Statistics Homework

Simon Xiang

Homework for Statistics (M 358 K).

Contents

1 Homework 1

Problem 1. *Write down the definition of* ***independence*** *of two* ***events****.*

Solution. Two events A, B are **independent** if $P(A \cap B) = P(A)P(B)$. ■

Problem 2. *Write down the definition of the* ***cumulative distribution function*** *of a random variable.*

Solution. For any random variable X, the **cumulative distribution function** of X is a function $F_X : \mathbb{R} \to [0,1]$, where $F_X(x) = \mathbb{P}[X \leq x]$ for all $x \in \mathbb{R}$. ■

Problem 3. *Let $\Omega = \{a_1, a_2, a_3, a_4, a_5\}$ be an outcome space, and let $\mathbb{P}$ be a probability on Ω. Assume that $\mathbb{P}[A] = 0.5, \mathbb{P}[B] = 0.4, \mathbb{P}[C] = 0.4$, and $\mathbb{P}[D] = 0.2$, where*

$$A = \{a_1, a_2, a_3\},\ B = \{a_2, a_3, a_4\},\ C = \{a_3, a_5\},\ D = \{a_4\}.$$

Are the events A and B independent? Why?

Solution. They are not independent, since their intersection in the outcome space is non-empty. ■

Problem 4. *Show that the probability of rolling two sixes twice is just as likely as rolling two threes twice.*

Solution. Rolling die are independent events as well as each double dice roll, so the probability of rolling two sixes twice is equivalent to the probability of rolling a dice 4 times in a row and getting 6 each time. By the multiplication rule for independent events we have

$$P(6 \cdot 6 \cdot 6 \cdot 6) = P(6)P(6)P(6)P(6) = \frac{1}{6^4} = \frac{1}{1296}.$$

Similarly,

$$P(3 \cdot 3 \cdot 3 \cdot 3) = P(3)P(3)P(3)P(3) = \frac{1}{3^4} = \frac{1}{1296}.$$

So the probabilities of the two events are equal. ■

Problem 5. *If you roll a pair of fair dice, what are the probabilities of*

(1) Getting a sum of 1?
(2) Getting a sum of 5?
(3) Getting a sum of 12?

Solution. Think of the sample space as the space of ordered pairs $S = \{(m,n) \mid m,n \in \{1, \cdots, 6\}\}$ representing the ordered die tosses. Then $|S| = 36$.

(1) There are no two integers in $\{1, \cdots, 6\}$ that sum up to 1. So this probability is zero.
(2) The pairs of integers summing up to 5 in $\{1, \cdots, 6\}$ are $(1,4)$ and $(2,3)$. Since order of tossing matters we have the four possibilities $(1,4), (4,1), (2,3), (3,2)$, and the probability of getting a sum of 5 is $\frac{4}{36} = \frac{1}{9}$.
(3) The only pair of integers summing up to 12 in $\{1, \cdots, 6\}$ are $(6,6)$. So the probability of getting a sum of 6 is $\frac{1}{36}$.

■

Problem 6. *Problem 3.8 from the textbook.*

Solution. (a) Living below the poverty line and speaking a foreign language are not disjoint. Let A represent living below the poverty line and B represent speaking a foreign language. Then $P(A \cap B) = 0.042 \neq 0$, so these two outcomes are not disjoint.

(b) The variables A and B are the same as in the previous part.

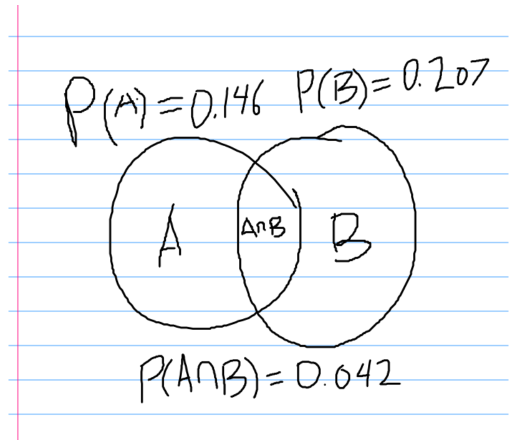

Figure 1: The events and their associated probabilities.

(c) The percentage of Americans living below the poverty line only speaking English is $14.6 - 4.2 = 10.4$ percent.

(d) The percentage of Americans living below the poverty line or speak a foreign language is $14.6 + 20.7 - 4.2 = 31.1$ percent.

(e) The percentage of Americans living above the poverty line only speaking English is $1 - 31.1 = 68.9$ percent.

