

Homework 3

Due 23:59 Nov 11

Problem 1

1. Find a recurrence relation for the number of bit strings of length n that contain three consecutive 0s.
2. What are the initial conditions?
3. How many bit strings of length seven contain three consecutive 0s?

Problem 2

Find the closed form expression for a_n given the recurrence and initial conditions as following:

Initial conditions: $a_0 = 3, a_1 = 6$.

Recurrence condition: $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$

Problem 3

Show that if n is an integer then $n^2 \equiv 0$ or $1 \pmod{4}$.

Problem 4

Compute the value of $7^{14} \pmod{47}$.

Problem 5

Solve the linear congruence $5x \equiv 12 \pmod{23}$.

Problem 6

Find all solutions to the system of congruences $x \equiv 2 \pmod{3}$, $x \equiv 1 \pmod{4}$, and $x \equiv 3 \pmod{5}$.

Problem 7

Show with the help of Fermat's little theorem that if n is a positive integer, then 42 divides $n^7 - n$.

Problem 8

Prove that there are infinitely many primes using the fundamental theorem of arithmetic.

Problem 9

Find the $\text{spc}(252, 198)$.

Problem 10

Show that if a , b , and m are integers such that $m \geq 2$ and $a \equiv b \pmod{m}$, then $\gcd(a, m) = \gcd(b, m)$.