EIE 2050 Digital Logic and Systems

Chapter 4 : Boolean Algebra and Logic Simplification

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Last Week

- Logic gates
  - Inverter, AND, OR, NAND, NOR, XOR, XNOR
  - Truth Tables, Distinctive Shape Symbols
- Performance Characteristics and Parameters
  - Propagation Delay Time
  - DC Supply Voltage
  - Power Dissipation
  - Input and Output Logic Levels
  - Speed-Power Product (SPP)
  - Fan-Out and Loading
- Troubleshooting
  - Internal Failures of IC Logic Gates
  - External Opens and Shorts

Boolean Operations and Expressions

- Boolean Addition : Equivalent to the OR operation

EXAMPLE 4–1

Determine the values of A, B, C, and D that make the sum term $A+\bar{B}+C+\bar{D}$ equal to 0.

Solution :

$A=0,\ \ \bar{B}=0,\ \ C=0,\ \ \bar{D}=0 \Rightarrow A=0,\ \ B=1,\ \ C=0,\ \ D=1$

- Boolean Multiplication : Equivalent to the AND operation

EXAMPLE 4–2

Determine the values of A, B, C, and D that make the product term $A\bar{B}C\bar{D}$ equal to 1.

Solution :

$A=1,\ \ \bar{B}=1,\ \ C=1,\ \ \bar{D}=1 \Rightarrow A=1,\ \ B=0,\ \ C=1,\ \ D=0$

Laws and Rules of Boolean Algebra

- Laws of Boolean Algebra
  - Commutative laws $A + B = B + A$ $AB = BA$
  - Associative laws $A + (B + C) = (A + B) + \mathrm{C}$ $A(BC) = (AB)\mathrm{C}$
  - Distributive law $A(B + C) = AB + \mathrm{AC}$
- Rules of Boolean Algebra

TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, B, or C can represent a single variable or a combination of variables.

Proof of Rule 10

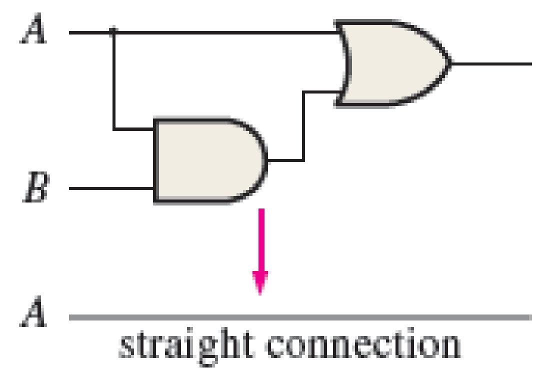
Rule 10: $A + AB = A$ This rule can be proved by applying the distributive law, rule 2, and rule 4 as follows:

$$
\begin{aligned}
A + AB = A \cdot 1 + AB = A(1 + B) &\quad \text{Factoring (distributive law)} \\
= A \cdot 1 &\quad \text{Rule 2: } (1 + B) = 1 \\
= A &\quad \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

TABLE 4–2

Rule 10: $A + AB = A$. Open file T04-02 to verify.

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

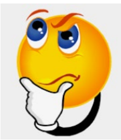


equal

Proof of Rule 11

Rule 11: $A + \bar{A}B = A + B$ This rule can be proved as follows:

$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B && \text{Rule 10: } A = A + AB \\
&= (AA + AB) + \overline{A}B && \text{Rule 7: } A = AA \\
&= AA + AB + A\overline{A} + \overline{A}B && \text{Rule 8: adding } A\overline{A} = 0 \\
&= (A + \overline{A})(A + B) && \text{Factoring} \\
&= 1 \cdot (A + B) && \text{Rule 6: } A + \overline{A} = 1 \\
&= A + B && \text{Rule 4: drop the 1}
\end{aligned}
$$



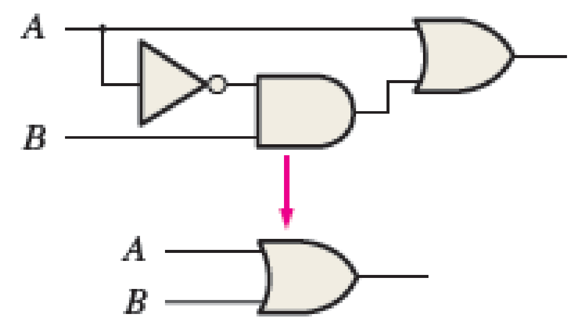
Any better ways?

TABLE 4–3

Rule 11: $A + \overline{A}B = A + B$. Open file T04-03 to verify.

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Proof of Rule 12

Rule 12: $(A + B)(A + C) = A + BC$ This rule can be proved as follows:

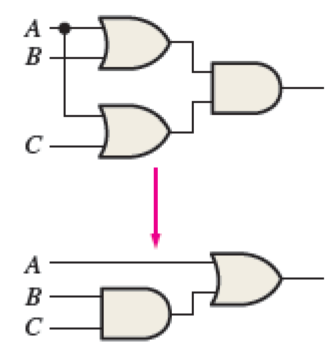
$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

TABLE 4–4

Rule 12: $(A + B)(A + C) = A + BC$. Open file T04-04 to verify.

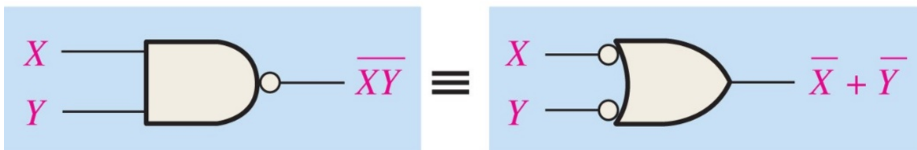
A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



DeMorgan's Theorems

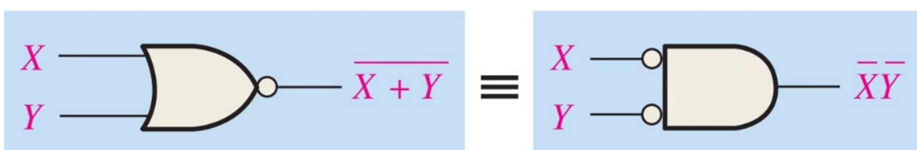
$$\overline{XY} = \bar{X} + \bar{Y}$$



NAND Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

$$\overline{X + Y} = \bar{X}\bar{Y}$$



NOR Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

DeMorgan’s Theorems : Examples

Example 4-6

Apply DeMorgan’s theorems to each expression:

