DATA STRUCTURES

WENYE LI
CUHK-SZ

BEST-FIRST GRAPH SEARCH

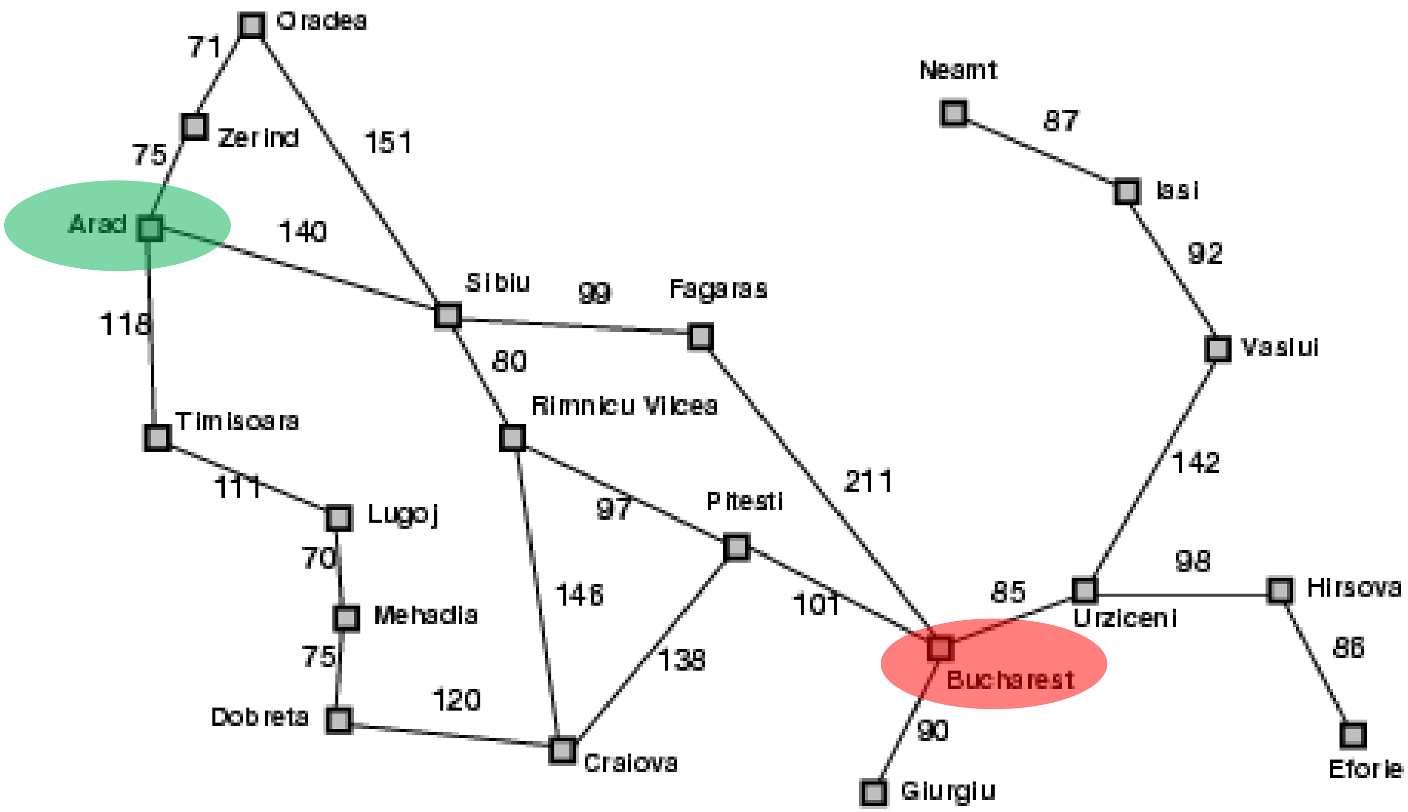
- A search strategy is defined by picking the order of node expansion

- Best-First Search: use an evaluation function *f(n)* for each node
  - estimate of "desirability"
  - Expand most desirable unexpanded node

- Special cases:
  - greedy best-first search
  - A^* search

This slides are designed for assignment 3, not included in final exam.

