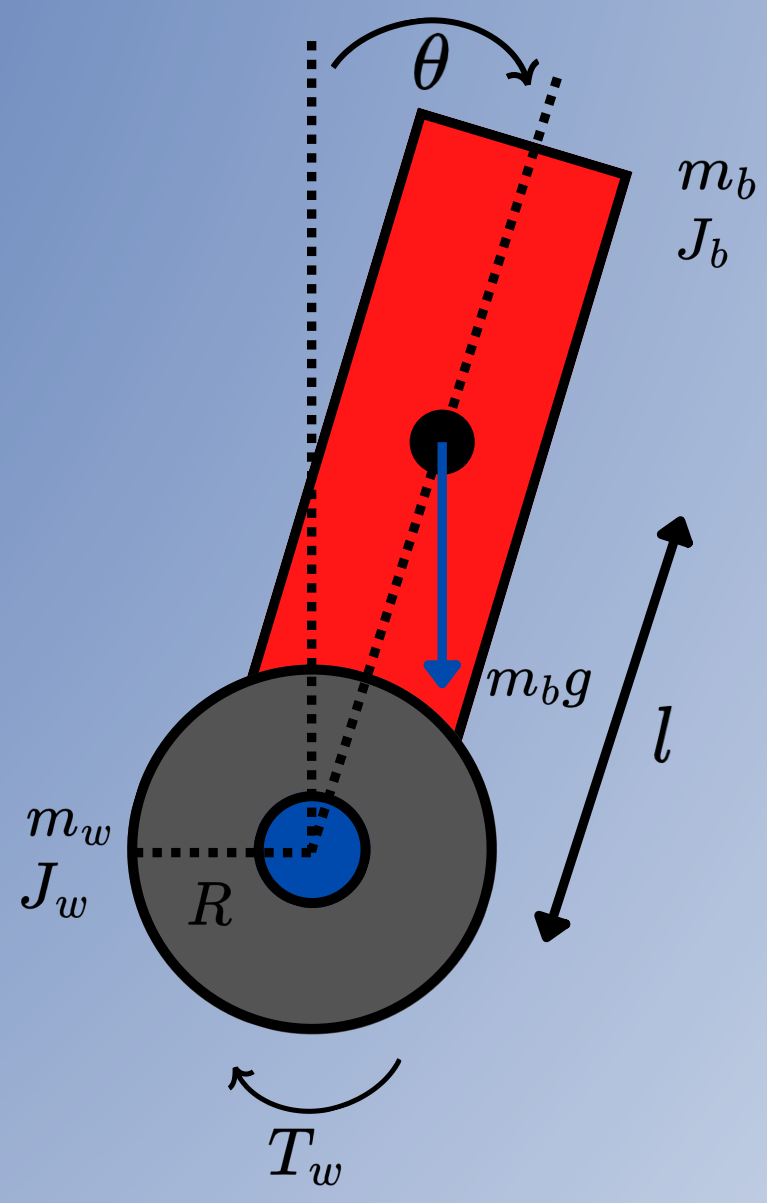
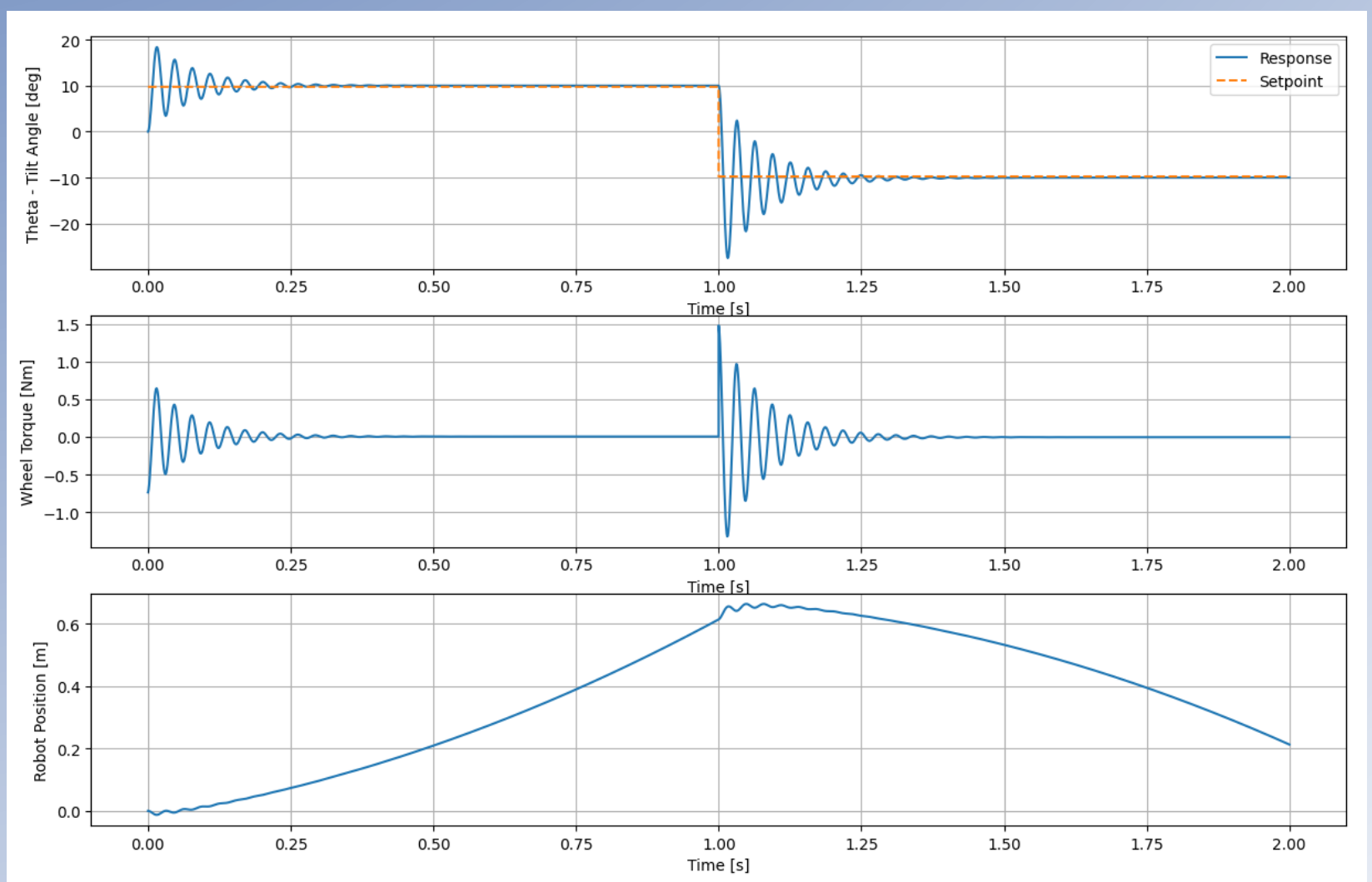
Self-Balancing Robot Modeling & Control





