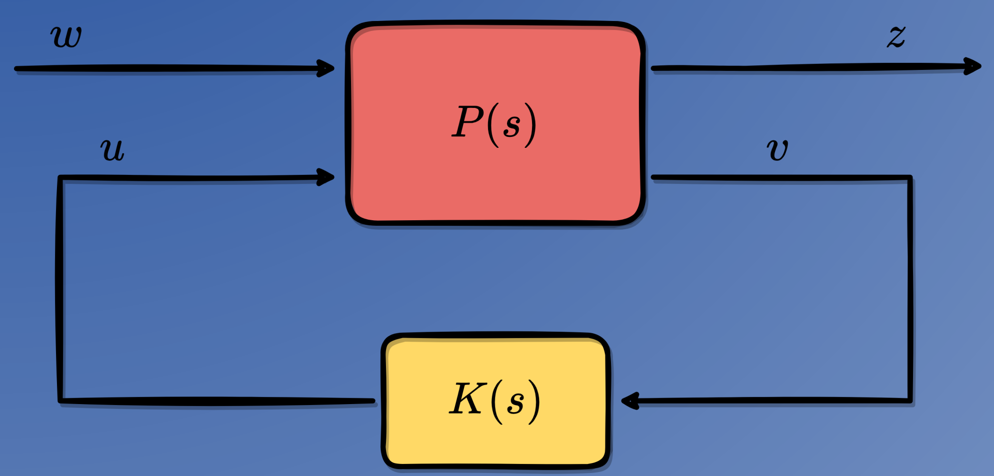
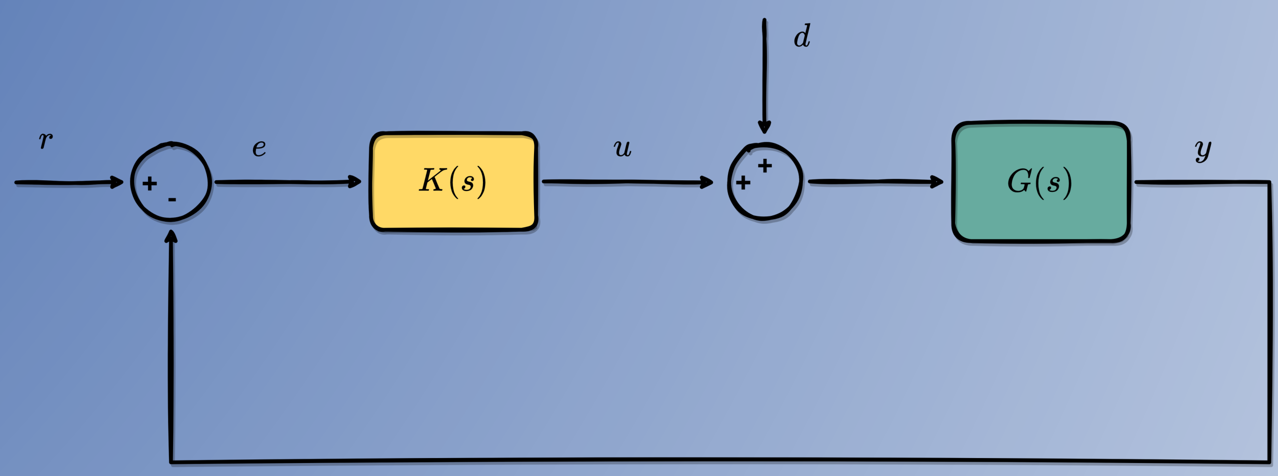
