gfpop: an R Package for Univariate Graph-Constrained Change-Point Detection

Machine Learning for Time Series

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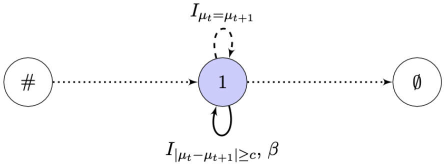
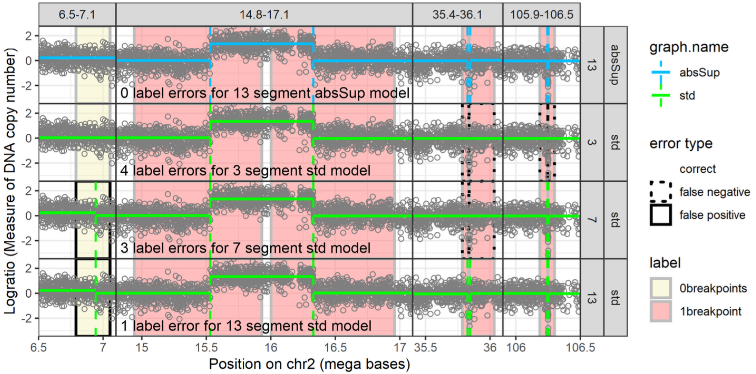
Introduction

$$Q_n^{std}(\mu) = \sum_{t=1}^{n} (y_t - \mu_t)^2 + \beta \sum_{t=1}^{n-1} I_{\mu_t \neq \mu_{t+1}}$$

$$Q_n^{iso}(\mu) = \sum_{t=1}^{n} (y_t - \mu_t)^2 \text{ such that } \mu_t \leqslant \mu_{t+1}$$

Introduction





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Introduction to graph of constraints

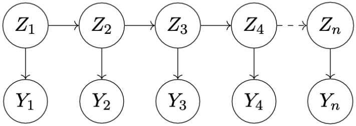

Figure: Hierarchical representation

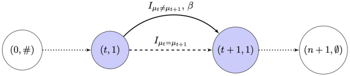

Figure: Graph of constraints

A simple example

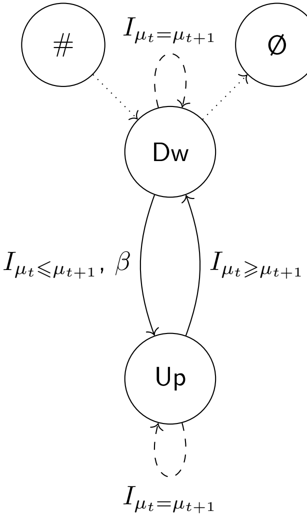

Figure: Collapsed graph for multi-modal regression

Optimization problem

Classical multiple change points detection problem :

$$(\hat{t}_1, ..., \hat{t}_K) = \underset{(t_1,...,t_k)}{\operatorname{argmin}} \sum_{k=0}^{K} c(x[t_k : t_{k+1}])$$

The article's problem :

$$Q_n^s(\theta) = \min_{\substack{p=(v,e)\in\mathcal{G}_n \\ \mu|p(\mu),\mu_n=\theta,v_n=s}} \sum_{t=1}^{n} (\gamma_{e_t}(y_t, \mu_t) + \beta_{e_t})$$

c and γ_e are always negative log-probabilities.

