1. a. Both true. The former, let $x=0$; the latter, let $y=0$.

b. False, true. The former, let $y=0, z=1$, then $xy=0\neq 1=z$

The latter, let $y=z=0$ then $xy=0=z$ for any x

2. a. $\forall x \forall y \forall z [((x<0)\wedge(y<0)\wedge(z<0)) \rightarrow (x+y+z<0)]$

b. $\forall x \in \mathbb{Z}\ \forall y \in \mathbb{Z}\ (2(a^2+b^2) \geq (a+b)^2)$

c. $\underset{\in R}{\forall x}\ \underset{\in R}{\exists a}\ \underset{\in R}{\exists b}\ (x=a+b)$

3. a. $A\oplus B=(A\cup B)-(A\cap B)$

$=(A\cup B)\cap(\overline{A\cap B})$

$=(A\cup B)\cap(\bar{A}\cup\bar{B})$

$=[A\cap(\bar{A}\cup\bar{B})]\cup[B\cap(\bar{A}\cup\bar{B})]$

$=[(A\cap\bar{A})\cup(A\cup\bar{B})]\cup[(B\cap\bar{A})\cup(B\cap\bar{B})]$

$=[\phi\cup(A\cup\bar{B})]\cup[(B\cap\bar{A})\cup\phi]$

$=(A\cup\bar{B})\cup(B\cap\bar{A})$

$=(A-B)\cup(B-A)$

b. $\bar{A}\oplus\bar{B}=(\bar{A}\cup\bar{B})-(\bar{A}\cap\bar{B})$

$=(\overline{A\cap B})-(\overline{A\cup B})$

$$= (\overline{A \cap B}) \cap (\overline{\overline{A \cup B}})$$

$$= (\overline{A \cap B}) \cap (A \cup B)$$

$$= (A \cup B) - (A \cap B)$$

$$= A \oplus B$$

c. $$A \oplus (B \oplus C) = (A - (B \oplus C)) \cup ((B \oplus C) - A)$$

$$= (A \cap \overline{B \oplus C}) \cup ((B \oplus C) \cap \bar{A})$$

$$= \{A \cap \overline{(B \cup C) - (B \cap C)}\} \cup \{[(B - C) \cup (C - B)] \cap \bar{A}\}$$

$$= \{A \cap \overline{(B \cup C) \cap \overline{B \cap C}}\} \cup \{[(B \cap \bar{C}) \cup (C \cap \bar{B})] \cap \bar{A}\}$$

$$= \{A \cap [\overline{B \cup C} \cup (B \cap C)]\} \cup \{(\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)\}$$

$$= \{A \cap [(\bar{B} \cap \bar{C}) \cup (B \cap C)]\} \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$$

$$= \{(A \cap \bar{B} \cap \bar{C}) \cup (A \cap B \cap C)\} \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$$

$$= \{(A \cap B \cap C) \cup (\bar{A} \cap \bar{B} \cap C)\} \cup \{(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C})\}$$

$$= \{C \cap [(A \cap B) \cup (\bar{A} \cap \bar{B})]\} \cup \{\bar{C} \cap [(A \cap \bar{B}) \cup (\bar{A} \cap B)]\}$$

$$= \{C - \overline{(A \cap B) \cup (\bar{A} \cap \bar{B})}\} \cup \{\bar{C} \cap [(A - B) \cup (B - A)]\}$$

$$= \{C - [\overline{A \cap B} \cap (A \cup B)]\} \cup \{\bar{C} \cap (A \oplus B)\}$$

$$= \{C - [(A \cup B) - (A \cap B)]\} \cup \{(A \oplus B) - C\}$$

$$= [C - (A \oplus B)] \cup [(A \oplus B) - C]$$

$$= (A \oplus B) \oplus C$$

4. Assume that are finitely many primes. $P_1, P_2, \cdots, P_{max}$ (Ascending)

Firstly, we proof "If p divides a, p doesn't divide $a+1$":

Assume that p also divides $a+1$.

p divides $a \Rightarrow a = p \cdot b$ for $b \in \mathbb{Z}$

p divides $a+1 \Rightarrow a+1 = pc$ for $c \in \mathbb{Z}$

$a+1-a = pc - pb = p(c-b) = 1$

$\because c-b \geq 1,\ p \geq 2 \therefore p(c-b) \geq 2$

contradiction.

So $P_1 \times P_2 \times \cdots \times P_{max} + 1$ isn't divided by any prime.

Because $P_1 \times P_2 \times \cdots \times P_{max}$ can divided by any prime,

According to the assumption, any prime isn't greater than P_{max}

But $P_1 \times P_2 \times \cdots \times P_{max} + 1 > P_{max}$. Contradiction.

$\therefore$ There are infinitely many primes.

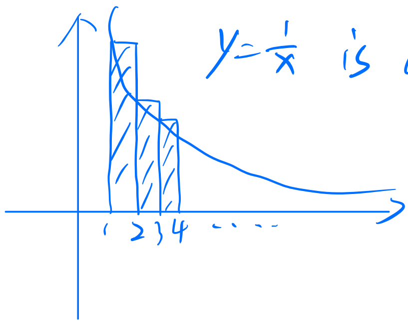
5. when $n=0$, $\sum_{i=0}^{n} 2^i = 1 = 2^{0+1} - 1$

Assume $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$ works for $n = k$.