(a) $\overline{\overline{(A + B)} + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Solution

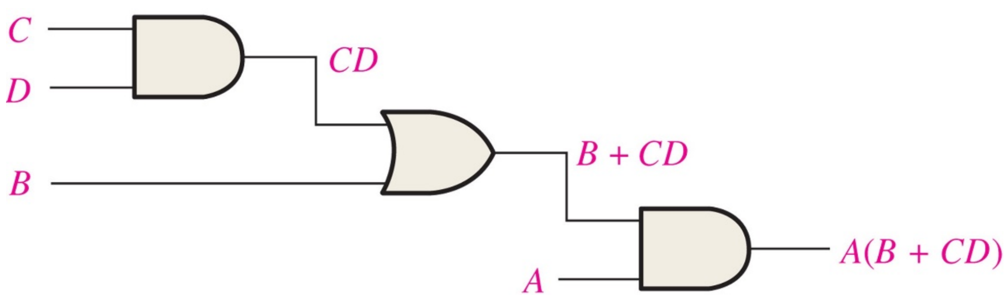
(a) $\overline{\overline{(A + B)} + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = ((\overline{A + B)\overline{C}\,\overline{D}})(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

Boolean Analysis of Logic Circuits

- First, derive the Boolean expression for a given combinational logic circuit;



- Then, construct the truth table.

TABLE 4–5

Truth table for the logic circuit in Figure 4–18.

Inputs				Output
A	B	C	D	$A(B + CD)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Logic Simplification Using Boolean Algebra

- Use the basic laws, rules, and theorems of Boolean algebra to manipulate and simplify an expression.

Example 4-7 Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

$$= \underbrace{AB + AB}_{\text{Rule 5}} + AC + \underbrace{BB}_{\text{Rule 7}} + BC$$ distributive law

$$= AB + AC + \underbrace{B + BC}_{\text{Rule 10}}$$

$$= \underbrace{\boxed{AB} + AC + \boxed{B}}_{\text{Rule 10}}$$

$$= AC + B$$

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Standard Forms of Boolean Expressions

❑ The Sum-of-Products (SOP) Form

$$(A\bar{B}+\bar{C}) = (A+\bar{C})(\bar{B}+\bar{C})$$

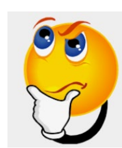
- **Domain** : the set of variables contained in the expression in either complemented or uncomplemented form.
- SOP form: 2+ product terms are summed by Boolean addition
- Overbar: $\bar{A}\bar{B}\bar{C}$ ✓ $\overline{ABC}$ ✗
- Conversion to SOP form: A(B+CD) = AB+ACD
- The **standard** SOP form : all the variables in the domain appear in each product term in the expression.

$$A\bar{B}C + ABC\bar{D} = A\bar{B}C(D+\bar{D}) + ABC\bar{D} = A\bar{B}CD + A\bar{B}C\bar{D} + ABC\bar{D}$$

❑ The Product-of-Sums (POS) Form

- POS form: 2+ sum terms are multiplied

$(A\bar{B}+\bar{C})\,(A+B)$



POS?

- The **standard** POS form:

Domain : A, B, C, D

$$A+\bar{B}+C = A+\bar{B}+C+D\bar{D}$$
$$= (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$$

The Standard POS Form

Recall Rule # **12.** $(A+B)(A+C) = A + BC$

$$P = \underbrace{P}_{} + DD = \underbrace{P + PD + P\bar{D}}_{} + D\bar{D}$$
$$= (P + D)(P + \bar{D})$$

Domain : A, B, C, D

$$A + \bar{B} + C = A + \bar{B} + C + D\bar{D}$$
$$= (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})$$

Now, back to our example $(A\bar{B} + \bar{C})\ (A + B)$

$$A\bar{B} + \bar{C} = (A + \bar{C})(\bar{B} + \bar{C}) = \boxed{(A + B + \bar{C})}\boxed{(A + \bar{B} + \bar{C})}\boxed{(A + \bar{B} + \bar{C})}(\bar{A} + \bar{B} + \bar{C})$$

$$A + B = (A + B + C)\boxed{(A + B + \bar{C})}$$

$$\Rightarrow (A\bar{B} + \bar{C})(A + B) = (A + B + C)\ (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + \bar{C})$$

Boolean Expressions and Truth Tables (I)

Example 4-20

Develop a truth table for the standard SOP expression : $\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

A	B	C	$\bar{A}\bar{B}C$	$A\bar{B}\bar{C}$	ABC	Output
0	0	0				0
0	0	1	**1**	Don’t Care		1
0	1	0				0
0	1	1				0
1	0	0	Don’t Care	**1**	Don’t Care	1
1	0	1				0
1	1	0				0
1	1	1	Don’t Care		**1**	1

Boolean Expressions and Truth Tables (II)

Example 4-21 Determine the truth table for the following standard POS expression:

$$(A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

A	B	C	$(A+B+C)$	$(A+\bar{B}+C)$	$(A+\bar{B}+\bar{C})$	$(\bar{A}+B+\bar{C})$	$(\bar{A}+\bar{B}+C)$	Output
0	0	0	0	Don't Care				0
0	0	1						1
0	1	0	Don't Care	0	Don't Care			0
0	1	1			0			0
1	0	0						1
1	0	1	Don't Care			0	Don't Care	0
1	1	0	Don't Care				0	0
1	1	1						1

From Truth Tables to Standard Expressions

Example 4-22 Determine the truth table for the following standard SOP expression:

TABLE 4–8

Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$\bar{A}BC$

$A\bar{B}\bar{C}$

$AB\bar{C}$

ABC

$X = \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

From Truth Tables to Standard Expressions

Example 4-22 Determine the truth table for the following standard POS expression:

TABLE 4–8

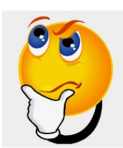
Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$A + B + C$

$A + B + \bar{C}$

$A + \bar{B} + C$

$\bar{A} + B + \bar{C}$



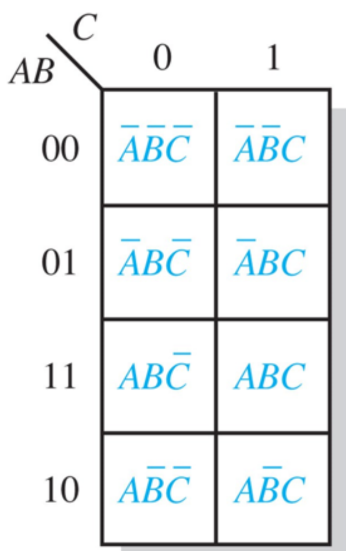
Any more solutions?