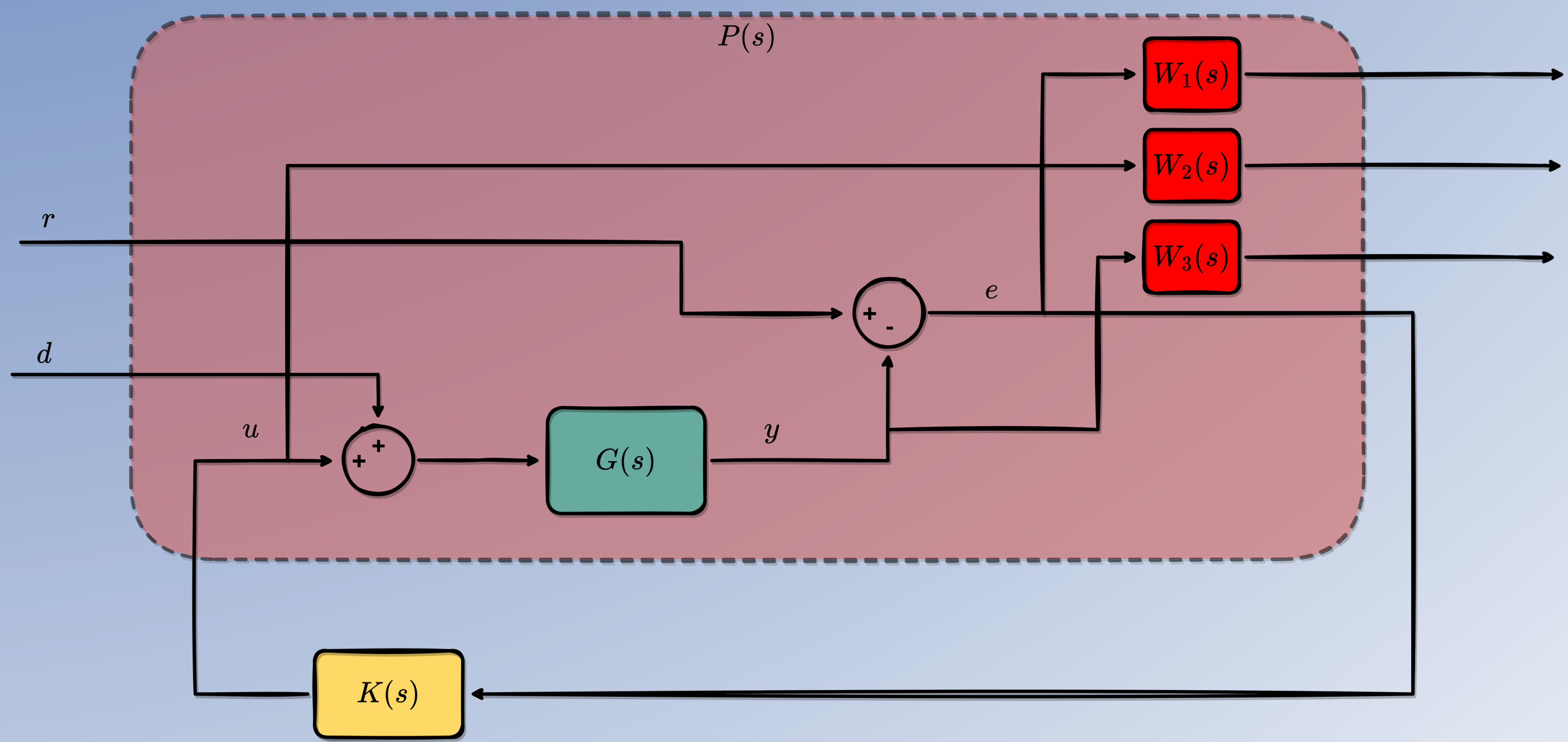
H Infinity Position Control





