The Cooper Union Department of Electrical Engineering
Prof. Fred L. Fontaine
ECE300 Communication Theory
Problem Set VIII: Block Codes
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1. The following is the parity check matrix of an (n, k) linear block code:

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) Specify n and k.
   (b) Find the generator matrix, G, that would corresponding to a systematic code.
   (c) List all the codewords.
   (d) Find the minimum Hamming distance of the code, and the error correcting capability (i.e., t such that it is a t-error correcting code).
   (e) Assume the all 1's vector is received: $r = 11 \cdots 1$. Compute the syndrome; the corresponding coset; the coset leader (if there is a tie, pick one); use the error vector determined this way to form the corrected code, and extract the decoded data bits.
   (f) Verify that n, k, t for this code satisfy the Hamming bound.

2. The following is used as the generator polynomial of a $(15, k)$ systematic cyclic code:

$$g(x) = x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1$$

   In this problem, you can perform long division in either polynomial or binary form, but you must show work.

   (a) Specify k.
   (b) Find the parity check matrix, h.
   (c) Generate the codeword corresponding to the data vector $d = 1010 \cdots$ (i.e., alternating 10 bits).
   (d) For the given codeword, make errors in the second and last bits to form a receive vector. Compute the checksum bits