CSC3001 Discrete Mathematics

Assignment 3

Deadline: 11:59 pm, Friday, Nov 11, 2022

1. Use the Euclidean algorithm to calculate $\gcd(102, 70)$. Use the extended Euclidean algorithm to write $\gcd(663, 234)$ as an integer linear combination of 663 and 234.

2. Prove that a number is divisible by 3 if and only if the sum of its digits is divisible by 3.

3. Prove that all numbers in the sequence

$$1007, 10017, 100117, 1001117, \ldots$$

   are divisible by 53.

4. A robot walks around a two-dimensional grid. It starts out at $(2, 0)$ and is allowed to take four different types of steps as:

   (a) $(+2, -1)$
   (b) $(+1, -2)$
   (c) $(+1, +4)$
   (d) $(-3, 0)$

   Prove that this robot can never reach $(0, -1)$.

5. NIM is a famous game in which two players take turns removing items from a pile of n items. For every turn, the player can remove one, two, or three items at a time. The player removing the last item loses. Prove that if each player plays the best strategy possible, the first player wins if $n \not\equiv 1 \pmod 4$ and the second player wins if $n \equiv 1 \pmod 4$. (For your interest, refer to the general NIM game at this link).

6. Find all solutions, if any, to the system:

$$\begin{cases} x \equiv 5 \pmod{6} \\ x \equiv 3 \pmod{10} \\ x \equiv 8 \pmod{15}. \end{cases}$$

7. Show with the help of Fermat's little theorem that if n is a positive integer, then $42|n^7 - n$.