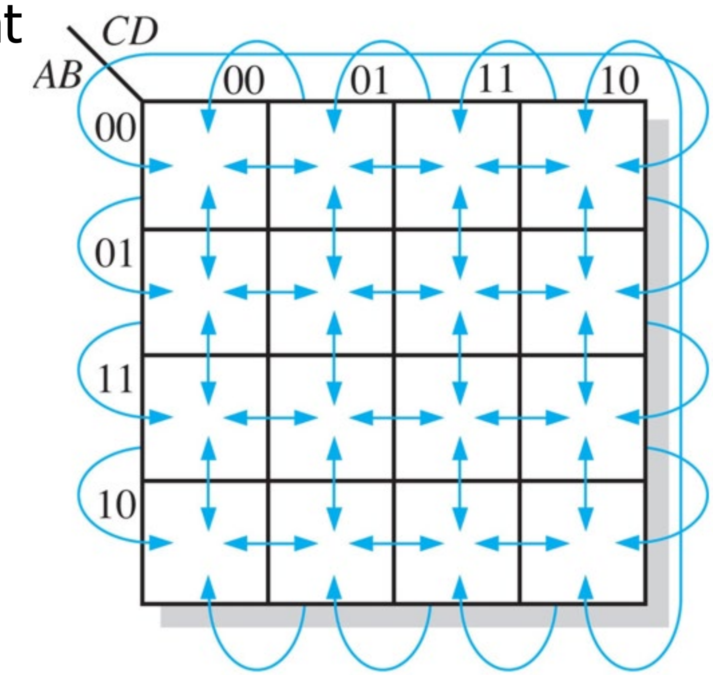
$$X = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + \bar{C})$$

The Karnaugh Map (K-Map)

- A systematic method to simply Boolean expressions to their simplest SOP/POS expressions, aka. the minimum expressions
- Cells : each represents a binary value of the input variables
  - # of **cells** : the total # of possible input variable combinations
  - Adjacent cells are indexed with the **Gray code**, i.e. only a single variable change between adjacent

A 3-variable K-map



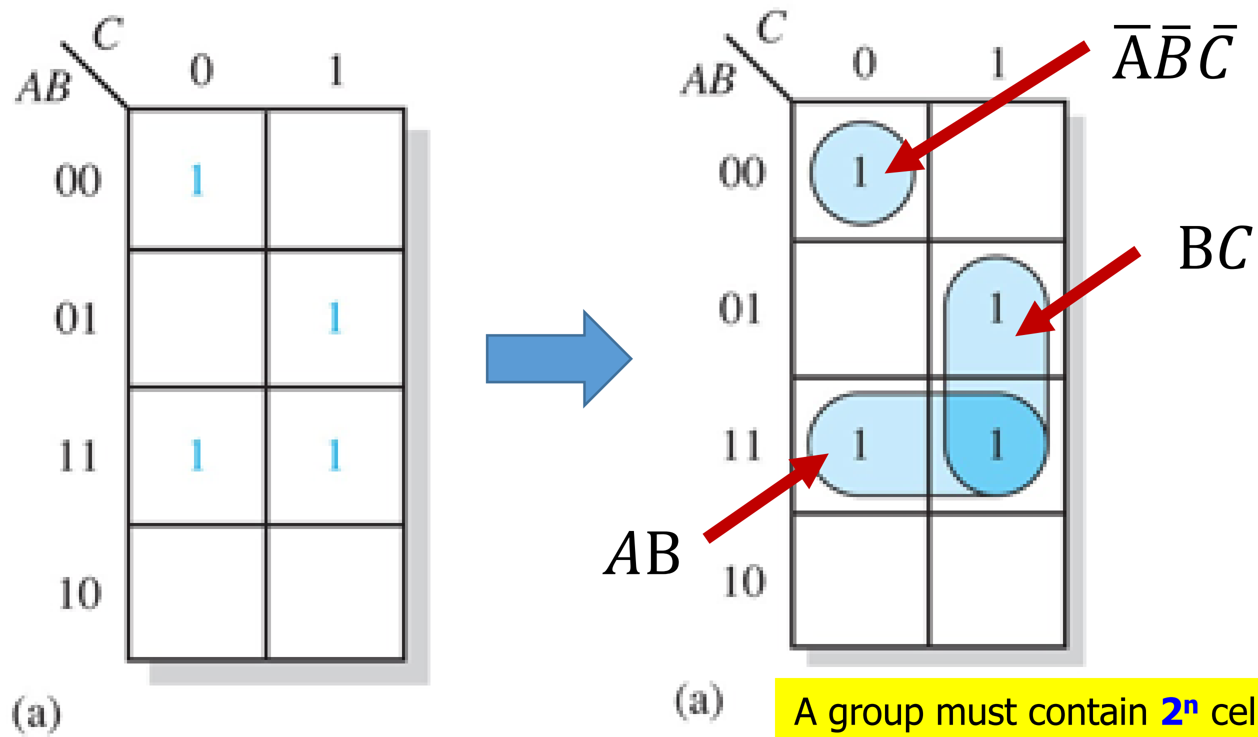


A 4-variable K-map

Karnaugh Map SOP Minimization (I)

Example 4-27 $\overline{A}\overline{B}\overline{C} + \overbrace{\overline{A}BC + \underbrace{ABC}}^{BC}\underbrace{{} + AB\overline{C}}_{AB}$ $= \overline{A}\overline{B}\overline{C} + \overline{A}BC + \boxed{ABC + ABC} + AB\overline{C}$

