narrowband

1) 16-QAM vs. 8-PSK

a) $\mathcal{E}_s = \frac{1}{M}\sum_{m=0}^{M-1} |\mathcal{E}_m|^2$ 16QAM, $M = 16$, $d = \sqrt{2}$, $\sqrt{1^2+3^2}$, $\sqrt{3^2+3^2}$

$\mathcal{E}_b = \frac{1}{\log_2 M}\mathcal{E}_s$ $\mathcal{E}_s = \frac{1}{16}\left(4(\sqrt{2})^2 + 8(\sqrt{10})^2 + 4(\sqrt{18})^2\right) =$

$\frac{1}{16}\left(4(2)+8(10)+4(18)\right) = 10$

$\mathcal{E}_b = \frac{40d^2}{16}\cdot\frac{1}{\log_2 M} = 10d^2 = \frac{10}{4}$

8-PSK, $M = 8$

$\mathcal{E}_s = A^2$

$\mathcal{E}_b = A^2 \cdot \frac{1}{\log_2 M} = \frac{A^2}{3}$

b) $d_{min} = A\,|1 - e^{j\pi/4}|$

$d_{min} = 2 = .7654A$

$A = 2.61\ldots$

c) Find $r_{16-QAM} - r_{8-PSK}$

$\mathcal{E}_{8-PSK} = \frac{A^2}{3}$, $d_{min} = 2$ $\mathcal{E}_{8PSK} = \frac{2.61^2}{3} = 2.2707$

$r_{8PSK} = 10\log_{10}(2.2707)$

$r_{16QAM} = 10\log_{10}\left[\frac{10}{4}\right]$, $\frac{10}{4} = 2.5$

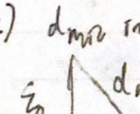
r_{16-QAM} takes more SNR per bit

$10\log_{10}[2.5] - 10\log[2.2707] = 0.4178$ dB

d) $\eta_{16QAM} = \frac{\log_2 16}{2} = 2$

$\eta_{8PSK} = \frac{\log_2 8}{2} = 1.5$ 16 QAM is more spectrally efficient

2) a) d_{min} in terms of $\mathcal{E}_0$



$d_{min} = \sqrt{2\mathcal{E}_0}$

b) $r_{8orth} = 10\log_{10}\left[\frac{d^2_{min}}{2\log_2 M\, N_0}\right] = 10\log_{10}\left[\frac{\mathcal{E}^2}{2(3)}\cdot\frac{1}{N_0}\right]$

$= 10\log_{10}\left[\frac{2}{3N_0}\right]$

c) $\eta_{orth} = \frac{\log_2 M}{M} = \frac{3}{8}$

d) Power vs Spectral

	Power	Spectral
Most eff.	8ary	16 QAM
	8PSK	8 PSK
Least eff.	16QAM	8ary

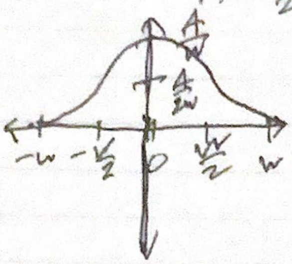
3) a) $X(f) = \frac{A}{2W}[1+\cos(\pi f/W)]\,\Pi\left(\frac{f}{2W}\right)$

$f = 0, \pm W/2, \pm W$

@ 0: $X(f) = \frac{A}{2W}[1+\cos(0)] = \frac{A}{W}$

@ $\pm\frac{W}{2}$: $X(f) = \frac{A}{2W}[1+\cos(\pm\frac{\pi}{2})] = \frac{A}{2W}$

@ $\pm W$: $X(f) = \frac{A}{2W}[1+\cos(\pm\pi)] = 0$



b) $X(f) = \frac{A}{2W}\left(1 + \frac{e^{j\pi \frac{f}{W}} + e^{-j\pi\frac{f}{W}}}{2}\right)\Pi\left(\frac{f}{2W}\right)$

$X(f) = \frac{A}{2W}\,\Pi\left(\frac{f}{2W}\right) + \frac{A}{4W}\,e^{-j2\pi f(\frac{1}{2W})}\,\Pi\left(\frac{f}{2W}\right)$

$x(t) = A\,\mathrm{sinc}(2Wt) + \frac{1}{2}A\,\mathrm{sinc}(2W(t-\frac{1}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(t+\frac{1}{2W}))$

c) $\mathrm{sinc}(\varepsilon) = \frac{\sin(\pi\varepsilon)}{\pi\varepsilon}$, $\mathrm{sinc}(0)$, $\mathrm{sinc}(n)$, $n \neq 0$

$\lim_{n\to 0}\frac{\sin(\pi n)}{\pi n} = \lim_{n\to 0}\frac{\pi\cos(\pi n)}{\pi}$

$= \lim_{n\to 0}\cos(\pi n)$

$\therefore \mathrm{sinc}(0) = 1$

$\mathrm{sinc}(n) = 0$ b/c $\mathrm{sinc}(n) = \frac{\sin(\pi n)}{\pi n} = 0$

d) $x(t) = A\,\mathrm{sinc}(2Wt) + \frac{1}{2}A\,\mathrm{sinc}(2W(t-\frac{1}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(t+\frac{1}{2W}))$

@ $t=0$, $x(0) = A\,\mathrm{sinc}(0) + \frac{1}{2}A\,\mathrm{sinc}(2W(\frac{-1}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(\frac{1}{2W}))$

$= A\,\mathrm{sinc}(0) + \frac{A\,\mathrm{sinc}(-1)}{2} + \frac{A\,\mathrm{sinc}(1)}{2} = A$

@ $t = -\frac{1}{2W}$, $A\,\mathrm{sinc}(2W(-\frac{1}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(-\frac{1}{2W}-\frac{1}{2W})) +$
$\frac{1}{2}A\,\mathrm{sinc}(2W(-\frac{1}{2W}+\frac{1}{2W})) = A\,\mathrm{sinc}(-1) + \frac{1}{2}A\,\mathrm{sinc}(-2)$
$+ \frac{1}{2}A\,\mathrm{sinc}(0) = \frac{1}{2}A$

@ $t = \frac{1}{2W}$, $A\,\mathrm{sinc}(2W(\frac{1}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(\frac{1}{2W}-\frac{1}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(\frac{1}{W}))$
$= A\,\mathrm{sinc}(1) + \frac{1}{2}A\,\mathrm{sinc}(0) + \frac{1}{2}A\,\mathrm{sinc}(2) = \frac{1}{2}A$

e) $x(\frac{k}{2W}) = A\,\mathrm{sinc}(2W(\frac{k}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(\frac{k}{2W}-\frac{1}{2W})) + \frac{1}{2}A\,\mathrm{sinc}(2W(\frac{k}{2W}-\frac{1}{2W}))$

$x(\frac{k}{2W}) = A\,\mathrm{sinc}(k) + \frac{1}{2}A\,\mathrm{sinc}(k-1) + \frac{1}{2}A\,\mathrm{sinc}(k+1)$

@ $k \in \mathbb{Z}$, $k \neq 0, \pm 1$, $\mathrm{sinc}() = 0$

$x(kT) = 0$