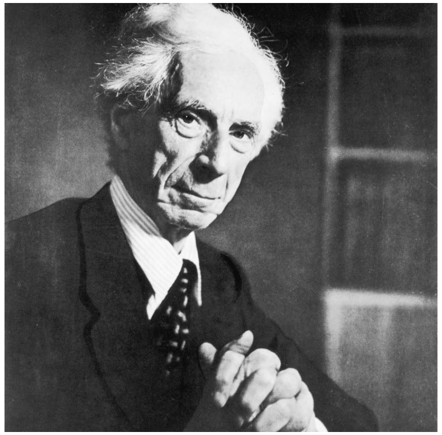

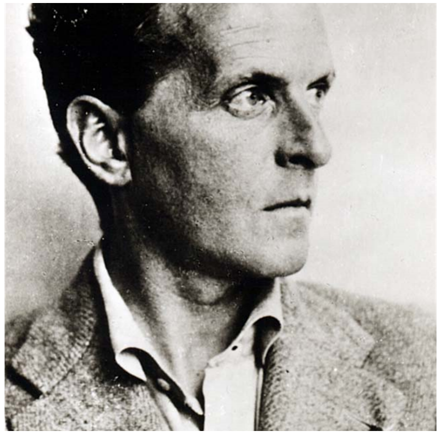

Wittgenstein & the Invention of Mathematics

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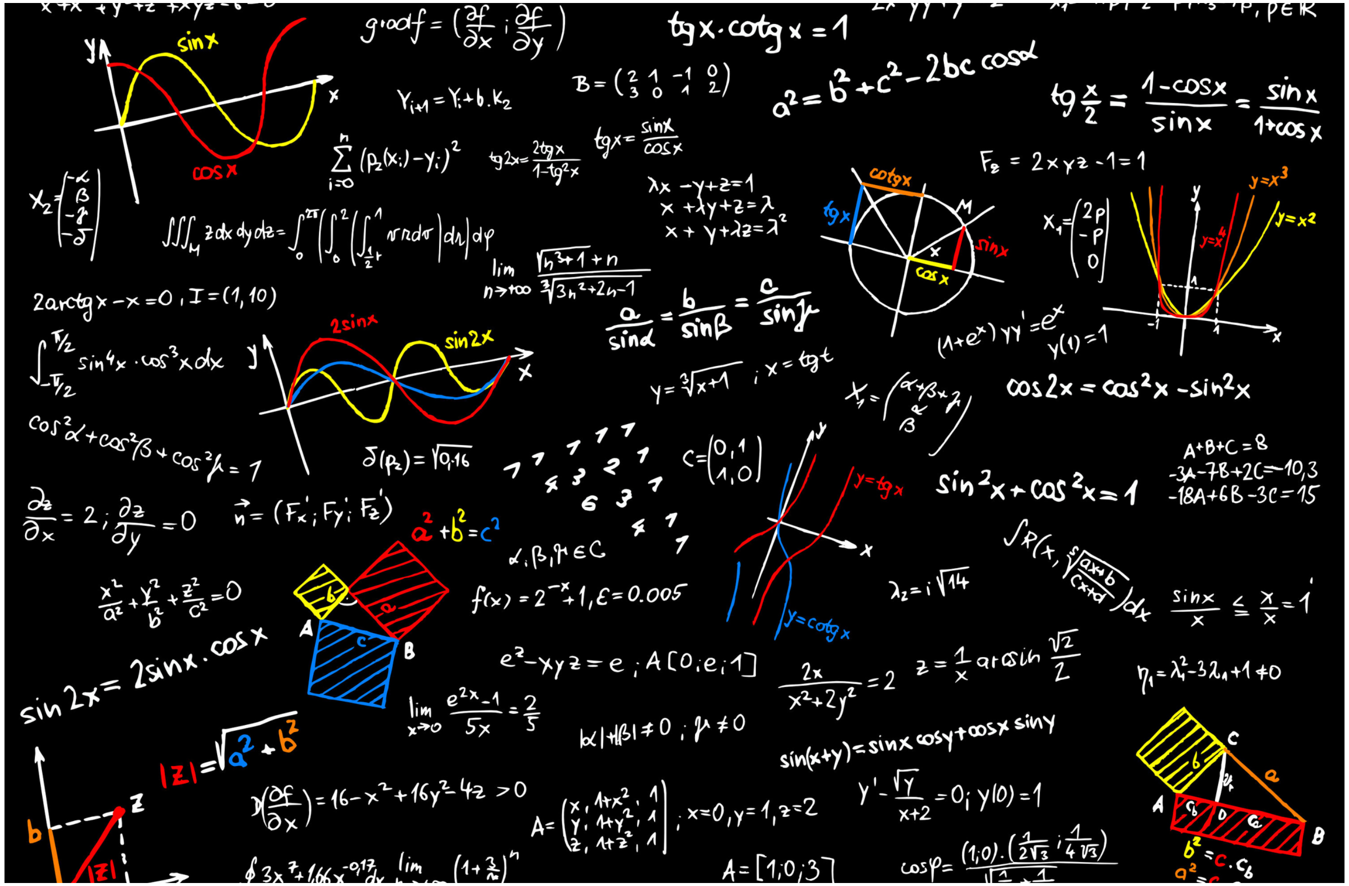
Maths in Science & Society Presentations 2014



