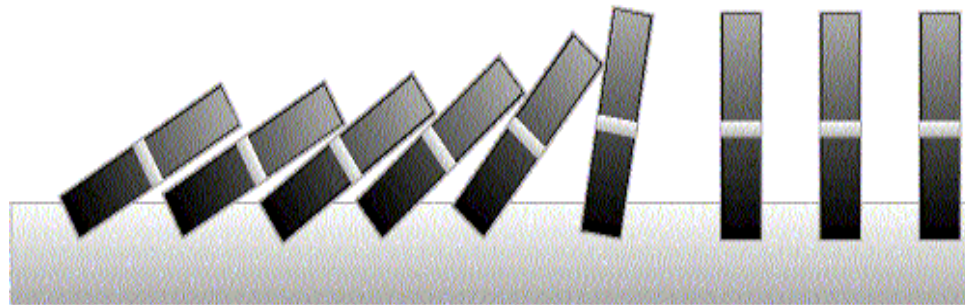
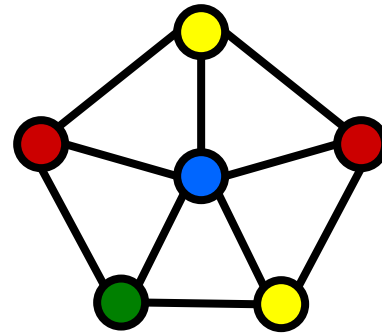
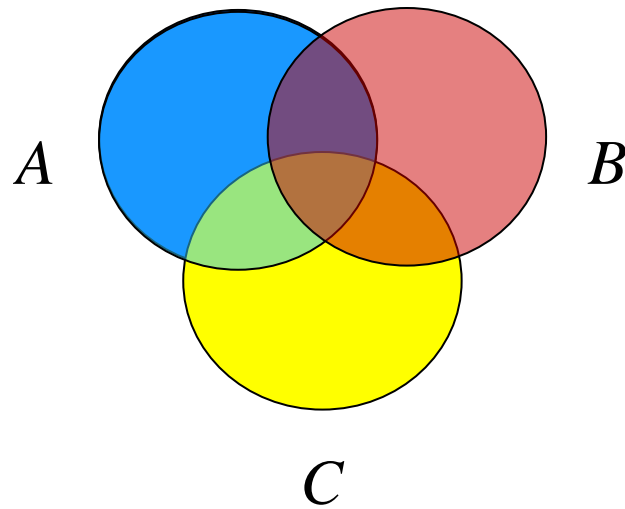


# About the Course



All lecture notes are based on Lau Lap Chi's notes from CUHK

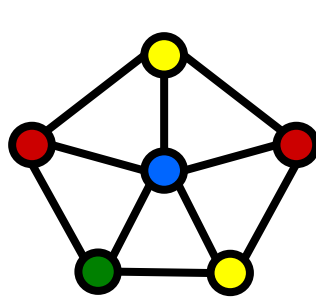
# Plan

- Course Information and Arrangement
- Topics of the Course
- Course Objective

Please read the course  
outline carefully!!!

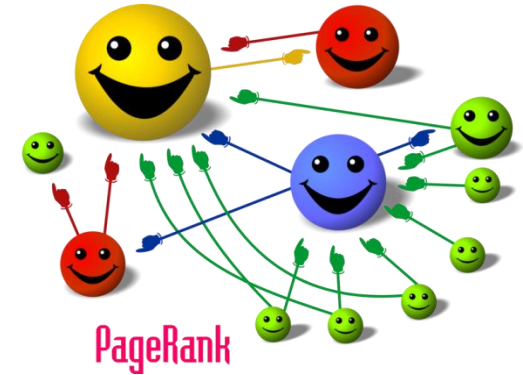
# Discrete Mathematics

What is discrete mathematics?

discrete mathematics	continuous mathematics
	
integers	real numbers
graphs	geometric space
induction	calculus
logic	

These two areas are not disjoint, e.g. calculus can be used to solve discrete problems (generating functions).

# Discrete Mathematics



Why discrete mathematics?

In computer science we usually deal with finite, discrete objects. For example,

- we cannot store a real number (infinite precision) in a computer but can only store bits (finite precisions).
- we often model a computer network as a graph, and use the knowledge and techniques in dealing with graphs to solve problems in networks.

The problems and the techniques are often different (e.g. induction, recursion).

# Topic 1: Logic and Proofs

How do computers (and humans) think?

