Chapter 2

Theorem 1 (De Morgan's Laws)**.** *Let S and T be sets. Then*

$$\overline{S \cap T} = \overline{S} \cup \overline{T}.$$

Proof. We want to show that $\overline{S \cap T} \subset \overline{S} \cup \overline{T}$ and $\overline{S} \cup \overline{T} \subset \overline{S \cap T}$. Let $x \in \overline{S \cap T}$, then $x \notin S \cap T$ which implies $x \notin S$ and $x \notin T$. Therefore $x \in \overline{S}$ and $x \in \overline{T}$ by definition, so $x \in \overline{S} \cup \overline{T}$.

Now let $x \in \overline{S} \cup \overline{T}$. By way of contradiction, assume that $x \notin \overline{S \cap T}$. We have $x \in \overline{S}$ or $x \in \overline{T}$ which implies that $x \notin S$ or $x \notin T$ by assumption. But since $x \notin \overline{S \cap T}$, $x \in S \cap T$ which implies that $x \in S$ and $x \in T$, a contradiction. So $x \in \overline{S \cap T}$, and we are done.

$\boxtimes$