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Robustness analysis for node multilateration localization in wireless sensor networks

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Abstract Node flip ambiguity is a key problem that needs to be addressed for range-based node localization in wireless sensor networks. In this paper we have implemented robustness analysis for node multilateration localization in wireless sensor networks. A robustness criterion, called orthogonal projection algorithm (OPA), to detect flip ambiguity for range-based nodes multilateration localization method is proposed. The basic idea of OPA comes from the orthogonal projection principle, and it has the low computational complexity. On the basis of OPA, we further derive an expression to quantify the probability of flip ambiguity occurrences. Theoretical analysis and numerical simulation results demonstrate that OPA has good detection results and the low computational complexity, the expression for calculating probability of flip ambiguity occurrences is feasible.

Keywords Flip ambiguity · Node localization · Orthogonal projection · Wireless sensor networks

1 Introduction

Because of the wide applications in environmental monitoring, natural disaster prediction, health care, manufacturing and transportation, wireless sensor network (WSN) has attracted enormous interests in recent years [1–4]. WSNs are composed of numerous tiny sensors nodes, which have limited sensing, computing, storage and communication capabilities [5]. Therefore, in order to make more efficient by using WSN, several basic issues must be solved. The most important ones include: limited energy resources, throughput, computational power and memory, poor quality of connection, dynamically changing network topology, limitations in sensor accuracy, or problems with ensuring secure network operation [6–8]. Thus, design and development of WSN is a non-trivial task. The main directions of current research in WSN include energy management [9, 10], routing protocol [11, 12], synchronization [13, 14], topology control [15, 16], information security [17] and applications [18].

As one of those directions of research, node localization has drawn great research interest [19–22]. Currently, node localization methods can be mainly divided into two categories: range-based and range-free [23]. Due to the high positioning accuracy, the range-based methods have been widely used. However, a primary problem for the range-based methods is whether a given sensor network is uniquely localizable [24, 25], that is, the flip ambiguities could occur in range-based node localization [26]. In fact, flip ambiguities do not always occur in every wireless sensor network, in case of occurrence, they can cause the avalanche effect, and result in significant degradation of the localization accuracy of the whole network. It is important for the localization process to identify possible flip ambiguities in the location estimates, and take some proper actions to prevent them from being used.

At present, many robust quadrilateral methods (RQMs) are mainly used to detect the node flip ambiguity [26–31]. The RQMs is to construct a quadrilateral that is composed of the unknown node and three reference nodes, and then judge the quadrilateral whether satisfies certain geometrical conditions which determine the robustness of the quadrilateral, i.e., the flip ambiguity will not occur in an unknown node localization. The literature [29] carries on geometric

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analysis of the quadrilateral and quantifies the likelihood that the quadrilateral satisfies the robustness. The methods in the literatures [27–29] have been conducted under the assumption of unknown nodes with internodes ranges from all three reference nodes, where only one distance measurement is considered to be in error, while the others are considered accurate. This is obviously not consistent with the practical situation. So, the literature [30] proposes an improved algorithm which permits having the errors in all three ranges. The literature [31] adds a reference node to form four quadrilaterals, and then uses the algorithm in literature [30] to detect the robustness of the four quadrilaterals. Only when all of these quadrilaterals satisfy the robustness, flip ambiguity will not occur in the unknown node localization.

The RQMs mainly are applied to trilateration localization with three reference nodes. Compared to trilateration localization, multilateration localization with at least three reference nodes generally can get higher positioning accuracy. Therefore, multilateration localization usually is used in range-based wireless sensor networks nodes localization. At present, the research on the flip ambiguity applied to multilateration localization is limited. Although a RQM proposed by the literature [30] is applied to multilateration localization, but its computational complexity is high, and detection effect is also poor. In the literature [32], node flip ambiguity is equal to determine whether exists a straight line intersecting with all range error circles (the centers at reference nodes, the radii are equal to the maximum absolute value of range errors between reference nodes and unknown nodes) of reference nodes, which is called the existence of intersecting line (EIL) problem in the literature [32]. This method is not only suitable for multilateration localization, but also the detection effect is well. The EIL problem is divided into two categories: equal radii and unequal radii. In the literature [32], EIL problem with equal radii is solved by the convex hull algorithm with low computational complexity. However, for EIL problem with unequal radii, the computational complexity of the common tangent algorithm (CTA) in the literature [32] is still high.

In order to address the high computational complexity of CTA, by using the orthogonal projection method, we prove that EIL problem in fact is equal to determine whether there is a straight line, which enable any two circles to have overlapping orthogonal projection onto the line. According to this evidence, we propose an orthogonal projection algorithm (OPA) to detect node flip ambiguity for unequal radii. OPA transforms EIL problem into an angle calculation problem, and uses the coordinate transformation to simplify the computational processes. The theoretical analysis shows that the computational complexity of OPA is $O(k^2)$, however, the computational complexity of CTA is $O(k^3)$. The simulation results demonstrate that both of OPA and CTA have exactly the same detection results.

On the basis of OPA, We further use an expression to define the probability of flip ambiguity occurrences. Simulation results show that the expression can correctly reflect the possibility size of the unknown node occurrences flip ambiguity.

