Motivating de Rham cohomology

why?

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Prerequisites

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- What groups are,

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- What groups are,
- And basic topology.

Motivation

Question

Does there exist a function that is the gradient of some other function? More precisely, when does $F: U \to \mathbb{R}^2$ for $U \subseteq \mathbb{R}^2$ satisfy

$$\frac{\partial F}{\partial x} = f_1, \quad \frac{\partial F}{\partial y} = f_2 \quad \text{for some } f = (f_1, f_2)?$$

(You could also think of this question as asking when vector fields have potential.)

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Answer

It depends on the topology of U!

Some vector calculus

Note that $\frac{\partial F}{\partial x} = f_1, \ \frac{\partial F}{\partial y} = f_2$ implies $\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$. Is this condition sufficient to show F is the gradient of some other function?

Example

easy example of something ssatisfying on $\mathbb{R}^2$.

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However, consider $f(x, y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$.

By a theorem, the above condition turns out to be sufficient if U looks like a ball (convex).

define div grad and curl

Define cohomology groups, do some theorem, show reason why our ealire rfucntion is false is because $H(\mathbb{R}^2 \setminus \{0\}) \neq 0$

Define the re Rham complexa nd bieng generated by the dx^i

calculate div grad curl using abstract, link between aglebra and calc:

testing testing

Definition

whatno

- test

testing testing

Definition

whatno

- test
- ok

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- te

what's the point?

text

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

1. what's the
2. point?

text

$$\nabla_\beta T^{\alpha\beta} = T^{\alpha\beta}_{\beta} = 0$$

1. of