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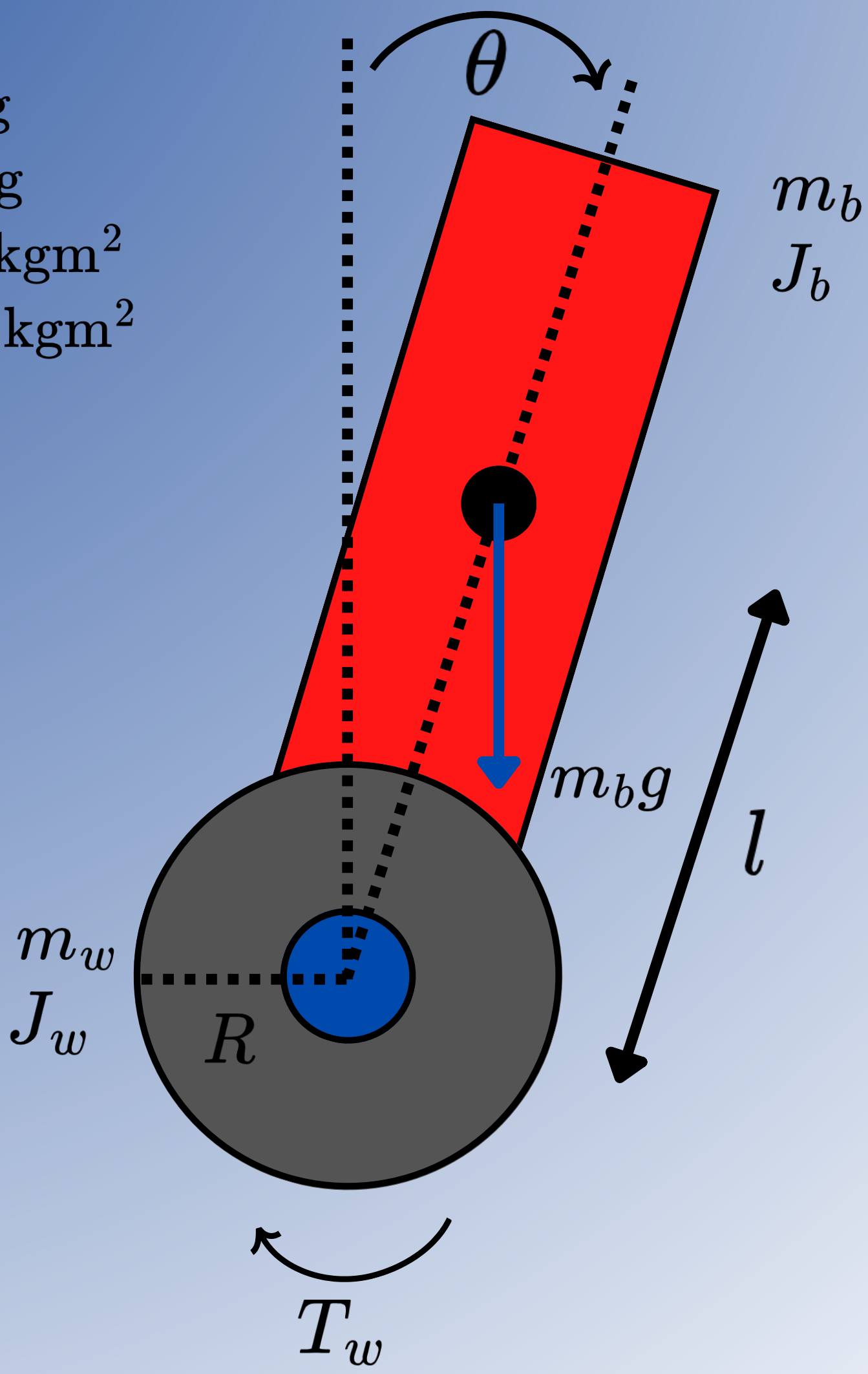
Code
https://github.com/simorxb/self-balancing-robot