2 Node flip ambiguity and EIL problem

2.1 Node flip ambiguity

For an unknown node, when a set of reference nodes are almost collinear, there is a probability that the unknown node can be reflected by a mirror formed from the reference nodes. The reflection process will cause a large localization error. It is just the flip ambiguity of node localization. In Fig. 1, nodes A, B and C are three reference nodes with known location information respectively, $\tilde{d}_{ad}$, $\tilde{d}_{bd}$ and $\tilde{d}_{cd}$ are the measured distances between the reference nodes and the unknown node respectively. We draw two circles (the centers are B and C, the radii are $\tilde{d}_{bd}$ and $\tilde{d}_{cd}$ respectively). The position of the unknown node must be at the intersection D or D′ of the two circles. D and D′ are symmetrical with respect to line BC. We assume that the real position of the unknown node is D, the distance of A to D or D′ is d_{ad} and d'_{ad} respectively. $\tilde{d}_{ad}$ is used to select the position of the unknown node, the selection criteria is closer to d_{ad} (selects D) or to d'_{ad} (selects D′). When nodes A, B and C are nearly collinear, the differences between d_{ad} and d'_{ad} are small. Because of the existence of the range errors, there is a possibility to select the wrong D′ as the estimated location of the unknown node. If the possibility has happened, and the node with wrong localization is used as reference node to locate other unknown nodes, the localization accuracy of either the

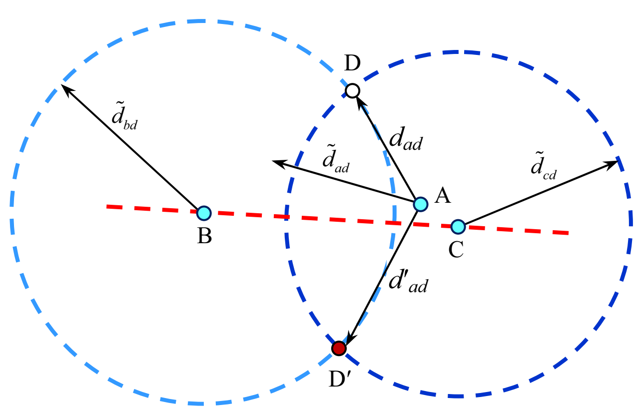

Fig. 1 The sketch map of node flip ambiguity

entire network or a large portion of the network may degrade.

The research shows that the frequency of unknown node occurrences flip ambiguity has a relationship with the number of reference nodes, the location of the reference nodes, and the range errors [33]. Generally speaking, the fewer the reference nodes are, the greater the likelihood of the reference nodes collinear is, and the larger the range error is, the greater the likelihood of unknown node occurrences flip ambiguity is.

2.2 EIL problem [32]

Multilateration localization for wireless sensor networks needs k ($k \geq 3$) reference nodes to locate an unknown node. Let a set M = $\{<p_i, \tilde{d}_i>\}$ ($i = 1, 2,\ldots, k$) denotes the known data, where p_i denotes the position of reference node i, and $\tilde{d}_i$ denotes the measured distance between the unknown node and the reference node i. Each $\tilde{d}_i$ contains two parts: the real distance d_i and the range error ε_i, i.e. $\tilde{d}_i = d_i + \varepsilon_i$. As shown in Fig. 2, we can translate the reference node ε_i distance along the direction of the range, and get the translated position $\tilde{p}_i$. After this step, we obtain a new set M′ = $\{<\tilde{p}_i, d_i>\}$ ($i = 1, 2, \ldots, k$). The difference between M and M′ is that there is no range error in M′. As a result, if all k translated positions $\tilde{p}_i$ in M′ are not likely to be collinear, there is no chance of flip ambiguity.

In practice, the range errors are not measurable. Hence, we cannot obtain the exact position of $\tilde{p}_i$. Since the value of ε_i is bounded, we can obtain the region scope where $\tilde{p}_i$ locates. For a reference node i in set M, we suppose the maximum absolute value of the measurement error is δ_i, i.e. $\delta_i = \max|\varepsilon_i|$ ($i = 1, 2, \ldots, k$). As shown in Fig. 2, the translated position $\tilde{p}_i$ must lie in the inside of a circle, which is defined as a circle centered at p_i with a radius δ_i.

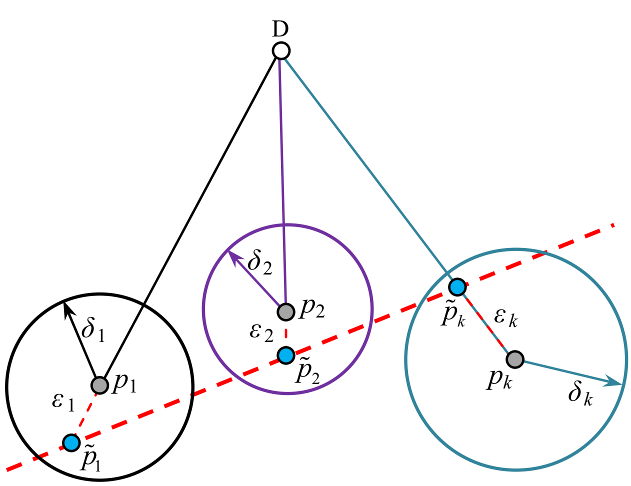

Fig. 2 The sketch map of EIL problem

Let a set S = $\{<p_i, \delta_i>\}$ ($i = 1, 2, \ldots, k$) denotes the k circles. If there exists a straight line that can intersect with all circles in set S, then all k translated positions $\tilde{p}_i$ are possible to be collinear, a flip ambiguity may occur. Otherwise, all of $\tilde{p}_i$ are not collinear certainly, no flip ambiguity will occur. Therefore, the flip ambiguity problem is equal to determine whether there is a straight line intersecting with all range error circles of the reference nodes [32].