$$\begin{bmatrix} z \\ v \end{bmatrix} = P(s) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$



Code

https://github.com/simorxb/H-Infinity-Position-Control

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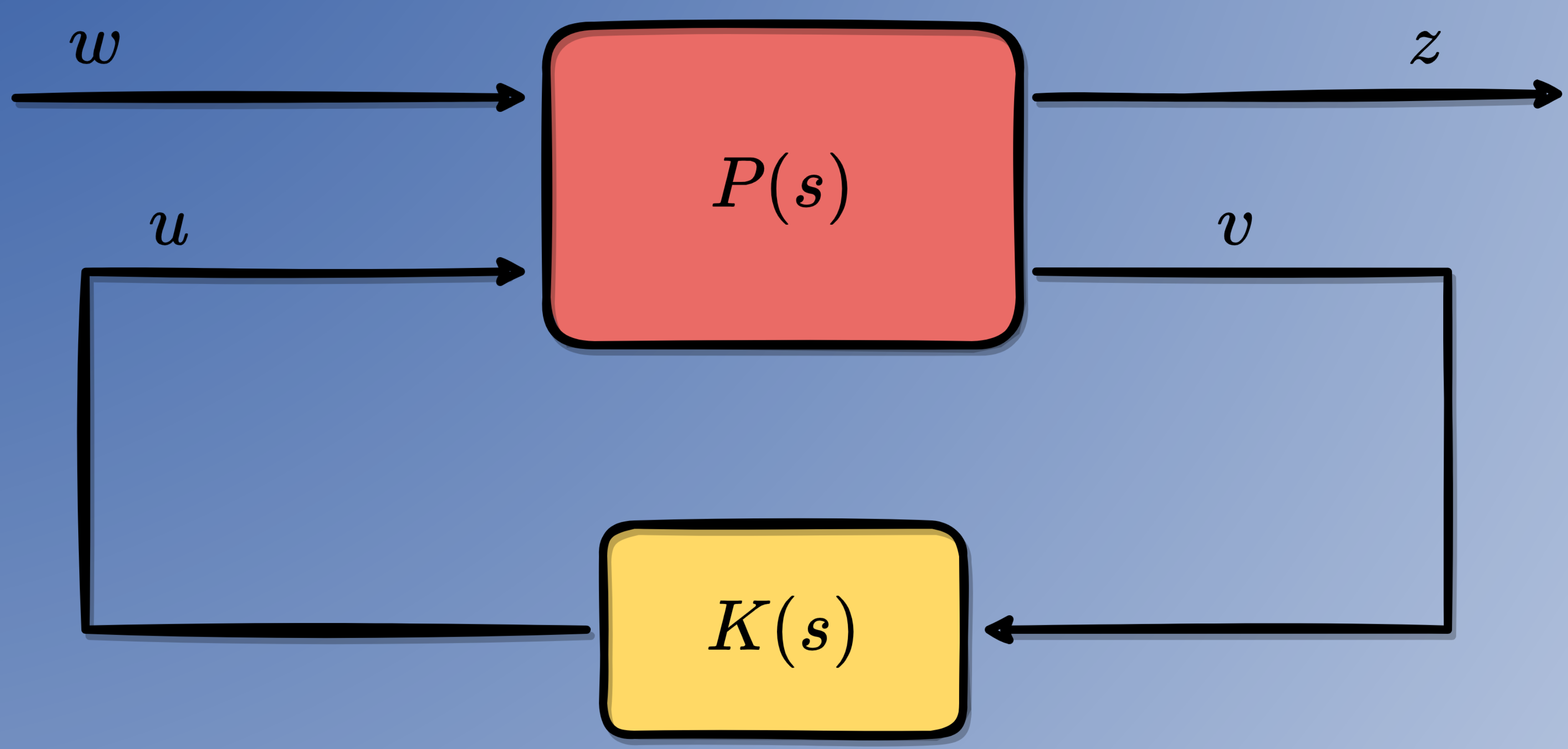
Control Problem Formulation