ROMANIA WITH STEP COSTS IN KM



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

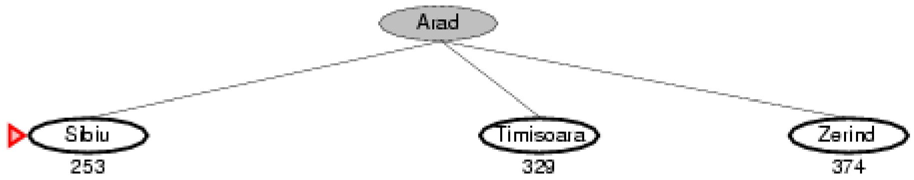
What is the shortest path from Arad to Bucharest?

GREEDY BEST-FIRST SEARCH

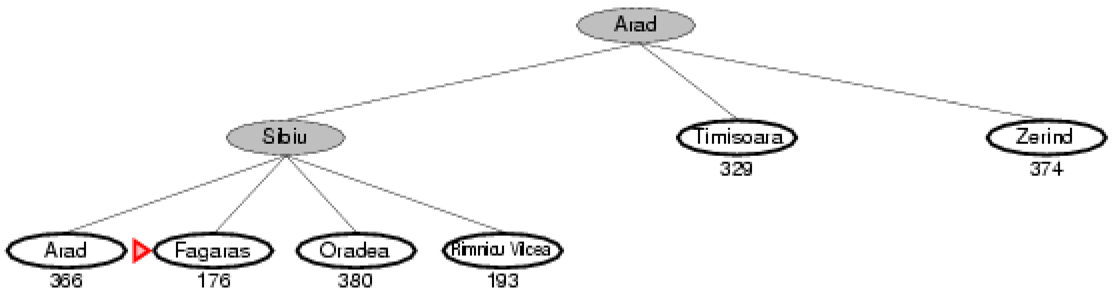
- Evaluation function $f(n) = h(n)$ (heuristic)
  - = estimate of cost from n to *goal*
- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal

GREEDY BEST-FIRST SEARCH EXAMPLE

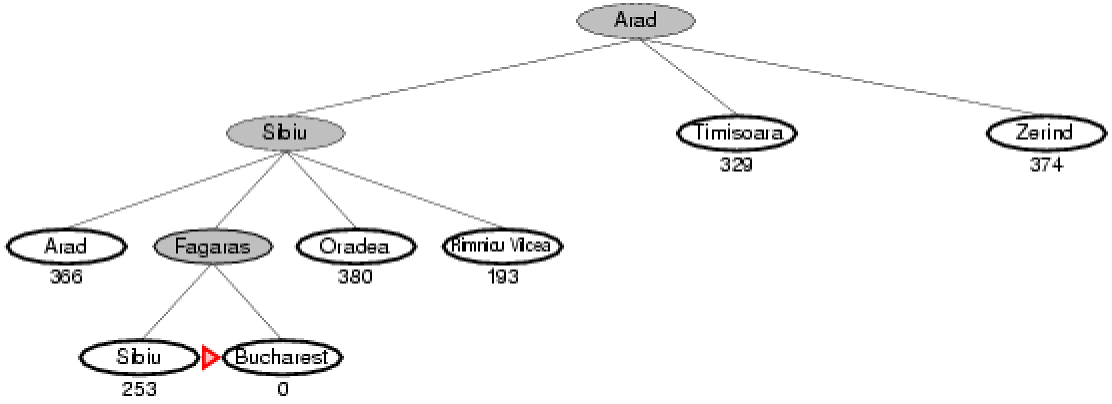
GREEDY BEST-FIRST SEARCH EXAMPLE



GREEDY BEST-FIRST SEARCH EXAMPLE



GREEDY BEST-FIRST SEARCH EXAMPLE



PROPERTIES OF GREEDY BEST-FIRST SEARCH

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time? Exponential, but a good heuristic can give dramatic improvement
- Space? keeps all nodes in memory
- Optimal? No