Robot Diagram

$m_b = 0.2445 \text{ kg}$
$m_w = 0.0343 \text{ kg}$
$J_b = 0.000255 \text{ kgm}^2$
$J_w = 0.000021 \text{ kgm}^2$
$R = 0.035 \text{ m}$
$l = 0.035 \text{ m}$



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Equations of Motion

https://fkeng.blogspot.com/2019/03/theory-and-design-of-two-wheels-self.html

$$\ddot{x} = \frac{m_b l \sin(\theta)\dot{\theta}^2 - \frac{(m_b l)^2 g \cos(\theta)\sin(\theta)}{m_b l^2 + J_b} + 2T_w\left(\frac{1}{R} + \frac{m_b l \cos(\theta)}{m_b l^2 + J_b}\right)}{m_b + 2\left(m_w + \frac{J_w}{R^2}\right) - \frac{(m_b l \cos(\theta))^2}{m_b l^2 + J_b}}$$

$$\ddot{\theta} = \frac{m_b l \sin(\theta)\left(g + \frac{m_b l \cos(\theta)\dot{\theta}^2}{m_b + 2\left(m_w + \frac{J_w}{R^2}\right)}\right) - 2\left(R + \frac{m_b l \cos(\theta)}{m_b + 2\left(m_w + \frac{J_w}{R^2}\right)}\right)\frac{T_w}{R}}{J_b + m_b l^2 - \frac{(m_b l \cos(\theta))^2}{m_b + 2\left(m_w + \frac{J_w}{R^2}\right)}}$$

