Algebraic Topology Homework

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This is my homework for the Fall 2020 section of Algebraic Topology (Math 382C) at UT Austin with Dr. Allcock. The course follows *Algebraic Topology* by Hatcher. Source files: `https://git.simonxiang.xyz/math_notes/files.html`

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§1 September 26, 2020: Homework 5

Hatcher Section 1.3 (p. 79): 5, 9, 14, 16, 20, 31
Hatcher Section 1.A (p. 86): 6, 7,

§1.1 Problem 5 Section 1.3

Problem. *Let X be the subspace of $\mathbb{R}^2$ consisting of the four sides of the square $[0,1] \times [0,1]$ together with the segments of the vertical lines $x = 1/2, 1/3, 1/4, \cdots$ inside the square. Show that for every covering space $\widetilde{X} \to X$ there is some neighborhood of the left edge of X that lifts homeomorphically to $\widetilde{X}$. Deduce that X has no simply-connected covering space.*

Solution. ok ■

§1.2 Problem 9

Problem. *Show that if a path-connected, locally path-connected space X has $\pi_1(X)$ finite, then every map $X \to S^1$ is nullhomotopic. [Use the covering space $\mathbb{R} \to S^1$.]*

§1.3 Problem 14

Problem. *Find all the connected covering spaces of $\mathbb{R}P^2 \vee \mathbb{R}P^2$.*

§1.4 Problem 16

Problem. *Given maps $X \to Y \to Z$ such that both $Y \to Z$ and the composition $X \to Z$ are covering spaces show that $X \to Y$ is a covering space if Z is locally path-connected, and show that this covering space is normal if $X \to Z$ is a normal covering space.*

§1.5 Problem 20

Problem. *Construct nonnormal covering spaces of the Klein bottle by a Klein bottle and by a torus.*

§1.6 Problem 31

Problem. *Show that the normal covering spaces of $S^1 \vee S^1$ are precisely the graphs that are Cayley graphs of groups with two generators. More generally, the normal covering spaces of the wedge sum of n circles are the Cayley graphs of groups with n generators.*

§1.7 Problem 6 Section 1.A

Problem. *Let F be the free group on two generators and let F' be its commutator subgroup. Find a set of free generators for F' by considering the covering space of the graph $S^1 \vee S^1$ corresponding to F'.*

§1.8 Problem 7

Problem. *If F is a finitely generated free group and N is a nontrivial normal subgroup of infinite index, show, using covering spaces, that N is not finitely presented.*