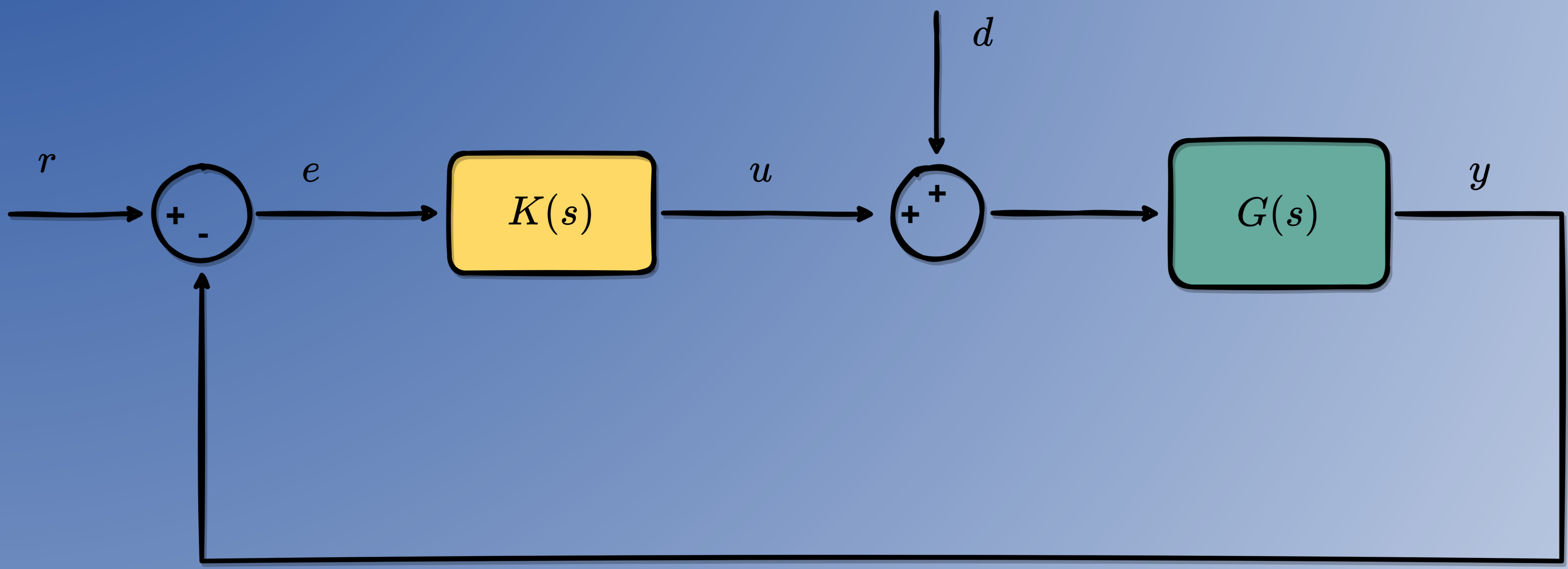


$$\begin{bmatrix} z \\ v \end{bmatrix} = P(s) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

w: Exogenous input (references signal and disturbances)
u: Control input
z: Error signals (that should be minimised)
v: Measured variables (used to control the system)

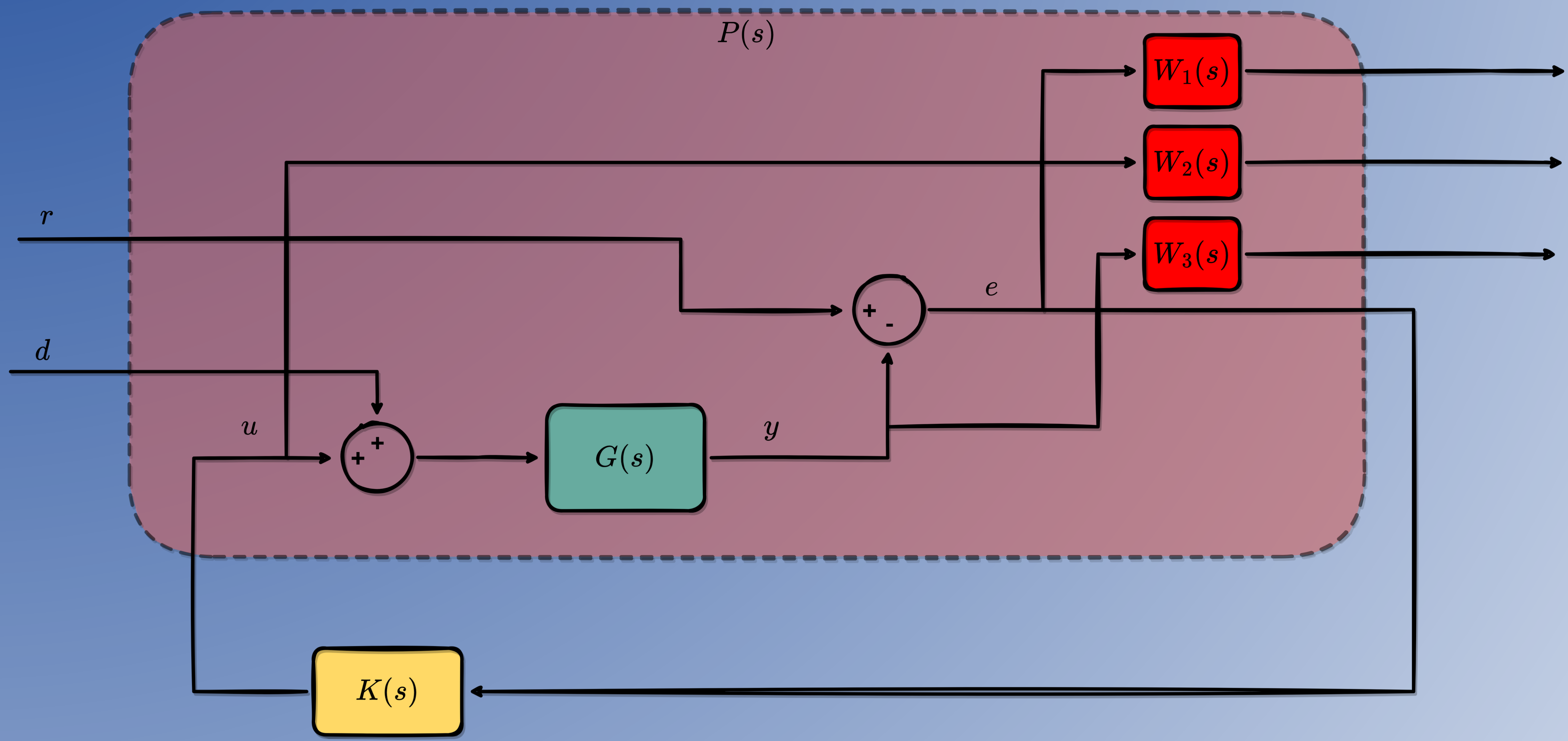
The objective of H_∞ control design is to find a controller $K(s)$ that minimises the H_∞ norm of the close loop system between w and z.