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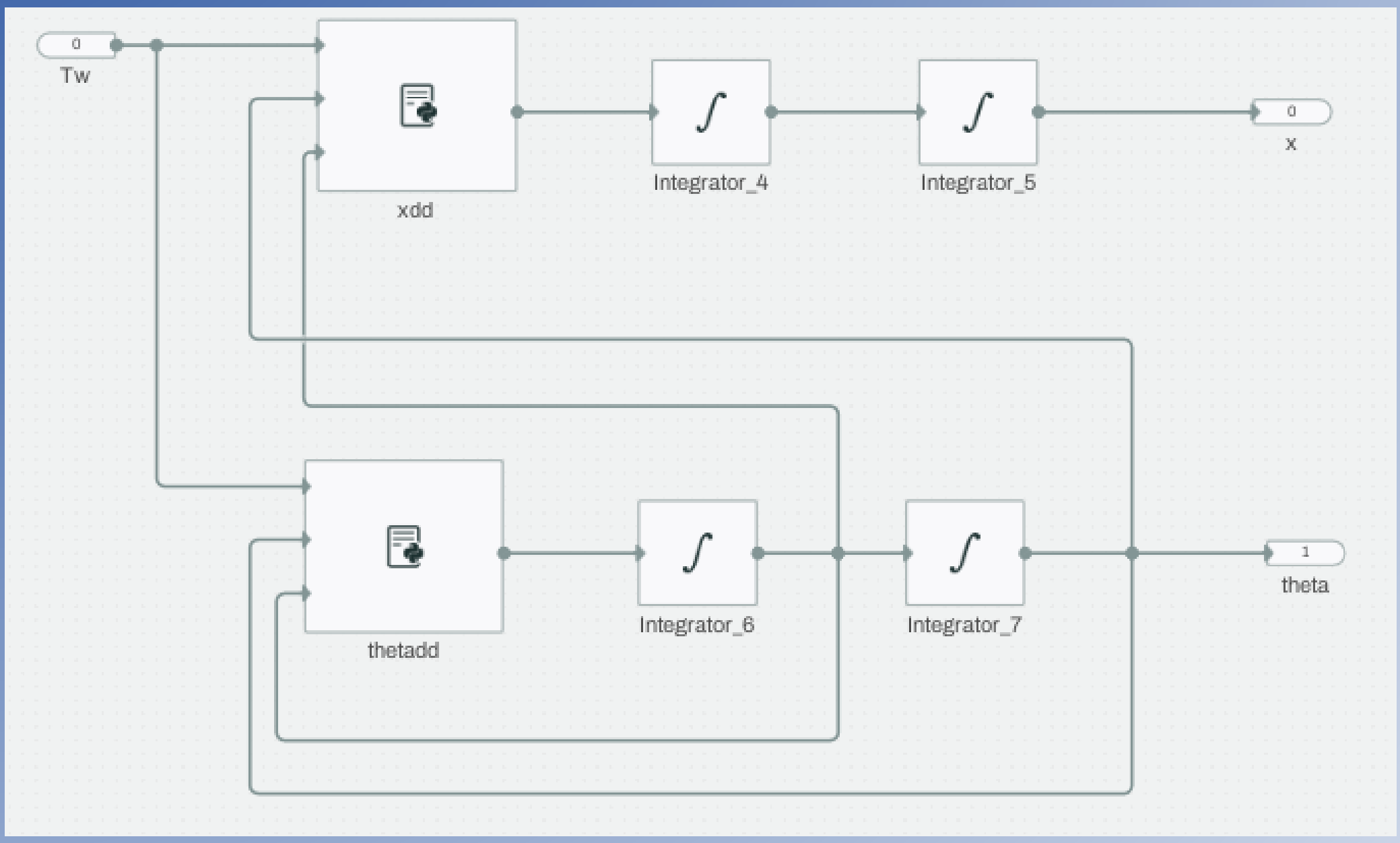
Collimator – Model



Self-Balancing Robot / xdd

Step | Init | Finalize

```
import math
xdd = (mb*l*math.sin(theta)*(thetad**2) - ((mb*l)**2)*g*math.cos(theta)*math.sin(theta)/(mb*(l**2)+Jb) +
2*Tw*(1/R+mb*l*math.cos(theta)/(mb*(l**2)+Jb)))/(mb + 2*(mw+Jw/(R**2)) - ((mb*l*math.cos(theta))**2)/(mb*(l**2) + Jb))
```

