3) \sum (*)$$

$$\hat{x}_3 = -\hat{x} \qquad \hat{y}_3 = -\hat{y}$$

$$= L_x L_y \sum_{\hat{x},\hat{y}} \underbrace{\hat{\psi}_{i_3}(-\hat{x},-\hat{y})}_{=\hat{\psi}_{i_3}(\hat{x},\hat{y})} e^{2\pi i \hat{t}_3 \cdot \begin{bmatrix}\hat{x}\\ \hat{y}\end{bmatrix}} \cdot \ldots$$

$$\cdot \sum_{\hat{x}',\hat{y}'} e^{-2\pi i (\vec{t}_1-\vec{t}_2)\cdot \begin{bmatrix}\hat{x}'\\ \hat{y}'\end{bmatrix}} e^{-2\pi i \hat{t}_2 \begin{bmatrix}\hat{x}\\ \hat{y}\end{bmatrix}} \hat{\psi}_{i_1}(\hat{x}',\hat{y}')\, \hat{\psi}_{i_2}(\hat{x}-\hat{x}',\hat{y}-\hat{y}')$$

$$= L_x L_y \sum_{\hat{x},\hat{y}} \hat{\psi}_{i_3}(\hat{x},\hat{y}) \underbrace{\sum_{\hat{x}',\hat{y}'} e^{-2\pi i (t_1-t_2)\cdot \begin{bmatrix}\hat{x}'\\ \hat{y}'\end{bmatrix}} e^{-2\pi i (\hat{t}_2-\hat{t}_3)\begin{bmatrix}\hat{x}\\ \hat{y}\end{bmatrix}} \hat{\psi}_{i_1}(\hat{x}',\hat{y}')\, \hat{\psi}_{i_2}(\hat{x}-\hat{x}',\hat{y}-\hat{y}')}_{K(\hat{x},\hat{y})}$$

$$? \; K(\hat{x},\hat{y}) = K(-\hat{x},-\hat{y})$$

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②

Collision tensor (in ridgelet basis)

$$\mathbb{D} = [0, L_x) \times [0, L_y)$$

cont...

$$\int_{\mathbb{D}} r_{i_1}(x)\, r_{i_2}(x)\, r_{i_3}(x)\, dx = L_x L_y \sum_{\hat{x},\hat{y}} \hat{q}_{i_3}(\hat{x},\hat{y}) \sum_{\hat{x}',\hat{y}'} \Big[\cos\Big(2\pi (\vec{t}_1 - \vec{t}_2) \cdot \begin{bmatrix} \hat{x}' \\ \hat{y}' \end{bmatrix} + 2\pi (\vec{t}_2 - \vec{t}_3) \cdot \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}\Big)$$

$$\hat{q}_{i_1}(\hat{x}',\hat{y}')\, \hat{q}_{i_2}(\hat{x}-\hat{x}', \hat{y}-\hat{y}') \Big]$$