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1. (a) true, true (let $x=0$; let $y=0$)

   (b) false, true (let $y=0, z=1$, then $xy=0 \neq 1 = z$; let $y=z=0$)

2. (a) $\forall x, \forall y\ \forall z\ ((x<0) \wedge (y<0) \wedge (z<0)) \iff (x+y+z<0)$

   (b) $\forall x, \forall y\ ((x \in \mathbb{Z}) \wedge (y \in \mathbb{Z})) \iff (2(x^2+y^2) \geq (x+y)^2)$

   (c) $\exists x, \exists y, \forall z\ ((x \in \mathbb{R}) \wedge (y \in \mathbb{R}) \wedge (z \in \mathbb{R})) \iff (x+y=z)$

3. (a) $A \oplus B = (A \cup B) - (A \cap B) = (A \cup B) \cap \overline{(A \cap B)} = (A \cup B) \cap (\bar{A} \cup \bar{B}) = [(A \cup B) \cap \bar{A}] \cup ((A \cup B) \cap \bar{B})$

$= ((\bar{A} \cap A) \cup (B \cap \bar{A})) \cup ((A \cap \bar{B}) \cup (B \cap \bar{B})) = (B \cap \bar{A}) \cup (A \cap \bar{B}) = (A-B) \cup (B-A)$

$(A-B) \cup (B-A) = (A \cap \bar{B}) \cup (B \cap \bar{A})$ $\therefore A \oplus B = (A-B) \cup (B-A)$

(b) $\bar{A} \oplus \bar{B} = (\bar{A} \cup \bar{B}) - (\bar{A} \cap \bar{B}) = (\bar{A} \cup \bar{B}) \cap \overline{(\bar{A} \cap \bar{B})} = (\bar{A} \cup \bar{B}) \cap (A \cup B) = (A \cup B) - (A \cap B)$

$= A \oplus B$

(c) $A \oplus (B \oplus C) = (A - (B \oplus C)) \cup ((B \oplus C) - A)$

$= (A \cap \overline{B \oplus C}) \cup ((B \oplus C) \cap \bar{A})$

$= (A \cap \overline{(B \cup C) - (B \cap C)}) \cup (((B-C) - (C-B)) \cap \bar{A})$

$= (A \cap \overline{(B \cup C) \cap \overline{(B \cap C)}}) \cup (((B \cap \bar{C}) \cup (C \cap \bar{B})) \cap \bar{A})$

$= (A \cap (\overline{(B \cup C)} \cup (B \cap C))) \cup ((B \cap \bar{C} \cap \bar{A}) \cup (C \cap \bar{B} \cap \bar{A}))$

$= ((A \cap \bar{B} \cap \bar{C}) \cup (A \cap B \cap C)) \cup ((\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C))$

$= (A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (A \cap B \cap C) \cup (\bar{A} \cap \bar{B} \cap C)$

$= (\bar{C} \cap ((A \cap \bar{B}) \cup (\bar{A} \cap B))) \cup (C \cap ((A \cap B) \cup (\bar{A} \cap \bar{B})))$

$= (\bar{C} \cap ((A-B) \cup (B-A))) \cup (C - \overline{(A \cup B) \cup (\bar{A} \cap \bar{B})})$

$= (\bar{C} \cap (A \oplus B)) \cup (C - (\overline{(A \cap B)} \cap (A \cup B)))$

$= (\bar{C} \cap (A \oplus B)) \cup (C - ((A \cup B) - (A \cap B)))$

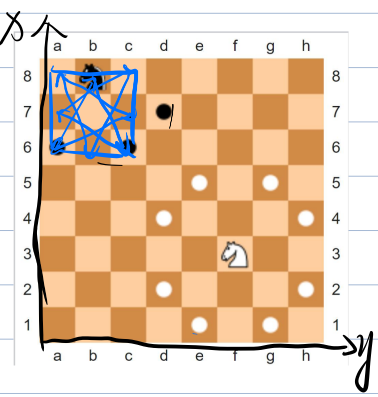
$= ((A \oplus B) - C) \cup (C - (A \oplus B))$

