Enhanced Timing Advanced Estimation With Symmetric Zadoff-Chu Sequences for Satellite Systems

Gaofeng Cui, *Member, IEEE*, Yizhou He, Pengxu Li, and Weidong Wang

***Abstract*—Timing advanced estimation with Zadoff-Chu sequences is sensitive to the frequency offset, especially for satellite systems with large Doppler shift and oscillator uncertainties. In this letter, Timing Advanced Estimation with Symmetric Zadoff-Chu sequences (TAE-SZC) is proposed to deal with the integer and fractional frequency offset, respectively. By adopting TAE-SZC, the effect of integer frequency offset can be resolved by the symmetric properties of Zadoff-Chu sequences, and an iterative compensation method is used to mitigate the effect of fractional frequency offset. Simulation results show that TAE-SZC outperforms the conventional timing advanced estimation methods without incurring additional consumption of time/ frequency resource.**

***Index Terms*—Timing advanced, Zadoff-Chu, frequency offset, satellite.**

I. Introduction

SATELLITE mobile communication plays an important role on providing coverage in the regions where the terrestrial network is not able to be deployed or work effectively [1]. For satellite mobile communication systems, as is shown in Fig. 1, Timing Advanced Estimation (TAE) is essential to avoid the inter-user interference due to timing misalignment in the return link. However, since TAE needs to be executed without frequency and time synchronization, the effectiveness of TAE is vulnerable to the frequency offset in consequence of the accumulated frequency uncertainties and Doppler shift.

Generally, Timing Advanced (TA) can be estimated by using either Pseudo-random Noise (PN) or Zadoff-Chu (ZC) sequences as a preamble in the random access channel. For PN sequence based TAE, methods evolved from Differential Post Detection Integration (DPDI) were proposed to mitigate the effect of frequency offset for satellite systems in [2]–[4]. In [5], a partial differential post-correlation method was developed for Global Positioning System. Recently, ZC sequences have gained more attention for its low Peak-to-Average Power Ratio (PAPR) and zero autocorrelation property [6]. However, since the correlation peak of ZC sequences can be shifted by the integer parts of relative frequency offset, ZC sequences is more vulnerable to frequency offset in the systems with small subcarrier bandwidth, such as the systems based on Orthogonal Frequency Division Multiplexing (OFDM) which is an candidate technique for next generation satellite systems [7], [8].

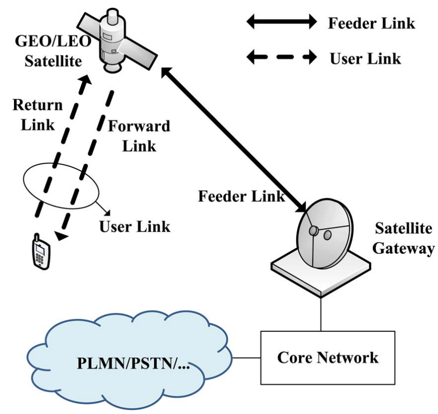

Fig. 1. The architecture of satellite systems.

For satellite mobile communication systems, authors in [9] developed a two step timing advanced estimation method to analyze the integer and fractional parts respectively. In [10], a modified frame alignment method was proposed by dividing User Equipments (UEs) into several groups according to the difference of their timing advanced. However, the above methods do not consider the effect of frequency offset which impacts the autocorrelation of ZC sequences. In [6], the effect of frequency offset due to accumulated frequency uncertainties and Doppler shift on ZC sequences was estimated with various root indices. Authors in [11] analyzes the timing accuracy with integer and fractional frequency offsets respectively. To deal with frequency offset, cyclic shift restriction and peak combining are used to mitigate the effect of frequency offset for terrestrial Long Term Evolution (LTE) network. However, the frequency offset in LTE is assumed to be less than one subcarrier bandwidth(1.25 KHz) with typical vehicle speed and compensation from downlink synchronization [11]. When the frequency offset is larger than one subcarrier bandwidth, the method for LTE cannot work effectively. For satellite mobile network, the frequency offset caused by the mobility of satellite and the frequency uncertainties can be multiple of subcarrier bandwidth. For example, the Doppler shifts caused by Low Earth Orbit (LEO) satellites can be more than 20 KHz [12]. In [13], integer frequency offset was estimated and utilized for timing advanced estimation by consuming doubled resource compared with method used in LTE. However, the residual fractional frequency offset in [13] affects the accuracy of the estimation seriously.

