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Short communication

An amendment to "Distributed synchronization under uncertainty: A fuzzy approach"

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Abstract

In a recent work we provided a framework for the synchronization of Critical Infrastructure Interdependency models with fuzzy state. In such a paper, however, a too restrictive condition is given, in that the possibility to synchronize the interdependency models without disclosing sensitive information is not completely feasible. In this note we provide an amendment providing a solution to the problem.

Keywords: Synchronization; Fuzzy systems; Critical infrastructures

1. Introduction

The synchronization of distributed systems is a challenging task with many engineering applications such as mobile robotics and distributed sensor networks (see [5] and the references therein). In the field of critical infrastructures, several attempts to provide an online framework have been made, where different control rooms synchronize in order to gain a shared point of view on the ongoing situation. However, due to the complexity of retrieving adequate quantitative data to tune the models, in many cases human experts are involved in the process and the models are set up based on the linguistic information elicited by the technicians and stakeholders. Among others in [4] a framework for the online synchronization of distributed *Input–Output Inoperability Models* (IIM) characterized by fuzzy state is provided. In the paper both a total information sharing and a partial information sharing are discussed, but the latter has proven to be ineffective due to a too restrictive condition, as highlighted in [1].

In this note we provide a solution to the problem showing how the systems may reach synchronization without disclosing sensitive information.

In [4] an array of p identical linear *Fuzzy Input–Output Inoperability Models* [2,3] are considered in the form:

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$$\begin{cases} \begin{bmatrix} q_i(k+1) \\ c_i(k+1) \end{bmatrix} = \begin{bmatrix} A^* & B \\ 0_n & I_n \end{bmatrix} \begin{bmatrix} q_i(k) \\ c_i(k) \end{bmatrix} + u_i(k) \\ y_i(k) = C \begin{bmatrix} q_i(k) \\ c_i(k) \end{bmatrix} \end{cases} \tag{1}$$

where $A^*, B \in \mathbb{R}^{n\times n}$, $q_i(k), c_i(k) \in \mathbb{E}^n$, $u_i(k) \in \mathbb{E}^{2n}$, where $\mathbb{E}$ is the set of fuzzy numbers defined in [4].

In [4] the following feedback is adopted:

$$u_i(k) = \Omega C \sum_{j=1}^{p} \gamma_{ij} \big(y_j(k) - y_i(k)\big) \tag{2}$$

where Ω is $n \times m$ and solves the synchronization problem and γ_{ij} are the coefficients of a $p \times p$ interconnection matrix that captures the ability of the systems to share information only with their neighbors.

In [4], Theorem 2.1 it is stated that, among other hypotheses, matrix C may be such that $m < n$ and $C^T C$ must be invertible in order to solve the problem in a simple way. However, as noted in [1], this condition implies that matrix C is square and invertible.

The approach in [4], however, although providing a framework for the synchronization of linear systems with fuzzy initial conditions, fails to provide a solution that avoids the disclosure of sensitive information.

2. Proposed solution

To overcome the above drawback, there is the need to share data in an encoded form that discloses as little information as possible.

This is the objective of the second case study provided in [4], where, unfortunately, an inappropriate C matrix is adopted. A correct formulation for such a problem can be obtained with the following $2n \times 2n$ matrix C:

$$C = \left[\begin{array}{c|c} -I_n & (I-A)^{-1}B \\ \hline \frac{1}{n}\cdots\frac{1}{n} & 0\cdots 0 \\ 1-1\ 0\cdots 0 & 0\cdots 0 \\ \vdots & \vdots \\ 1\ 0\cdots 0-1 & 0\cdots 0 \end{array}\right] \tag{3}$$

which is based on the EIR index [4], a suitable way to encode the data to be exchanged among the different subsystems.

We want to show that matrix C is invertible. To this end, let

$$H = \left[\begin{array}{c|c} \frac{1}{n} & \frac{1}{n}1_{n-1}^T \\ \hline 1_{n-1} & -I_{n-1} \end{array}\right]$$

It is a standard result that if P is a square matrix and P is invertible then

$$\det\left(\begin{bmatrix} P & Q \\ R & S \end{bmatrix}\right) = \det(P)\det\big(S - RP^{-1}Q\big) \tag{4}$$

therefore

$$\det(C) = -\det\big(H(I-A)^{-1}B\big) = -\det(H)\det\big((I-A)^{-1}B\big) \tag{5}$$

$\det((I-A)^{-1}B)$ is nonzero due to the structure of the IIM model [2], while, using the expression in Eq. (4) $\det(H)$ is given by:

$$\det(H) = \frac{1}{n}\det\big(-I_{n-1} - 1_{n-1}1_{n-1}^T\big) \tag{6}$$

which is nonzero by simple inspection. Hence C is nonsingular.

Fig. 1 shows the result of the encoding considering the same example as the one provided in [4]. The left column represents the agreement on some of the state variables (the α-level for $\alpha = 1$) for the 3 IIM systems; the right column contains some of the output variables exchanged among the agents. As it is possible to see from the figures the values are very different.

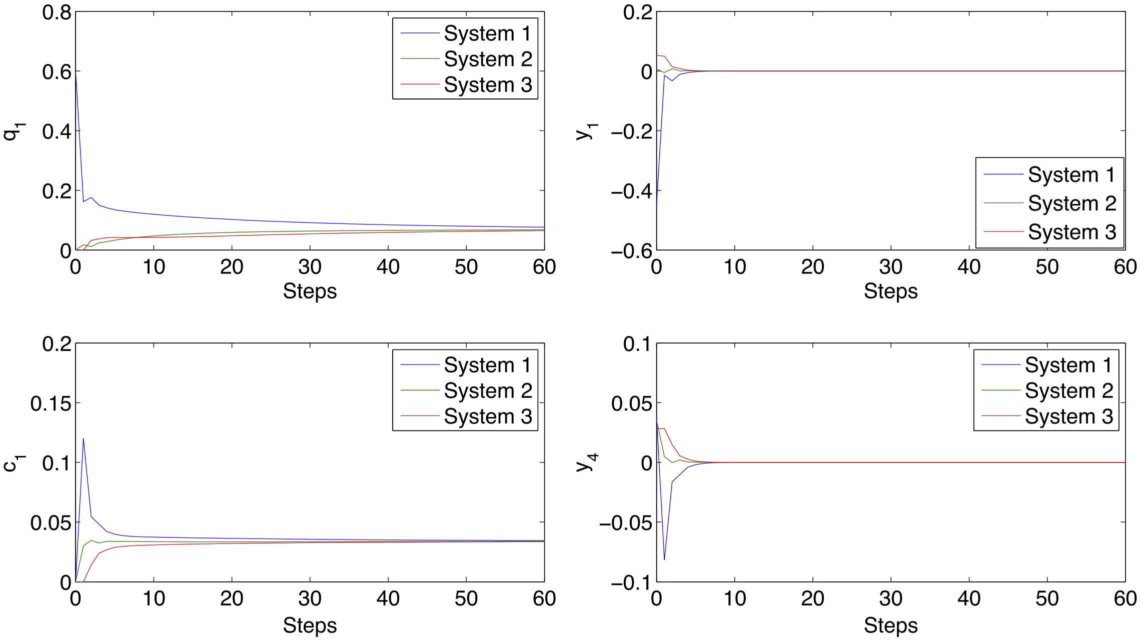

Fig. 1. Comparison between some of the state variables (left column) and some of the corresponding encoded output of the three systems (right column).

3. Conclusions

In this note we provide an amendment to the issue in [4] highlighted in [1]. Specifically we provide a way to synchronize an array of IIM models without disclosing sensitive information.

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