Cost functions

$$\text{L2 decomposition}: \ f_1 : \theta \mapsto 1, f_2 : \theta \mapsto \theta, f_3 : \theta \mapsto \theta^2$$
$$\text{Lin-log decomposition}: \ f_1 : \theta \mapsto 1, f_2 : \theta \mapsto \theta, f_3 : \theta \mapsto \log \theta$$

$$\gamma_{\ell^2}(y, \mu) = (y - \mu)^2 = y^2 - 2y\mu + \mu^2 = y^2 f_1(\mu) - 2y f_2(\mu) + f_3(\mu)$$

$$\gamma_{\text{Poisson}}(y, \mu) = -\log\left(e^{-\mu}\frac{\mu^y}{y!}\right) = \log(y!) f_1(\mu) + f_2(\mu) - y f_3(\mu)$$

Algorithm to solve the problem

Algorithm 1 Backtracking $\hat{s}$ and $\hat{\mu}$

procedure BACKTRACK$((Q_1^1, ..., Q_1^S), ..., (Q_n^1, ..., Q_n^S))$

$\hat{\mu} \leftarrow$ empty vector of size n

$\hat{s} \leftarrow$ empty vector of size n

$(\hat{s}_n, \hat{\mu}_n) = \underset{(s,\mu)}{\text{argmin}}\{Q_n^s(\mu)\}$

$\triangleright$ We can impose a subset of arrival states $\tilde{\mathcal{S}} \subset \{1, ..., S\}$ by $(\hat{s}_n, \hat{\mu}_n) \leftarrow \{\underset{(s,\mu)}{\text{argmin}}\{Q_n^s(\mu)\}, \, s \in \tilde{\mathcal{S}}\}$

for $t = n-1$ **to** $t = 1$ **do**

$\quad \hat{s}_t = \underset{s | \exists \, edge \, (s, \hat{s}_{t+1})}{\text{argmin}} \left\{ O_t^{s, \hat{s}_{t+1}}(\hat{\mu}_{t+1}) + \gamma_{(s, \hat{s}_{t+1})}(y_{t+1}, \hat{\mu}_{t+1}) + \beta_{(s, \hat{s}_{t+1})} \right\}$

$\quad \hat{\mu}_t = \underset{\mu | I_{(\hat{s}_t, \hat{s}_{t+1})}(\mu, \hat{\mu}_{t+1})}{\text{argmin}} \left\{ Q_t^{\hat{s}_t}(\mu) \right\}$ $\triangleright$ If $\hat{\mu}_t$ is such that the constraint is active, we have 'forced = TRUE' in ″gfpop() response

end for

return $(\hat{s}, \hat{\mu})$

Figure: Backtrack algorithm

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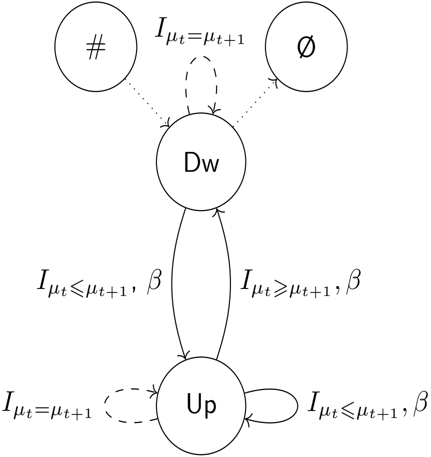
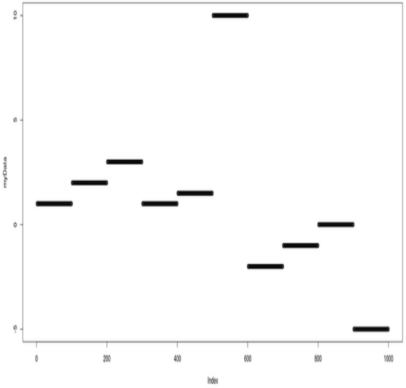
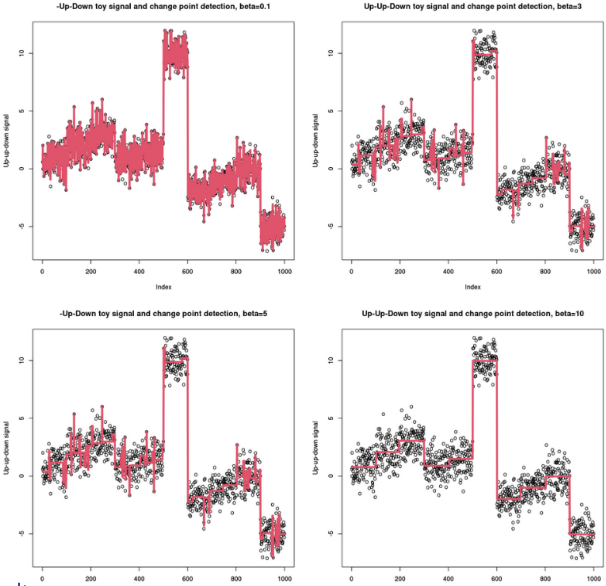
Up-up-down signal

