# Report on Project 4: Driverless Car

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## 1 Overview

In this project, we use HMM to solve driverless car problems.

## 2 Problem 1

### 2.1 Problem 1a

According to the constraints given by the instruction,

$$\begin{aligned}\mathbb{P}(C_2 = 1|D_2 = 0) &= \frac{\mathbb{P}(C_2 = 1, D_2 = 0)}{\mathbb{P}(D_2 = 0)} \\ &= \frac{\mathbb{P}(D_2 = 0|C_2 = 1)\mathbb{P}(C_2 = 1)}{\mathbb{P}(D_2 = 0)}\end{aligned}$$

And we have

$$\begin{aligned}\mathbb{P}(C_2 = 1) &= \mathbb{P}(C_1 = 0)\mathbb{P}(C_2 = 1|C_1 = 0) + \mathbb{P}(C_1 = 1)\mathbb{P}(C_2 = 1|C_1 = 1) \\ &= 0.5\epsilon + 0.5(1 - \epsilon) \\ &= 0.5\end{aligned}$$

So we have

$$\begin{aligned}\mathbb{P}(C_2 = 1|D_2 = 0) &= \frac{0.5\eta}{\mathbb{P}(D_2 = 0)} \\ \mathbb{P}(C_2 = 0|D_2 = 0) &= \frac{0.5(1 - \eta)}{\mathbb{P}(D_2 = 0)}\end{aligned}$$

So the answer is

$$\begin{aligned}\mathbb{P}(C_2 = 1|D_2 = 0) &= \frac{0.5\eta}{0.5\eta + 0.5(1 - \eta)} \\ &= \eta\end{aligned}$$

Notice that $\mathbb{P}(D_2 = 0)$ is not 1 since $\mathbb{P}(D_2)$ is related to $\mathbb{P}(C_2)$.

## 2.2 Problem 1b

According to the HMM graph, the result could be written as

$$\begin{aligned}
&\mathbb{P}(C_2 = 1 | D_2 = 0, D_3 = 1) \\
\propto\ &\mathbb{P}(C_2 = 1, D_2 = 0, D_3 = 1) \\
=\ &\{\sum_{C_1=\{0,1\}} \mathbb{P}(C_1)\mathbb{P}(C_2 = 1|C_1)\} \cdot \mathbb{P}(D_2 = 0|C_2 = 1) \cdot \{\sum_{C_3=\{0,1\}} \mathbb{P}(C_3|C_2 = 1)\mathbb{P}(D_3 = 1|C_3)\} \\
=\ &0.5\eta(\epsilon\eta + (1-\epsilon)(1-\eta))
\end{aligned}$$

Also,

$$\begin{aligned}
&\mathbb{P}(C_2 = 0 | D_2 = 0, D_3 = 1) \\
\propto\ &\mathbb{P}(C_2 = 0, D_2 = 0, D_3 = 1) \\
=\ &\{\sum_{C_1=\{0,1\}} \mathbb{P}(C_1)\mathbb{P}(C_2 = 0|C_1)\} \cdot \mathbb{P}(D_2 = 0|C_2 = 0) \cdot \{\sum_{C_3=\{0,1\}} \mathbb{P}(C_3|C_2 = 0)\mathbb{P}(D_3 = 1|C_3)\} \\
=\ &0.5(1-\eta)((1-\epsilon)\eta + \epsilon(1-\eta))
\end{aligned}$$

After normalization, the result is

$$\mathbb{P}(C_2 = 1 | D_2 = 0, D_3 = 1) = \frac{\epsilon\eta^2 + \eta(1-\epsilon)(1-\eta)}{\epsilon\eta^2 + 2\eta(1-\epsilon)(1-\eta) + \epsilon(1-\eta)^2}$$

## 2.3 Problem 1c

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