Math 226 **Assignment #5**

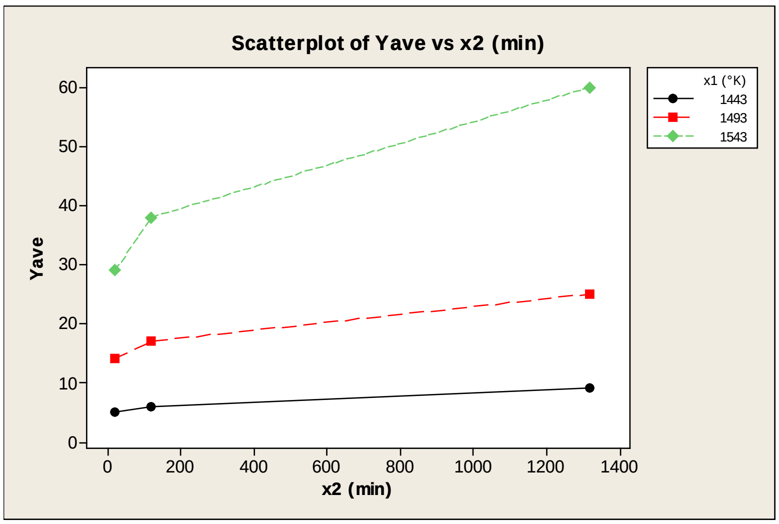
Topic: Factorial effects; Introduction to probability;
Reading Assignment: Chapter sections 4.1-4.5

*** REMINDER: Exam #1 is on Monday 9/26**

P1.

a. There are three levels of *temperature*, three levels of *time*, and 2 replicates;
This is a full factorial structure with 2 factors, 3 levels each, i.e. 3x3 with 2 replicates;

b. Interaction plot.



c. The non-parallelism of the slopes indicates that there may be an interaction between the temperature and time. Therefore, the main effects model alone is not appropriate to summarize the data.

d. *Sample mean responses* appearing in the table below are computed by averaging the 2 replicates at each treatment combination, i.e. $\overline{Y}_{ij} = \frac{1}{2}\sum_{i=1}^{2} Y_{ij}$

		B: Time			**Row**
		1	**2**	**3**	**Ave:**
A: Temp	**1**	5	6	9	**6.7**
	2	14	17	25	**18.7**
	3	29	38	60	**42.3**
Colm Ave:		**16.0**	**20.3**	**31.3**	**22.6**

e. Fitted main effects

$$a_1 = \bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot} = -15.89$$
$$a_2 = \bar{y}_{2\cdot} - \bar{y}_{\cdot\cdot} = -3.89$$
$$a_3 = \bar{y}_{3\cdot} - \bar{y}_{\cdot\cdot} = 19.78$$
$$b_1 = \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot} = -6.56$$
$$b_2 = \bar{y}_{\cdot 2} - \bar{y}_{\cdot\cdot} = -2.22$$
$$b_3 = \bar{y}_{\cdot 3} - \bar{y}_{\cdot\cdot} = 8.78$$

f. Compute the fitted interaction effects.

$$ab_{11} = \bar{y}_{11} - (\bar{y}_{\cdot\cdot} + a_1 + b_1) = 4.89$$
$$ab_{12} = \bar{y}_{12} - (\bar{y}_{\cdot\cdot} + a_1 + b_2) = 1.56$$
$$ab_{13} = \bar{y}_{13} - (\bar{y}_{\cdot\cdot} + a_1 + b_3) = -6.44$$
$$ab_{21} = \bar{y}_{21} - (\bar{y}_{\cdot\cdot} + a_2 + b_1) = 1.89$$
$$ab_{22} = \bar{y}_{22} - (\bar{y}_{\cdot\cdot} + a_2 + b_2) = .56$$
$$ab_{23} = \bar{y}_{23} - (\bar{y}_{\cdot\cdot} + a_2 + b_3) = -2.44$$
$$ab_{31} = \bar{y}_{31} - (\bar{y}_{\cdot\cdot} + a_3 + b_1) = -6.78$$
$$ab_{32} = \bar{y}_{32} - (\bar{y}_{\cdot\cdot} + a_3 + b_2) = -2.11$$
$$ab_{33} = \bar{y}_{33} - (\bar{y}_{\cdot\cdot} + a_3 + b_3) = 8.89.$$

g. **Interaction Model:** $\hat{Y}_{ij} = \overline{Y}_{\bullet\bullet} + a_i + b_j + ab_{ij}$

$\hat{Y}_{11} = 22.6 - 15.89 - 6.56 + 4.89 = 5$

$\hat{Y}_{12} = 22.6 - 15.89 - 2.22 + 1.56 = 6$

$\hat{Y}_{13} = 22.6 - 15.89 + 8.78 - 6.44 = 9$

$\hat{Y}_{21} = 22.6 - 3.89 - 6.56 + 1.89 = 14$

$\hat{Y}_{22} = 22.6 - 3.89 - 2.22 + 0.56 = 17$

$\hat{Y}_{23} = 22.6 - 3.89 + 8.78 - 2.44 = 25$

$\hat{Y}_{31} = 22.6 + 19.78 - 6.56 - 6.78 = 29$

$\hat{Y}_{32} = 22.6 + 19.78 - 2.22 - 2.11 = 38$

$\hat{Y}_{33} = 22.6 + 19.78 + 8.78 + 8.89 = 60$

		B: Time			**Row**
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Because the fitted interaction terms compute how "out of parallel" the fitted *main effects* model is from the actual data, the fitted response from the *interaction model* is equal to the mean response from the data.

