Complex Analysis Lecture Notes

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These are my lecture notes for the Fall 2020 section of Complex Analysis (Math 361) at UT Austin with Dr. Radin. These were taken live in class, usually only formatting or typo related things were corrected after class. You can view the source code here: `https://git.simonxiang.xyz/math_notes/file/freshman_year/complex_analysis/master_notes.tex.html`. I was also unhappy with the textbook, so some supplementary notes from different texts are found at the bottom of the document.

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I want to do some real math! These notes will follow Stein and Shakarchi §1.2.

§1.1 Continuous functions

We've already seen the standard epsilon-delta definition of continuity. An equivalent definition is the sequential definition, that is, for every sequence $\{z_1, z_2, \cdots\} \subseteq \Omega \subseteq \mathbb{C}$ such that $\lim z_n = z_0$, then f is continuous at z_0 if $\lim f(z_n) = f(z_0)$. Since the notions for convergence of complex numbers and $\mathbb{R}^2$ is the same, f of $z = x + iy$ is continuous iff it's continuously viewed as a function of two real variables x and y. If f is continuous, then the real valued function defined by $z \mapsto |f(z)|$ is clearly continuous (by the triangle inequality).

We say f attains a *maximum* at the point $z_0 \in \Omega$ if

$$|f(z)| \leq |f(z_0)| \text{ for all } z \in \Omega.$$

The definition of a minimum is what you think it is.

Theorem 1.1. *A continuous function on a compact set Ω is bounded and attains a maximum and minimum on Ω.*

Proof. Same as the any one you'd find in a Real Analysis course. $\boxtimes$

§1.2 Holomorphic functions

Let's talk about the good stuff.