Standard Feedback Control Architecture



This is a typical way to represent a feedback control system. How can we express it in a framework that allows us to find $K(s)$ using the H_∞ optimisation?

H Infinity Optimisation Framework



$$w = \begin{bmatrix} r \\ d \end{bmatrix} \qquad z = \begin{bmatrix} W_1(s)e \\ W_2(s)u \\ W_3(s)y \end{bmatrix}$$

$W_1(s)$, $W_2(s)$ and $W_3(s)$ are the optimisation weight functions.

$$v = e$$
$$e = r - y$$

$$P(s) = \begin{bmatrix} W_1(s) & -W_1(s)G(s) & -W_1(s)G(s) \\ 0 & 0 & W_2(s) \\ 0 & W_3(s)G(s) & W_3(s)G(s) \\ 1 & -G(s) & -G(s) \end{bmatrix}$$

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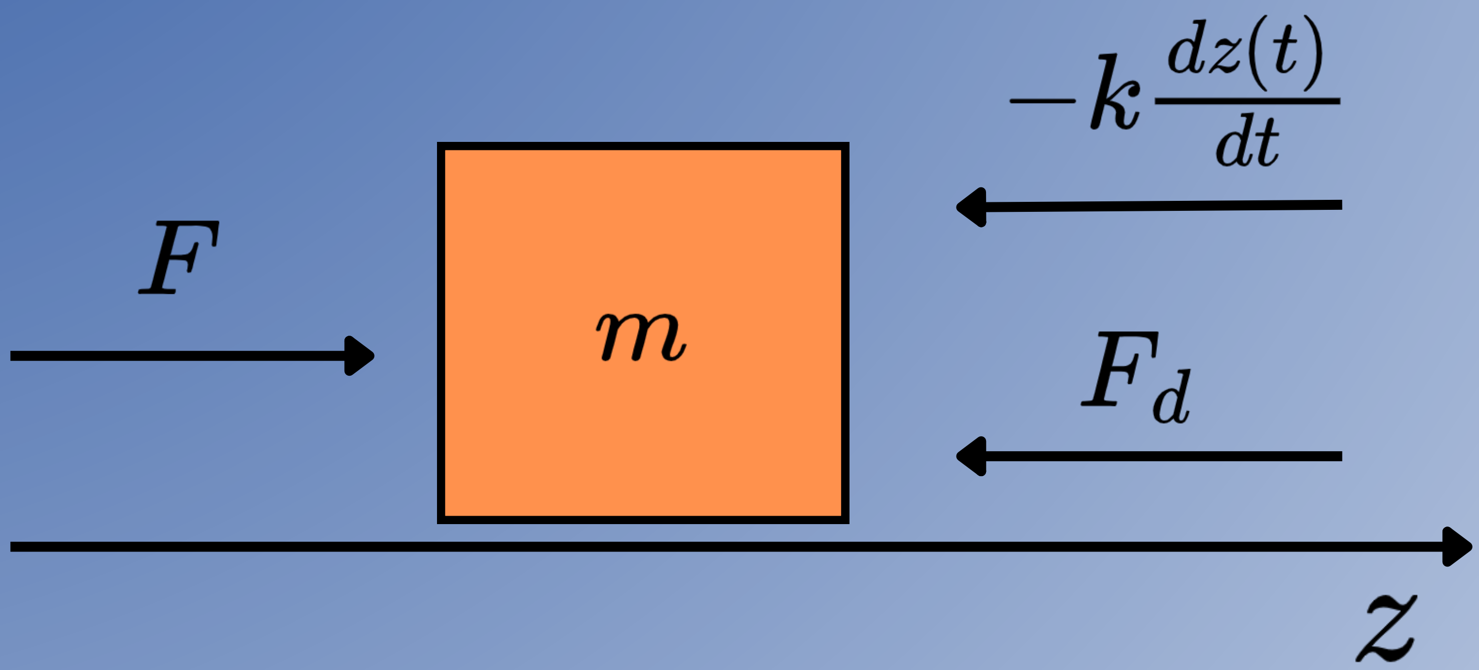
Plant



