

Algebraic Topology Homework

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This is my homework for the Fall 2020 section of Algebraic Topology (Math 382C) at UT Austin with Dr. Allcock. The course follows *Algebraic Topology* by Hatcher. Source files: https://git.simonxiang.xyz/math_notes/files.html

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§1 September 14, 2020: Homework 2

Hatcher Section 1.2 (p. 52): 1, 10, 14, 16, 21,

Hatcher Section 1.3 (p. 79): 30,

Hatcher Section 1.A (p. 86): 5.

§1.1 Problem 1 Section 1.2

Problem. Show that the free product $G * H$ of nontrivial groups G and H has trivial center, and that the only elements of $G * H$ of finite order are the conjugates of finite-order elements of G and H .

§1.2 Problem 10

Problem. Consider two arcs α and β embedded in $D^2 \times I$ as shown in the figure. The loop γ is obviously nullhomotopic in $D^2 \times I$, but show that there is no nullhomotopy of γ in the complement of $\alpha \cup \beta$.

Hint (from Dr. Allcock): this can be done directly with Van Kampen's, but it becomes easier if you manipulate $(D^2 \times I) \setminus (\alpha \cup \beta)$ first, being careful not to change the homotopy type, and carrying along the loop γ .

§1.3 Problem 14

Problem. Consider the quotient space of a cube I^3 obtained by identifying each square face with the opposite square face via the right-handed screw motion consisting of a translation by one unit in the direction perpendicular to the face combined with a one-quarter twist of the face about its center point. Show this quotient space X is a cell complex with two 0-cells, four 1-cells, three 2-cells, and one 3-cell. Using this structure, show that $\pi_1(X)$ is the quaternion group $\{\pm 1, \pm i, \pm j, \pm k\}$ of order eight.

§1.4 Problem 16

Problem. Show that the fundamental group of the surface of infinite genus shown below is free on an infinite number of generators.

§1.5 Problem 21

Problem. Show that the join $X * Y$ of two nonempty spaces X and Y is simply-connected if X is path-connected.

Hint (from Dr. Allcock): If you are not comfortable with the join of spaces then wrap your mind around the following examples in order:

1. Join of two points
2. Join of a point and an interval

3. *Join of a point and a circle*
4. *Join of 2 copies of the interval*
5. *Join of a circle and an interval*
6. *Join of two circles (doesn't embed in \mathbb{R}^3 , but still understandable).*

That might be enough: if not, work out examples using the figure 8 or S^2 .

§1.6 Problem 30 Section 1.3

Problem. *Draw the Cayley graph of the group $\mathbb{Z} * \mathbb{Z}_2 = \langle a, b \mid b^2 \rangle$.*

§1.7 Problem 5 Section 1.A

Problem. *Construct a connected graph X and maps $f, g: X \rightarrow X$ such that $fg = \mathbb{1}$ but f and g do not induce isomorphisms on π_1 . [Note that $f_*g_* = \mathbb{1}$ implies that f_* is surjective and g_* is injective.]*