$\sum_{i=0}^{k+1} 2^i = 2^{k+1} + \sum_{i=0}^{k} 2^i = 2^{k+1} + (2^{k+1} - 1) = 2^{k+2} - 1$

we proof "$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$ works for $n = k+1$".

$\therefore$ It works for any $n \in \mathbb{N}$

6. 

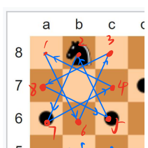
$y=\frac{1}{x}$ is decreasing $\Rightarrow \frac{1}{a}=\int_a^{a+1}\frac{1}{a}dx > \int_a^{a+1}\frac{1}{x}dx$ for $0<a<b$

$$\Rightarrow \sum_{i=1}^{x}\frac{1}{i} > \sum_{i=1}^{x}\int_i^{i+1}\frac{1}{x}dx = \sum_{i=1}^{x}(\ln(i+1)-\ln i)$$
$$= \ln(x+1)-\ln 1$$
$$= \ln(x+1)$$

When $y\le 1$, let $x=2$, we have $\sum_{i=1}^{x}\frac{1}{i}=1+\frac{1}{2}=\frac{3}{2}>y$.

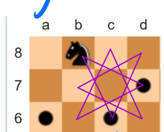
When $y>1$, we let $x=\lceil e^y-1\rceil\ge 2$, we have $\sum_{i=1}^{x}\frac{1}{i}>\ln(\lceil e^y-1\rceil+1)$
$$=\ln\lceil e^y\rceil \ge \ln e^y = y$$

$$\therefore \forall_{y\in\mathbb{R}}\ \exists_{x\in\mathbb{Z}^+}\left(\sum_{i=1}^{x}\frac{1}{i}>y\right)$$

7. 

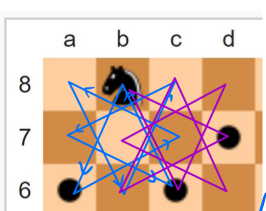
pic.1

According to the picture, the route is a closed cycle. once knight move to any point of the 8 points, then it can move to another 7 points without using other points. (In this proof, point means position.)



pic.2

Similarly, we can construct another cycle. And this cycle overlaps the first cycle, which means that once knight move to any point of the two cycle, then it can move to any other point of the two cycle.



pic.3

we find that the two cycle cover all points of pic.2, namely, a 3x4 rectangle.
It is stressed that if two 3x4 rectangles overlap each other, then knight who moves to any point of the two rectangles can move to all points of the two rectangles. It's easy to know the chessboard must be covered completely by many 3x4 rectangles which overlaps others to keep as a whole.
So knight can move to any point of the chessboard from any starting point.

8. Assume there are $a, b, c \in \mathbb{Z}^+$ s.t. $\frac{1}{4}a^6+\frac{1}{16}b^6=c^6$

$$4a^6+b^6=16c^6$$

$$b^6=16c^6-4a^6 \Rightarrow b \text{ is even, let } b=2m \quad m\in\mathbb{Z}^+$$

$$64m^6=16c^6-4a^6$$

$$16m^6=4c^6-a^6$$

$$a^6=4c^6-16m^6 \Rightarrow a \text{ is even, let } a=2n \quad n\in\mathbb{Z}^+$$

$$64n^6=4c^6-16m^6$$

$$16n^6=c^6-4m^6$$

$$c^6=4m^6+16m^6 \Rightarrow c \text{ is even. let } c=2l \quad l\in\mathbb{Z}^+$$

we have $\frac{1}{4}a^6+\frac{1}{16}b^6=c^6$

$$\frac{1}{4}(2n)^6+\frac{1}{16}(2m)^6=(2l)^6$$

$$\frac{1}{4}n^6+\frac{1}{16}m^6=l^6$$

namely, if there're a, b, c s.t. $\frac{1}{4}a^6+\frac{1}{16}b^6=c^6$, then there're $\frac{a}{2}, \frac{b}{2}, \frac{c}{2}$ s.t. $\frac{1}{4}(\frac{a}{2})^6+\frac{1}{16}(\frac{b}{2})^6=(\frac{c}{2})^6$, then there're $\frac{\frac{a}{2}}{2}, \frac{\frac{b}{2}}{2}, \frac{\frac{c}{2}}{2}$ s.t. ...

we know when $x\in\mathbb{Z}^+$, $x>0 \Leftrightarrow 2x>x \Leftrightarrow x>\frac{x}{2}$

But all positive integers are greater than 1, they can't fall infinitely.

So there are no positive integer solutions.

Supplementary proof of " a^6 is even $\Rightarrow$ a is even"

Assume a is not even i.e. odd (in this situation)

then $a = 2k+1 \quad k \in \mathbb{Z}$

$a^2 = (2k+1)^2 = \underbrace{(2k+1)\cdot 2k}_{\text{even}} + \underbrace{2k+1}_{\text{odd}}$ is odd.

$a^3 = a^2(2k+1) = \underbrace{a^2\cdot 2k}_{\text{even}} + \underbrace{a^2}_{\text{odd}}$ is odd

Similarly, if a^{n-1} is odd, $a^n = a^{n-1}(2k+1) = \underbrace{a^{n-1}\cdot 2k}_{\text{even}} + \underbrace{a^{n-1}}_{\text{odd}}$ is odd.

It's easy to know a^6 is odd which is contradictory to a^6 is even.

$\therefore$ a^6 is even $\Rightarrow$ a is even.