Simon Yoon
ECE300 HW #8

1) a) Parity check matrix is n-k by n

$n=7$

$7-k=4$ (7, 3)

$k=3$

b) $H=[P^T|I]$

$P^T=\begin{bmatrix}0&1&1&0\\1&1&0&0\\1&0&1&0\end{bmatrix}$

$P=\begin{bmatrix}0&1&1&0\\1&1&0&0\\1&0&1&0\end{bmatrix}$

$G=[I|P]$

$G=\begin{bmatrix}1&0&0&0&1&1&0\\0&1&0&0&1&1&0\\0&0&1&0&1&0&1\end{bmatrix}$

c) $000 \rightarrow 0000000$

$001 \rightarrow 1000110$

$010 \rightarrow 0101110$

$011 \rightarrow 1000110 \rightarrow 1101000$

0101110

1101000

$100 \rightarrow 0011011$

$101 \rightarrow 0011011 \rightarrow 1011101$

$+1000110$

1011101

$110 \rightarrow 0011011 \rightarrow 0110101$

$+0101110$

0110101

$111 \rightarrow 1000110 \rightarrow 1110011$

0101110

$+0011011$

1110011

d) d_{min} = min hamming weight of nz codewords

$d_{min} = 3$

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$t = 1$

e) $S = rH^T$

$= [1111111] \begin{bmatrix} 0&1&1&0 \\ 1&1&1&0 \\ 1&0&1&1 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} = [1100]$

$S = 1100$

Slepian's Array:

S	coset			
1100	0001100	1001010	0100010	1100100
	0010111	1010001	0111001	1111111

$e = 0100010$

$r = c + e$

$1111111 = c + 0100010$

$c = 1011101$; $d = 101$

f) since $t=1$:

$$\binom{7}{0} + \binom{7}{1} \leq 2^{n-k}$$

$$1 + n \leq 2^{n-k}$$

$1 + 7 \leq 2^{7-3}$ Hamming Bound ✓

$8 \leq 16$

2. a) degree(g) = n − k

$10 = 15 - k$

$k = 5$

b) $h(x) = \frac{x^n + 1}{g(x)}$

$x^{10}+x^9+x^5+x^4+x^2+x+1 \;\big|\; x^{15}+1$

$x^5 + x^3 + x + 1$

$x^{15} + x^{13} + x^{10} + x^9 + x^7 + x^6 + x^5$

$x^{13} + x^{10} + x^9 + x^7 + x^6 + x^5$

$x^{13} + x^{11} + x^8 + x^7 + x^5 + x^4 + x^3$

$x^{11} + x^{10} + x^9 + x^8 + x^6 + x^4 + x^3$

$x^{11} + x^9 + x^6 + x^5 + x^3 + x^2 + x$

$x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1$

$x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1$

0

$h(x) = x^5 + x^3 + x + 1$

c) $d = 10101$

$d = x^4 + x^2 + 1$

$x^{10}+x^8+x^5+x^4+x^2+x+1 \;\big|\; x^{14}+x^{12}+x^{10}$

$x^4 + 1$

$x^{14} + x^{12} + x^9 + x^8 + x^6 + x^5 + x^4$

$x^{10} + x^9 + x^8 + x^6 + x^5 + x^4$

$x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1$

$r(x) = x^9 + x^6 + x^2 + x + 1$

$r(x) = \frac{x^{n-k} d(x)}{g(x)}$

$c = x^{14} + x^{12} + x^{10} + x^9 + x^6 + x^2 + 1$

101011001000111

d) change second, last bit

r = 0010110010000101

Checksum is last 10 digits

Checksum = 1001000101