EIL problem is divided into two categories: equal radii and unequal radii. For equal radii, the convex hull algorithm is used in the literature [32]. The convex hull algorithm firstly obtains the convex hull of the reference nodes, and then calculates the width [34] of the convex hull. If the width of the convex hull is not larger than the diameter, there is a straight line that can intersect with all circles. The convex hull algorithm with low computational complexity is only $O(k\log k)$, where k is the number of circles. For unequal radii, the CTA is used in the literature [32]. CTA firstly calculates the common tangents of every pair circles, and then determines whether the common tangents intersect with the remaining circles. Therefore, CTA has high computational complexity $O(k^3)$.

3 Orthogonal projection algorithm (OPA)

3.1 Basic idea of the orthogonal projection algorithm

The basic idea of OPA comes from the following theorem that we propose. The theorem cleverly gives sufficient and necessary conditions for EIL problem. Its detailed proof is followed.

Theorem *The sufficient and necessary condition for EIL problem is that there is a straight line, which enables any two circles to have overlapping orthogonal projection onto the straight line.*

3.1.1 The necessity proof

As shown in Fig. 3, there are following known conditions: there are k circles and a straight line l. Any two circles have overlapping orthogonal projection onto the straight line l.

Let the left endpoint of the ith ($i = 1, 2,\ldots, k$) circle's orthogonal projection line segment is A_i, the right endpoint is B_i. Suppose the rightmost left endpoint is A_m, the leftmost right endpoint is B_n. Then all the right endpoint cannot be on the left side of A_m, otherwise there are at least two circles whose orthogonal projection line segments are not overlapping, which is inconsistent with the known conditions. Similarly, all the left endpoint cannot be on the right side of B_n. Therefore, the line segment A_mB_n is the common part of all circles' orthogonal projection line segments onto the straight line l. We draw a perpendicular line l_0 of the straight line l through any point P on the line segment A_mB_n. Obviously, the straight line l_0 inevitably intersects with all circles.

3.1.2 The sufficient proof

See Fig. 3 from another view, there are following known conditions: there are k circles and a straight line l_0. The straight line l_0 intersects with all k circles.

We draw a perpendicular line l of the straight line l_0 through any point P on the straight line l_0. So the point P is the orthogonal projection point of the straight line l_0 onto the straight line l. Because the straight line l_0 intersects with the k circles, the orthogonal projection line segments of all k circles onto the straight line l must contain point P. In other words, any two circles have overlapping orthogonal projection onto the straight line l.

3.2 The process of the orthogonal projection algorithm

According to the theorem in Sect. 3.1, we propose an OPA to solve EIL problem with unequal radii. Since each straight line direction can be denoted by the angle θ between the straight line and the horizontal axis, therefore, EIL problem is equal to determine whether there is an angle θ that satisfies the conditions (any two circles have overlapping orthogonal projection onto the straight line).

In EIL problem, let (x_i, y_i) and r_i denote the center's coordinate and the radius of the ith circle respectively. A straight line l intersects with x-axis at point P whose coordinate is $(a, 0)$. The angle between the straight line l and the x-axis is θ $(0 \le \theta < \pi)$. To simplify the calculation, we adopt a coordinate transformation method to determine the orthogonal projection line segment of all circles onto the straight line l. The aim of the coordinate transformation is that the horizontal axis of the new coordinate system coincides with the straight line l, which requires two step transformations. Step 1: Translate the coordinate origin O of the original coordinate system xOy in Fig. 4(a) to point P, which obtains the coordinate system $x'O'y'$ as in Fig. 4(b). Step 2: Rotate the coordinate system $x'O'y'$ an angle θ around O′, which obtains the coordinate system $x''O''y''$ as in Fig. 4(c).

According to the coordinate transformation formula, the coordinate of the ith ($i = 1, 2, \ldots, k$) circle's center in the coordinate system $x''O''y''$ is:

$$((x_i - a)\cos\theta + y_i \sin\theta, -(x_i - a)\sin\theta + y_i\cos\theta).$$

The coordinate of the orthogonal projection point of the ith circle's center onto the straight line l in the coordinate system $x''O''y''$ is $((x_i - a)\cos\theta + y_i\sin\theta, 0)$. Therefore, if any two circles have overlapping orthogonal projection onto the straight line l, it must satisfy the following formula:

$$\begin{aligned} &|(x_m - x_n)\cos\theta + (y_m - y_n)\sin\theta| \le r_m + r_n \\ &\quad (m = 1, 2, \ldots, k; n = 1, 2, \ldots, k; \quad m \ne n). \end{aligned} \tag{1}$$

Formula (1) contains g ($g = k(k - 1)/2$) inequalities.