**Logic:** propositional logic, first order logic

**Proof:** induction, contradiction

$$\forall x \exists y, z \quad x = y + z$$

$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$

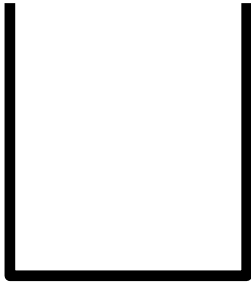
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5	6	7	8
9	10	11	12
13	14	15	

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

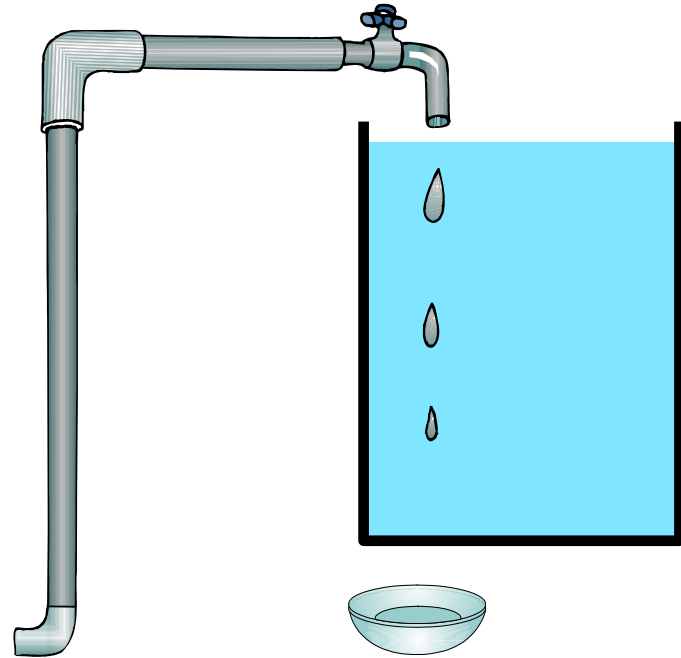
Applications: artificial intelligence, database, circuit, algorithms

Objective: to reason rigorously and learn basic proof techniques (e.g. induction)

## Topic 2: Number Theory



3 Gallon Jug

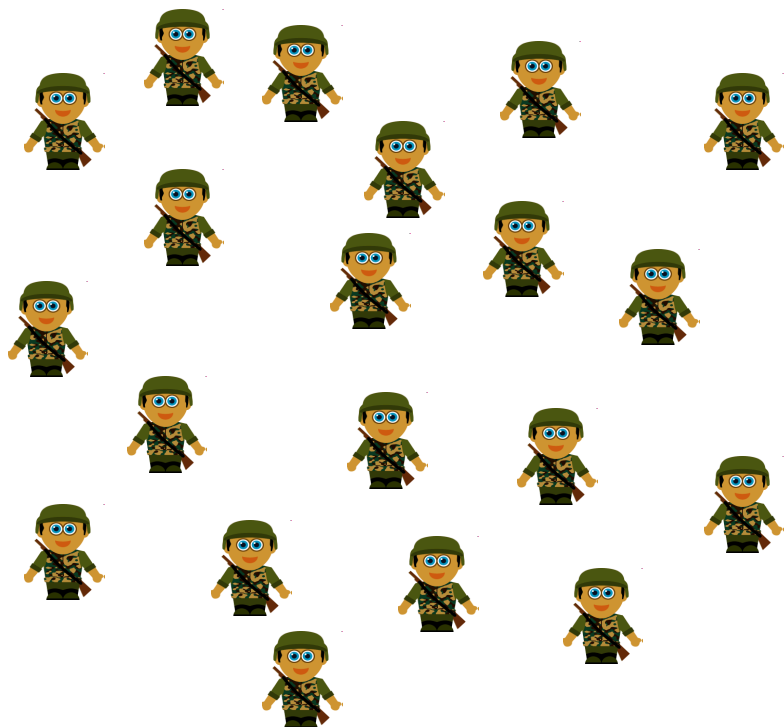


5 Gallon Jug

How to get 4 gallons?



## Topic 2: Number Theory



We have 1073 soliders.



