Rayleigh Distribution

$$f(x) = \frac{x}{\alpha^2} e^{\frac{-x^2}{2\alpha^2}}$$

ML Estimator

$$f(x_i / \alpha) = \prod_{i=1}^{n} f(x_i, \alpha)$$

★Set $\alpha^2 = \theta$, ln( ) both sides★

$$\ln(f(x_i/\theta)) = \sum_{i=1}^{n} \ln\left(\frac{x}{\theta} e^{\frac{-x^2}{2\theta}}\right)$$

$$\ln(f(x_i/\theta)) = \sum_{i=1}^{n} \ln(x) - \ln(\theta) + \left(\frac{-x_i^2}{2\theta}\right)$$

★ take derivative, set $=0$, solve for θ ★

$$0 = \sum_{i=1}^{n} \left(-\frac{1}{\theta} + \frac{x_i^2}{2\theta^2}\right)$$

$$0 = -\frac{n}{\theta} + \sum_{i=1}^{n} x_i^2 \left(\frac{1}{2\theta^2}\right)$$

$$\theta = \frac{1}{2n} \sum_{i=1}^{n} x_i^2$$

★ substitute alpha back in ★

$$\boxed{\alpha = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} x_i^2}}$$

Exponential Distribution

$$f(x) = \lambda e^{-\lambda x}$$

ML estimator (same process as before)

$$f(x_i / \lambda) = \prod_{i=1}^{n} f(x_i, \lambda)$$

$$\ln f(x_i / \lambda) = \sum_{i=1}^{n} \ln\left(\lambda e^{-\lambda x}\right)$$

$$\ln f(x_i / \lambda) = \sum_{i=1}^{n} \ln(\lambda) - \lambda x_i$$

$$0 = \sum_{i=1}^{n} \frac{1}{\lambda} - x_i$$

$$-\frac{n}{\lambda} = -\sum_{i=1}^{n} x_i$$

$$\lambda = \frac{n}{\sum_{i=1}^{n} x_i}$$