Homework 4

Due 23:59 Nov. 30

Problem 1 (10 points)

For each of the following sequences, determine whether there exists a simple graph with such a degree sequence. Draw such a graph if it exists.

(a) 5, 4, 3, 2, 1, 0 (b) 6, 5, 4, 3, 2, 1 (c) 2, 2, 2, 2, 2, 2 (d) 3, 3, 2, 2, 2, 2

Problem 2 (10 points)

Determine the number of non-isormorphic simple graphs with five vertices and three edges.

Problem 3 (10 points)

Construct an example in which there is more than one stable matching. (You only need two boys and two girls to do this.)

Problem 4 (20 points)

Suppose preferences are given by the following tables:

BOY	1	2	3	4	5
Adam	Beth	Amy	Diane	Ellen	Cara
Bill	Diane	Beth	Amy	Cara	Ellen
Carl	Beth	Ellen	Cara	Diane	Amy
Dan	Amy	Diane	Cara	Beth	Ellen
Eric	Beth	Diane	Amy	Ellen	Cara

Table 1: Boys' Preferences

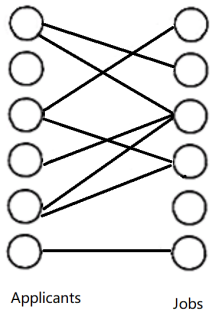
GIRL	1	2	3	4	5
Amy	Eric	Adam	Bill	Dan	Carl
Beth	Carl	Bill	Dan	Adam	Eric
Cara	Bill	Carl	Dan	Eric	Adam
Diane	Adam	Eric	Dan	Carl	Bill
Ellen	Dan	Bill	Eric	Carl	Adam

Table 2: Girls' Preferences

(1) Find a stable matching using the Gale-Shapley algorithm with boys making proposals.
(2) Find a stable matching using the Gale-Shapley algorithm with girls making proposals.

Problem 5 (20 points)

Suppose we have a set $\{J_1, J_2, \ldots, J_r\}$ of r jobs to be filled by a pool of s applicants $\{A_1, A_2, \ldots, A_s\}$. Each job can be filled by at most one applicant and each applicant be assigned to at most one job. Also each job can be filled by only a subset of applicants qualified for the jobs. It is known in advance if a job J_i can be filled by applicant A_j. We would like to find the maximum number of jobs that can be filled. Formulate this as a maximum matching problem, and solve the following example problem. You need to give the maximum number of jobs that can be filled and plot the corresponding maximum matching graph.



Problem 6 (20 points)

A graph is called d-regular, if all vertices have degree d. Prove that any bipartite d-regular graph contains d disjoint perfect matchings.

Problem 7 (20 points)

Use Hall's theorem to solve the partial Latin Square which some columns have been filled as follows.

1	2	6	3		
2	3	1	4		
3	4	2	5		
4	5	3	6		
6	1	5	2		
5	6	4	1		

Problem 8 (20 points)

Corncob College elects 10 students to serve as officers on 8 committees. The list of the members of each of the committees is:

- Corn Feed Committee: Darcie, Barb, Kyler
- Dorm Policy Committee: Barb, Jack, Anya, Kaz
- Extracurricular Committee: Darcie, Jack, Miranda
- Family Weekend Committee: Kyler, Miranda, Jenna, Natalie
- Homecoming Committee: Barb, Jenna, Natalie, Skye
- Off Campus Committee: Kyler, Jenna, Skye
- Parking Committee: Jack, Anya, Miranda

- Student Fees Committee: Kaz, Natalie

They need to schedule meetings for each of these committees, but two committees cannot meet at the same time if they have any members in common.

1. Draw a graph representing this situation. *(Hint: Let the vertices represent the committees.)*
2. Determine the minimum different meeting times we will need.