CSC3001 Discrete Mathematics

Assignment 2

Deadline: 11:59pm Friday, Oct28,2022

1.Let f_n be the n-th Fibonacci sequence,$f_{n+2} = f_{n+1} + f_n$

a.$f_1 = f_2 = 1$,prove that $f_1 + f_2 + \cdots + f_n = f_{n+2} - 1$
b.$f_1 = a, f_2 = b$,prove that $f_1 + f_2 + \cdots + f_n = f_{n+2} - b$

2.Find and prove closed form formulas for generating functions
$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$
of the following sequences

(a) $a_n = a^n$,where $a \in \mathbf{R}$;

(b)$a_n = \binom{m}{n}$,where $m \in N$;

(c)$a_n = f_n$, where f_n is the n-th Fibonacci number(assume $f_0 = 0, f_1 = f_2 = 1$)

3.Using the formlar

$$\binom{n}{m} = \frac{n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-m+1)}{m \cdot (m-1) \cdot (m-2) \cdot \dots \cdot 2 \cdot 1}$$

p is a prime number.
Prove that $\binom{p}{k}$is divisible by p for $0 < k < p$;
Deduce by induction on n that $n^p \equiv_p n$.($_p n$ means $n (mod\, p)$)

4.Using the identity

$$(1+x)^n (1+x)^n = (1+x)^{2n}$$

Prove that

$$\sum_{m=0}^{n} \binom{n}{m} \binom{n}{n-m} = \binom{2n}{n}$$

Deduce that

$$\sum_{m=0}^{n} \binom{n}{m}^2 = \binom{2n}{n}$$

5.Find all solutions,if any,solutions to the system

$$x \equiv 5(mod6)$$
$$x \equiv 3(mod10)$$
$$x \equiv 8(mod15)$$

6.Show steps to find;
(a) the greatest common divisor of 1234567 and 7654321.
(b)the greatest common divisor of $2^3 3^5 5^7 7^9 11$ and $2^9 3^7 5^5 7^3 13$

7.Label the first prime number 2 asP_1.Label the second prime number 3 as P_2.Similarly,label the n-th prime number as P_n.Prove that $P_n < 2^{2^n}$ for an arbitrary $n \in N^+$. (Hint:consider $P_1 P_2 P_2 \dots P_{n-1} + 1$.)

8.In a round-robin tournament,every team plays every other team exactly once and each match has a winner and a loser.We say that the team $p_1, p_2, \dots, p_m$form a cycle if p_1 beats p_2,p_2 beats p_3,and p_m beats p_1.Show that if there is a cycle of length $m(m > 3)$among the players in a round-robin tournament,there must be a cycle off three of these players.(Hint:Use well-ordering principle.)