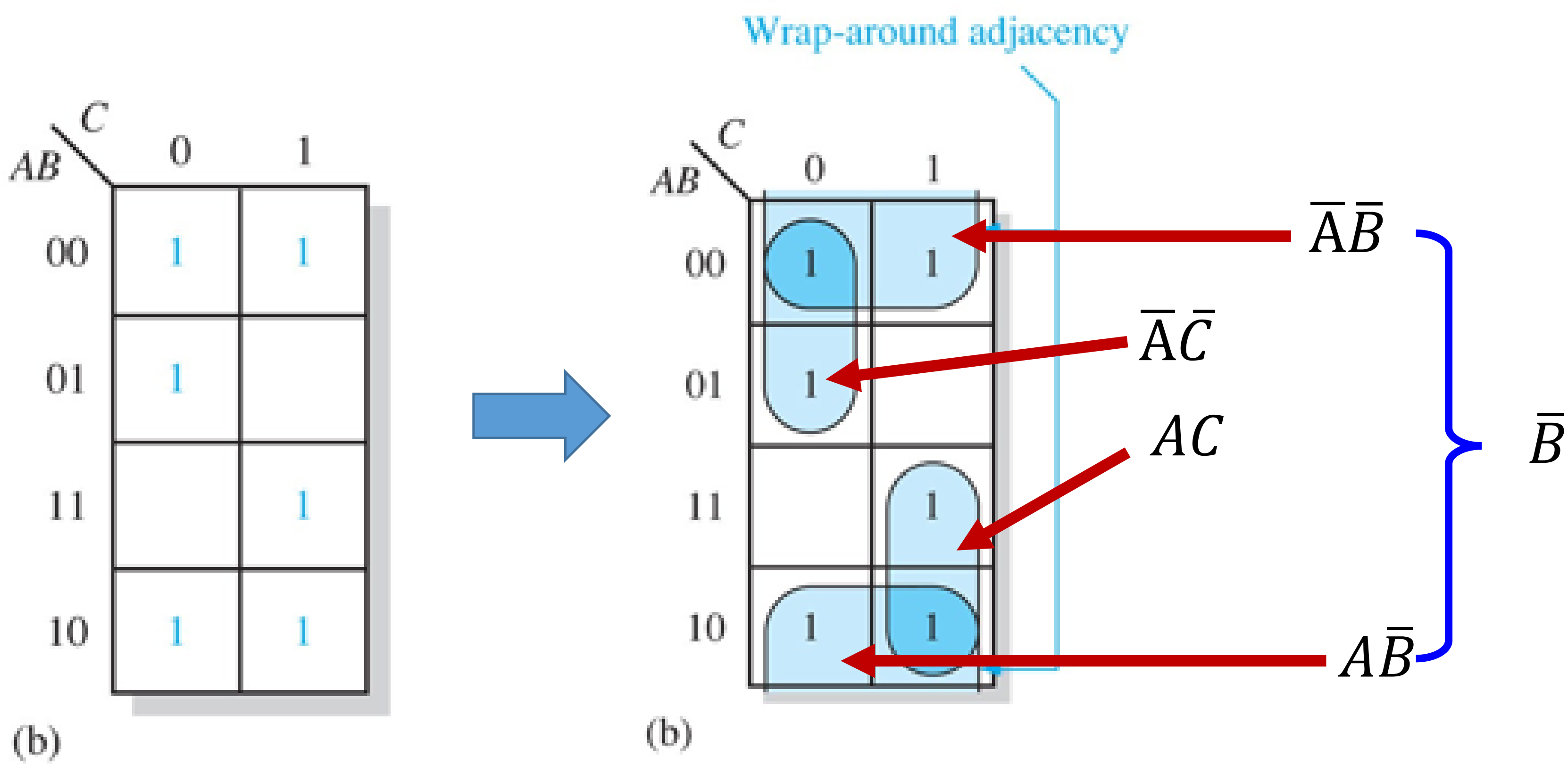
$= \overline{A}\overline{B}\overline{C} + BC + AB$



A group must contain 2^n cells, n=0,1,2,…

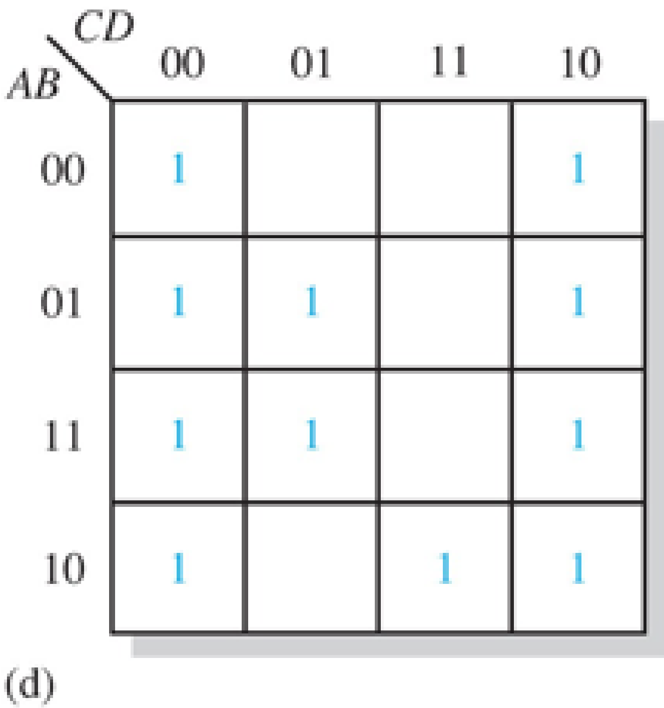
Karnaugh Map SOP Minimization (II)

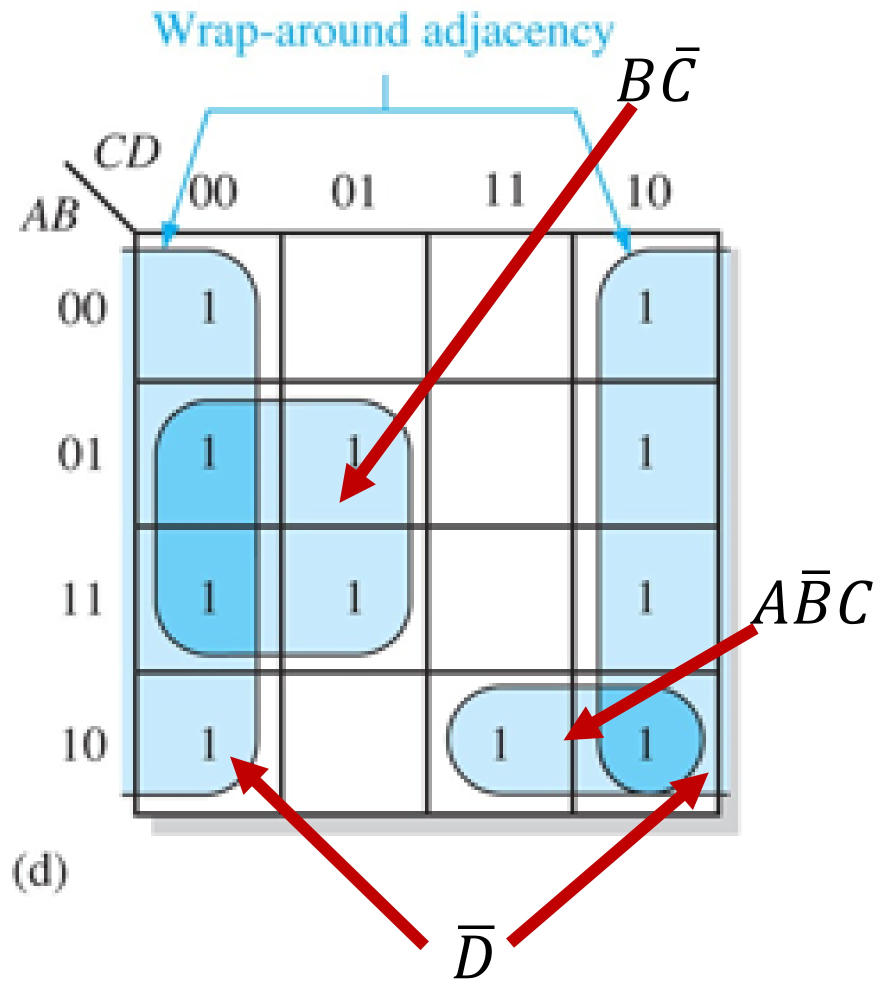
Example 4-27 $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC + A\bar{B}\bar{C} + A\bar{B}C = \bar{A}\bar{C} + AC + \bar{B}$



