Math 226 **Assignment 12**

Topic: Interval estimation of process parameters; Prediction intervals; Hypothesis testing;

Reading Assignment: 9.2, 9.4, 10.1, 10.2

Assigned Problems:

Chapter 9: **12, 15, 16, 22, 58**

Chapter 10: **16, 17, 18, 24**

9.12 We seek a 98% large sample confidence interval for the mean percent of shrinkage given that $\bar{x} = 18.4, \quad s = 1.2$, and $n = 45$. Thus,

$x + z_{0.01}\sigma/\sqrt{n} \approx 18.4 \pm 2.33(1.2)/\sqrt{45} = (17.98, \quad 18.82)$.

9.15 We need to estimate the percent of shrinkage to within $B = 0.2$ with confidence coefficient $1 - \alpha = 0.98$. From Exercise 9.12, the standard deviation is given as 1.2. Using this information, we get

$n = (z_{0.01}\sigma/B)^2 \approx (2.33(1.2)/0.2)^2 = (13.98)^2 = 195.44$ or $n = 196$.

9.16 With $\hat{p} = y/n = 12/100$, an approximate 95% confidence interval for the population proportion of resistors that fail to meet the tolerance specification is as follows:

$\hat{p} \pm z_{0.0252}\sqrt{\hat{p}(1-\hat{p})/n} = 0.12 \pm 1.96\sqrt{(0.12)(0.88)/100} = (0.0563, \quad 0.1837)$.

9.22 We are given that $\bar{x} = 180, \quad s = 5$, and $n = 5$. Assuming a normal population of warpwise breaking-strength measurements, a 95% confidence interval for the true mean warpwise breaking-strength is

$\bar{x} \pm \dfrac{t_{0.05}s}{\sqrt{n}} = 180 \pm \dfrac{2.776(5)}{\sqrt{5}} = (173.7927, \quad 186.2073)$ where $t_{0.025}$ is found from t-table, with 4 degrees of freedom.

9.58 We calculate $\bar{x} = \dfrac{25.5}{6} = 4.25$ and $s = \left(\dfrac{110.25 - \dfrac{(25.5)^2}{6}}{5}\right)^{1/2} = 0.6124$. A 90% prediction interval for the compressive strength is (degrees of freedom $= 5$)

$\bar{x} \pm t_{0.05}s\sqrt{1 + \dfrac{1}{n}} = 4.25 \pm 2.015(0.6124)\sqrt{1 + \dfrac{1}{6}} = (2.9171, 5.5829)$.

10.16 Hypotheses: $H_0 : \mu \geq 64 \quad H_a : \mu < 64$

Test Statistics: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \approx \dfrac{62 - 64}{8/\sqrt{50}} = -1.77$

Rejection Region: $z < -z_{0.01} = -2.33$

Conclusion: Fail to reject H_0 at $\alpha = 0.01$; i.e., there is insufficient evidence to conclude that the mean hardness is less than 64 at $\alpha = 0.01$.

P-value: $P(Z \leq -1.77) = 0.5 - 0.4616 = 0.0384$

10.17 $\beta = P(\text{fail to reject } H_0 \text{ given that } H_a \text{ is true})$

$$= P\left(\frac{\bar{x} - 64}{8/\sqrt{50}} > -2.33 | \mu = 60\right)$$

$$= P(\bar{x} > 61.3639 | \mu = 60)$$

$$= P\left(\frac{\bar{x} - 60}{8/\sqrt{50}} > \frac{61.3639 - 60}{8/\sqrt{50}}\right)$$

$$= P(Z > 1.21) = 0.5 - 0.3869 = 0.1131$$

10.18 $n \geq \dfrac{(z_{0.01} + z_{0.05})^2\sigma^2}{(\mu_a - \mu_0)^2} = \dfrac{(2.33 + 1.645)^2(8)^2}{(60 - 64)^2} = 63.2025$ or $n = 64$

10.24 Hypotheses: $H_0 : \mu = 1.5$ $H_a : \mu > 1.5$

Test Statistics: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{1.9 - 1.5}{0.04/\sqrt{10}} = 6.32$

Rejection Region: $t > t_{0.05} = 1.833$ (degrees of freedom $= 9$)

Conclusion: Reject H_0 at $\alpha = 0.05$; i.e., there is sufficient evidence to conclude that the mean porosity is significantly greater than 1.5 at $\alpha = 0.05$.