In this letter, a signature format based on symmetric ZC sequences is proposed to deal with the frequency offset being multiple of subcarrier bandwidth. System model is introduced

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in Section II. Section III presents the proposed method. Simulation results are shown in Section IV. Section V concludes the letter.

II. Signal Model and Problem Analysis

Fig. 1 shows a satellite system with Line Of Sight (LOS) channel between the satellite and earth station. The ZC sequence of odd length being transmitted via the return link can be expressed as follow,

$$ZC_u(n) = e^{-j\pi un(n+1)/N} \tag{1}$$

where u denotes the root index for ZC sequences. Generally, u is an integer and satisfies that u and the preamble length N are co-prime. Note that, although we assume odd preamble length in this letter, the proposed method can be extended to the scenarios of arbitrary ZC sequences with u and N being co-prime. Based on the ZC sequence listed in (1), the post-processed access signal at the receiver with one user can be expressed as,

$$y_k(n) = \rho_k S_k(n - \tau_k) e^{j2\pi f_k n/(f_{RA}N)} + w_k(n) \tag{2}$$

where $y_k(n)$ represents the received signal for user k. ρ_k is the received Signal to Noise Ratio (SNR) for user k, S_k represents the transmitted signal of user k. We denote $\bar{\tau}_k$ as the delay caused by the uncertain location of the user, the normalized uncertain delay $\tau_k = round(\bar{\tau}_k/T_s)$ is normalized by the sampling interval T_s and rounded to an integer for simplicity, $0 \le \tau_k < N$. For the scenario with $\bar{\tau}_k/T_s$ as a non-integer, the proposed method is also applicable, but the fractional part of $\bar{\tau}_k/T_s$ due to sampling misalignment will result in energy leakage and decrease SNR. f_k is the frequency offset due to Doppler shift and oscillator uncertainties, f_{RA} is the subcarrier bandwidth of random access channel, and $w_k(n)$ is the complex Gaussian noise with zero mean and unit variance.

When ZC sequence in (1) is adopted by S_k in (2), τ_k can be derived by using the auto-correlation properties of ZC sequences. However, the frequency offset will affect the accuracy of the timing advacnced estimation. With Δf_1 and Δf_2 being denoted as the integer and fractional parts of f_k/f_{RA}, respectively, Δf_1 will make the correlation peak shifted and Δf_2 will make the value of the peak decreased [11].

III. Enhanced Timing Advanced Estimation Based on Symmetric ZC Sequences

With the ZC sequence expressed in (1), the effect of integer frequency offset on the $ZC_u(n)$ and its conjugation $ZC_u^*(n)$ has the symmetric property elaborated in property 1 with $(A)^*$ denoted as the conjunction of sequence A.

Property 1: Given $ZC_u(n)$ and $ZC_u^*(n)$, the integer frequency offset Δf_1 makes the corrlation peak of $ZC_u(n)$ and $ZC_u^*(n)$ cyclicly shifted right(left) and left(right) symmetrically.

Proof: The proof can be found in Appendix A. ■

Moreover, the ZC sequences also has the symmetric property expressed in property 2.

Property 2: When the root index of two ZC sequences satisfies $u + v = N$ and N is an odd number, the two ZC sequences satisfy $ZC_v(n) = ZC_u^*(n)$.

Proof: The ZC sequence in (1) with $u + v = N$ can be expressed as,

$$\begin{aligned} ZC_u(n) &= e^{-j\pi un(n+1)/N} \\ &= e^{-j\pi(N-v)n(n+1)/N} \\ &= e^{-j\pi n(n+1)} e^{-j\pi(-v)n(n+1)/N} \\ &= e^{j\pi vn(n+1)/N} \\ &= ZC_v^*(n) \end{aligned}$$

■

In property 2, since n is an integer, $n(n+1)$ is an even integer and $e^{-j\pi n(n+1)} = 1$.

With the symmetric properties of the ZC sequences, a novel signal format based on symmetric ZC sequences is given as,

$$S_k(n) = \frac{1}{\sqrt{2}} \left(ZC_u(n) + ZC_u^*(n) \right) \tag{3}$$

Since the effect of Δf_1 and Δf_2 on the ZC sequences is different, two methods are proposed in this section to deal with them respectively.