Karnaugh Map SOP Minimization (III)

Example 4-27

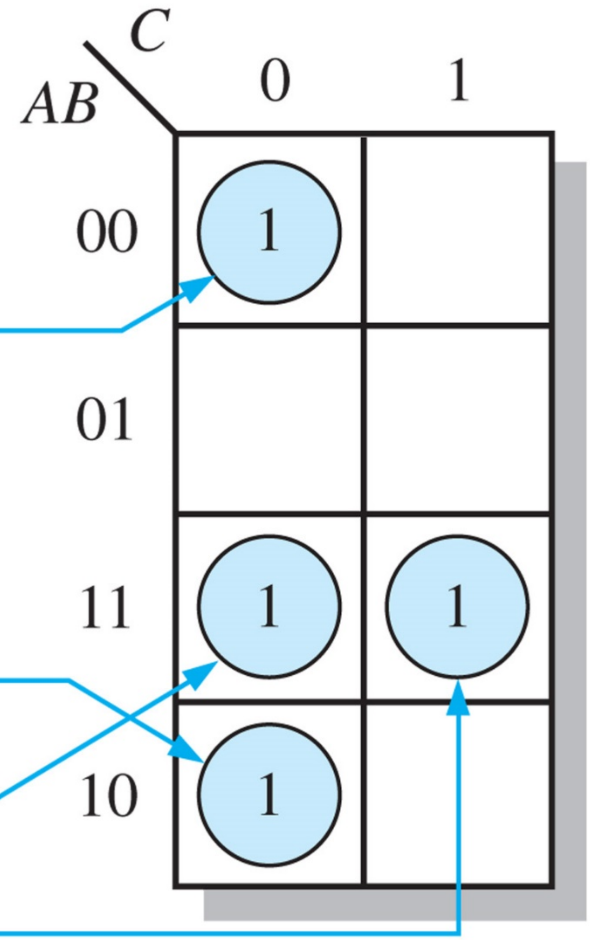




Mapping Directly from a Truth Table

$$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

Inputs			Output
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



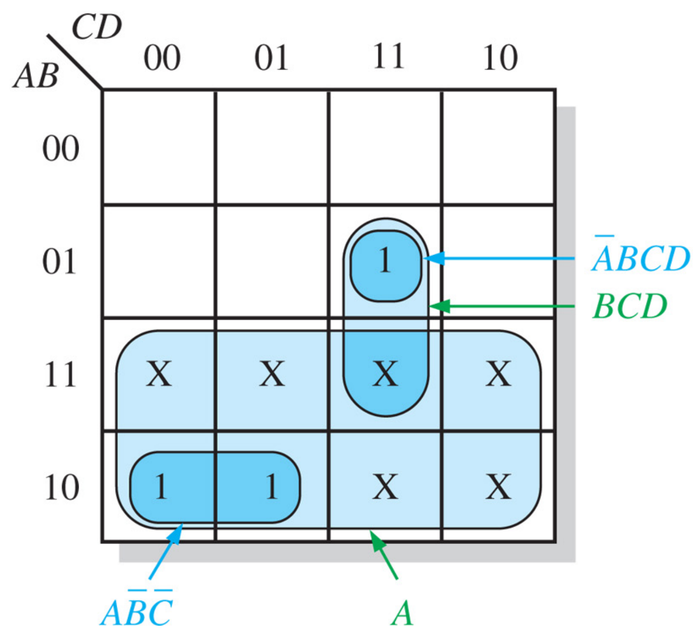
“Don’t Care” Conditions

Inputs				Output
A	*B*	*C*	*D*	*Y*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don’t cares

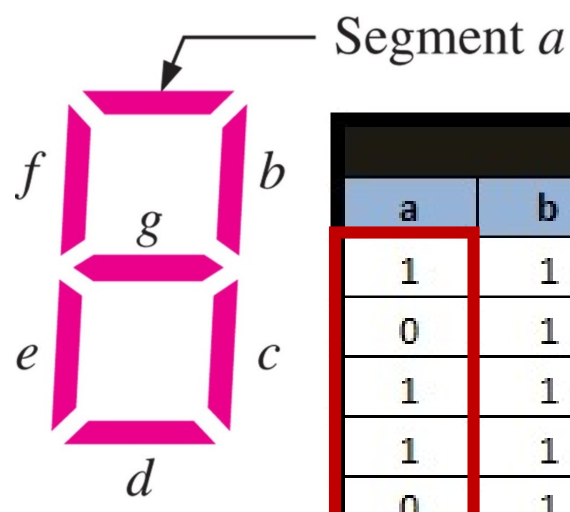
(a) Truth table

- “don’t care” can be either a 1 or a 0
- When grouping the 1s, the “don’t care” terms (X’s) can be treated as 1s to make a larger grouping



(b) Without “don’t cares” $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With “don’t cares” $Y = A + BCD$

EXAMPLE 4–32 (I)



Segments Inputs							7 Segment Display Output	
a	b	c	d	e	f	g		A B C D
1	1	1	1	1	1	0	0	0 0 0 0
0	1	1	0	0	0	0	1	0 0 0 1
1	1	0	1	1	0	1	2	0 0 1 0
1	1	1	1	0	0	1	3	0 0 1 1
0	1	1	0	0	1	1	4	0 1 0 0
1	0	1	1	0	1	1	5	0 1 0 1
1	0	1	1	1	1	1	6	0 1 1 0
1	1	1	0	0	0	0	7	0 1 1 1
1	1	1	1	1	1	1	8	1 0 0 0
1	1	1	1	0	0	1	9	1 0 0 1

TABLE 2–5 Each digit can be represented by a BCD code

Decimal/BCD conversion.

