3170 assn4 121040051

Question 1

(a)

$$E(X_N)_1 = 0p_0 + 1p_1 + 2p_2 + 3p_3 + \ldots + Np_N$$
$$= \frac{C}{1} + 2\frac{C}{2} + 3 * \frac{C}{3} + \ldots + N * \frac{C}{N}$$
$$= NC$$

$\because$ this is a distribution

$\therefore \sum_N p_k = \sum_N \frac{C}{k} = 1$

$\Rightarrow C(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{N}) = 1$

$\Rightarrow C = \frac{1}{\sum_{k=1}^{N} \frac{1}{k}}$

Therefore, $E(X_N)_1 = NC = \frac{N}{\sum_{k=1}^{N} \frac{1}{k}}$

(b)

The average $E(X_N)$ of uniform distribution:

$$E(X_N)_2 = 0 * p_0 + 1 * p_1 + 2 * p_2 + 3 * p_3 + \ldots + N * p_N$$
$$= \frac{1}{N} + 2 * \frac{1}{N} + 3 * \frac{1}{N} + \ldots + N * \frac{1}{N}$$
$$= \frac{(1+N)*N}{2} * \frac{1}{N}$$
$$= \frac{1+N}{2}$$

When N=10:

$$E(X_N)_1 = \frac{10}{\sum_{k=1}^{10} \frac{1}{k}} = 3.414,$$

$$E(X_N)_2 = \frac{1+10}{2} = 5.5$$

(c)

When $E(X_{N^*})_1 = E(X_{N^*})_2$, we have: $\frac{N^*}{\sum_{k=1}^{N^*} \frac{1}{k}} = \frac{1+N^*}{2}$

$\Rightarrow N^* = 1$

Since $E(X_{N^*})_1$ is divergent, and the difference with $E(X_{N^*})_2$ will get larger after intersection, N*=1 is a unique solution for equality in a range.

(d)

<i> if it's sequential search:

For each element, we'll keep searching until we find a element `greater than the one we're searching for.

$\Rightarrow$ average number of comparisons: $\frac{N}{2}$

<ii> if it's binary search:

For each element, the average number of comparisons is the total layer number of binary tree, which is $log(N)$

(e)

<i>when the required record is present in the file:

average number of comparisons is $\frac{N}{2}$

<ii> when required record is not present in the file:

average number of comparisons are the number of layers of this heap is N

Question 2

we have n=23,

with average storage utility 0.69, we have 23*0.69=15.87 $\approx$ 16

(i) Level1:

average number of nodes: 16

average number of key entries: 16*(16-1) = 16*15 = 240

average number of children pointers: 16*16 = 256

(ii) Level3:

average number of nodes: 16^3 = 4096

average number of key entries: $16^3 * 15$ = 61440

average number of children pointers: 16^4 = 65536

(iii) Level4:

average number of nodes: 16^4 = 65536

average number of key entries: $16^4 * 15$ = 983040

average number of children pointers: 16^5 = 1048576

(iv) height of tree is 2:

average number of entries = level0_key + level1_key + level2_key = 15 + 16*15 + 16*16*15 = 4095

(v) height of tree is 3:

average number of entries = level0_key + level1_key + level2_key + level3_key= 15 + 16*15 + 16*16*15 + 16*16*16*15 = 65535

(vi) height of tree is 4:

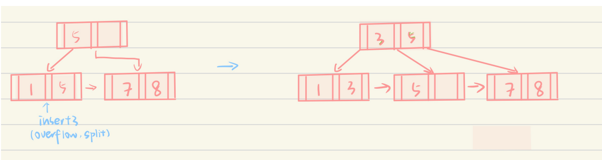
average number of entries = level0_key + level1_key + level2_key + level3_key + level4_key = 15 + 16*15 + 16*16*15 + 16*16*16*15 + 16*16*16*16*15 = 1048575

$\Rightarrow$ average total number of entries hat such a tree:

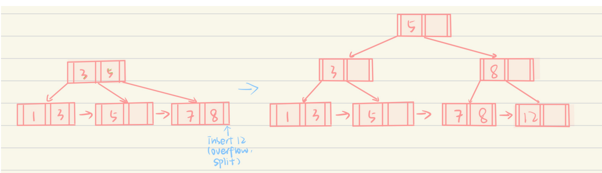
- we first have each node have a maximum of n children → 23 here
- with average storage utility of B tree 0.69, we have average children we have is 0.69n → 0.69*23 $\approx$ 16 here
- for i^{th} layer, we have $(0.69n)^i * (0.69n-1) = 16^i * 15$ key entries
- Therefore, for the tree with height h, we have: $\sum_{i=0}^{h}((0.69n)^i * (0.69n-1)) = \sum_{i=0}^{h}(16^i * 15)$
- $\Rightarrow \sum_{i=0}^{h}((0.69n)^i * (0.69n-1))$
  $= (0.69n-1)\sum_{i=0}^{h}(0.69n)^i$
  $= (0.69n-1) \times (1-(0.69n)^{h+1})/(1-0.69n)$
  $= (0.69n)^{h+1} - 1$