4.4 Let J denote the event that Jim gets one of the jobs and D denote the event that Don gets one of the jobs; similarly define M, S, and N for Mary, Sue, and Nancy, respectively.

a Considering unordered outcomes (i.e., JD is the same outcome as DJ), all possible selections of two applicants from the five are:

JD JM
JS JN
DM DS
DN MS
MN SN

b $A = \{JD, \quad JM, \quad JS, \quad JN, \quad DM, \quad DS, \quad DN\}$; i.e., there are 7 elements in A.

c $B = \{JM, \quad JS, \quad JN, \quad DM, \quad DS, \quad DN\}$; i.e., there are 6 elements in B.

d The set containing two females is the complement of the set containing at least one male; i.e., $\overline{A} = \{MS, \quad MN, \quad SN\}$.

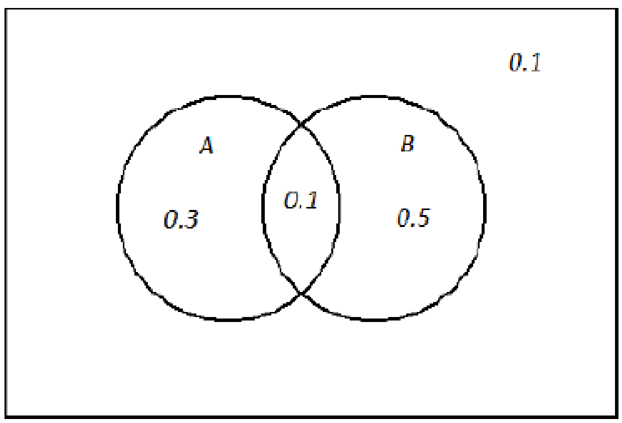
e $\overline{A} = \{MS, \quad MN, \quad SN\}$

$AB = B = \{JM, \quad JS, \quad JN, \quad DM, \quad DS, \quad DN\}$

$A \cup B = A = \{JD, \quad JM, \quad JS, \quad JN, \quad DM, \quad DS, \quad DN\}$

$\overline{AB} = \{JD, \quad MS, \quad MN, \quad SN\}$

4.10 Venn diagram of proportion of customers at outlet I and I



Using the above Venn diagram, we have the following:

a $P(A) = 0.3 + 0.1 = 0.4$

b $P(A \cup B) = 0.9$

c $P(\overline{B}) = 0.4$

d $P(AB) = 0.1$

e $P(A \cup \overline{B})$ 0.5

f $P(\overline{A} \cap \overline{B}) = 0.1$

g $P(\overline{A \cup B}) = 0.1$

4.24 a $P\begin{pmatrix}\text{one blue, one white,}\\ \text{and one green ordered}\end{pmatrix} = \dfrac{\begin{pmatrix}\text{number of ways one blue,}\\ \text{one white, and one green}\\ \text{can be ordered}\end{pmatrix}}{\begin{pmatrix}\text{total number of ways}\\ \text{a sample of three}\\ \text{can be chosen}\end{pmatrix}}$

$$= \frac{3!}{4^3} = \frac{6}{64} = \frac{3}{32} = 0.09375$$

b $P\begin{pmatrix}\text{two blues}\\ \text{are ordered}\end{pmatrix}$

$$= \frac{\begin{pmatrix}\text{number of ways of}\\ \text{ordering two blues}\\ \text{out of an order of three}\end{pmatrix}\begin{pmatrix}\text{number of ways (colors)}\\ \text{the remaining car}\\ \text{can be ordered}\end{pmatrix}}{\begin{pmatrix}\text{total number of ways}\\ \text{a sample of three}\\ \text{can be chosen}\end{pmatrix}}$$

$$= \frac{\binom{3}{2}\binom{3}{1}}{4^3} = \frac{9}{64} = 0.140625$$

c Note that if no black are chosen, then each order may be any of the three remaining colors, and this may be done in $3^3 = 27$ ways. Thus P(at least one black) = 1−P(none are black) $= 1 - \frac{3^3}{4^3} = 1 - \frac{27}{64} = \frac{37}{64} = 0.578125$.

d Multiplying the answer to (b) by the number of ways to choose a duplicated color, we have $\frac{\binom{4}{1}\binom{3}{2}\binom{3}{1}}{4^3} = \frac{36}{64} = 0.5625$.

4.33 a $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{0.15}{0.3} = 0.5$

b No, since $P(A|B) \neq P(A)$, we can conclude that A and B are not independent.