$$m\frac{d^2z(t)}{dt^2} = F - k\frac{dz(t)}{dt} - F_d$$

$$m = 10kg$$
$$k = 0.5\frac{Ns}{m}$$

Optimisation

The optimisation was run using the h_inf command in Scilab. The weight functions chosen are:

$$W_1(s) = \frac{100}{s+1}$$

$$W_2(s) = \frac{0.1s}{s+1}$$

$$W_3(s) = \frac{0.1s}{s+1}$$

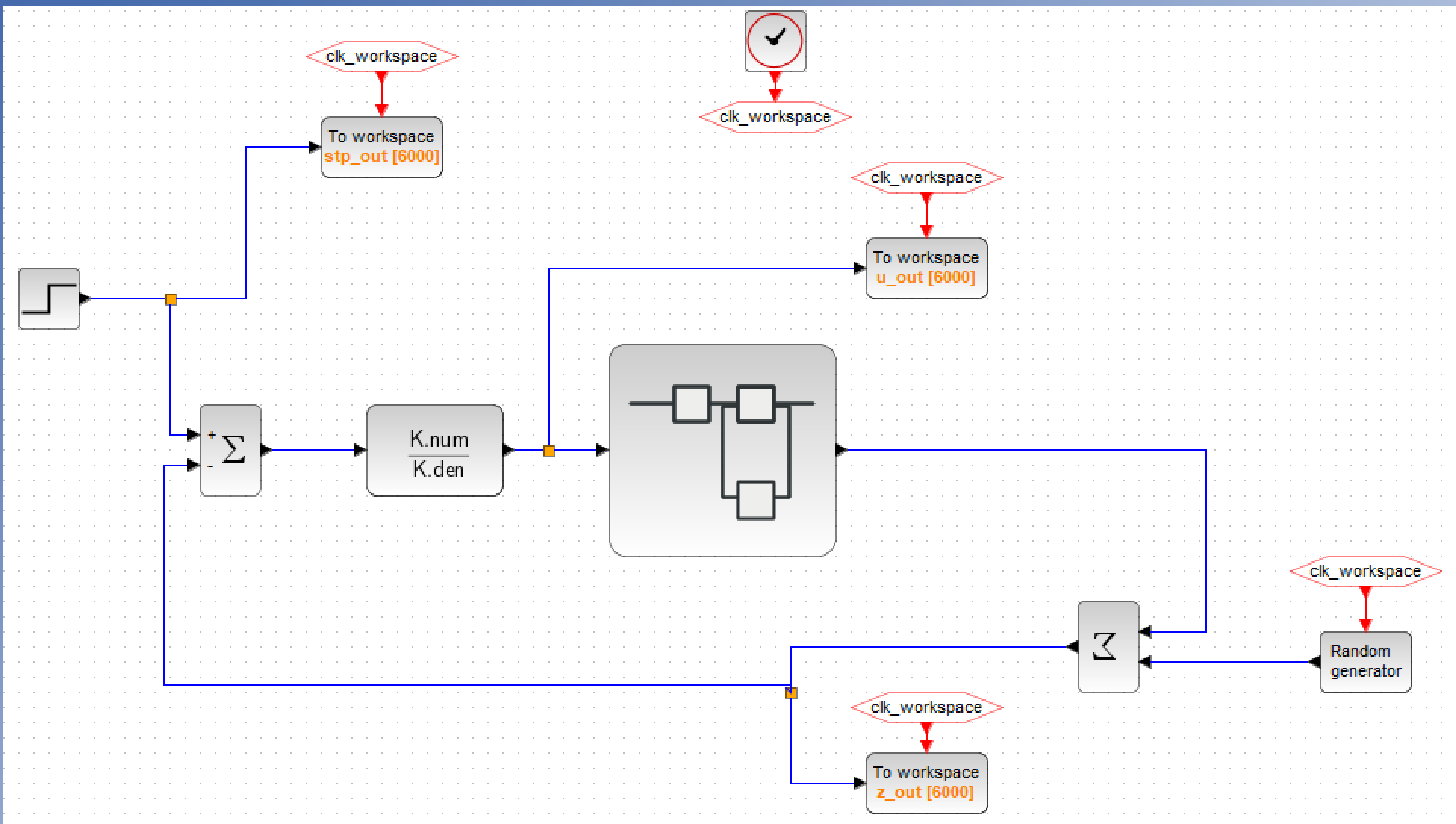
The idea is to have W_1 dominant in the low frequency band (at steady state we want to minimise the control error) and have W_2 and W_3 dominant at high frequency, where we want to minimise chattering and have measurement disturbance rejection.