Question 3

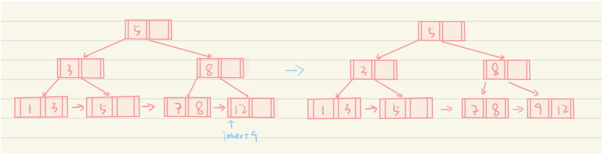
Insert Key3:



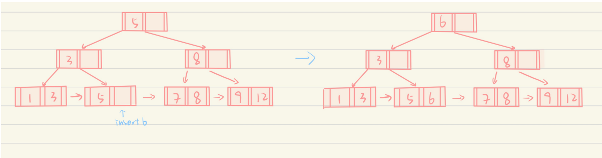
Insert Key12:



Insert Key9:

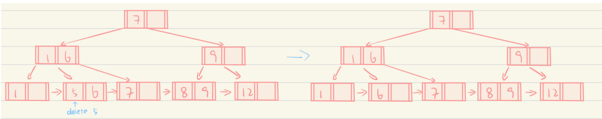


Insert Key6:

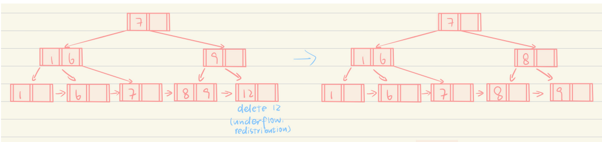


Question 4

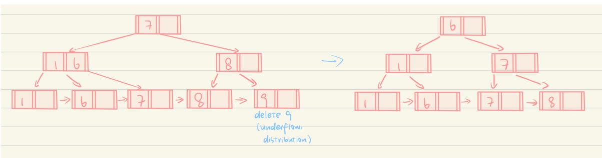
Delete Key5:



Delete Key12:



Delete Key9:



Question 5

With K: total number of items in tree,

n: maximum capacity of a node

N: random number of nodes in the tree

ρ: random storage utilization of the tree

f: minimum fullness factor, and f= $\frac{3}{4}$ here

we have:

$\rho = \frac{K}{Nn}$

minimum number of nodes is $\frac{K}{n}$

maximum number of nodes is $\frac{K}{\frac{3}{4}n} = \frac{4K}{3n}$

$\Rightarrow N \sim U(\frac{K}{n}, \frac{4K}{3n})$

(1)

$$E(\rho) = E(\frac{K}{Nn}) = \frac{K}{n}E(\frac{1}{N})$$
$$= (\frac{K}{n}) \times (\frac{3n}{K}) \int_{\frac{K}{n}}^{\frac{4K}{3n}} (\frac{1}{t})dt$$
$$= 3 * (ln(\frac{4K}{3n}) - ln(\frac{K}{n}))$$

$= 3 * ln(\frac{4}{3})$
$= 86.3\%$

(2)

$\sigma_f^2 = f - (\frac{f}{f'})^2[ln(\frac{1}{f})]^2$
$= \frac{3}{4} - (\frac{\frac{3}{4}}{\frac{1}{4}})^2[ln(\frac{4}{3})]^2$
$= 0.00515$

$\Rightarrow$ sd(ρ) = $\sqrt{0.00515}$ = 0.072

Question 6

(1)

Record#	2305	1168	2580	4871
Hash Index	1	0	4	7
	1620	2428	3943	4750
	4	4	7	6
Record#	5659	1821	1074	7115
Hash Index	3	5	2	3
	6975	4981	9280	
	7	5	0	

$\Rightarrow$

Hash Index	0	1	2	3
Records#	1168	2305	1074	5659
	9280			7115
Hash Index	4	5	6	7
Records#	2580	1821	4750	4871
	1620	4981		3943
	2428			6975

$\Rightarrow$ we have: bucket4 and bucket7 are overflow

$\Rightarrow$ Therefore, average number of block accesses for a random record retrieval on Part# is:

$$2 * \frac{2}{15} + 1 * \frac{13}{15} = \frac{17}{15} = 1.133$$

(2)

<step i> insert 2305, 1168

(2305) mod 128 = 1 = $(00001)_2$

(1168) mod 128 = 16 = $(10000)_2$

we have d=0, d'=0 with 2 elements in one local buckets

<step ii> insert 2580, 4871

(2580) mod 128 = 20 = $(10100)_2$

(4871) mod 128 = 7 = $(00111)_2$

we have d=1, d'=1 with 4 elements in two local buckets

<step iii> insert 5659, 1821

(5659) mod 128 = 27 = $(11011)_2$

(1821) mod 128 = 29 = $(11101)_2$

we have d=2,

Record#	2305	1168	2580	4871	5659	1821

Hash Index	1	16	20	7	27	29
Hash Value	00001	10000	10100	00111	11011	11101

⇒ global depth: d=2

global index:	Records:		local depth:
00 / 01	Record1: 2305	Record4: 4871	d’ = 1
10	Record2: 1168	Record3: 2580	d’ = 2
11	Record5: 5659	Record6: 1821	d’ = 2