Outline

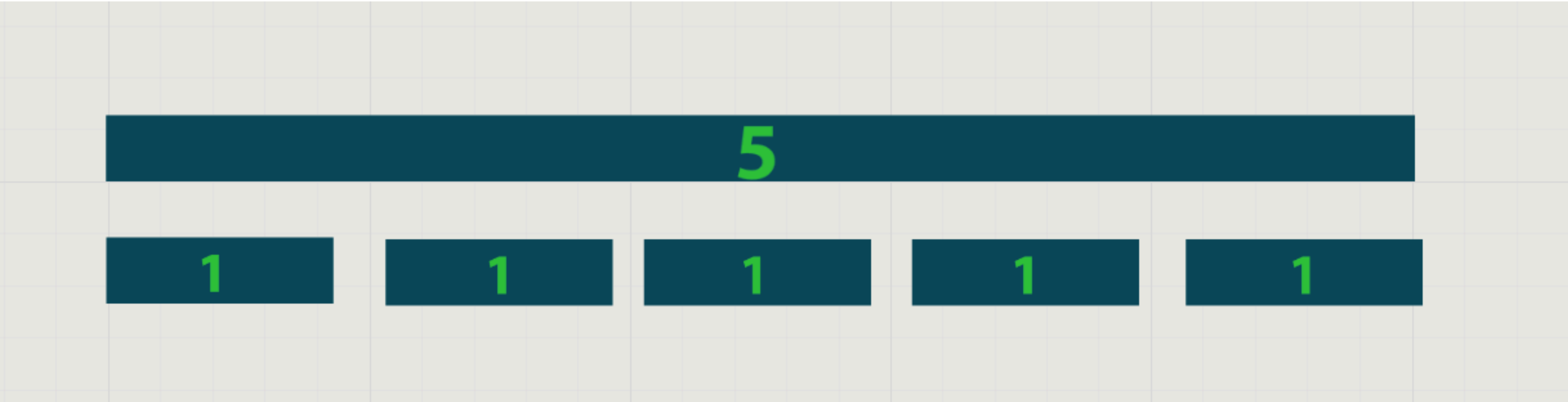
- ✧ Are numbers invented?
- ✧ Can we have mathematics without language?

1: Number

- 1.1: Ancient Greek Definition
- 1.2: Russell & Frege's Definition
- 1.3: Wittgenstein's Logical Definition
- 1.4 Numbers in the Animal Kingdom

1.1 Ancient Greece

- Thought of numbers as lengths
- For example a number was of ‘prime length’ if it couldn’t be broken down into non-unit integer pieces.
- Trouble with irrational numbers clearly follows.



1.2 Russell & Frege

- "The Cardinal Number of a is defined as the class of all classes similar to a. This definition is due to Frege." [Whitehead, A., Russell, B. 1912, Vol. 2, p. 4].
- "A cardinal number appertains to a class, not to the members of the class." [Whitehead, A., Russell, B. 1912, Vol. 2, p. 4].
- Two sets have the same 'number' if we can show a bijection.
- Then move on to more complicated operations.
- Set theoretic notion of number extends to transfinite numbers.

1.3 Wittgenstein's Logical Definition

- Number is the exponent of an operation (in logic).
- "The general form of the cardinal number is: [0, ξ , ξ + 1]" [Wittgenstein, L. 1922, 6.03]

$$\Omega'\Omega^{\nu\prime}x = \Omega^{\nu+1}x$$

- We see the inductive step shown above.
- Along with the basis case (zero), this implies a strict ordering straight away.

1.3 Wittgenstein's Logical Definition

- "The so-called law of induction cannot in any case be a logical law, for it is obviously a significant proposition. And therefore it cannot be a law a priori either." [Wittgenstein, L. 1922, 6.31].

- "The concept number is nothing else than that which is common to all numbers, the general form of number." [Wittgenstein, L. 1922, 6.022]

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1.4 Numbers in the Animal Kingdom

- "*We succeeded in bringing out the abstract property of number by varying the size, shape, arrangement and other physical properties of the objects presented, so that the chimpanzee could give the correct answer consistently only if it singled out the number of objects as the important property. (Ferster, 1964, p. 105).*" [Davis, H., Memmott, J. 1982, p. 553]
- Does this mean numbers are *a priori* ? If so, numbers have a 'good' set theoretic definition.