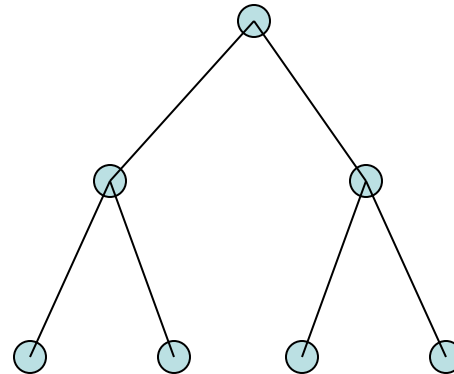
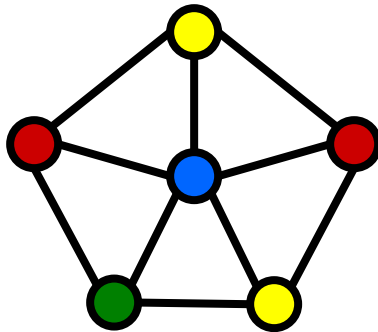
How could he figure it out?!

Applications: divisibility, cryptography

Objective: to learn elementary number theory and classical results

# Topic 3: Graph Theory

- Graphs
- Degree sequence, Eulerian graphs, isomorphism
- Trees
- Matching
- Coloring



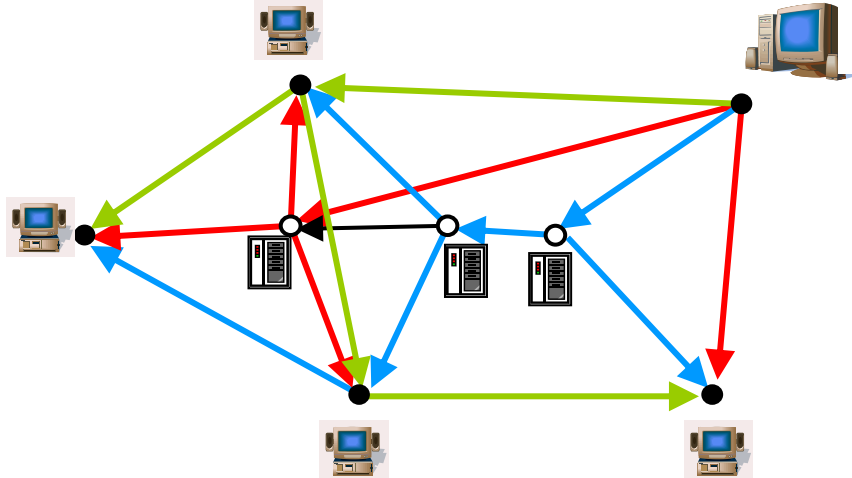
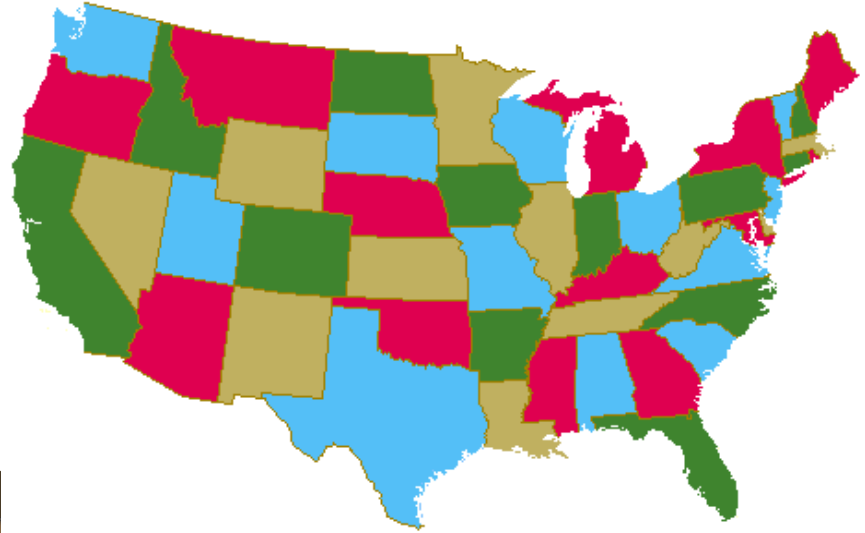
Applications: Computer networks, circuit design, data structures



# Topic 3: Graph Theory

How to color a map?

How to schedule exams?