A. Timing Advanced Estimation With Integer Frequency Offset Mitigation

When we assume the fractional frequency offset $\Delta f_2 = 0$, the received access signal expressed in (2) can be rewritten as,

$$y_k(n) = \rho_k S_k(n - \tau_k) e^{j2\pi \Delta f_1 n/N} + w_k(n) \tag{4}$$

Subsitute (3) into (4), (4) can be expressed as,

$$\begin{aligned} y_k(n) &= \frac{\rho_k}{\sqrt{2}} \left(ZC_u(n-\tau_k) + ZC_u^*(n-\tau_k) \right) e^{j2\pi \Delta f_1 n/N} \\ &\quad + w_k(n) \\ &= y_k^1(n) + y_k^2(n) + w_k(n) \end{aligned} \tag{5}$$

where $y_k^1(n) = \frac{\rho_k}{\sqrt{2}}(ZC_u(n-\tau_k))e^{j2\pi\Delta f_1 n/N}$ and $y_k^2(n) = \frac{\rho_k}{\sqrt{2}}(ZC_u^*(n-\tau_k))e^{j2\pi\Delta f_1 n/N}$. (5) can be regarded as the additive of two separate ZC sequences. By applying the cyclic correlation with $ZC_u^*(n)$, the correlation output can be expressed as,

$$\begin{aligned} Corr_k^{(1)}(n) &= \frac{1}{N} \sum_{m=0}^{N-1} \left(y_k^1(m) + y_k^2(m) + w_k(m) \right) ZC_u^*(m-n) \\ &= Corr_k^{(11)}(n) + Corr_k^{(21)}(n) + Corr_w^*(n) \end{aligned} \tag{6}$$

In (6), $Corr_k^{(11)}(n)$ can be considered as the auto-correlation output, while $Corr_k^{(21)}(n)$ is the interference. According to the property 2, $Corr_k^{(21)}(n)$ can be treated as the cross-correlation of two ZC sequences, and the average value of $Corr_k^{(21)}(n)$ is $1/\sqrt{N}$ [11]. Thus, we can conclude that the peak of $Corr_k^{(1)}(n)$ is the peak of $Corr_k^{(11)}(n)$. The cyclic shift n_1 corresponding to the peak of the $Corr_k^{(1)}(n)$ can be derived as,

$$(un_1 - (u\tau_k + \Delta f_1))_N = 0 \tag{7}$$

In (7), $((\bullet))_N$ denotes modulo-N. n_1 is the index of the peak and $n_1 < N$. The proof for (7) can be found in Appendix A. Since Δf_1 is unknow, the τ_k cannot be determined by (7). To get τ_k, the symmetric property expressed in property 1 is adopted and the cyclic correlation output of $y_k(n)$ with $ZC_u(n)$ is given by,

$$\begin{aligned} Corr_k^{(2)}(n) &= \frac{1}{N}\sum_{m=0}^{N-1}\big(y_k^1(m)+y_k^2(m)+w_k(m)\big)ZC_u(m-n) \\ &= Corr_k^{(21)}(n) + Corr_k^{(22)}(n) + Corr_w(n) \end{aligned} \tag{8}$$

According to property 2, $Corr_k^{(21)}(n)$ can be considered as the cross-correlation of two sequences, while the $Corr_k^{(22)}(n)$ is the auto-correlation output. Thus, the peak of $Corr_k^{(22)}(n)$ can be found as the peak of correlation output $Corr_k^{(2)}(n)$. The cyclic shift n_2 corresponding to the peak of the $Corr_k^{(2)}(n)$ can be expressed as,

$$(-un_2 + (u\tau_k - \Delta f_1))_N = 0 \tag{9}$$

In (9), n_2 is the index of the peak and $n_2 < N$. The proof for (9) can be found in Appendix A. With (7) and (9), the index of peak n_1 and n_2 are shown as,

$$n_1 = \tau_k + \Delta f_1 u^{-1} + aN \tag{10}$$

$$n_2 = \tau_k - \Delta f_1 u^{-1} + bN \tag{11}$$

where u^{-1} satisfy $((uu^{-1}))_N = 1$, a and b are integer that determined by Δf_1. By applying (10) + (11), we can obtain,

$$n_1 + n_2 = 2\tau_k + cN \tag{12}$$

where c is an integer and $c = a + b$. With (12), τ_k can be expressed as,

$$\tau_k = \frac{n_1 + n_2 - cN}{2} \tag{13}$$

With the constraint of $0 \le \tau_k < N$, the unique τ_k can be derived by using (13).