Decimal Digit	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

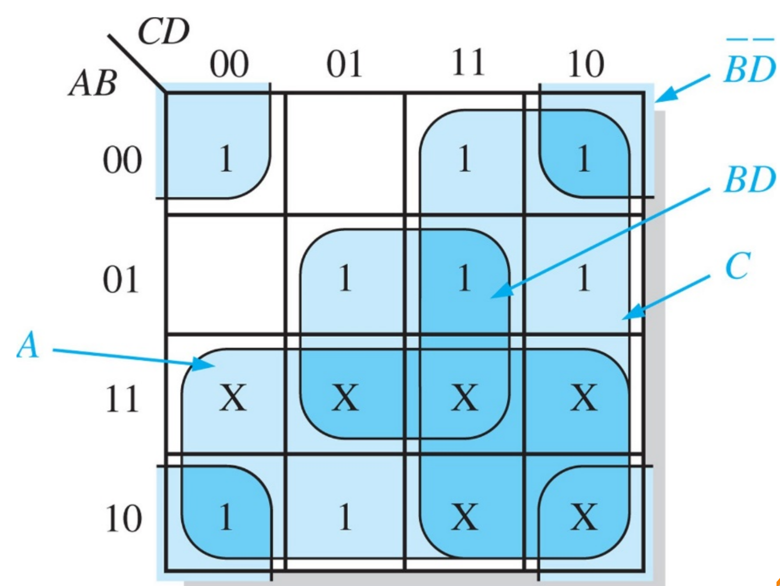
EXAMPLE 4–32 (II)

Truth table for Segment a

A	B	C	D	a
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	1
11	X	X	X	X
10	1	1	X	X

$$a = A + C + BD + \bar{B}\bar{D}$$



Karnaugh Map POS Minimization (I)

- ❑ In SOP minimization, we focus on those 1's
- ❑ In **POS** minimization, we focus on those **0's**

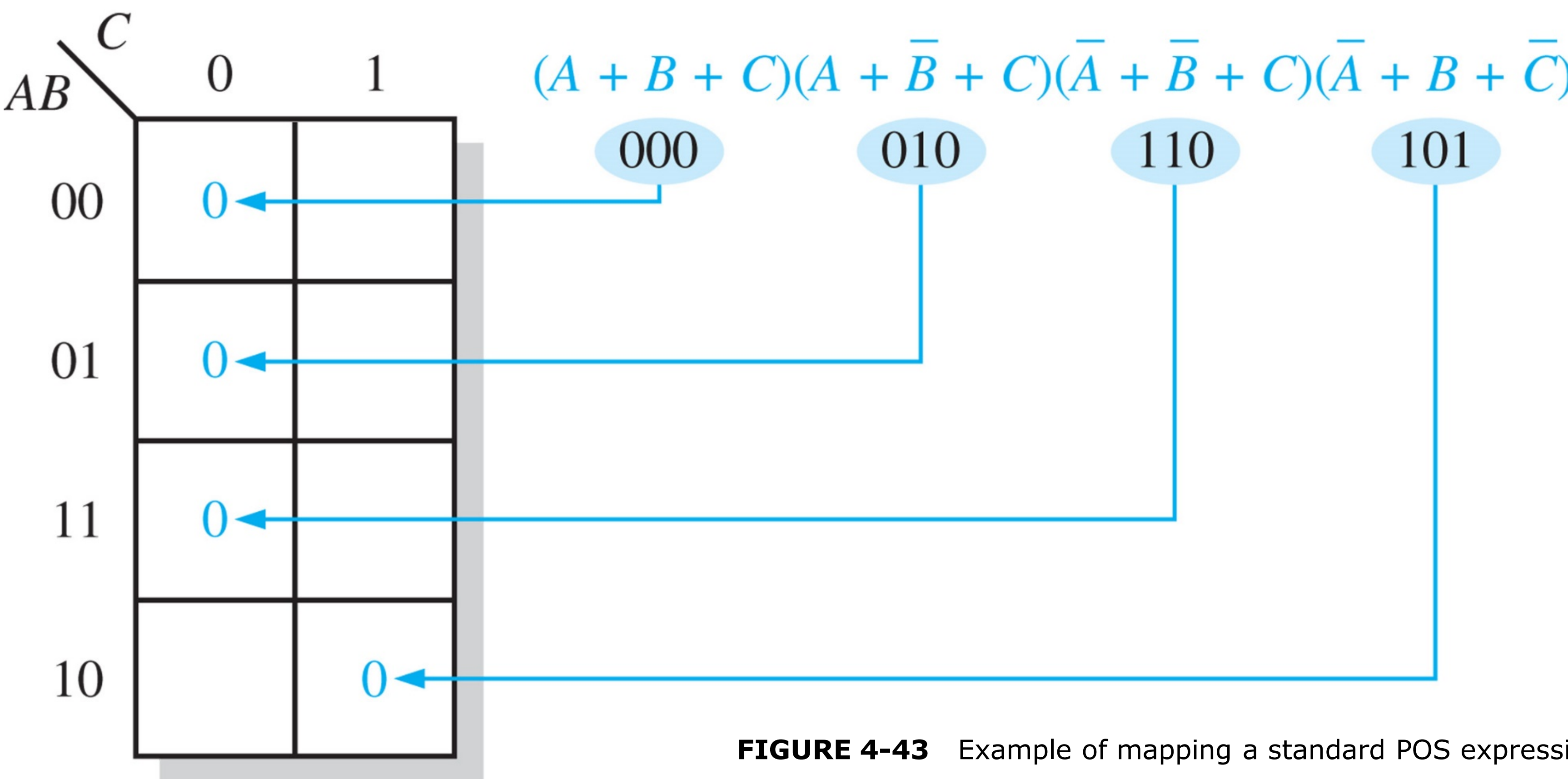


FIGURE 4-43 Example of mapping a standard POS expression.

