The Cooper Union Department of Electrical Engineering
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ECE300 Communication Theory
Problem Set IV: Analog Communications
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1. This problem reviews basic parameters for FM. Consider a tone signal with amplitude $2V$ at frequency $5kHz$ that is frequency modulated using a VCO with frequency sensitivity $10kHz/V$. Compute: (1)the frequency deviation; (2)the β; (3)the bandwidth estimate given by Carson's rule. Also specify whether this should be considered narrowband or wideband FM.

2. In this problem, you will be synthesizing and studying various analog modulations of a specific signal:
$$m(t) = \frac{1}{1 + (t-10)^2}$$
which you will represent over the time span $0 \le t \le 20\,\text{sec}$. The spectrum is:
$$M(f) = \pi e^{-j40\pi f} e^{-2\pi |f|}$$
The 90% energy containment bandwidth B (i.e., 90% of the energy is contained in the range $|f| \le B$) is given by:
$$B = \frac{\ln 10}{4\pi} = 0.1832Hz$$
Some other information you will find useful for this problem: the Hilbert transform is:
$$\hat{m}(t) = \frac{t-10}{1 + (t-10)^2}$$
and the integral is:
$$v(t) = \int_{\tau=-\infty}^{t} m(\tau)\, d\tau = \frac{\pi}{2} + \tan^{-1}(t-10)$$
where here $\tan^{-1}$ is the principal branch, returning values in the range $-\pi/2$ to $\pi/2$. For example, $v(-\infty) = 0$, $v(10) = \pi/2$, $v(\infty) = \pi$.

We are going to use a carrier frequency of $f_c = 10Hz$, and a sampling rate of $f_s = 100Hz$.

   (a) In MATLAB, set up a time vector t with values $0 \le t \le 20$ at sample times (i.e., integer multiples of $1/f_s$). All your time domain signals should use this t vector. You will note t has length approximately (but not exactly) 2000.

   (b) The way you are going to compute spectra is to use *fft* of length $N = 4096$, via the usual zero padding. After applying *fftshift,* the proper frequency vector should span roughly from $-f_s/2$ to $f_s/2$. Generate the *correct* frequency vector f. All you frequency domain plots should use this f vector (unless specified otherwise below). Also, for any of the frequency plots in decibels (not all will be in dB), make sure the vertical bounds are set so interesting features are visible (i.e., don't let is go down to $-1million$ dB or something).

(c) First, plot $|M(f)|$ on a decibel scale, but only over the frequency range $|f| \leq 1Hz$. Show the full values (i.e., don't limit the vertical axis) so you can observe, in particular, how small the magnitude spectrum becomes at $1Hz$.

(d) Next, generate two AM signals, at 10% and 90% modulation index, respectively. Plot them both (using *subplot)* in the time domain, with the envelope superimposed for each (in a different color). Then plot the spectra on a *decibel scale.* Also plot the zoomed-in spectra over the subrange $8 < f < 12Hz$.

(e) Generate DSB-SC, USSB and LSSB. Plot the modulated signals in the time domain (use *subplot*). Also, for each signal, compute the envelope and superimpose the envelope (in a different color) on top of the modulated signal.

(f) It turns out the envelopes for USSB and LSSB have a simple formula. *Find the formula for the envelopes!*

(g) Compute the spectra of DSB-SC, USSB and LSSB, and plot the magnitude spectra of each (use subplot) on a decibel scale, first over the full frequency range, and then over the subrange $8 < f < 12Hz$.

(h) Generate the FM signal using $k_f = 0.5$. Plot the FM signal in the time domain. It may be a little hard to see, so you may also want to provide a plot for a subset of the time. It may not be immediately obvious that the frequency is shifting since the frequency deviation is so small compared to the carrier frequency.

(i) Plot the FM magnitude spectrum in decibels, again over the full range and then over $8 < f < 12Hz$.

(j) In order to see the bandwidth expansion of FM, superimpose plots of the magnitude spectra of the DSB-SC and FM signals on a *linear* scale (i.e., not in dB), *normalized* so each has peak magnitude 1, over the range $8 < f < 12Hz$.