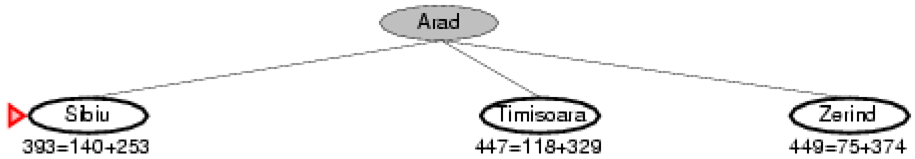
A^* SEARCH

- Idea: avoid expanding paths that are already expensive

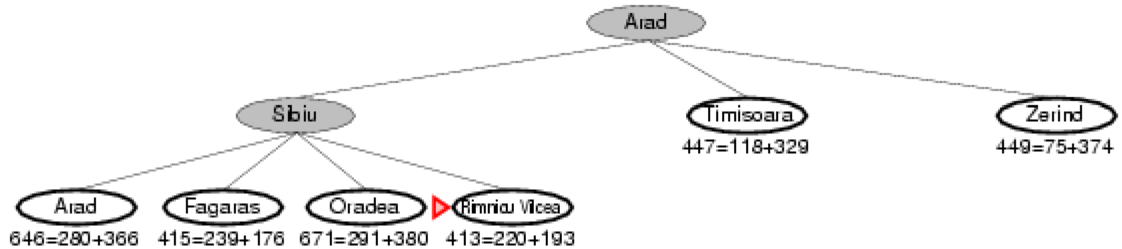
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost so far to reach n
  - $h(n)$ = estimated cost from n to goal
  - $f(n)$ = estimated total cost of path through n to goal

