Homework 5

Due: 23:59 Dec 9

Problem 1 (10 Points)

Show that if G is a bipartite simple graph with v vertices and e edges, then $e \leq v^2/4$.

Problem 2 (10 Points)

The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. More precisely, let $G = (V, E)$ be a simple graph and let K be the set of all 2-element subsets of V. The complement graph $\overline{G}$ is defined as $\overline{G} = (V, K \backslash E)$. If G is a simple graph with 15 edges and $\overline{G}$ has 13 edges, how many vertices does G have?

Problem 3 (15 Points)

Suppose that there are four employees in the computer support group of the School of Engineering. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software.

a) Use a bipartite graph to model the four employees and their qualifications.

b) Use Hall's theorem to determine whether there is an assignment so that all support areas are taken care of while everyone is responsible for at most one area.

c) If there exists an assignment so that all support areas are taken care of while everyone is responsible for at most one area, find one.

Problem 4 (10 Points)

Radio stations broadcast their signal at certain frequencies. However, there are a limited number of frequencies to choose from, so nationwide many stations use the same frequency. This works because the stations are far enough apart that their signals will not interfere; no one radio could pick them up at the same time.

Suppose six new radio stations are to be set up in a currently unpopulated (by radio stations) region. The distances among stations are recorded in the table below. How many different channels are needed for six stations located at the distances shown in the table, if two stations cannot use the same channel when they are within 150 miles of each other?

Table 1: Distances in miles among stations

	1	2	3	4	5	6
1	——	85	175	200	50	100
2	85	——	125	175	100	160
3	175	125	——	100	200	250
4	200	175	100	——	210	220
5	50	100	200	210	——	100
6	100	160	250	220	100	——

Problem 5 (15 Points)

Determine whether the following graphs are planar. If so, please draw the given planar graph without any crossings. If not, please try to explain it.

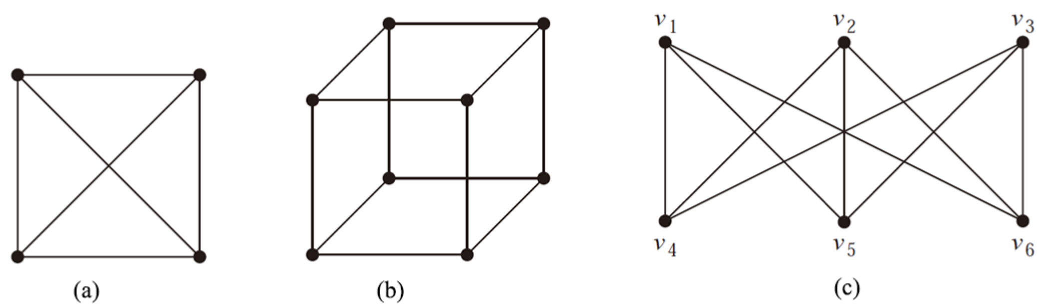

Figure 1: Graphs

Problem 6 (10 Points)

The graph G has six vertices with degrees $(5, 4, 4, 3, 2, 2)$. How many edges does G have? Could G be planar? If so, how many faces would it have. If not, please explain it.

Problem 7 (10 Points)

Let m, n, and r be nonnegative integers with r not exceeding either m or n. Prove the following formula by a combinatorial proof.

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k}\binom{n}{k} \tag{1}$$

Problem 8 (20 Points)

A 0-1 sequence $\{a_n\}$ with $2m$ terms is said to be normal if the following two conditions are satisfied.

1) There exist m terms equal to 0 and the other m terms equal to 1 in $\{a_n\}$.

2) For arbitrary $k \leq 2m$, the number of terms equal to 0 is not less than that of terms equal to 1 in the first k terms $\{a_1, a_2, \cdots, a_k\}$.

Please complete the following questions.

a) Show that the number of abnormal 0-1 sequences $\{a_n\}$ with $2m$ terms equals that of sequences $\{a_n\}$ of which $(m+1)$ terms are 0s and $(m-1)$ terms are 1s.

b) For $m = 4$, determine the number of different normal 0-1 sequences $\{a_n\}$.

Note: An abnormal 0-1 sequence $\{a_n\}$ is a 0-1 sequence that does not satisfy the properties of normal 0-1 sequences.