# Assignment 1

**Due 23:59, Oct 15**
**Only hard copy accepted unless with specific reasons**

**Question 1** Let $p, q, r$ be the propositions:
$p$: You get an A on the final exam.
$q$: You do every exercise in this book.
$r$: You get an A in this class.
Write these propositions using $p, q, r$ and logical connectives (including negations).

i. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

ii. You will get an A in this class if and only if you finish at least one of them: do every exercise in this book, get an A on the final exam.

*(5 points for each question, totally 10)*

**Question 2** Construct a truth table for the following compound proposition.

$$p \oplus \neg q$$

*(10 points)*

**Question 3** Show that the following statement is a tautology by using truth tables and by using logical equivalencies.

$$[p \wedge (p \rightarrow q)] \rightarrow q$$

*(10 points)*

**Question 4** Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.
*Hint: Find an assignment of truth values that makes one of these propositions true and the other false.*
*(5 points)*

**Question 5** Let $C(x)$ be the statement $x$ has a cat, let $D(x)$ be the statement $x$ has a dog, and let $F(x)$ be the statement $x$ has a ferret. Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives. Let the domain consists of all students in your class.

i. A student in your class has a cat, a dog, and a ferret.

ii. No student in your class has a cat, a dog, and a ferret.

iii. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

*(3 points, 3 pts, 4pts)*

**Question 6** Express the negations of each of these statements so that all negation symbols immediately precede the predicates.

i. $\exists x \exists y (Q(x, y) \Leftrightarrow Q(y, x))$

ii. $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

*(5 points for each question, totally 10)*

**Question 7** Find two sets $A$ and $B$ such that $A \in B$ and $A \subseteq B$. Justify your answer.
*(5 points)*

**Question 8** Let $A = \{a, b, c\}, B = \{x, y\}, C = \{0, 1\}$. Find

i. $A \times B \times C$

ii. $C \times A \times B$

iii. If $D = A \times B$, find $D \times C$

*(3 points, 3 pts, 4 pts)*

**Question 9** Let $A, B$ be sets. Prove that $A \cup (A \cap B) = A$.
*(10 points)*

**Question 10** Let $A, B, C$ be sets. Show that

i. $(A \cap B \cap C) \subseteq (A \cap B)$

ii. $(A - B) - C \subseteq A - C$

iii. $(B - A) \cup (C - A) = (B \cup C) - A$

*(5 points for each question, totally 15)*