A* SEARCH EXAMPLE

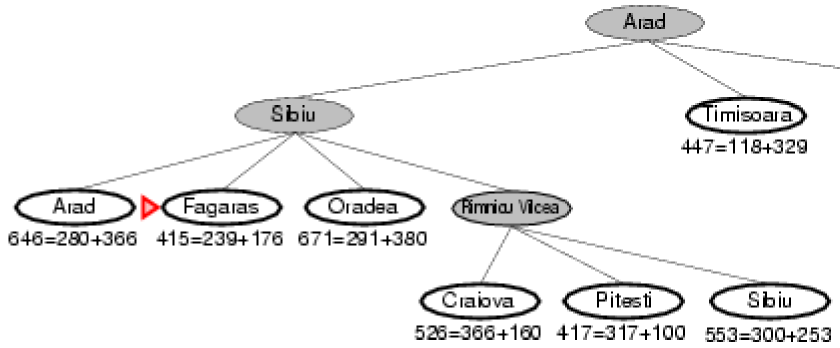
A* SEARCH EXAMPLE



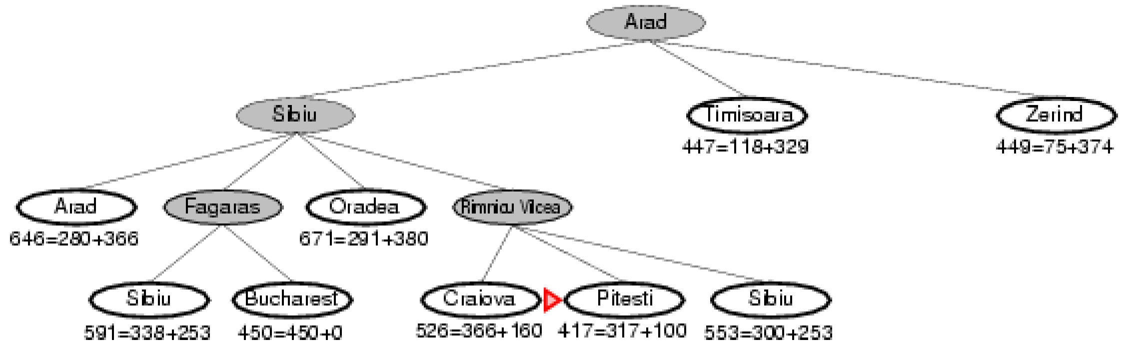
A* SEARCH EXAMPLE



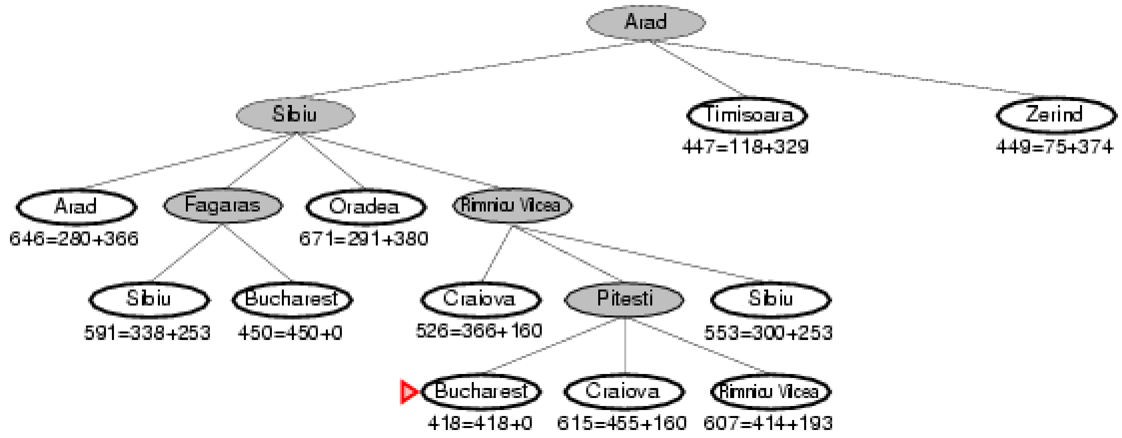
A* SEARCH EXAMPLE



A* SEARCH EXAMPLE



A* SEARCH EXAMPLE



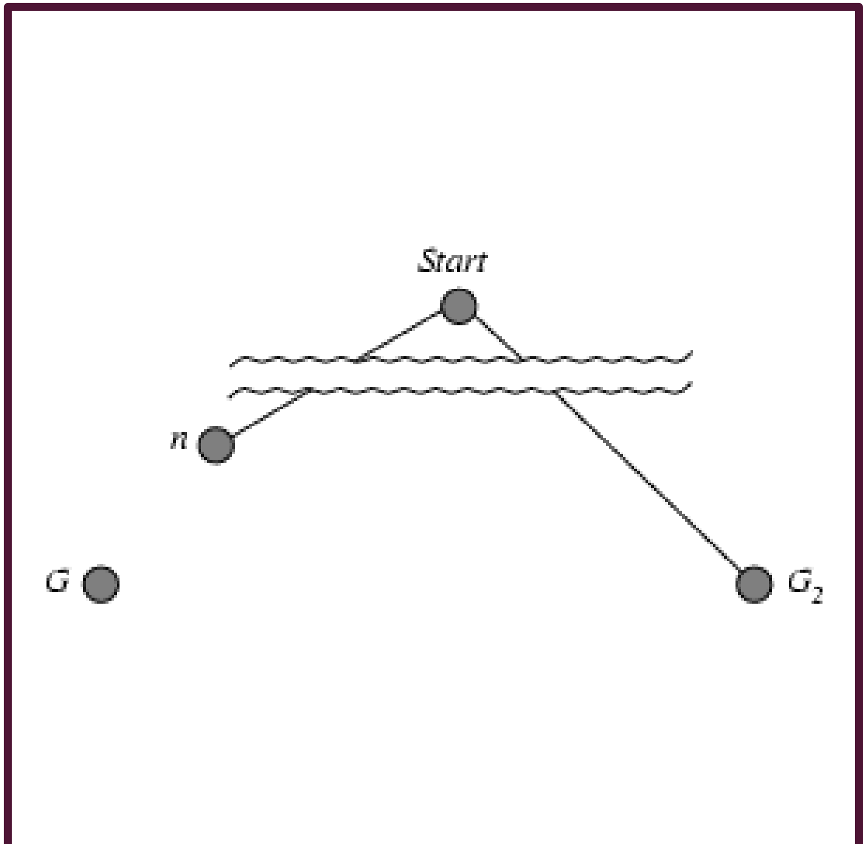
ADMISSIBLE HEURISTICS

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^{*}(n)$, where $h^{*}(n)$ is the true cost to reach the goal state from n.

