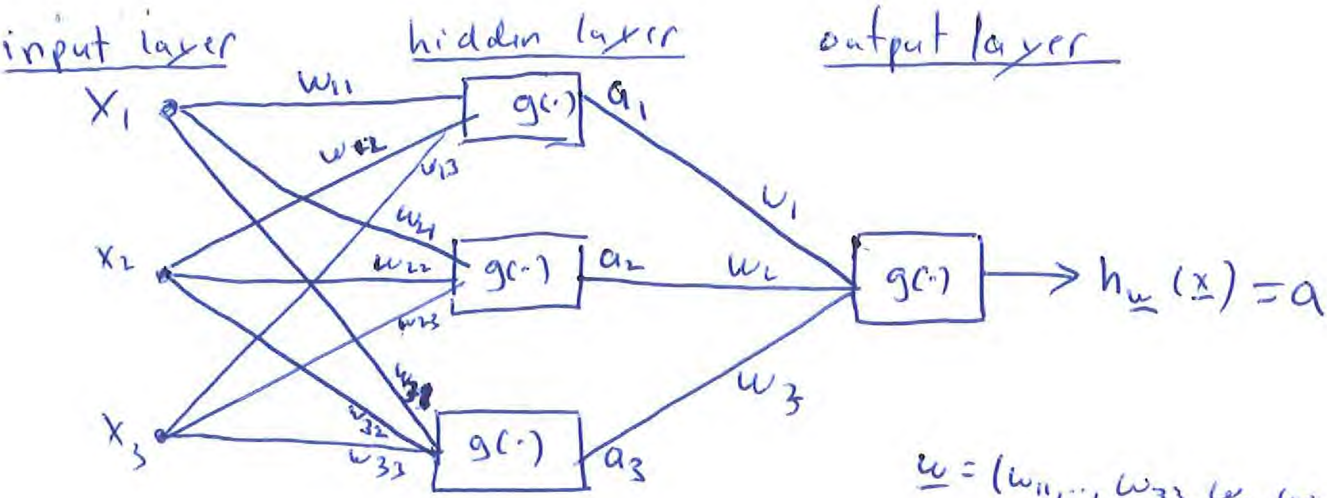

$\underline{w} = (w_{11}, \dots, w_{33}, w_1, w_2, w_3)$

$$a_1 = g(\overbrace{w_{11}x_1 + w_{12}x_2 + w_{13}x_3}^{z_1}) = g(z_1)$$

$$a_2 = g(\overbrace{w_{21}x_1 + w_{22}x_2 + w_{23}x_3}^{z_2}) = g(z_2)$$

$$a_3 = g(\overbrace{w_{31}x_1 + w_{32}x_2 + w_{33}x_3}^{z_3}) = g(z_3)$$

$$h_{\underline{w}}(\underline{x}) = g(\overbrace{w_1 a_1 + w_2 a_2 + w_3 a_3}^{z}) = g(z)$$

$$= \frac{1}{1 + \exp(-w_1 a_1 - w_2 a_2 - w_3 a_3)}$$

Computational effort:

- Suppose N features $\underline{x} = (x_1, \dots, x_N)$
- Suppose L hidden units
- Effort to calculate a_ℓ is $O(N)$
- Effort to calculate all a_ℓ's is $O(LN)$
- Effort to calculate $h_{\underline{w}}(\underline{x})$ from a_ℓ's is $O(L)$
- Total effort $O(LN)$. Acceptable.

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$$g(x) = \frac{1}{1+e^{-x}} = \left(1+e^{-x}\right)^{-1}$$

$$g'(x) = -1\left(1+e^{-x}\right)^{-2}\left(e^{-x}\right)(-1)$$

$$= \frac{e^{-x}}{\left(1+e^{-x}\right)^{2}} = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$= g(x)\left(1-g(x)\right)$$

Calculating the Gradient

In order to apply gradient descent, need to calculate gradients. Consider calculating gradient of $h_{\underline{w}}(\underline{x})$. For example,

$$\frac{\partial h_{\underline{w}}(\underline{x})}{\partial w_2} = \frac{\partial}{\partial w_2} g(w_1 a_1 + w_2 a_2 + w_3 a_3)$$

$$= g'(w_1 a_1 + w_2 a_2 + w_3 a_3) \cdot a_2$$

$$= g(w_1 a_1 + w_2 a_2 + w_3 a_3) \cdot [1 - g(w_1 a_1 + w_2 a_2 + w_3 a_3)]\, a_2$$

$$= h_{\underline{w}}(\underline{x})\, [1 - h_{\underline{w}}(\underline{x})]\, a_2$$

More generally,

$$\frac{\partial h_{\underline{w}}(\underline{x})}{\partial w_i} = h_{\underline{w}}(\underline{x})\, [1 - h_{\underline{w}}(\underline{x})]\, a_i$$

$$= a(1-a)\, a_i$$

$$\boxed{\frac{\partial h_{\underline{w}}(\underline{x})}{\partial w_i} = a(1-a)\, a_i}$$

$$\frac{\partial h_{\underline{w}}(\underline{x})}{\partial w_{32}} = \frac{\partial}{\partial w_{32}} g(z)$$

$$= g'(z) \frac{\partial z}{\partial w_{32}}$$

$$= g(z)(1-g(z)) \frac{\partial z}{\partial w_{32}} = a(1-a) \frac{\partial z}{\partial w_{32}}$$

$$\frac{\partial z}{\partial w_{32}} = \frac{\partial}{\partial w_{32}} \left[w_1 a_1 + w_2 a_2 + w_3 a_3 \right]$$

$$= w_3 \frac{\partial a_3}{\partial w_{32}}$$

$$\frac{\partial a_3}{\partial w_{32}} = \frac{\partial}{\partial w_{32}} g(w_{31} x_1 + w_{32} x_2 + w_{33} x_3)$$

$$= g'(z_3)\, x_2 = g(z_3)(1-g(z_3))\, x_2$$

$$= a_3(1-a_3)\, x_2$$

$$\frac{\partial h_{\underline{w}}(\underline{x})}{\partial w_{32}} = a(1-a) \cdot a_3(1-a_3)\, w_3\, x_2$$

$$\boxed{\frac{\partial h_{\underline{w}}(\underline{x})}{\partial w_{ij}} = a(1-a)\, a_i(1-a_i)\, w_i\, x_j}$$

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$$J(\underline{w}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log h_{\underline{w}}(\underline{x}^{(i)}) + (1-y^{(i)})\log\left(1-h_{\underline{w}}(\underline{x}^{(i)})\right)\right]$$

So need to calculate partials for $\log h_{\underline{w}}(\underline{x}^{(i)})$ and $\log(1-h_{\underline{w}}(\underline{x}^{(i)}))$.

$$\frac{\partial \log h_{\underline{w}}(\underline{x})}{\partial w_2} = \frac{1}{h_{\underline{w}}(\underline{x})}\frac{\partial}{\partial w_2}h_{\underline{w}}(\underline{x})$$

$$= \frac{1}{a}a(1-a)a_2 = (1-a)a_2$$

$$\frac{\partial \log h_{\underline{w}}(\underline{x})}{\partial w_{32}} = \frac{1}{h_{\underline{w}}(\underline{x})}\frac{\partial}{\partial w_{32}}h_{\underline{w}}(\underline{x})$$

$$= \frac{1}{a}a(1-a)a_3(1-a_3)w_3x_2$$

$$= (1-a)a_3(1-a_3)w_3x_2$$

~~$\frac{\partial \, cost(h_w(x), y)}{\partial w_j}$~~