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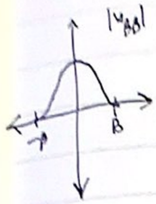
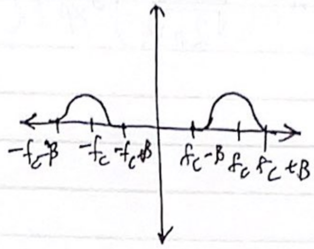
1. $X(f) = \frac{1}{2} x_{BB}(f - f_c) + \frac{1}{2} x_{BB}^*(-f - f_c)$



amplitude by $\frac{1}{2}$
shifted f_c
reflected



$$\mathcal{E}_s = \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df$$

$$\mathcal{E}_{BP} = \int_{\infty}^{\infty} |X(f)|^2 = \int_{f_c-B}^{f_c+B} \left(\frac{|X_{BB}^*(f)|}{2}\right)^2 df + \int_{f_c-B}^{f_c+B} \left(\frac{|x_{BB}(f)|}{2}\right)^2 df$$

$$= \frac{\mathcal{E}_{BB}}{4} + \frac{\mathcal{E}_{BB}}{4} = \frac{\mathcal{E}_{BB}}{2}$$

2. $f(x_{n+1} | x_n, x_{n-1}, \dots, x_1) = f(x_{n+1} | x_n), \quad n \geq 1$

two steps

show $f(x_{n+2} | x_{n+1}, x_n) = f(x_{n+2} | x_{n+1})$

$$f(x_{n+2}, x_{n+1} | x_n) = f(x_{n+1} | x_n) \cdot f(x_{n+2} | x_{n+1})$$

$$\frac{f(x_{n+2}, x_{n+1} | x_n)}{f(x_{n+1} | x_n)} = f(x_{n+2} | x_{n+1})$$

$$\frac{f(x_{n+2}, x_{n+1}, x_n)}{f(x_n)} \cdot \frac{f(x_n)}{f(x_{n+1}, x_n)} = f(x_{n+2} | x_{n+1})$$ via conditional probability eq.

$$f(x_{n+2} | x_{n+1} + x_n) = f(x_{n+2} | x_{n+1})$$

Chapman-Kolmogorov: $\int_{x_{n+1}=-\infty}^{\infty} f(x_{n+2} | x_{n+1}, x_n) f(x_{n+1} | x_n) \, dx_{n+1} = f(x_{n+2} | x_n)$

$f(x_{n+2}) = \int_{x_{n+1}=-\infty}^{\infty} f(x_{n+2} | x_{n+1}) f(x_{n+1}) \, dx_{n+1}$ → Equivalence in notes

$$f(x_{n+2}) = f(x_{n+2})$$

3) a) unconditional pdf of x, $f(x)$

$f(x) = \sum_{i=1}^{M} f(\theta) f(\frac{x}{\theta}) \rightarrow \sum_{i=1}^{M} \pi_i N(\mu_i, \theta_i^2) = \sum_{i=1}^{M} \pi_i \left(\frac{1}{\sqrt{2\pi\theta_i}}\right) \cdot \exp\left[\left(\frac{-1}{2\theta_i^2}\right)(x-\mu_i)^2\right]$

b) $P(\theta | x) = \frac{f(x|\theta) P(\theta)}{f(x)}$ discrete

$$P(\theta | x) = \frac{N(\mu_0, \theta_0^2) \pi_0}{\sum_{i=1}^{M} \pi_i N(\mu_i, \theta_i^2)} \Rightarrow$$

$$P(\theta \mid x) = \frac{\pi_0 \left(\frac{1}{\sqrt{2\pi}\theta_0}\right) \exp\left[\left(-\frac{1}{2\theta_0^2}\right)(x-\mu_0)^2\right]}{\sum_{i=1}^{M} \pi_i \left(\frac{1}{\sqrt{2\pi}\theta_i}\right) \exp\left[\left(-\frac{1}{2\theta_i^2}\right)(x-\mu_i)^2\right]}$$