# CSC3001 Discrete Mathematics

Assignment 1

Deadline: 11:59 pm, Friday, Oct 14, 2022

1. Let $x \in \mathbb{R}$, $y \in \mathbb{R}$, and $z \in \mathbb{R}$.

   (a) Let $F(x, y)$ be the statement $xy = 0$. Are the following quantification $\exists x \forall y F(x, y)$ and $\forall x \exists y F(x, y)$ true or false?

   (b) Let $G(x, y, z)$ be the statement $xy = z$. Are the following quantification $\forall y \forall z \exists x G(x, y, z)$ and $\exists y \exists z \forall x G(x, y, z)$ true or false, respectively?

2. Use predicates, quantifiers, logical connectives, and mathematical operators to translate the following statements. For instance, "The sum of two positive numbers is always positive" into $\forall x \forall y((x > 0) \wedge (y > 0)) \iff (x + y > 0)$.

   (a) The sum of three negative numbers is always negative.

   (b) Two times the sum of the squares of two integers is greater than or equal to the square of their sum.

   (c) Every real number is the sum of two real numbers.

3. Let $S$ be a set and let $A, B, C \subseteq S$. Define the symmetric difference as $A \oplus B := (A \cup B) - (A \cap B)$ which denotes the set containing those elements in either $A$ or $B$, but not in both $A$ and $B$. prove that:

   (a) $A \oplus B = (A - B) \cup (B - A)$.

   (b) $\overline{A} \oplus \overline{B} = A \oplus B$.

   (c) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$.

4. Prove that there are infinitely many primes.

5. Prove by induction that for any $n \in \mathbb{N}$, $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$.

6. Prove $\underset{y \in \mathbb{R}}{\forall} \underset{x \in \mathbb{Z}^+}{\exists} (\sum_{i=1}^{x} \frac{1}{i} > y)$.

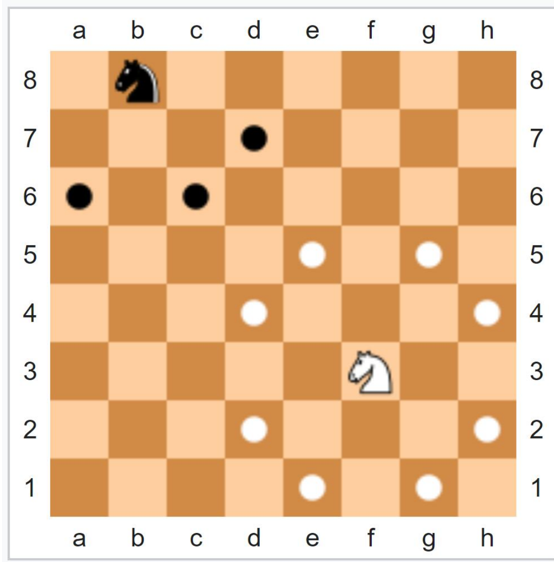


Figure 1: Knights on different squares, with dots representing possible knight moves.

7. Given an $8 \times 8$ chessboard, prove that a knight (the movement of a knight is shown in Fig. 1) can move to an arbitrary position starting from any starting position.

8. Prove that there are no positive integer solutions for $\frac{1}{4}a^6 + \frac{1}{16}b^6 = c^6$.