We obtain:

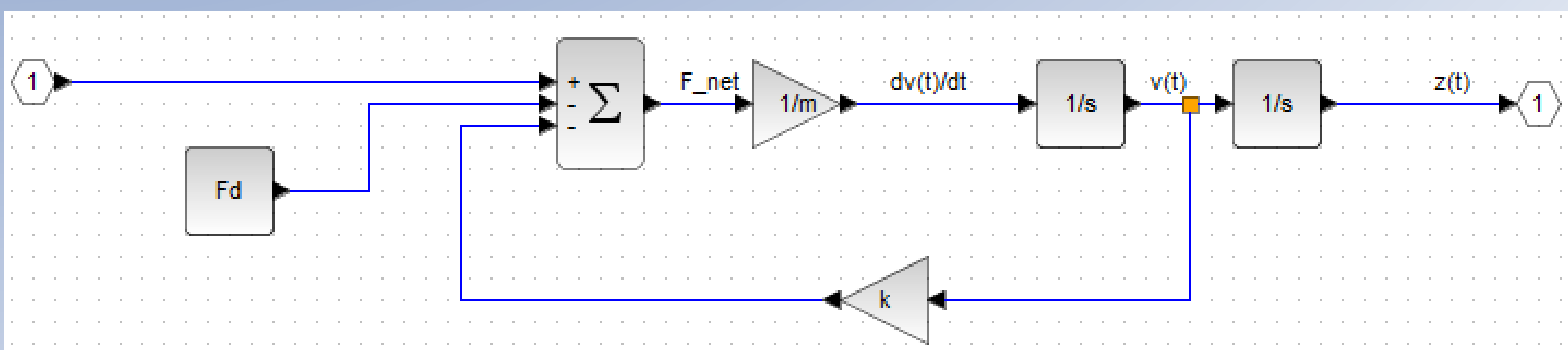
$$K(s) = \frac{30.8+143s+262s^2}{8.07+9.07s+s^2}$$

Xcos

Top Level



Plant



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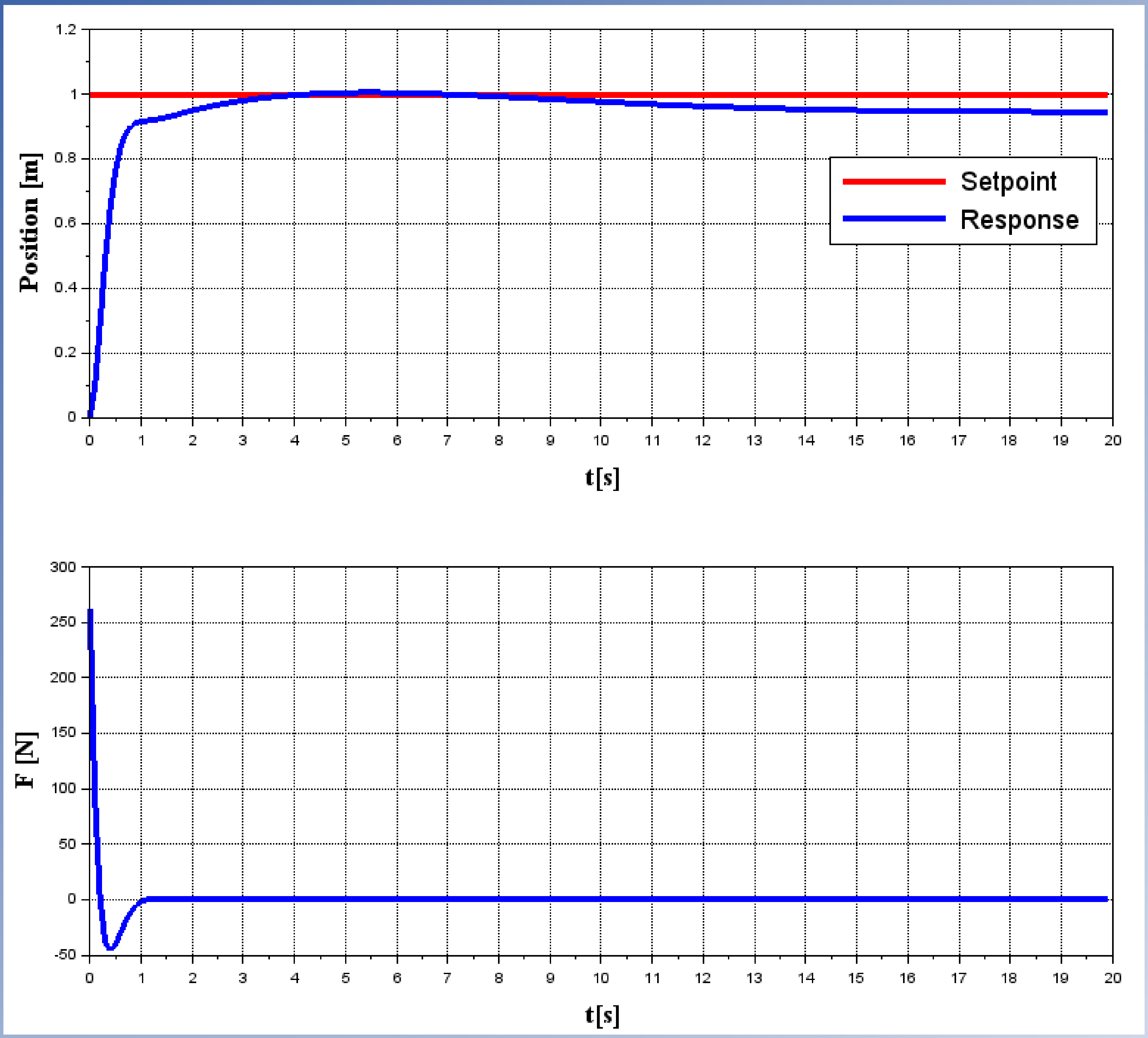
Simulation