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic

- Theorem: If $h(n)$ is admissible, A^{*} using `TREE-SEARCH` is optimal

OPTIMALITY OF A* (PROOF)

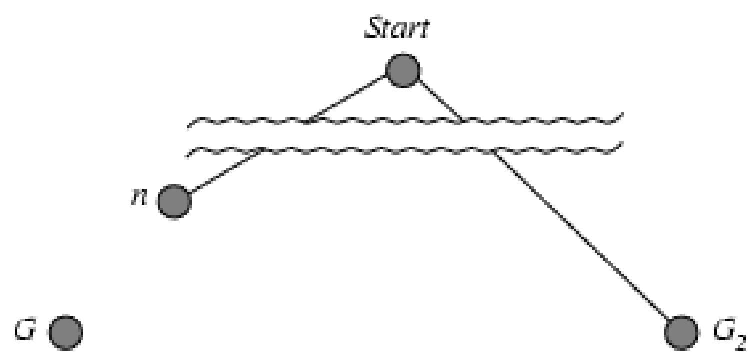


- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

OPTIMALITY OF A* (PROOF)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.

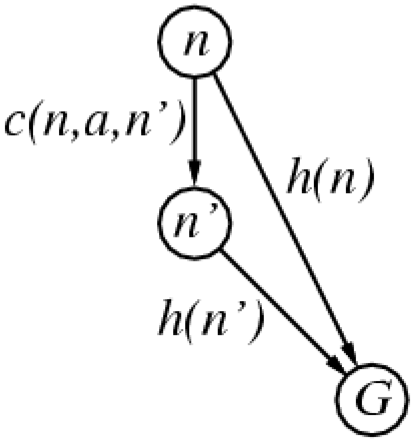


- $f(G_2)$ $> f(G)$ from above
- $h(n)$ $\leq h^*(n)$ since h is admissible
- $g(n) + h(n)$ $\leq g(n) + h^*(n)$
- $f(n)$ $\leq f(G)$
- Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion

CONSISTENT HEURISTICS

- A heuristic is consistent if for every node n, every successor n' of n generated by any action a,

  $h(n) \leq c(n,a,n') + h(n')$

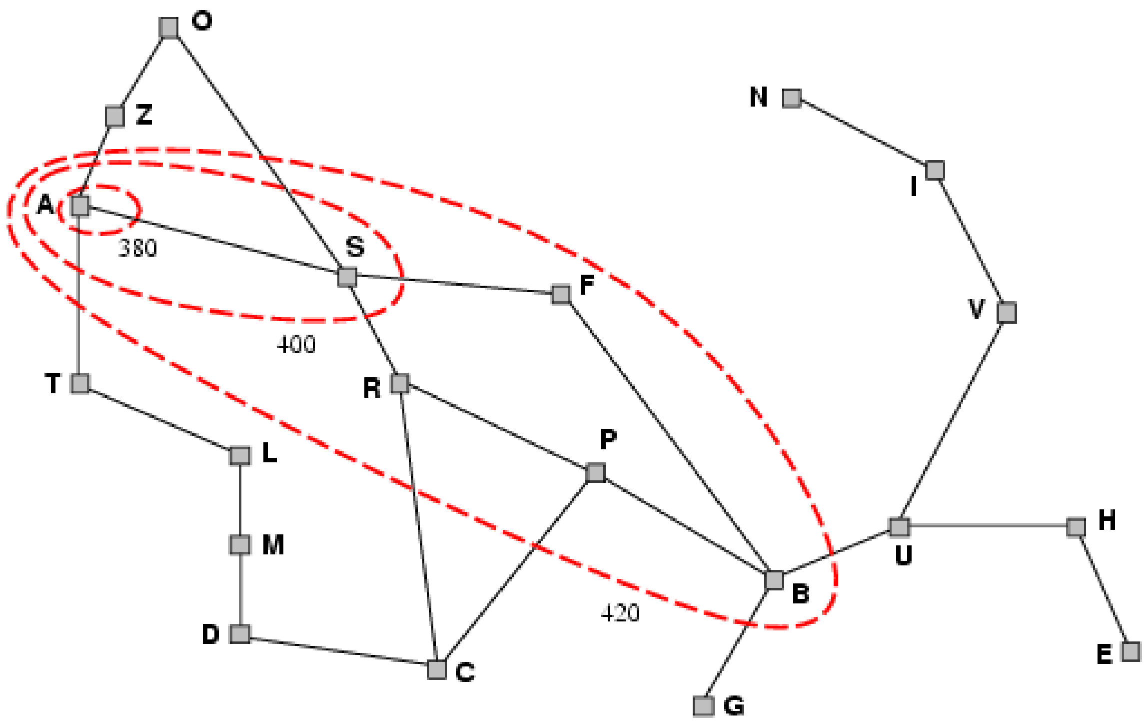


- If h is consistent, we have

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\geq g(n) + h(n) \\
&= f(n)
\end{aligned}
$$

- i.e., $f(n)$ is non-decreasing along any path.