Karnaugh Map POS Minimization (II)

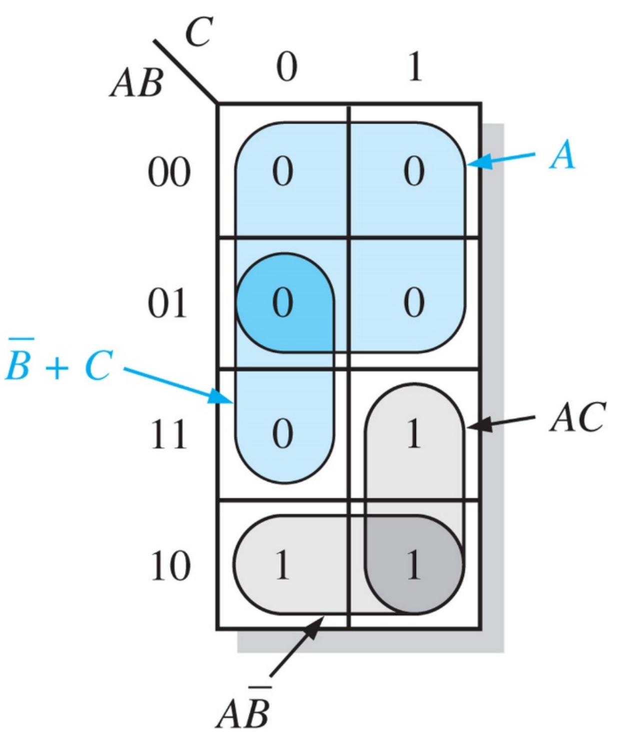
Example 4-34 $(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$

000 001 010 011 110

Example 4-35 $(B+C+D)(A+B+\bar{C}+D)(\bar{A}+B+C+\bar{D})(A+\bar{B}+C+D)(\bar{A}+\bar{B}+C+D)$

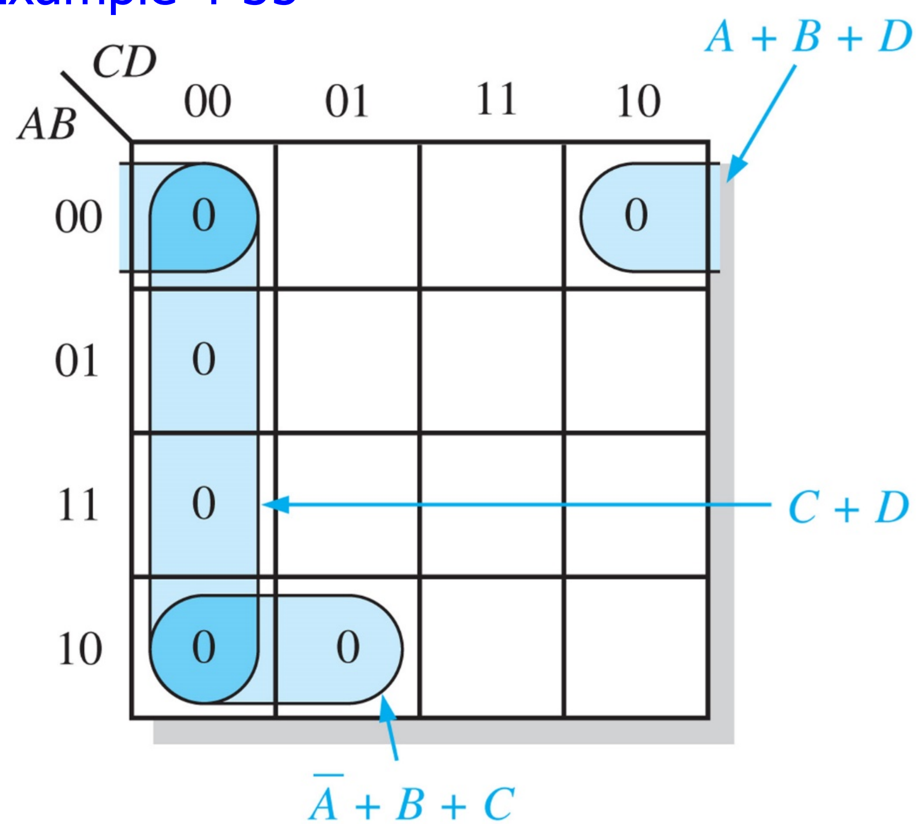
X000 0010 1001 0100 1100

Example 4-34



$AC+A\bar{B}=A(\bar{B}+C)$

Example 4-35



Converting btw POS & SOP Using K-Map

Example 4-35 Using a Karnaugh map, convert the following standard POS expression into a min. POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\bar{A}+\bar{B}+C+D)(A+\bar{B}+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+\bar{D})(\bar{A}+B+C+\bar{D})(A+B+\bar{C}+D)$$