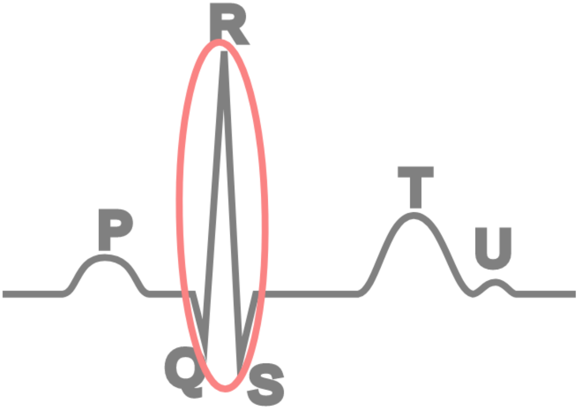




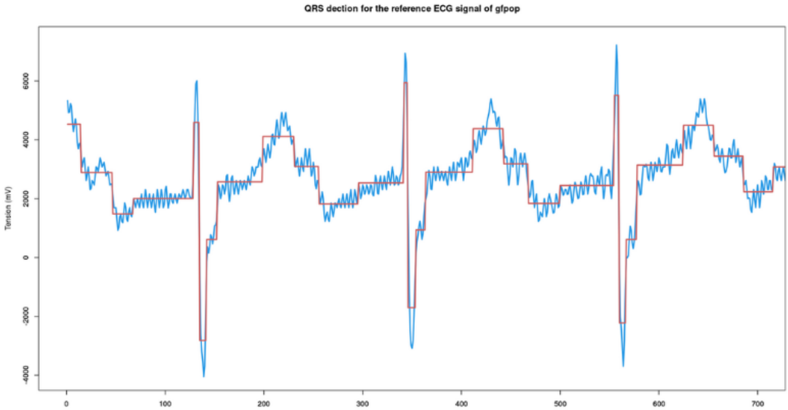
Up-up-down signal



QRS complex in ECG signals

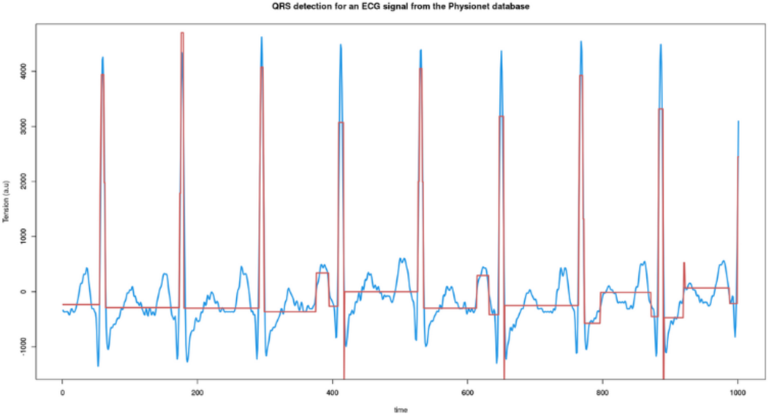


QRS complex in ECG signals



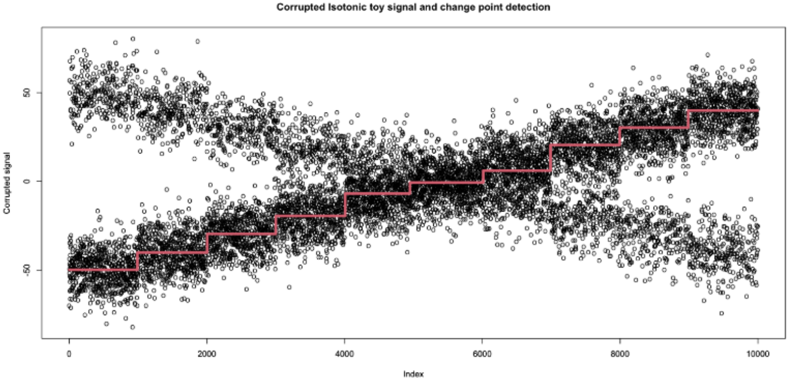
Sample rate : 250 Hz

QRS complex in ECG signals

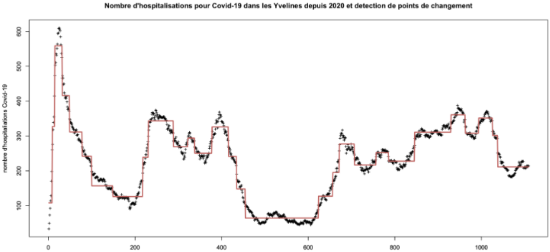


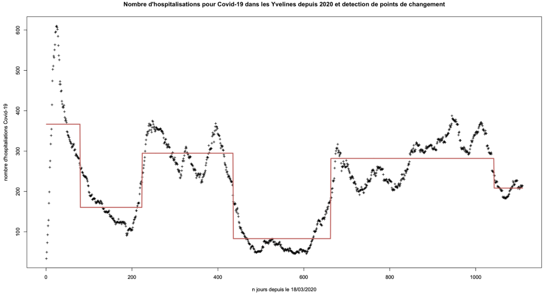
Sample rate : 250 Hz

Isotonic regression

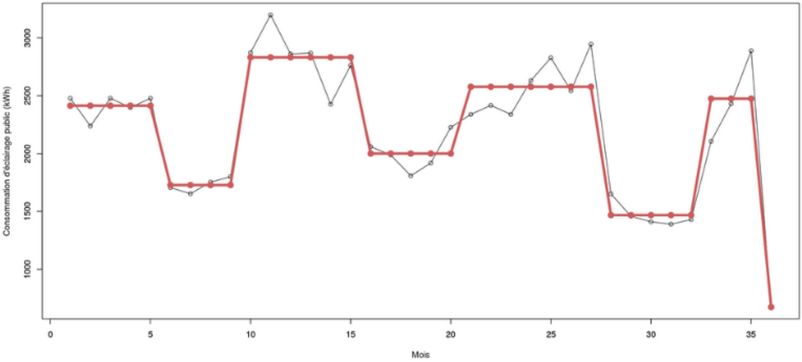


Number of Covid-19 hospitalisations





Public lighting energy consumption



We fit a simple Up-Down model.

Discussion

Strengths

1. Generalize the change point detection problem
2. Easy to use R package
3. Flexible
4. Reproducibility : github available
   `https://github.com/vrunge/gfpop/tree/master`

Weaknesses

1. Hard to tune hyperparameters and perform model selection
2. Rigid

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References

1. Toby Dylan Hocking et al. "Constrained Dynamic Programming and Supervised Penalty Learning Algorithms for Peak Detection in Genomic Data". In: Journal of Machine Learning Research 21.87 (2020), pp. 1–40. url: http://jmlr.org/papers/v21/18-843.html
2. Vincent Runge et al. gfpop: an R Package for Univariate Graph-Constrained Change- Point Detection. 2022.
3. Charles Truong, Laurent Oudre, and Nicolas Vayatis. "A review of change point detection methods". In: CoRR abs/1801.00718 (2018). url: http://arxiv.org/abs/1801.00718.