Exercise ind. 1

(1) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Base case: $n = 1$

$$\sum_{i=1}^{1} i^2 = \frac{1 \cdot (1+1)(2 \cdot 1+1)}{6} = \frac{2 \cdot 3}{6} = 1 \quad \text{ok}$$

Inductive step: Assume that it holds for $n = k > 1$

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6},$$

Now show that it holds for $k+1$

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2 = [\text{assumption}] =$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} =$$

$$= \frac{(k+1)(k(2k+1) + 6(k+1))}{6} = \frac{(k+1)(2k^2 + k + 6k + 6)}{6} =$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6} = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

Q.E.D

Exercise ind. 1

(2) $\sum_{j=1}^{n} (2j-1) = n^2$

Base Case: $n=1$: $\sum_{j=1}^{1} (2j-1) = (2\cdot1-1) = 1 = 1^2$ ok

Inductive Step: Assume that it holds for $n=k>1$

$$\sum_{j=1}^{k} (2j-1) = k^2$$

Now show this for $k+1$

$$\sum_{j=1}^{k+1} (2j-1) = \sum_{j=1}^{k} (2j-1) + (2\cdot(k+1)-1) = [\text{assumption}] =$$

$$= k^2 + (2\cdot(k+1)-1) = k^2 + 2k + 1 =$$

$$= (k+1)^2$$

Q.E.D