Algebraic Topology Homework

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This is my homework for the Fall 2020 section of Algebraic Topology (Math 382C) at UT Austin with Dr. Allcock. The course follows *Algebraic Topology* by Hatcher. Source files: `https://git.simonxiang.xyz/math_notes/files.html`

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§1 October 7, 2020: Homework 6

Hatcher Section 2.1: 1, 2, 3, 5, 10a, 23, 24.

§1.1 Question 1

Problem. *What familiar space is the quotient* Δ*-complex of a* 2*-simplex* $[v_0, v_1, v_2]$ *obtained by identifying the edges* $[v_0, v_1]$ *and* $[v_1, v_2]$*, preserving the ordering of vertices?*

§1.2 Question 2

Problem. *Show that the* Δ*-complex obtained from* Δ^3 *by performing the order preserving edge identifications* $[v_0, v_1] \sim [v_1, v_3]$ *and* $[v_0, v_2] \sim [v_2, v_3]$ *deformation retracts onto a Klein bottle. Also, find other pairs of identifications of edges that product* Δ*-copmlexes deformation retracting onto a torus, a* 2*-sphere, and* $\mathbb{R}P^2$*.*

§1.3 Question 3

Problem. *Construct a* Δ*-complex structure on* $\mathbb{R}P^n$ *as a quotient of* Δ*-complex structure on* S^n *having vertices the two vectors of length* 1 *along each coordinate axis in* $\mathbb{R}^{n+1}$*.*

§1.4 Question 5

Problem. *Compute the simplicial homology groups of the Klein bottle using the* Δ*-complex structure described at the beginning of this section.*

§1.5 Question 10a

Problem. *Show the quotient space of a finite collection of disjoint* 2*-simplices obtained by identifying pairs of edges is always a surface, locally homeomorphic to* $\mathbb{R}^2$*.*

§1.6 Question 23

Problem. *Show that the second barycentric subdivision of a* Δ*-complex is a simplicial complex. Namely, show that the first barycentric subdivision produces a* Δ*-complex with the property that each simplex has all its vertices distinct, then show that for a* Δ*-complex with this property, barycentric subdivision produces a simplicial complex.*

§1.7 Question 24

Problem. *Show that each n-simplex in the barycentric subdivision of* Δ^n *is defined by n inequalities* $t_{i_0} \leq t_{i_1} \leq \cdots \leq t_{i_n}$ *in its barycentric coordinates, where* $(i_0, \cdots, i_n)$ *is a permutation of* $(0, \cdots, n)$*.*