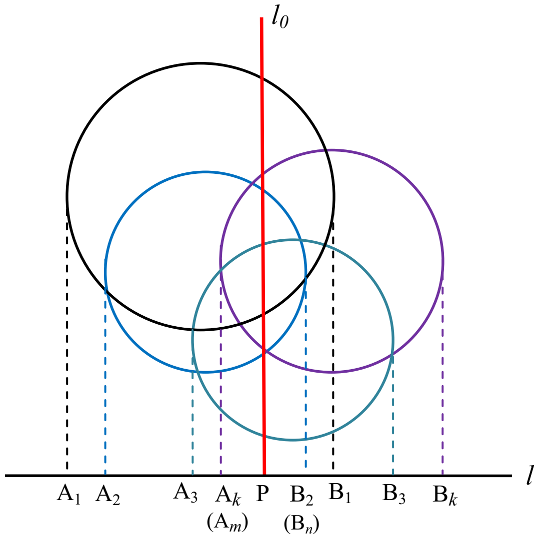

Fig. 3 The relationship between EIL problem and OPA

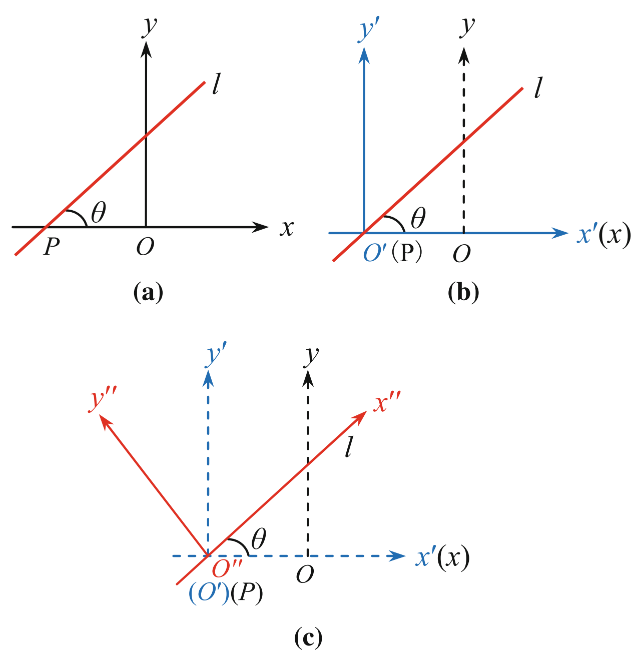

Fig. 4 The coordinates transformation process. **a** The original system, **b** the translated coordinate system, **c** the rotated coordinate system

Let:

$c_j = x_m - x_n, d_j = y_m - y_n,$
$e_j = r_m + r_n,\ (j = 1, 2, \cdots g).$

Formula (1) can be simplified as:

$$|c_j \cos\theta + d_j \sin\theta| \le e_j (\mathrm{j} = 1, 2, \ldots,\ \mathrm{g}). \tag{2}$$

By formula (2) and the initial condition $0 \le \theta < \pi$, we can obtain:

$$\theta \in \left(\bigcap_{j=1}^{g} A_j\right) \bigcup \left(\bigcap_{j=1}^{g} B_j\right) \bigcap (0,\ \pi). \tag{3}$$

where

$$A_j = \left[-\arcsin\frac{e_j}{\sqrt{c_j^2 + d_j^2}} - \arctan\frac{c_j}{d_j},\ \arcsin\frac{e_j}{\sqrt{c_j^2 + d_j^2}} - \arctan\frac{c_j}{d_j}\right],$$

$$B_j = \left[\pi - \arcsin\frac{e_j}{\sqrt{c_j^2 + d_j^2}} - \arctan\frac{c_j}{d_j},\ \pi + \arcsin\frac{e_j}{\sqrt{c_j^2 + d_j^2}} - \arctan\frac{c_j}{d_j}\right].$$

When the set denoted in formula (3) is empty, formula (1) has no solution, i.e. there is no any straight line which can intersect with the k circles, no flip ambiguity of the unknown node will occur. Otherwise, there are straight lines which can intersect with the k circles, a flip ambiguity may occur.

Figure 5 is a flowchart for OPA that detects node flip ambiguity. The core of the whole process is to determine whether there is a straight line to satisfy EIL condition. For each pair circles of the set S, OPA lists an inequality according to formula (1), and then solve it. Therefore, the computational complexity of OPA is $O(k^2)$, however, the computational complexity of CTA is $O(k^3)$. So, for unequal radii, the computational complexity of the proposed OPA significantly reduces.

3.3 The probabilities of flip ambiguity occurrences

Compared with CTA, OPA that we proposed can quantify the probability of flip ambiguity occurrences. According to the discussion in Sect. 3.2, in formula (3), θ is actually a union set of a finite number of closed interval, that is:

$$\theta = \bigcup_{i=1}^{k} [L_i, H_i]. \tag{4}$$

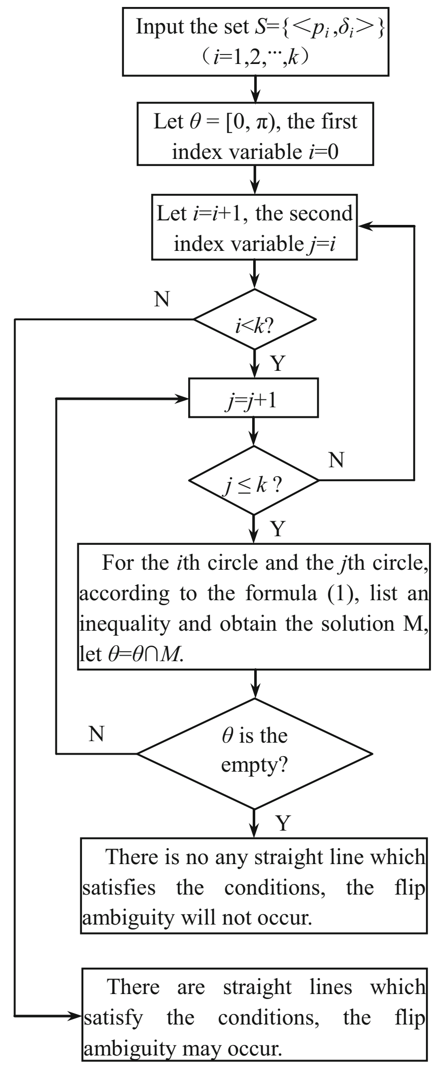

Fig. 5 The flowchart for OPA

where k denotes the number of closed interval, L_i and H_i denote lower and upper bounds of the ith closed interval respectively.