$= (A \oplus B) \oplus C$

4. Assume there are only finitely many primes.
Let $p_1, p_2, \cdots p_k$ be all the primes.
Consider $p_1 p_2 \cdots p_k + 1 = X$
from p_1 to p_k can not divide X. X is not in $p_1, p_2 \cdots p_k$.
so, there is an X which is prime forever.
There are infinitely many primes.

5. when $n=0$ $\sum_{i=0}^{n} 2^0 = 2^1 - 1 = 1$ Base Case $(n=0)$ is true.
Base Case (n) $\sum_{i=0}^{n} 2^i = 2^0 + 2^1 + \cdots 2^n = 2^{n+1} - 1$
Base Case $(n+1)$ $\sum_{i=0}^{n+1} 2^i = 2^0 + 2^1 + \cdots 2^n + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1} = 2^{n+2} - 1$
Base Case $(n+1)$ is true
$\therefore \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

6. we want to prove question stem, must prove (when $n \to \infty$) $\sum_{i=1}^{x} \frac{1}{i}$ is infinite.
$\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$
consider $z = \frac{1}{x}$ $\sum_{i=1}^{n} \frac{1}{i} > \int_1^{\infty} z$
$\int_1^{\infty} z = \int_1^{\infty} \frac{1}{x} = \ln x \Big|_1^{\infty} = +\infty$ $\therefore \sum_{i=1}^{n} \frac{1}{i} \to +\infty$ (when $n \to \infty$)
now, $y \in R$, $\sum_{i=1}^{x} \frac{1}{i} \in [1, +\infty)$
$\therefore \forall_{y \in R} \exists_{x \in Z^+} \left(\sum_{i=1}^{x} \frac{1}{i} > y\right)$ is true.

7. 

Firstly, we take out randomly 9 postions from picture.
take (8,a) (8,b) (8,c) (7,a) (7,b) (7,c) (6,a) (6,b) (6,c) for example.
The start position is (8,b).
(8,b) → (6,c) → (7,a) → (8,c) → (6,b) → (8,a) → (7,c) → (6,a) → (8,b)
So, we can let this example is the base case.
The same as from the center is (7,b) to (7,c)
the flow is (8,c) → (6,d) → (7,b) → (8,d) → (6,c) → (8,b) → (7,d) → (7,b) → (8,c) so that the track recovers 3×4 rectangle.
we can move this rectangle to all the chessboard. Now we will find a knight can move to an arbitrary position from any starting position.

8. assume there are positive integer solutions, let a.b.C are the least solutions

$$4a^6 + b^6 = 16c^6$$

$\because 4a^6$ and $16c^6$ are even $\therefore b^6$ is even

$\because b^6$ is even $\therefore b$ is even

let $b = 2b_0$

$4a^6 + 64b_0^6 = 16c^6$ $\quad a^6 + 16b_0^6 = 4c^6$

$\because 16b_0^6$ and $4c^6$ are even

$\therefore a^6$ is even let $a = 2a_0$

$64a_0^6 + 16b_0^6 = 4c^6$ $\quad c^6 = 4b_0^6 + 16a_0^6$

$\because 4b_0^6$ and $16a_0^6$ are even

$\therefore c^6$ are even let $C = 2C_0$

$4b_0^6 + 16a_0^6 = 64C_0^6$ $\quad \frac{1}{4}a_0^6 + \frac{1}{16}b_0^6 = C_0^6$

a_0, b_0, C_0 are also the solutions of the function.

$a_0 < a,\ b_0 < b,\ C_0 < C$

so it is contradicting minimality of a.b.C

Therefore there is no positive integer solution for $4a^6 + b^6 = 16c^6$