CS-AD-216: Foundations of Computer Graphics

Assignment 4, Due: October 6

Instructions:

- Assignments can be submitted in groups of at most three. The purpose of groups is to learn from each other, not to divide work. Each member should participate in solving the problems and have a complete understanding of the solutions submitted.
- Submit your assignments as a zip file (one per group).

Problem 1 (10 points: 3 + 4 + 3)**.**
All coordinates of the points and vectors below are in the standard orthonormal basis: unit vectors along the positive x, y and z directions.

Let a, b and c be three points in three dimensions with coordinates $(1, 2, 3)$, $(1, 3, 2)$ and $(2, -1, 3)$ respectively.

1. Write down the equation of the plane h passing through the three points.
2. Does the point $(\frac{7}{6}, \frac{11}{6}, \frac{16}{6})$ lie in the triangle abc? If so, what are its barycentric coordinates w.r.t. a, b and c?
3. Consider the ray $p + t\vec{v}$, $t \geq 0$, where p is the point $(1, 1, 1)$ and v is the vector $(0, 2, -1)$. Does the ray intersect the plane h? If so, at which point?

Problem 2 (10 points: 3+3+4)**.**
Write the 3×3 homogeneous transformation matrices for the following transformations in two dimensions:

1. translation by 1 along positive x axis and 2 along the positive y axis, followed by scaling by a factor 3 along both x and y axes.
2. clockwise rotation by 30 degrees around the point $(-1, 2)$.
3. reflection about the line $y = x + 2$.

Problem 3 (10 points)**.**
The goal of this exercise is to produce drawings of Mandelbrot and Julia sets.

The Mandelbrot set is defined as follows. For each complex number $x + iy$, we construct a sequence of complex numbers $z_0, z_1, \cdots$ where $z_0 = 0$ and $z_{n+1} = z_n^2 + c$, where $c = x + iy$ and $i = \sqrt{-1}$. The complex number $x + iy$ belongs to the Mandelbrot set if and only if all the complex numbers in the sequence have magnitude at most 2. The magnitude of a complex number $a + ib$ is $\sqrt{a^2 + b^2}$.

For drawing such a set, we associate with each point (x, y) in the plane, the complex number $x + iy$ and we assign a color black or white to it depending on whether $x + iy$ belongs to the Mandelbrot set. However, a more beautiful drawing is obtained if we color each point depending on how quickly in the sequence corresponding to it we get a complex number of magnitude more than 2.

For the purpose of programming, we can compute the first (at most) N complex numbers in the sequence for each point and find the smallest i such that z_i has magnitude more than 2. We can then give the point a color based on i. If none of the complex numbers numbers among the first N in the sequence have magnitude more than 2, then we can give the point a default color (say black). N can be set to around 300.

Julia sets are define in a very similar way. This time, the sequence for the point (x, y) starts with $z_0 = x + iy$ and c is a fixed complex number that belongs to the Mandelbrot set. Every choice of c gives a different drawing.

We would also like Pan and Zoom functionalities in our program i.e., we should be able to drag the picture using the mouse and zoom into or out of the picture using the mouse wheel. Please write the appropriate event handlers for the same.