We define the length $L(\theta)$ of θ, as shown in formula (5):

$$L(\theta) = \sum_{i=1}^{k} (H_i - L_i). \tag{5}$$

The greater $L(\theta)$ is, the more number of the straight line which satisfies EIL condition is, the greater the likelihood of the unknown node occurrences flip ambiguity is. Therefore, we can use an expression to quantify the probability of flip ambiguity occurrences.

As shown in formula (6), we define the probability of flip ambiguity occurrences by using the normalization method.

$$P_b = L(\theta)/\pi. \tag{6}$$

At present, for the unknown nodes that may occur flip ambiguity, the pessimistic approach mostly is used, i.e., discard those positioning results with possible flip ambiguities. This approach improves the positioning accuracy, but also reduces the number of the positioning nodes. Obtaining the probability P_b of flip ambiguity occurrences, we can set a threshold. Only when P_b is greater than the threshold, we discard the positioning results. In this way, according to the actual requirements, we can make a tradeoff between the positioning accuracy and the number of the positioning nodes.

4 The simulation analysis

In order to verify the performance of the proposed OPA, a great deal of numeric simulations is implemented in MATLAB on a computer with Intel Core2 Duo E8400 processor (3.00 GHz), and 4 GB memory. We use received signal strength indication (RSSI) method to measure the distances, the range error can be expressed [35]:

$$e = d \times 10^{\sigma/10n}. \tag{7}$$

where e denotes the range error, d denotes the real distance between nodes, n denotes the path loss exponent (n is 4 in this paper), σ denotes the Gauss white noise (the mean is 0, the standard deviation is 4 in this paper). The formula (7) shows that there is a relationship between the range error of RSSI method and the real distance, the range error of RSSI method is suitable for EIL problem with unequal radii.

4.1 The detection result comparison

The network areas adopt square and C shape in the simulation. The node distributions adopt the uniform distribution and the random distribution. The side length of the square area is 50 m. The C shape area is formed by removing a 30 m × 30 m area from the square area. The distances between nodes are 10 m (there are 0–2 m random errors) in the uniform distribution. The simulation experiments only compare the detection results of the proposed OPA with CTA. The numbers of anchor nodes are both 5 in Figs. 6 and 7. Figure 6 is the detection results of the square area, where Fig. 6(a), (b) are the uniform distribution and the random distribution respectively, and the number of nodes is 25. Figure 7 is the detection results of the C shape, where Fig. 7(a), (b) area are the uniform distribution and the random distribution respectively, and the number of nodes is 16. As can be seen from these figures, for different network area and different node distribution, the detection results of OPA and CTA are completely consistent.

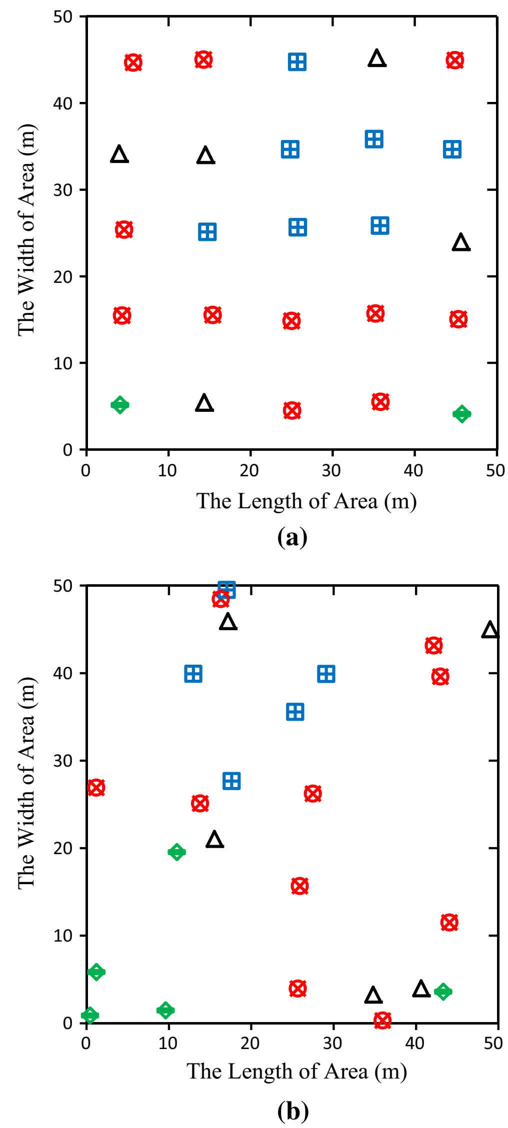

Fig. 6 The comparison for square network area. **a** Uniform distribution nodes, **b** random distribution nodes. *Triangle* the anchor nodes, *plus symbol* no occur flip ambiguity nodes using OPA detection, *times symbols* occur flip ambiguity nodes using OPA detection, *solid line* unlocatable nodes using OPA detection, *square* no occur flip ambiguity nodes using CTA detection, *circle* occur flip ambiguity nodes using CTA detection, *diamond* unlocatable nodes using CTA detection

4.2 The average localization error comparison

To compare the ability of detection nodes flip ambiguity, we measure the average localization error in different node communication radius by using OPA and CTA in network localization respectively.