The simulation is carried out with the disturbance $F_d = 0.2\ N$. As the controller does not have integral action, we have steady state error non-zero.

There are various approaches to force an integrator in the controller, which are topics for future posts.

PID Control

Interested in PID Control? Check out my digital course:

https://simonebertoni.thinkific.com/

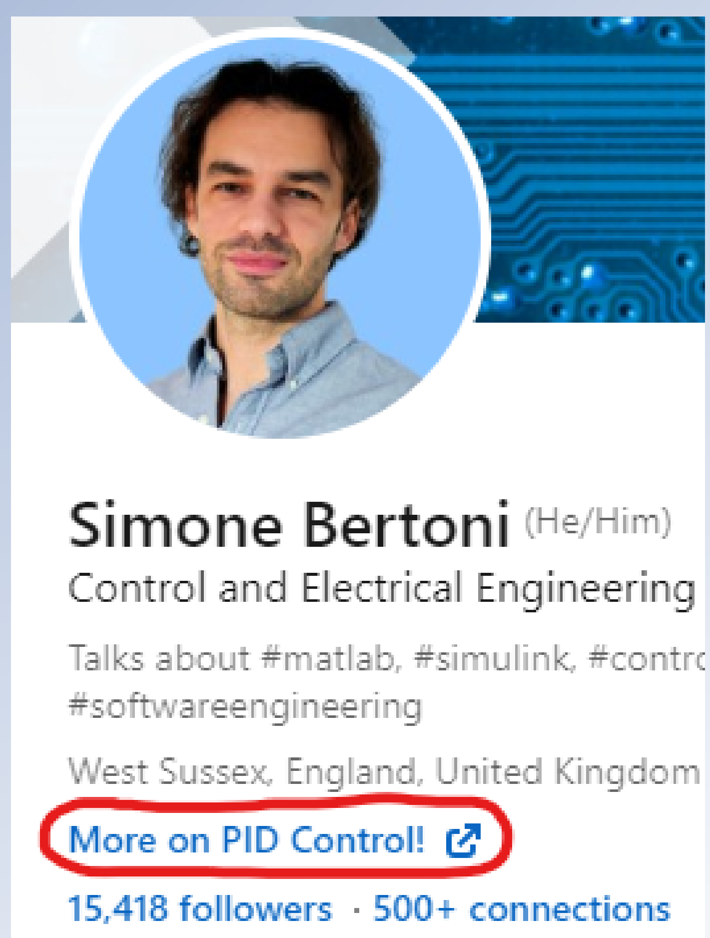
★★★★★ Very helpful and practical

★★★★★ A different way to learn PID !

★★★★★ Very good sharing of experience

★★★★★ Great course

Find the link here!



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