Computer Graphics: Assignment #3

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Problem 1

1. Compute all vectors of length 1 perpendicular to the vectors (1, 2, 3) and (2, 3, 4).

$$\begin{aligned}\boldsymbol{v} &= (1,2,3) \times (2,3,4) \\ &= (-1,2,-1) \\ ||\boldsymbol{v}|| &= \sqrt{6} \\ \boldsymbol{u} &= \pm\frac{\boldsymbol{v}}{||\boldsymbol{v}||}\end{aligned}$$

thus $\boldsymbol{u} = (-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}})$ or $\boldsymbol{u} = (\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$

2. Compute the angle between the vectors (2, 3, 1) and (1, 2, 3). You can express your answer using inverse trigonometric functions.

Using the definition of dot product: $\boldsymbol{u} \cdot \boldsymbol{v} = ||\boldsymbol{u}|| \times ||\boldsymbol{v}|| \cos\theta$,

$$\begin{aligned}-2+6-3 &= \sqrt{4+9+1} \times \sqrt{1+4+9} \cos\theta \\ 1 &= 14\cos\theta \\ \theta &= \arccos(\frac{1}{14})\end{aligned}$$

3. Let $\boldsymbol{u} = (7, 4, 22)$ and $\boldsymbol{v} = (7, \sqrt{3}, 9.9)$. Compute $(\boldsymbol{u}+\boldsymbol{v}) \cdot (\boldsymbol{u} \times \boldsymbol{v})$.

$$\begin{aligned}\boldsymbol{u}+\boldsymbol{v} &= (0,\ 4+\sqrt{3},\ -12.1) \\ \boldsymbol{u} \times \boldsymbol{v} &= (39.6 + 22\sqrt{3},\ 154 - 69.3,\ 7\sqrt{3} + 28) \\ (\boldsymbol{u}+\boldsymbol{v}) \cdot (\boldsymbol{u} \times \boldsymbol{v}) &= 0\ +\ 84.7(4+\sqrt{3})\ -\ 84.7(4+\sqrt{3}) \\ &= 0\end{aligned}$$

4. Consider a plane in three dimensions passing through the point (2, 0, 1) and having normal (1, 1, 2). What is the implicit equation of the plane?

$$\begin{aligned}\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) &= 0 \\ 1(x-2) + 1(y-0) + 2(z+1) &= 0 \\ x + y + 2z &= 0\end{aligned}$$

5. Is the point (2, 5) an affine combination of the points (6, 3) and (9, 11)? Explain your answer.

   No. If (2,5) was an affine combination of the two points, it would lie on the line that connects the two points. However, We can easily see that it does not, by comparing the vector (6, 3) - (9, 11) = (15, -8) with the vector (6, 3) - (2, 5) = (4, -2) and observing that the direction does not match.

Problem 2

Please see the attached files '*HW_3/hw3_interaction.html*' and '*HW_3/hw3_interaction.js*'.