Advanced & Intelligent Control

Digital PID Tunning tables

Written by: Juan Camilo Gómez C

$$C(z^{-1}) = \frac{U(z^{-1})}{E(z^{-1})} = K_e \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} \quad ; \; 1/10 \leq K_e \leq 1 \rightarrow recomended \rightarrow \begin{cases} 1 & Fast \\ 3/4 & Moderate \\ 1/2 & Slow \end{cases}$$

For $q_0 > 0 \rightarrow \; q_1 < -q_2 \, , -(q_0 + q_1) < q_2 < q_0$

Table 1. Digital Controllers

Parameter	Forward rectangular method FRM	Backward rectangular method BRM	Trapezoidal method TRAP
q_0	$K_c\left(1+\frac{\tau_d}{T}\right)$	$K_c\left(1+\frac{T}{\tau_i}+\frac{\tau_d}{T}\right)$	$K_c\left(1+\frac{T}{2\tau_i}+\frac{\tau_d}{T}\right)$
q_1	$-K_c\left(1-\frac{T}{\tau_i}+\frac{2\tau_d}{T}\right)$	$-K_c\left(1+\frac{2\tau_d}{T}\right)$	$-K_c\left(1-\frac{T}{2\tau_i}+\frac{2\tau_d}{T}\right)$
q_2	$\frac{K_c\tau_d}{T}$	$\frac{K_c\tau_d}{T}$	$\frac{K_c\tau_d}{T}$

Table 2. Ziegler-Nichols/Critic Gain

Controller	K_c	τ_i	τ_d
P	$(1/2)K_u$		
PI	$(9/20)K_u$	$(83/100)T_u$	
PID	$(3/5)K_u$	$(1/2)T_u$	$(1/8)T_u$

$\rightarrow$ Robust Ziegler-Nichols tunning

$$K_c = K_u G_m \cos(\emptyset)$$

$$\tau_i = \frac{T_u}{4\pi\alpha}\left(\tan(\emptyset) + \sqrt{4\alpha + \tan^2(\emptyset)}\right)$$

$$\tau_d = \alpha\,\tau_i$$

G_m : Desired inverse of gain margin (default = 0.5)

$\emptyset$: Desired phase margin

α : Desing seletion of the ration τ_d: τ_i

Tables for FOR systems auto-tunning

$$G_p(s) = \frac{Ke^{-\alpha s}}{\tau s + 1} \rightarrow T \rightarrow G_p(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} z^{-d}$$

$$\theta = \alpha + {}^{T}/_{2}$$

$$K = \lim_{z\to 1} G_p(z) = \frac{b_0 + b_1}{1 + a_1}$$

$$\tau = -\frac{T}{ln(|a_1|)} \qquad \theta = dT$$

Table 3. Ziegler-Nichols/Reaction Curve

Controller	K_c	τ_i	τ_d
P	$\frac{\tau}{K\theta}$		
PI	$\frac{9\tau}{10K\theta}$	$\frac{10\theta}{3}$	
PID	$\frac{6\tau}{5K\theta}$	2θ	$\frac{1}{2}\theta$

Table 4. IMC

Controller	K_c	τ_i	τ_d	λ
PI	$\frac{\tau}{K\lambda}$	τ		$\lambda \geq 1.7\alpha$ $\lambda \geq 0.2\tau$
PID	$\frac{2\tau+\alpha}{K(2\lambda+\alpha)}$	$\frac{2\tau+\alpha}{2}$	$\frac{\tau\alpha}{2\tau+\alpha}$	$\lambda \geq 0.8\alpha$ $\lambda \geq 0.2\tau$

Table 5. 3C

Controller	K_c	τ_i	τ_d
PI	$\frac{0.928}{K}\left(\frac{\theta}{\tau}\right)^{-0.946}$	$0.928\tau\left(\frac{\theta}{\tau}\right)^{0.503}$	
PID	$\frac{1.3070}{K}\left(\frac{\theta}{\tau}\right)^{-0.950}$	$0.740\tau\left(\frac{\theta}{\tau}\right)^{0.7380}$	$0.365\tau\left(\frac{\theta}{\tau}\right)^{0.950}$

Table 6. Cohen-Coon/Reaction Curve

Controller	K_c	τ_i	τ_d
P	$\frac{\tau}{K\theta}\left(1+\frac{\theta}{3\tau}\right)$		
PI	$\frac{\tau}{K\theta}\left(\frac{9}{10}+\frac{\theta}{12\tau}\right)$	$\frac{\theta(30\tau+3\theta)}{9\tau+20\theta}$	
PID	$\frac{\tau}{K\theta}\left(\frac{4}{3}+\frac{\theta}{4\tau}\right)$	$\frac{\theta(32\tau+6\theta)}{13\tau+8\theta}$	$\frac{4\theta\tau}{11\tau+2\theta}$