At present, for the unknown nodes that may occurrences flip ambiguity, there are two kinds of processing method. One is the pessimistic method mainly adopted, i.e., abandon the localization of the unknown node, in order to avoid their effect on positioning accuracy of subsequent nodes

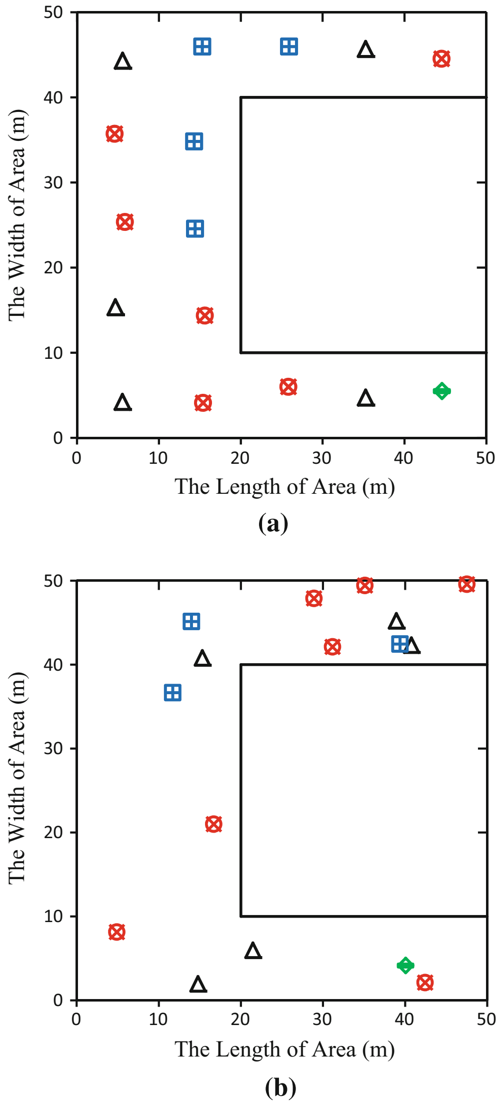

Fig. 7 The comparison for C shape network area. **a** Uniform distribution nodes, **b** random distribution nodes. *Triangle* the anchor nodes, *plus symbol* no occur flip ambiguity nodes using OPA detection, *times symbols* occur flip ambiguity nodes using OPA detection, *solid line* unlocatable nodes using OPA detection, *square* no occur flip ambiguity nodes using CTA detection, *circle* occur flip ambiguity nodes using CTA detection, *diamond* unlocatable nodes using CTA detection

[24, 26, 30]; the other is optimistic method, i.e., other additional information (the connectivity between nodes, etc.) used to further detect flip ambiguity of the unknown node and then estimate the position of them [36, 37]. In this paper, we mainly compare the detection results of TCA and OPA, so the relatively simple pessimistic method is used in the simulation.

As in Sect. 4.1, the network areas adopt square and C shape respectively. The node distributions adopt uniform distribution and random distribution respectively. The side length of the square area is 100 m, and the number of nodes is 100. The C shape area is formed by removing a 50 m × 40 m area from the square area, and the number of nodes is 80. The distances between nodes are 10 m (there are 0–2 m random errors) in the uniform distribution. The number of anchor nodes is 10 and the unit detection error is 0.5 m in the simulation.

The average localization error is calculated by the following formula:

$$e = \frac{1}{N}\sum_{n=1}^{N}\frac{\sum_{i\in V_n}\sqrt{(x_i-\hat{x}_i)^2+(y_i-\hat{y}_i)^2}}{|V_n|R}\times 100\,\%. \tag{8}$$

where e denotes the average localization error, N denotes the number of networks (N is 100 in the paper), (x_i, y_i) and $(\hat{x}_i, \hat{y}_i)$ denote the real coordinates and the estimated coordinate of the node i respectively, V_n denotes the set of nodes that can be positioned in the nth network, $|V_n|$ denotes the number of V_n, R denotes the node communication radius.

Figures 8 and 9 respectively are the average localization error in different node communication radius for the square network area and C shape network area, where the nodes in Figs. 8(a) and 9(a) both are uniform distribution, the nodes in Figs. 8(b) and 9(b) both are random distribution.

It can be seen from Figs. 8 and 9, for different network areas and different nodes distribution, the average localization error in OPA detection and CTA detection are completely overlapping, which does further demonstrate that the detection results of two algorithms are the same. In addition, we can see that, the average localization error by adding flip ambiguity detection in the localization process is obviously smaller compared without flip ambiguity detection.

4.3 The calculation time comparison

To compare the computational complexity of OPA and CTA, we simulate the calculation time of them. The simulation parameters are same as the Sect. 4.2.

The average calculation time of 100 different networks are shown in Tables 1, 2, 3 and 4. The simulation network areas adopt square and C shape. The node distributions adopt uniform distribution and random distribution. In Tables 1, 2, 3 and 4, R and N denote the node communication radius and the average number of the reference nodes respectively. T_{CTA} and T_{OPA} denote the average detection time for CTA and OPA respectively. β denotes the ratio of the two times, i.e. $\beta = T_{CTA}/T_{OPA}$.