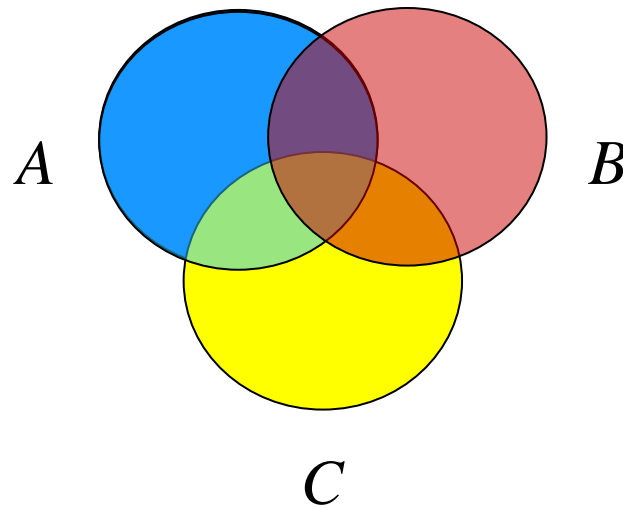


How to send data efficiently?

Objective: to model problems and learn basic concepts and knowledge

# Topic 4: Counting

- Sets and Functions
- Combinations, Permutations, inclusion-exclusion
- Counting by mapping, pigeonhole principle
- Recursions



Applications: probability, data structures, algorithms

Objective: to learn basic concepts (set, functions) and fundamental techniques.

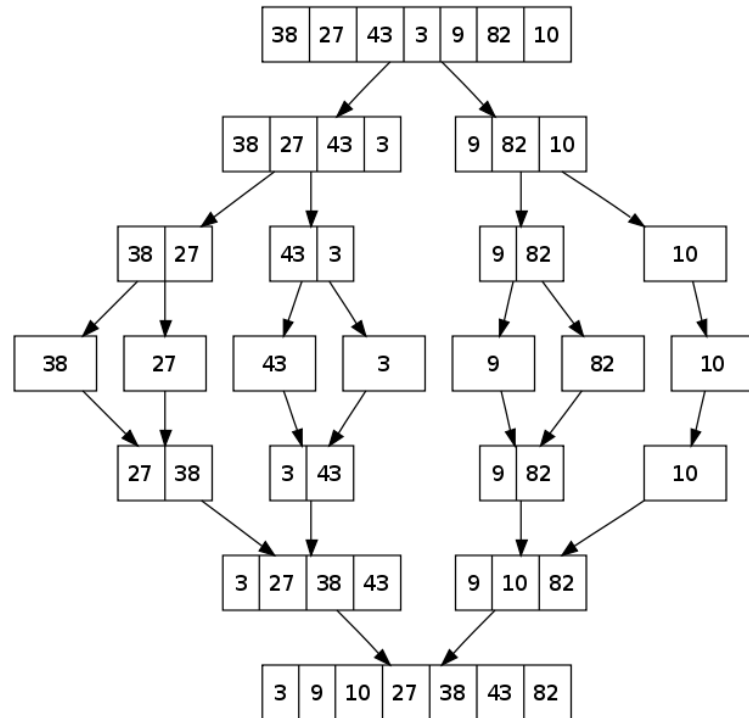
# Topic 4: Counting

How many steps are needed to sort  $n$  numbers?

## Algorithm 1 (Bubble Sort):

Every iteration moves the  $i$ -th smallest number to the  $i$ -th position

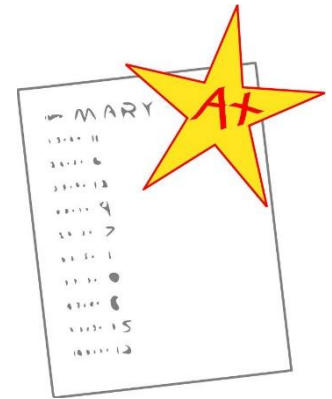
## Algorithm 2 (Merge Sort):



Which algorithm runs faster?

Solving the recursion

# How to Perform Well?



Which of the following is correct?

1.  $\emptyset \in \mathbb{R}$
2.  $x^2 \neq y^2 \Leftrightarrow x \neq y$
3.  $a > b, c > d \Rightarrow ac > bd$
4. The sum of odd number of consecutive positive integers is odd.

Correction

$$\emptyset \subseteq \mathbb{R}$$

$$x^2 \neq y^2 \Rightarrow x \neq y$$

"ac" can be  $>$  or  $<$  or  $=$  "bd"

$1+2+3$  is odd??

The most important thing in this course is NOT theory BUT a **CLEAR MIND!!!**

**Think twice for every step you have done!**

Sometimes, completing the solution maybe far away from being correct..