Table 7. Ciancone-Marlin

	PI				***PID***					
	K_c		τ_i		K_c		τ_i		τ_d	
	$\frac{\beta}{K}$		$\delta(\tau+\theta)$		$\frac{\beta}{K}$		$\delta(\tau+\theta)$		$\varphi(\tau+\theta)$	
$\emptyset$	*Set-Point*		*Disturbance*		*Set-Point*			*Disturbance*		
$\alpha(\tau+\alpha)$	β	δ	β	δ	β	δ	φ	β	δ	φ
0.0	*1.417*	*0.748*	*1.259*	*0.241*	*1.520*	*0.732*	*0.000*	*1.392*	*0.245*	*0.000*
0.1	*1.417*	*0.748*	*1.259*	*0.235*	*1.520*	*0.732*	*0.000*	*1.392*	*0.245*	*0.000*
0.2	*1.193*	*0.964*	*1.626*	*0.518*	*1.430*	*0.941*	*0.017*	*1.823*	*0.513*	*0.000*
0.3	*1.032*	*0.881*	*1.377*	*0.789*	*1.110*	*0.864*	*0.035*	*1.423*	*0.714*	*0.008*
0.4	*0.918*	*0.818*	*1.000*	*0.746*	*0.862*	*0.773*	*0.060*	*1.130*	*0.691*	*0.036*
0.5	*0.861*	*0.756*	*0.858*	*0.702*	*0.687*	*0.672*	*0.087*	*0.859*	*0.641*	*0.070*
0.6	*0.722*	*0.693*	*0.599*	*0.659*	*0.545*	*0.596*	*0.118*	*0.670*	*0.600*	*0.107*
0.7	*0.648*	*0.631*	*0.464*	*0.615*	*0.460*	*0.545*	*0.155*	*0.521*	*0.550*	*0.148*
0.8	*0.608*	*0.568*	*0.383*	*0.572*	*0.398*	*0.505*	*0.215*	*0.406*	*0.505*	*0.203*
0.8	*0.594*	*0.506*	*0.359*	*0.528*	*0.370*	*0.468*	*0.139*	*0.335*	*0.468*	*0.132*
1.0	*0.558*	*0.443*	*0.316*	*0.485*	*0.370*	*0.432*	*0.047*	*0.284*	*0.432*	*0.050*

Table 8. Error criterions

				ICE		**IAE**		**IAET**	
Controller	K_c	τ_i	τ_d	a	b	a	b	a	b
P	$\frac{a}{K}\left(\frac{\theta}{\tau}\right)^b$			*1.411*	*-0.917*	*0.902*	*-0.985*	*0.940*	*-1.084*
PI	$\frac{a}{K}\left(\frac{\theta}{\tau}\right)^b$			*1.305*	*-0.959*	*0.984*	*-0.986*	*0.859*	*-0.977*
		$\frac{\tau}{a}\left(\frac{\theta}{\tau}\right)^b$		*0.492*	*0.739*	*0.608*	*0.707*	*0.674*	*0.680*
PID	$\frac{a}{K}\left(\frac{\theta}{\tau}\right)^b$			*1.495*	*-0.945*	*1.435*	*-0.921*	*1.357*	*-0.947*
		$\frac{\tau}{a}\left(\frac{\theta}{\tau}\right)^b$		*1.101*	*0.771*	*0.878*	*0.749*	*0.842*	*0.738*
			$a\tau\left(\frac{\theta}{\tau}\right)^b$	*0.560*	*1.006*	*0.482*	*1.137*	*0.381*	*0.995*

Table 7. Parallel-Series PID Conversion	
Series to Parallel	***Parallel to Series***
$K_c = K_c \frac{(\tau_i + \tau_d)}{\tau_i}$	$K_c = \frac{K_c}{2}\left(1 + \sqrt{1 - 4\frac{\tau_d}{\tau_i}}\right)$
$\tau_i = \tau_i + \tau_d$	$\tau_i = \frac{\tau_i}{2}\left(1 + \sqrt{1 - 4\frac{\tau_d}{\tau_i}}\right)$
$\tau_d = \frac{\tau_i \tau_d}{\tau_i + \tau_d}$	$\tau_d = \frac{2\tau_d}{1 + \sqrt{1 - 4\frac{\tau_d}{\tau_i}}}$