(f) The event that someone lives below the poverty line is not independent of the event that the person speaks a foreign language at home. If they were, we would have

$$P(A \cap B) = P(A)P(B) = 0.030222.$$

However, we know that $P(A \cap B) = 0.042$. So the assumption that the two conditions are independent must be false.

■

Problem 7. *Problem 3.20 as in the textbook.*

Solution. We know that $P(\text{condition} \mid +) = P(\text{condition} \cap +)/P(+)$. Drawing out the tree, we have $P(\text{condition} \cap +) = 0.03 * 0.99 = 0.0297$. Then we have $P(+) = 0.0297 + 0.97 * 0.02 = 0.0491$. Dividing, we get $P(\text{condition} \mid +) = 0.0297/0.0491 = 0.604888$. ■

Problem 8. *Problem 3.22 as in the textbook.*

Solution. The answer is 0.4867. See Figure 2 for the steps.

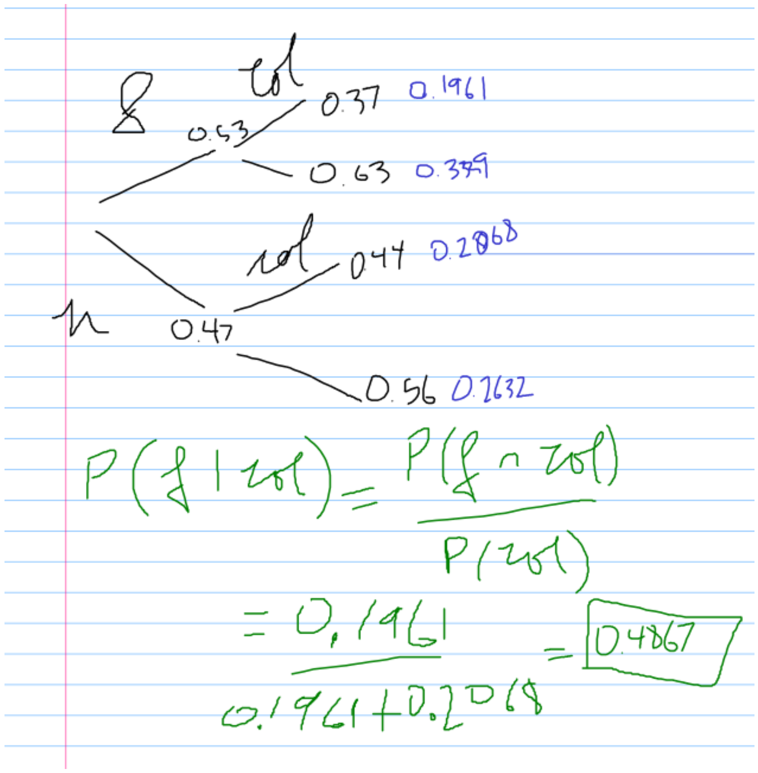

Figure 2: Problem 3.22 work.

∎

2 Homework 2

Problem 9. *Let E and F be any two events. If $P[E] = P[F] = \frac{2}{3}$, then E and F cannot be mutually exclusive. T/F, and why?*

Solution. True. If E, F were mutually exclusive, then $P(E \cup F) = P(E) + P(F) = 4/3$. This is a contradiction since probabilities cannot exceed one. ∎

Problem 10. *If events A and B are mutually exclusive, they are necessarily independent. T/F and why?*

Solution. True. In general, we have $P(A \cup B) = P(A) + P(B) + P(A \cap B)$. If A, B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B) = P(A) + P(B) + P(A \cap B).$$

This implies that $P(A \cap B) = 0$, and the events are necessarily independent. ∎

Problem 11. *A test is used to determine whether people exhibiting green spots have the* duckpox *or not. It is believed that at any given time 4% of the people exhibiting green spots actually have the* duckpox*. The test is 99% accurate if a person actually has the* duckpox*. The test is 96% accurate if a person does* **not** *have the* duckpox*. What is the probability that a randomly selected person who tests positive for the* duckpox *actually has the* duckpox*?*

Solution. The probability that someone randomly selected tests positive for duckpox is 50.77%. See the image attached for work.

