The complete scene is described by the amplitude and phase offset of the wavefield at each point, $s(x, y, z)$. The antenna records the wavefield at position $z = 0$, for each frequency ω, $s(x, y, z = 0, \omega)$. To reconstruct the wavefield at depth, z, we *downwards continue* the recorded wavefield by integrating over the signals received at each antenna after multiplying by an appropriate phase offset.

$$\begin{aligned} s(x, y, z) &= \int s(x, y, z, \omega) d\omega \\ &= \int (\int \int s(x', y', z = 0, \omega) e^{-j\frac{2\omega}{c}||x'-x, y'-y, z||} dx' dy') d\omega \end{aligned}$$

To perform the above operation efficiently, we can avoid the double integral by first pre-transforming the recorded signal to the frequency domain. Let $S(k_x, k_y, z = 0, \omega) = \mathrm{FT}_{x,y}\{s(x, y, z = 0, \omega)\}$ be the frequency domain representation of the recorded signal, then

$$S(k_x, k_y, z) = \int S(k_x, k_y, z = 0, \omega) e^{-jk_z z} d\omega$$

where $k_z = \sqrt{(\frac{2\omega}{c})^2 - k_x^2 - k_y^2}$. By resampling $S(k_x, k_y, z = 0, \omega)$ along evenly spaced intervals of k_z, the above operation can be transformed into an Inverse Fourier transform and computed efficiently using an IFFT. This step is known in the literature as Stolt interpolation.

$$\begin{aligned} S(k_x, k_y, z) &= \int S(k_x, k_y, z = 0, k_z) e^{-jk_z z} dk_z \\ &= \mathrm{IFT}_{k_z}\{S(k_x, k_y, z = 0, k_z) e^{-jk_z z_0}\}, \end{aligned}$$

where z_0 is the minimum distance to the target. Thus the complete algorithm is

$$s(x, y, z) = \mathrm{IFT}_{k_x,k_y}\{\mathrm{IFT}_{k_z}\{\mathrm{Stolt}\{\mathrm{FT}_{x,y}\{s(x', y', z = 0, \omega)\}\} e^{-jk_z z_0}\}\}.$$