B. Compensation for Fractional Frequency Offset

Although symmetric ZC sequences can eliminate the effect of integer frequency offset, it cannot resolve the fractional frequency offset Δf_2. Moreover, the correlation peak will be decreased by Δf_2 which can deteriorate the accuracy of the integer frequency offset mitigation. According to (A.2), the fractional frequency offset Δf_2 can decrease the real peak and increase the false peak simultaneously. With the increment of Δf_2, the real peak will be decreased and the false peak will be increased. When $\Delta f_2 = 0.5$, the real peak and the false peak reach the same value. To ease the effect of Δf_2, an iterative compensation method is elaborated as below,

1) Initialize $q = 0$, $y_k^{temp}(n) = y_k(n)$;
2) If $q \le Iter$, compensate $1/2^q$ frequency offset via multiplying $y_k^{temp}(n)$ by $e^{j2\pi n/(2^q N)}$ and get $y_k^{comp}(n) = y_k^{temp}(n)e^{j2\pi n/(2^q N)}$; Else, turn to 6);
3) Calculate cyclic correlation of $y_k^{temp}(n)$ and $y_k^{comp}(n)$ with $ZC_u(n)$;
4) Get the value of the peak P_1 and P_2 for the correlation output of $y_k^{temp}(n)$ and $y_k^{comp}(n)$ respectively;
5) If $P_1 < P_2$, $y_k^{temp}(n) = y_k^{comp}(n)$, $q = q + 1$, return to 2); Else, $q = q + 1$, return to 2);
6) $y_k(n) = y_k^{temp}(n)$, end;

The $Iter$ is used to control the compensation precision. Since the peak value is inversely proportional to the Δf_2, the iterative compensation method is used to compensate the fractional frequency offset by searching for the greater peak. Although the compensated frequency can affect the value of Δf_1, the mixed frequency offset has same effect on the symmetric ZC sequences and will not affect the accuracy of τ_k.

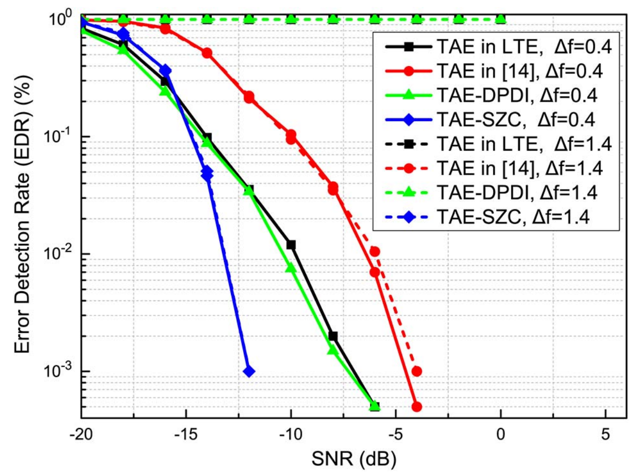

Fig. 2. Performance comparison with $\Delta f = 0.4$ and $\Delta f = 1.4$.

IV. Simulation

In this section, Error Detection Rate (EDR) is used as a metric to analyze the performance of several methods, including TAE-SZC, TAE in LTE, TAE in [13] and TAE based on DPDI. For TAE in [13], two ZC sequences with half power are transmitted in two access symbols. While for TAE-DPDI, three ZC sequences are transmitted continuously in three access symbols. Δf is used as the frequency offset. The length and the root index of ZC sequences is expressed as (N, u).

Fig. 2 shows the EDRs of different schemes with $\Delta f = 0.4$ and $\Delta f = 1.4$, respectively. When the SNR is below -15 dB and $\Delta f = 0.4$, TAE in LTE and TAE-DPDI can achieve better performance, because the power allocated for TAE-SZC and TAE in [13] is divided into two parts. With the increment of SNR, the effect of fractional frequency offset become the dominant factor and TAE-SZC outperforms other schemes. Moreover, since TAE-DPDI combines the correlation outputs which have been already shifted by the integer frequency offset, it is not applicable to the ZC sequences with large frequency offset.

With $SNR = -16$ dB, the comparison of different schemes is given in Fig. 3. With the increment of frequency offset, the EDR of TAE-SZC remains constant. In addition, the EDR for (N, u) being (839,4) and (838,5) implies that TAE-SZC is applicable to arbitrary ZC sequences with u and N being co-prime. Moreover, since two symmetric ZC sequences are transmitted in one access slot, it doesn't need more time/frequency resource to implement the TAE-SZC.

