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ECE 300 Problem Set V: Information Theory

1. DMS: {a b c d e f g h}, P_i: {.24, .21, .18, .15, .07, .06, .05, .04}

a) VLC where $l_i = \lceil -\log_2 P_i \rceil$, l_i: {3, 3, 3, 3, 4, 5, 5, 5} via calculator

Kraft Inequality: $\sum_{i=1}^{M} 2^{-l_i} \leq 1$

$$\sum_{i=1}^{M} 2^{-\lceil -\log_2 P_i \rceil} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32} + \frac{1}{32} \overset{?}{\leq} 1$$

$\frac{21}{32} \leq 1$ ∴ Lengths chosen satisfy the Kraft inequality

b) $\bar{L} = \sum l_i P_i$, $H = \sum_{i=1}^{M} P_i \log_2 \frac{1}{P_i}$

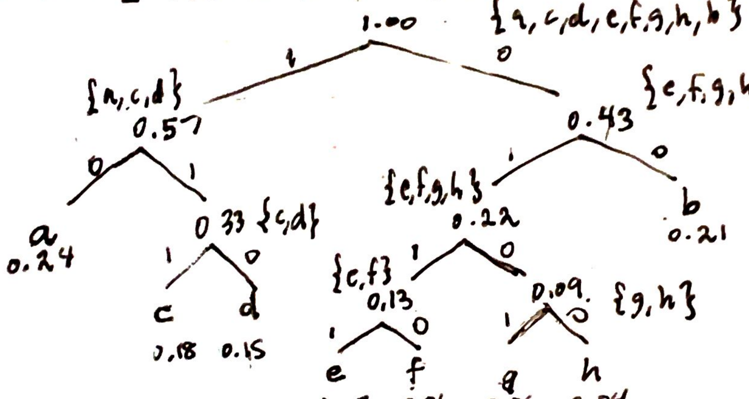
$H(x) = 2.73674\ldots$ (via calculator) bits/symb

$\bar{L} = 3.37$ bits/symb (via calculator)

$H(x) \leq \bar{L} < H(x) + 1$

$2.73674\ldots \leq 3.37 \leq 3.73674\ldots$ ✓

2.



Symbol	Codewords
A	10
B	00
C	111
D	110
E	0111
F	0110
G	0101
H	0100

$l_{i\,\{a,b,c,d,e,f,g,h\}} = \{3,3,3,3,4,5,5,5\}$

They differ.

new l: {2, 2, 3, 3, 4, 4, 4, 4}

Kraft Inequality: $\sum_{i=1}^{M} 2^{-l_i} \leq 1$

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{16}{16}$$

$\frac{16}{16} \leq 1$ ✓

$H(x) \leq \bar{L} \leq H(x) + 1$

$\bar{L} = \sum l_i P_i = 2.77$ bits/symb

$2.73674\ldots \leq 2.77 \leq 3.73674$

3. a) $H = \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i}$ via MATLAB

compute $H(Y_1)$, $H(Y_2)$, $H(Y_2|Y_1)$

$H(Y_1)$ = entropy of first binit

$\sum_{i=1}^{2} \vec{P}_i \log_2 \frac{1}{\vec{P}_i}$, $\vec{P}_i = [\frac{.57}{1}, \frac{.43}{1}]$

$H(Y_1) = .9858$ bits

$H(Y_2)$ = entropy of second binit

$\sum_{i=1}^{M} \vec{P}_i \log_2 \frac{1}{\vec{P}_i}$, $\vec{P}_i = [.45, .55] \rightarrow H(Y_2) = 0.9928$ bits

second binit : 1 $\quad .55 \quad .22 \quad .33$ second binit : 0 $\quad .45 \quad .21 \quad .24$

$H(Y_2|Y_1)$ = entropy of second binit given first binit

$\vec{P}_i = [\frac{0.22}{0.43}, \frac{0.21}{0.43}, \frac{0.24}{0.57}, \frac{0.33}{0.57}]$

$H(Y_2|Y_1) = 0.9895$ bits

5. $N_0 = 10^{-8}$ Joules

$B = 1$ MHz

$C = 1$ Mbps

$$C = B \log_2 (1 + \frac{P}{N_0 B}) \text{ [bits/sec]}$$

$$1 \times 10^6 = 1 \times 10^6 \log_2 (1 + \frac{P}{10^{-2}})$$

$$1 = \log_2 (1 + P \cdot 10^2)$$

$$P = \frac{1}{100} = 10^{-2} \text{ watts}$$

3b) In a DMS, the X_i's are independent and therefore, X_1 should not have an impact on X_2 aka X_{n+1}. Since $H(Y_2|Y_1) \neq H(Y_2)$ given $Y_1 = 0$ nor 1.

$0.9895 \neq .9928$

3c) In a stationary process, the unconditional joint probability distributions does not change over time. In a stationary source, the random variables are iid. So, the probability vector and its statistics should not depend on n. Because the entropy is only determined by P_i. If $H(Y_1)$, $H(Y_2)$... aren't equal, then the corresponding probabilities aren't either. And therefore, it wouldn't be iid nor stationary.