2021 Summer Minicourses, UT Austin

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Notes for some of the Summer 2022 minicourses, taught by various graduate students here at UT Austin. If you taught one of these courses and don't want the notes to be viewed publicly (or if you have any comments or suggestions in general), please contact me at simonxiang@utexas.edu! Homepage: https://web.ma.utexas.edu/SMC/Minicourses.html

Contents

1 Lie groupoids and differentiable stacks

An overview: this minicourse will define Lie groupoids, and give examples and morphisms of special classes of Lie groupoids. Then we talk about an equivalence relation between Lie groupoids, and some geometric structures on Lie groupoids. The second part of course focuses on stacks and differentiable stacks, and some geometric structures on differentiable stacks.

Definition 1.1. A Lie groupoid $G_1 \rightrightarrows G_0$ consists of smooth maps $s\colon G_1 \to G_0$, $t\colon G_1 \to G_0$, $m\colon G_1 \times_{G_0} G_1 \to G_1$, $i\colon G_1 \to G_1$, $e\colon G_0 \to G_0$. More compactly, it is a groupoid where the sets G_1 of objects and G_0 are manifolds and the maps are smooth.

Example 1.1. Smooth manifolds M give a Lie groupoid $M \rightrightarrows M$, and $G \rightrightarrows *$ is simply a Lie group. There is also a Lie group action on the manifold $M \times G \to M$, resulting in a Lie groupoid $M \times G \rightrightarrows M$.

Example 1.2. Another example is the principal G-bundle over a manifold. Let G be a Lie group, M be a smooth manifold, $P \to M$ be a principal G-bundle. Then we have the Gauge groupoid $[(P \times P)/G \rightrightarrows M]$. We also have a vector bundle over a manifold $E \to M$, where the Lie groupoid structure is given by $[\mathrm{GL}(E) \rightrightarrows M]$. Finally, we have the tangent groupoid associated to a Lie groupoid, where $[G \rightrightarrows M] \rightsquigarrow [TG \rightrightarrows TM]$.