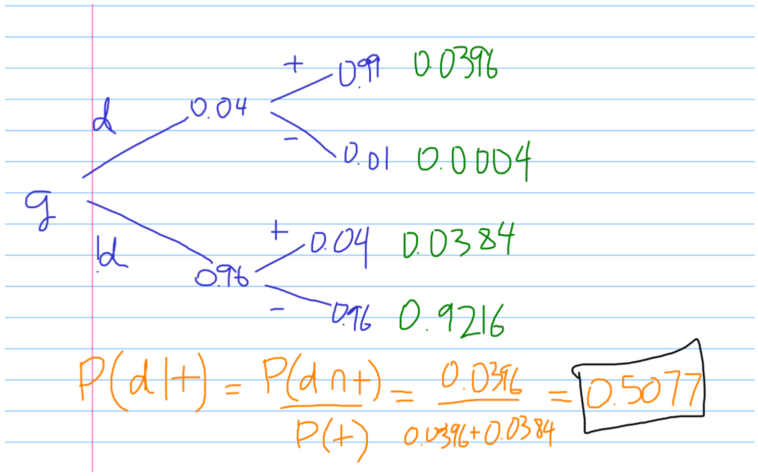

Figure 3: Probability of someone randomly tested having duckpox.

■

Problem 12. *Problem 4.18 as in the textbook.*

Solution. (a) Yes. Each adult is a trial with constant probability of success $p = 0.9$, and there are 100 independent trials.

(b) The probability that 97 out of 100 sampled Americans had chickenpox is given by

$$\binom{100}{97}(0.9)^{97}(0.1)^3 = 0.0059.$$

(c) The probability that 3 out of 100 sampled Americans have *not* had chickenpox is given by

$$\binom{100}{3}(0.1)^3(0.9)^{97} = \binom{100}{97}(0.9)^{97}(0.1)^3 = 0.0059.$$

(d) The probability that 1 in 10 sampled Americans have had chickenpox is

$$\binom{10}{1}(0.9)^1(0.1)^9 = 9 \cdot 10^{-9}.$$

(e) The probability that 3 in 10 sampled Americans have *not* had chickenpox is

$$\binom{10}{3}(0.1)^3(0.9)^7 = 0.0574.$$

■

Problem 13. *Problem 4.22 as in the textbook.*

Solution. We have $n = 10$, $p = 0.07$.

(a) Here the number of scenarios where at least one has arachnaphobia is $2^{10} - 1 = 1023$, since this holds in every scenario except the one where no one has arachnaphobia. So the probability of at least one having arachnaphobia is given by

$$\sum_{i=1}^{10} \text{Binom}(i, 10, 0.07) = 0.516.$$

(b) This is given by
$$\text{Binom}(2, 10, 0.07) = 0.123.$$

(c) This is given by
$$\text{Binom}(0, 10, 0.07) + \text{Binom}(1, 10, 0.07) = 0.848.$$

(d) Yes, the probability is pretty high (more than 80%).

■

Problem 14. *Problem 4.24 as in the textbook.*

Solution. We have $n = 3$.

(a) Here $k = 2, p = 0.25$. This is given by $\text{Binom}(2, 3, 0.25) = 0.141$.

(b) Here $k = 0, p = 0.25$. This is given by $\text{Binom}(0, 3, 0.25) = 0.422$.

(c) This is given by
$$\text{Binom}(1, 3, 0.25) + \text{Binom}(2, 3, 0.25) + \text{Binom}(3, 3, 0.25) = 0.578.$$

(d) There is one scenario in which this happens, so the probability is given by $1 \cdot (0.25)^1 \cdot (0.75)^2 = 0.141$.

■

Problem 15. *Problem 4.26 as in the book.*

Solution. We have $n = 3, p = 0.51$.

(a) Here $k = 2$. This is given by $\text{Binom}(2, 3, 0.51) = 0.382$.

(b) All possible orderings of two boys in three children are as follows:

1	B	B	G
2	B	G	B
3	G	B	B

There are three possibilites. Then the probility of two boys in three children is given by
$$3 \cdot (0.51)^2 \cdot (0.49) = 0.382,$$
which matches up with the answer from (a).

(c) This would be a lot harder since we would need to write out $\binom{8}{3} = 56$ possibilities, which is a lot harder than writing out three possibilities. Therefore it is much easier to use the binomial distribution calculation.

■

3 Homework 3

testset this word is *italicized* and this word is **bolded**.

Definition 3.1. Define this thing as **this**

This math $x^2 - ab = 10$ mode. This also math mode (equation)

$$x^2 = \sqrt{25}$$

This math $x^2 - ab = 10$ mode. THis also math mode:

$$x^2 = \sqrt{25}$$

Enter math mode $x^2 = \sqrt{10}$.

$$x^2 = \frac{1}{2} + \pm 3$$

Problem 16. *ok*