Computer Graphics Assignment #4. Oct. 6, 2016.

#1.

1. $\vec{v}_1 = (1,3,2) - (1,2,3) = (0,1,-1)$

$\vec{v}_2 = (2,-1,3) - (1,2,3) = (1,-3,0).$

$\vec{n} = \vec{v}_1 \times \vec{v}_2 = (-3,-1,-1) \rightsquigarrow \vec{n}' = (3,1,1)$

$P_0 = (1,2,3).$

$$\boxed{3(x-1) + (y-2) + (z-3) = 0}$$

2. $\frac{1}{6}(7,11,6) = \alpha(1,2,3) + \beta(1,3,2) + \gamma(2,-1,3)$

$(7,11,6) = \alpha(6,12,18) + \beta(6,18,12) + \gamma(12,-6,18)$

$$\left[\begin{array}{ccc|c} 6 & 6 & 12 & 7 \\ 12 & 18 & -6 & 11 \\ 18 & 12 & 18 & 6 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -23/24 \\ 0 & 1 & 0 & 11/8 \\ 0 & 0 & 1 & 3/8 \end{array}\right]$$

$(\alpha, \beta, \gamma) = (-23/24, 11/8, 3/8)$. Since $\alpha < 0$, the point does not ~~lie on~~ lie in the triangle $\triangle abc$.

3. ~~$h$ can be expressed as:~~ $p + t\vec{v} = (1, 1+2t, 1-t) = (x^*, y^*, z^*)$

$3(1-1) + (1+2t-2) + (1-t-3) = t - 3 = 0$

The equation is true for $t=3$. $\Rightarrow \boxed{p + 3\vec{v} = (1,7,-2)}$

#2

1. $$M\cdot p = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} p. = (\text{Scale by } 3)(\text{translate})\ p.$$

$$M = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

2. $$Mp = \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translate back}} \underbrace{\begin{bmatrix} \cos(\pi/6) & -\sin(\pi/6) & 0 \\ \sin(\pi/6) & \cos(\pi/6) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Rotate by } 30^\circ} \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translate } (-1,2) \text{ to origin.}} p$$

$$M = \begin{bmatrix} \sqrt{3}/2 & -1/2 & \sqrt{3}/2 \\ 1/2 & \sqrt{3}/2 & 5/2 - \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

#2. 3.

$$Mp = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translate back}} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{reflection across } y=x} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translate } y=x+2 \text{ to } y=x.} p$$

$$M = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$