Statistics Homework

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Homework for Statistics (M 358 K).

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Problem 1. *As in the homework.*

Solution. Note that $|T_1| = 2|T_2| = 4|T_3|$ and $|T_2| = 2|T_3|$.

(a) We have

- $p_1 = \frac{T_1}{(T_1+T_2+T_3)} = \frac{4T_3}{(4T_3+2T_3+T_3)} = \frac{4T_3}{7T_3} = \frac{4}{7}$.
- $p_2 = \frac{T_2}{(T_1+T_2+T_3)} = \frac{2T_3}{7T_3} = \frac{2}{7}$.
- $p_3 = \frac{T_3}{(T_1+T_2+T_3)} = \frac{T_3}{7T_3} = \frac{1}{7}$.

(b) The probability that the result of the coin toss is heads is 27/70. See the figure for work.

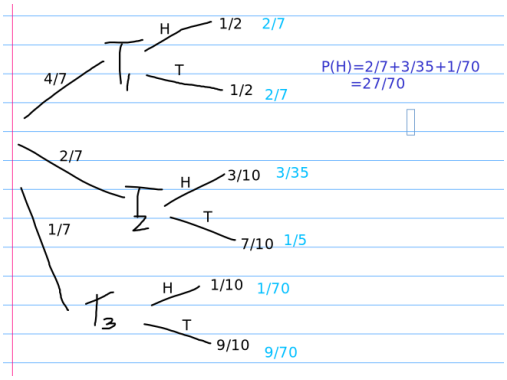

Figure 1: Probability tree for the coin toss. ■

Problem 2. *As in the homework.*

Solution. The probability that someone who contracts goosepox is susceptible is 34%. See the figure for work.

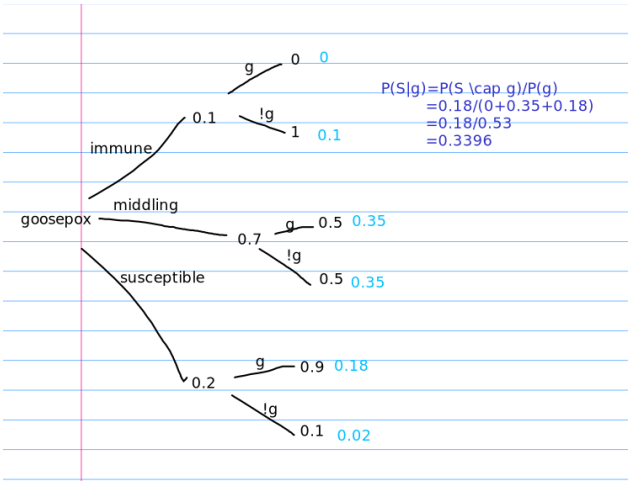

Figure 2: Probability tree for goosepox and susceptibility. ■

Problem 3. *As in the homework.*

Solution. Using R, we have `dbinom(2, 3, 0.5)=0.375`. Using analytical methods, we have

$$\text{Binom}(2,3,0.5) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = 3\cdot\frac{1}{4}\cdot\frac{1}{2} = \frac{3}{8} = 0.375. \qquad \blacksquare$$

Problem 4. *As in the homework.*

Solution. Using R, we have `sapply(0:2, function(num) dbinom(num, 4, 0.5)) |> sum()=0.6875`. Using analytical methods, we have

$$\begin{aligned}\sum_{i\in\{0,1,2\}} \text{Binom}(i,4,0.5) &= \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 + \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 \\ &= \frac{1}{16} + \frac{1}{4} + \frac{3}{8} \\ &= \frac{11}{16} \\ &= 0.6875. \qquad \blacksquare\end{aligned}$$

Problem 5. *As in the homework.*

Solution. Using R, we have `dinom(4, 5, 0.2)=0.0064`. Using analytical methods, we have

$$\text{Binom}(4,5,0.2) = \binom{5}{4}\left(\frac{1}{5}\right)^4\left(\frac{1}{5}\right)^1 = 5\cdot\frac{1}{625}\cdot\frac{4}{5} = \frac{4}{625} = 0.0064. \qquad \blacksquare$$