As can be seen from Tables 1, 2, 3 and 4, for different network areas and different nodes distribution, the average detection time for OPA is less than that for CTA. With the increasing number of the reference nodes, the ratio is increasing. Therefore, for a larger number of reference

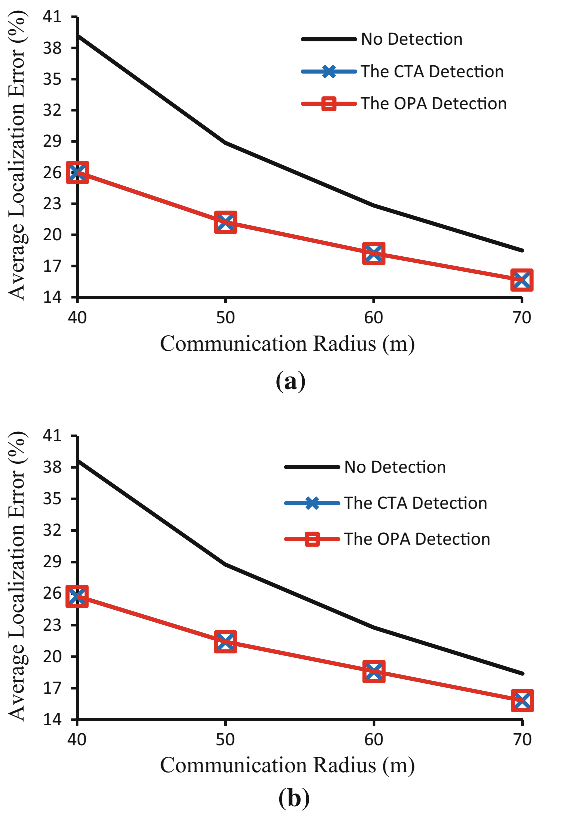

Fig. 8 The average localization error for square network area. **a** The nodes uniform distribution, **b** the nodes random distribution

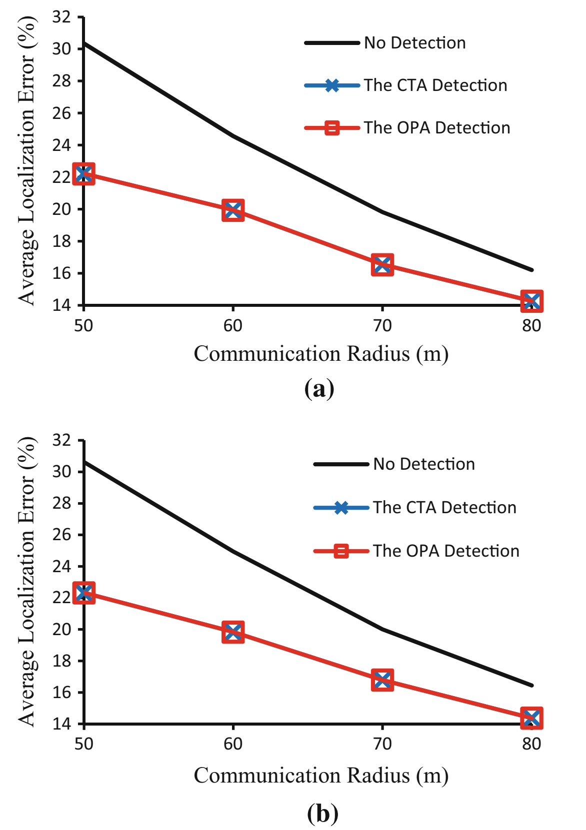

Fig. 9 The average localization error for C shape network area. **a** The nodes uniform distribution, **b** the nodes random distribution

nodes, obviously, OPA is more suitable. Compared with CTA, the proposed OPA can obtain the same detection result, and greatly reduce the computational complexity.

4.4 The verification about the probability expression

In order to verify the feasibility of the probability expression of flip ambiguity occurrences by we proposed, we will carry out the relevant simulation. Firstly we set up three intervals (0, 0.1], (0.1, 0.2] and (0.2, 0.3]. Secondly, for each unknown nodes, we calculate the probability P_b of flip ambiguity occurrences by using the formula (6). Lastly, for the unknown nodes whose probabilities P_b are located in the above three intervals, we simulate their average localization error in different node communication radius. As in Sect. 4.2, the network areas adopt square and C shape respectively. The node distributions adopt uniform distribution and random distribution respectively. The simulation parameters are same as the Sect. 4.2.

Figures 10 and 11 respectively are the average localization error of the unknown nodes with the different probability P_b in different node communication radius for the square network area and C shape network area, where the nodes in Figs. 10(a) and 11(a) both are uniform distribution, the nodes in Figs. 10(b) and 11(b) both are random distribution. The simulation results come from the average value of 100 different networks.

It can be seen from Figs. 10 and 11, for different network areas and different nodes distribution, with the increase of

Table 1 Time comparison for square network area and uniform distribution nodes

R(m)	40	50	60	70
N	6.347	6.843	7.248	8.258
T_{CTA} (ms)	0.733	0.880	0.966	1.354
T_{OPA} (ms)	0.203	0.243	0.250	0.254
β	3.606	3.614	3.863	5.330

Table 2 Time comparison for square network area and random distribution nodes

R(m)	40	50	60	70
N	5.254	6.228	7.059	8.072
T_{CTA} (ms)	0.689	0.742	0.854	1.138
T_{OPA} (ms)	0.203	0.228	0.251	0.292
β	3.295	3.328	3.794	4.964

