#1 $\vec{v}' = \vec{v}\cos\theta + (1-\cos\theta)(\vec{v}\cdot\vec{u})\hat{u} + \sin\theta(\hat{u}\times\vec{v})$

represent in vector form

$$= \begin{pmatrix} v_1\cos\theta \\ v_2\cos\theta \\ v_3\cos\theta \end{pmatrix} + \begin{pmatrix} (v_1u_1+v_2u_2+v_3u_3)u_1 \\ (v_1u_1+v_2u_2+v_3u_3)u_2 \\ (v_1u_1+v_2u_2+v_3u_3)u_3 \end{pmatrix}(1-\cos\theta) + \sin\theta\begin{pmatrix} \cancel{u_2v_3 - u_3v_2} \\ -u_1v_3 + u_3v_1 \\ u_1v_2 - u_2v_1 \end{pmatrix}$$

simplify.

$$= \begin{pmatrix} v_1\cos\theta + (v_1u_1+v_2u_2+v_3u_3)u_1(1-\cos\theta) + \sin\theta(u_2v_3 - u_3v_2) \\ v_2\cos\theta + (v_1u_1+v_2u_2+v_3u_3)u_2(1-\cos\theta) + \sin\theta(u_3v_1 - u_1v_3) \\ v_3\cos\theta + (v_1u_1+v_2u_2+v_3u_3)u_3(1-\cos\theta) + \sin\theta(u_1v_2 - u_2v_1) \end{pmatrix}$$

organize with regards to v.

$$= \begin{pmatrix} v_1[\cos\theta + u_1^2(1-\cos\theta)] + v_2[u_1u_2(1-\cos\theta) - u_3\sin\theta] + v_3[u_1u_3(1-\cos\theta) + u_2\sin\theta] \\ v_1[u_1u_2(1-\cos\theta) + u_3\sin\theta] + v_2[\cos\theta + u_2^2(1-\cos\theta)] + v_3[u_2u_3(1-\cos\theta) - \sin\theta u_1] \\ v_1[u_1u_3(1-\cos\theta) - u_2\sin\theta] + v_2[u_2u_3(1-\cos\theta) + u_1\sin\theta] + v_3[\cos\theta + u_3^2(1-\cos\theta)] \end{pmatrix}$$

take out v.

$$= \begin{pmatrix} \cos\theta + u_1^2(1-\cos\theta) & u_1u_2(1-\cos\theta) - u_3\sin\theta & u_1u_3(1-\cos\theta) + u_2\sin\theta \\ u_1u_2(1-\cos\theta) + u_3\sin\theta & \cos\theta + u_2^2(1-\cos\theta) & u_2u_3(1-\cos\theta) - u_1\sin\theta \\ u_1u_3(1-\cos\theta) - u_2\sin\theta & u_2u_3(1-\cos\theta) + u_1\sin\theta & \cos\theta + u_3^2(1-\cos\theta) \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

#3 Given a rotation matrix M, multiplying it to a set of vectors/points should not change their relationships, or their magnitude/distance.

This also holds for rotating a set of standard basis vectors, which would be represented as I (identity matrix) whose columns are each a vector in the standard basis.

Thus ~~multiplying~~ $M \cdot I = M$ must also have columns that are orthonormal to eachother, since it is a result of rotating the standard basis.