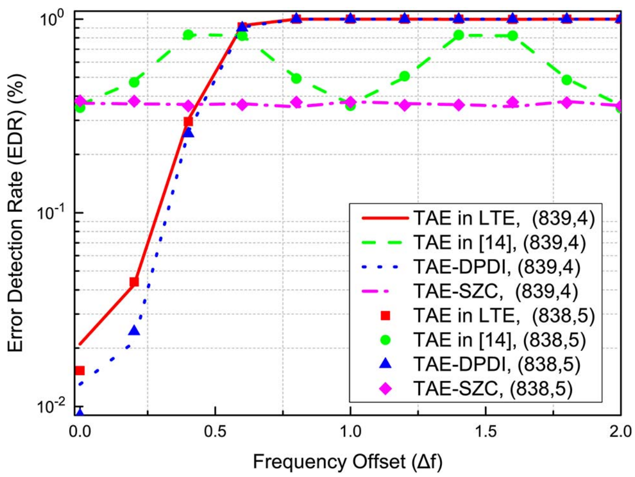

Fig. 3. Performance comparison with $SNR = -16$ dB.

V. Conclusion

Timing advanced estimation with large frequency offset in satellite communication was investigated in this letter. To tackle the effect of frequency offset on ZC sequences, TAE-SZC was proposed to deal with the integer frequency offset and fractional frequency offset, respectively. With the symmetric ZC sequences, the effect of integer frequency offset can be mitigated. For the fractional frequency offset, an iterative compensation method was proposed. Simulation results show that TAE-SZC can achieve better performance without additional consumption of time/frequency resource.

Appendix A
Proof of the Property 1

Assume the frequency offset being expressed as Δf that includes the integer and fractional part. The correlation output of ZC sequence expressed in (1) with the delay τ_k and frequency offset Δf can be denoted as,

$$\begin{aligned} corr_1(m) &= \sum_{n=0}^{N-1} ZC_u(n-\tau_k)e^{j2\pi\Delta f n/N} ZC_u^*(n-m) \\ &= \sum_{n=0}^{N-1} e^{j2\pi u n\tau_k/N} e^{-j\pi u(-\tau_k+\tau_k^2)/N} e^{j2\pi\Delta f n/N} \\ &\quad e^{-j2\pi unm/N} e^{j\pi u(-m+m^2)/N} \\ &= \sum_{n=0}^{N-1} e^{-j2\pi(um-u\tau_k-\Delta f)n/N} e^{j\phi_1} \end{aligned} \tag{A.1}$$

where $\phi_1 = e^{\frac{-\pi u(-\tau_k+\tau_k^2)}{N}} e^{\frac{\pi u(-m+m^2)}{N}}$. The amplitude of (A.1) can be expressed as,

$$|corr_1(m)| = \begin{cases} N, & ((um - u\tau_k - \Delta f))_N = 0 \\ 0, & ((um - u\tau_k - \Delta f))_N = p_1 \\ S_1(\Delta f, m), & \Delta f \text{ is a non-integer} \end{cases} \tag{A.2}$$

where $p_1 \neq 0$, p_1 is an integer. $S_1(\Delta f, m)$ can be written as,

$$S_1(\Delta f, m) = \frac{sin\left[\pi\left(um - (u\tau_k + \Delta f)\right)\right]}{sin\left[\pi\left(um/N - (u\tau_k + \Delta f)/N\right)\right]}$$

According to (A.2), when $((um - u\tau_k - \Delta f))_N = 0$, $|corr_1(m)|$ get the maximum value. Similarly, the correlation output for the $ZC_u^*(n)$ with the delay τ_k and frequency offset Δf can be got as,

$$|corr_2(m)| = \begin{cases} N, & ((-um + u\tau_k - \Delta f))_N = 0 \\ 0, & ((-um + u\tau_k - \Delta f))_N = p_2 \\ S_2(\Delta f, m), & \Delta f \text{ is a non-integer} \end{cases} \tag{A.3}$$

where $p_2 \neq 0$, p_2 is an integer. $S_2(\Delta f, m)$ can be got as,

$$S_2(\Delta f, m) = \frac{sin\left[\pi\left(-um + (u\tau_k - \Delta f)\right)\right]}{sin\left[\pi\left(-um/N + (u\tau_k - \Delta f)/N\right)\right]}$$

According to (A.3), when $((-um + u\tau_k - \Delta f))_N = 0$, $|corr_2(m)|$ get the maximum value. Moreover, it is shown from (A.2) and (A.3) that the correlation peak is shifted right and left from τ_k respectively. Similarly, the same conclusion can be got for the ZC sequences of even length.

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