Table 3 Time comparison for C shape network area and uniform distribution nodes

R(m)	50	60	70	80
N	5.992	6.918	7.761	8.813
T_{CTA} (ms)	0.503	0.742	0.988	1.378
T_{OPA} (ms)	0.212	0.226	0.237	0.264
β	2.378	3.290	4.176	5.228

Table 4 Time comparison for C shape network area and random distribution nodes

R(m)	50	60	70	80
N	6.258	7.039	7.851	8.830
T_{CTA} (ms)	0.638	0.873	1.154	1.558
T_{OPA} (ms)	0.212	0.229	0.238	0.268
β	3.002	3.810	4.849	5.816

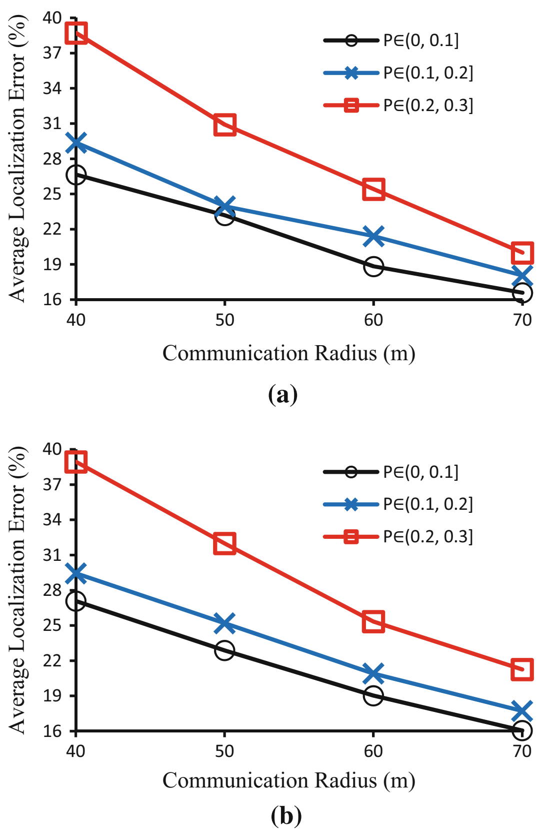

Fig. 10 The average localization error of the unknown nodes with the different probability P_b for square network area. **a** The nodes uniform distribution, **b** the nodes random distribution

P_b, the average localization error of the unknown nodes is increasing. In other words, for the unknown node, the probability of flip ambiguity occurrences is increasing with the increase of P_b. This proves that the expression for

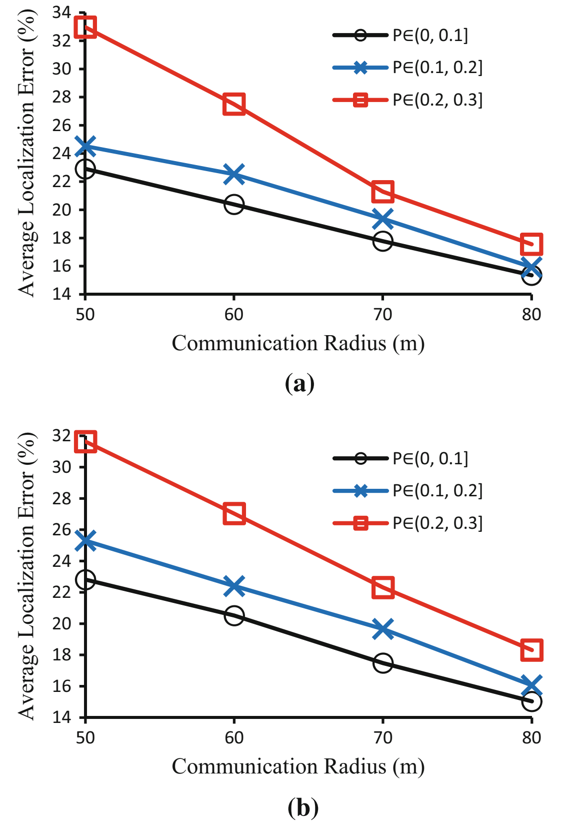

Fig. 11 The average localization error of the unknown nodes with the different probability P_b for C shape network area. **a** The nodes uniform distribution, **b** the nodes random distribution

calculating probability of flip ambiguity occurrences is feasible. It can correctly reflect the possibility size of the unknown node occurrences flip ambiguity.

5 Conclusions

Node flip ambiguity is a key problem that needs to be addressed for the range-based node localization in wireless sensor networks. Let the node flip ambiguity problem translate into EIL problem, which is currently the better method. In order to address the high computational complexity of the existing method for EIL problem with unequal radii, we propose the OPA to detect flip ambiguity. Theoretical analysis and a great deal of data simulations show that the proposed OPA has exactly the same detection results with CTA, but the computational complexity is greatly reduced.

Currently, the research on node flip ambiguity in wireless sensor network mainly focuses on the two-dimensional space. However, in practical applications, the nodes are often distributed in three-dimensional space. We often need the three-dimensional information of the nodes, such as the ocean, mountains, and forests and so on. Due to the computational complexity and other reasons, if we simply extend the most current detection methods from two-dimensional space to three-dimensional space, it is not appropriate. Therefore, it is necessary for us to further study the detection method of node flip ambiguity in the three-dimensional space.

At present, for the unknown nodes that may occur flip ambiguity, the pessimistic processing approaches mostly are used. However, they reduce the number of the positioning nodes. Although there are a few optimistic processing approaches, they are either too complex or inefficient. Thus, another research direction is the processing method of node flip ambiguity.

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