- Theorem: If $h(n)$ is consistent, A* using `GRAPH-SEARCH` is optimal



OPTIMALITY OF A*

- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$

PROPERTIES OF A*

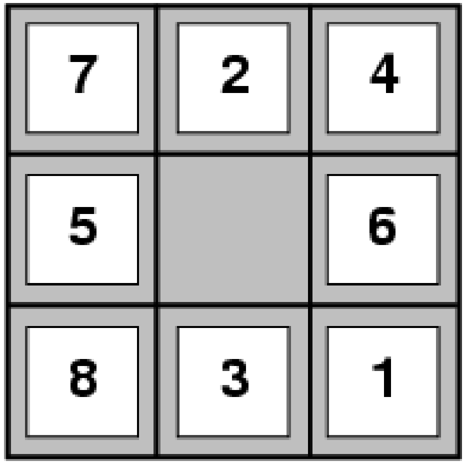
- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

ADMISSIBLE HEURISTICS

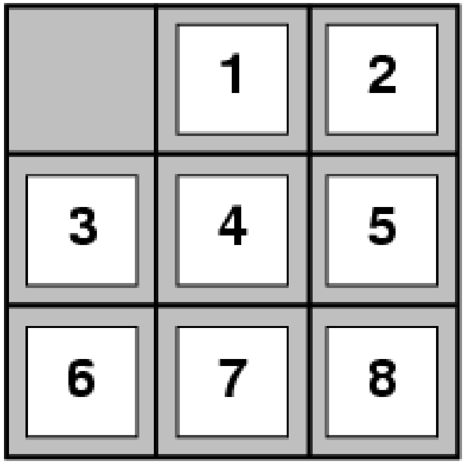
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State



Goal State

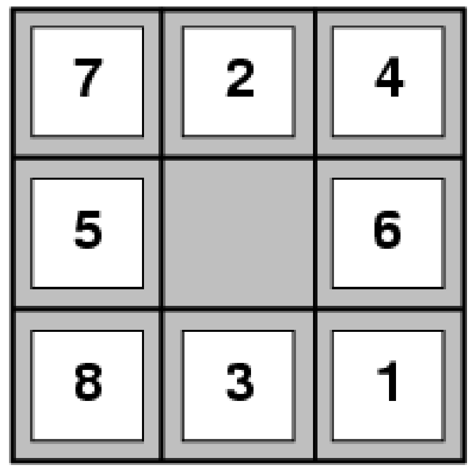
- $h_1(S)$ = ?
- $h_2(S)$ = ?

ADMISSIBLE HEURISTICS

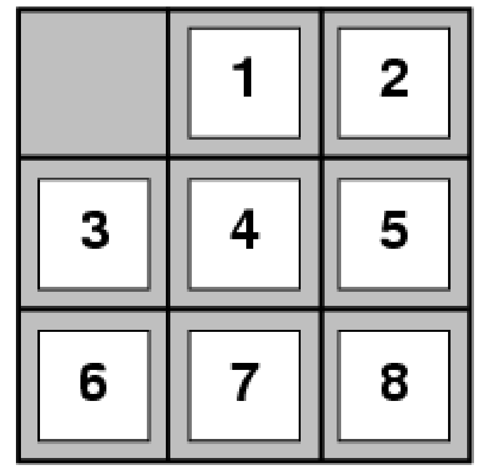
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State



Goal State

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

DOMINANCE

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1 *, and* h_2 is better for search

- Typical search costs (average number of nodes expanded):
- *d=12*
  - IDS = 3,644,035 nodes
  - $A^*(h_1)$ = 227 nodes
  - $A^*(h_2)$ = 73 nodes
- *d=24*
  - IDS = too many nodes
  - $A^*(h_1)$ = 39,135 nodes
  - $A^*(h_2)$ = 1,641 nodes

THANKS