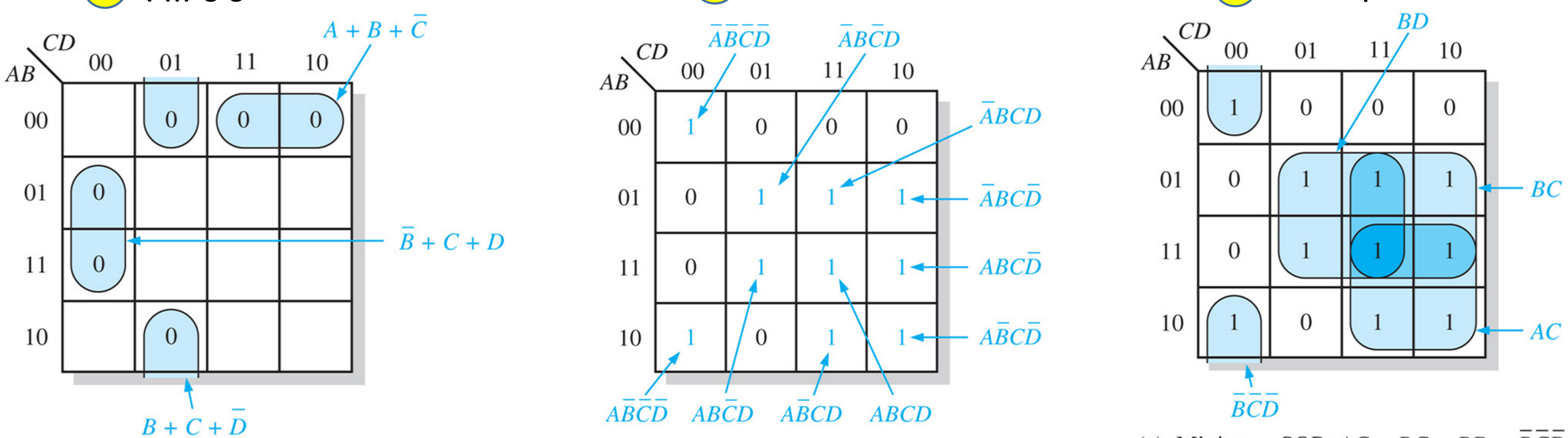
1 Find all 0's

1100 0100 0001 0011 1001 0010

2 Fill 0's

3 Fill 1's

4 K-Map



(a) Minimum POS: $(A+B+C)(\bar{B}+\bar{C}+D)(B+C+\bar{D})$

Standard SOP:
$\bar{A}\bar{B}\bar{C}\bar{D}+\bar{A}B\bar{C}D+\bar{A}BCD+\bar{A}BC\bar{D}+ABC\bar{D}+A\bar{B}C\bar{D}+$
$A\bar{B}\bar{C}\bar{D}+AB\bar{C}D+A\bar{B}CD+ABCD$

(c) Minimum SOP: $AC+BC+BD+\bar{B}\bar{C}\bar{D}$

The Quine-McCluskey Method (I)

- A formal tabular method for applying the Boolean distributive law to find the minimum SOP
- K-map can handle up to 4 or 5 variables but QM method can handle more
- **Step 1**: Write the function in standard minterm form and construct the truth table

TABLE 4–9

ABCD	*X*	Minterm
0000	0	
0001	1	m_1
0010	0	
0011	1	m_3
0100	1	m_4
0101	1	m_5
0110	0	
0111	0	
1000	0	
1001	0	
1010	1	m_{10}
1011	0	
1100	1	m_{12}
1101	1	m_{13}
1110	0	
1111	1	m_{15}

$$X = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD$$

The Quine-McCluskey Method (II)

- **Step 2**: Group according to the number of 1's in each minterm

TABLE 4–10

Number of 1s	Minterm	*ABCD*
1	m_1	0001
	m_4	0100
2	m_3	0011
	m_5	0101
	m_{10}	1010
	m_{12}	1100
3	m_{13}	1101
4	m_{15}	1111

- **Step 3**: Compare adjacent groups to find minterms are the same in every position except one.

TABLE 4–11

Number of 1s in Minterm	Minterm	ABCD	First Level
1	m_1	0001 ✓	(m_1, m_3) 00x1
	m_4	0100 ✓	(m_1, m_5) 0x01
2	m_3	0011 ✓	(m_4, m_5) 010x
	m_5	0101 ✓	(m_4, m_{12}) x100
	m_{10}	1010	(m_5, m_{13}) x101
	m_{12}	1100 ✓	(m_{12}, m_{13}) 110x
3	m_{13}	1101 ✓	(m_{13}, m_{15}) 11x1
4	m_{15}	1111 ✓	

The Quine-McCluskey Method (III)

- **Step 4**: Repeat Step 5 until all groups are done

TABLE 4–12

First Level	Number of 1s in First Level	Second Level
(m_1, m_3) 00x1	1	$(m_4, m_5, m_{12}, m_{13})$ x10x
(m_1, m_5) 0x01		$(m_4, m_5, m_{12}, m_{13})$ x10x
(m_4, m_5) 010x ✓		
(m_4, m_{12}) x100 ✓		
(m_5, m_{13}) x101 ✓	2	
(m_{12}, m_{13}) 110x ✓		
(m_{13}, m_{15}) 11x1	3	

- **Step 5**: Write down the reduced expression

TABLE 4–13

	Minterms							
Prime Implicants	m_1	m_3	m_4	m_5	m_{10}	m_{12}	m_{13}	m_{15}
$B\overline{C}$ $(m_4, m_5, m_{12}, m_{13})$			✓	✓		✓	✓	
$\overline{A}\,\overline{B}D$ (m_1, m_3)	✓	✓						
$\overline{A}\,\overline{C}D$ (m_1, m_5)	✓			✓				
ABD (m_{13}, m_{15})							✓	✓
$A\overline{B}C\overline{D}$ (m_{10})					✓			

$$X = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD$$
$$= B\bar{C} + \bar{A}\bar{B}D + \bar{A}\bar{C}D + ABD + A\bar{B}C\bar{D}$$

Chapter Review

- Boolean Operations and Expressions :
  - Addition → OR; Multiplication → AND
- Laws and Rules of Boolean Algebra :
  - Commutative, Associative and Distributive
  - 12 Rules
- DeMorgan's Theorems
- Standard Forms of Boolean Expressions
  - SOP and POS
- Boolean Expressions and Truth Tables
- The Karnaugh Map
  - Karnaugh Map SOP Minimization
  - Karnaugh Map POS Minimization
- The Quine-McCluskey Method

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

DeMorgan's Theorems

$$\overline{XY} = \bar{X} + \bar{Y}$$

$$\overline{X + Y} = \bar{X}\bar{Y}$$

True/False Quiz

- Variable, complement, and literal are all terms used in Boolean algebra.
- Addition in Boolean algebra is equivalent to the NOR function.
- Multiplication in Boolean algebra is equivalent to the AND function.
- The commutative law, associative law, and distributive law are all laws in Boolean algebra.
- The complement of 0 is 0 itself.
- When a Boolean variable is multiplied by its complement, the result is the variable
- "The complement of a product of variables is equal to the sum of the complements of each variable" is a statement of DeMorgan's theorem.
- SOP means sum-of-products.
- Karnaugh maps can be used to simplify Boolean expressions.
- A 3-variable Karnaugh map has six cells.

True/False Quiz

- ✓ Variable, complement, and literal are all terms used in Boolean algebra.
- ✗ Addition in Boolean algebra is equivalent to the NOR function.
- ✓ Multiplication in Boolean algebra is equivalent to the AND function.
- ✓ The commutative law, associative law, and distributive law are all laws in Boolean algebra.
- ✗ The complement of 0 is 0 itself.
- ✗ When a Boolean variable is multiplied by its complement, the result is the variable
- ✓ "The complement of a product of variables is equal to the sum of the complements of each variable" is a statement of DeMorgan's theorem.
- ✓ SOP means sum-of-products.
- ✓ Karnaugh maps can be used to simplify Boolean expressions.
- ✗ A 3-variable Karnaugh map has six cells.