2: Language & Intuition

- 2.1: The Language of Mathematics & Mathematical Prose
- 2.2: Mechanics
- 2.3: Kurt Gödel & Mathematical Intuition
- 2.4: Can Intuition Be Wrong?

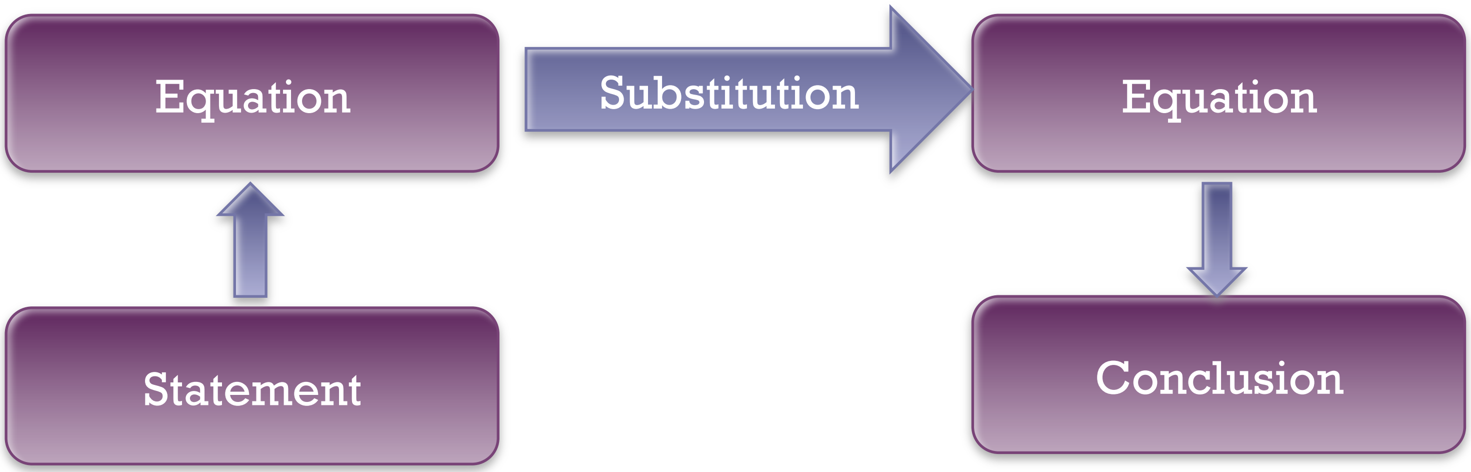
2.1: The Language of Mathematics & Mathematical Prose

- Wittgenstein argues infinity is not a ‘number word’.
- He also argues mathematics and language are both just logical propositions, and that we can’t do mathematics without using language.
- A suitably complex proposition may have a completely different meaning when stated in another language. Idioms, context, etc.

2.2: Mechanics

- "Mechanics is an attempt to construct according to a single plan all true propositions which we need for the description of the world." [Wittgenstein, L. 1922, 6.343].

- "In life it is never a mathematical proposition which we need, but we use mathematical propositions only in order to infer from propositions which do not belong to mathematics to others which equally do not belong to mathematics." [Wittgenstein, L. 1922, 6.211].



2.3: Kurt Gödel & Mathematical Intuition

- “Intuition is equivalent of what perception is for physical objects” – Gödel.
- Important part of mathematician’s arsenal.

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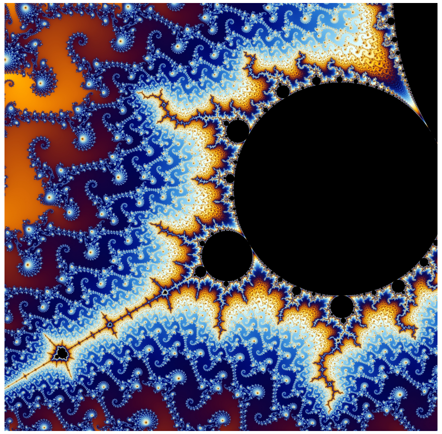
2.4: Can Intuition Be Wrong?

- Non-Euclidean Geometry
- Complex Numbers
- Decimal System
    - Irrational Numbers



3: Summary

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- Distinction the mathematics we have today and the ideal, whether invented or discovered.
- Mathematics can only tell us about mathematics, but we can infer statements about the real world through 'translation'.
- If mathematics and language cannot be separated, and language is invented, then mathematics is invented too.
- If numbers are invented, then any mathematics using numbers is invented too.