Simon Queric
simon.queric@telecom-paris.fr

Lab session # 6
ALTEGRAD 2023

11/28/23

Question 1

For a given node v in the Erdős-Rényi random graph $G(n,p) = (V,E)$, the degree of v, is equal to $d(v) = \sum_{u \in V \setminus \{v\}} X_u$ where X_u is a Bernoulli random variable which is equal to 1 if $u \sim v$ in G and is equal to 0 otherwise. For all u we have $\mathbb{P}(X_u = 1) = p$. Hence, $\mathbb{E}[d(v)] = (n-1) \times p$. In the case where $n = 25, p = 0.2$ we have $\mathbb{E}[d(v)] = 24 \times 0.2 = 4.8$. In the case where $n = 25, p = 0.4$ we have $\mathbb{E}[d(v)] = 24 \times 0.4 = 9.6$.

Question 2

The sum operation is invariant by permutation which is a property that we like because nodes in graph are not ordered. A graph is a purely combinatorial object. The output of the GNN shouldn't change for two isomorphic graphs. However, fully connected layers are order-sensitive, this is why we use sum or mean operators as readout functions.
Furthermore, from a practical point of view, it would be tricky to implement because the number of nodes can change from one graph to another. In the figure 1, the ouput would be different if we change the order of the graphs (e.g G_2, G_3, G_1).

Question 3

When we use the mean operator for readout's model, all the cycle graphs have same embeddings, while when we use the sum operator for readout's model, all embeddings are different although they're strongly correlated.

Question 4

The graph C_9 and the graph constituted of three cycle graphs C_3. Although these graphs aren't isomorphic, they have same embeddings with a GNN model which uses the sum operator and therefore cannot be distinguished by the GNN.

References