Self-Balancing Robot / thetadd

Step | Init | Finalize

```
import math
thetadd = (mb*l*math.sin(theta)*(g + mb*l*math.cos(theta)*(thetad**2)/(mb + 2*(mw + Jw/(R**2)))) -
2*(R + mb*l*math.cos(theta)/(mb + 2*(mw + Jw/(R**2)))*Tw/R)/(Jb + mb*(l**2) - ((mb*l*math.cos(theta)))**2/(mb + 2*(mw + Jw/(R**2))))
```

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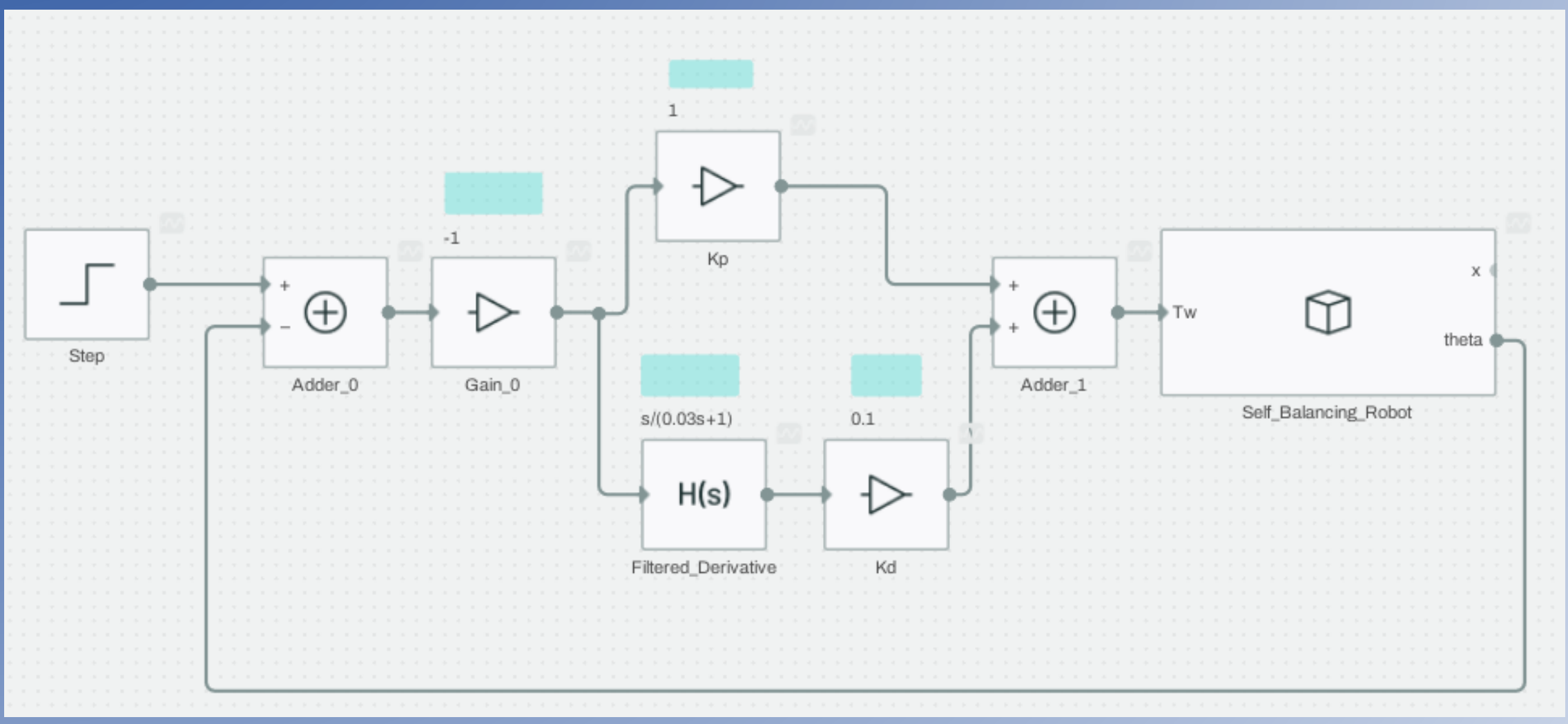
Collimator – PD Control



$$PD \rightarrow C(s) = k_p + k_d \frac{s}{\tau s + 1}$$

$$k_p = 1$$
$$k_d = 0.1$$
$$\tau = 0.03$$

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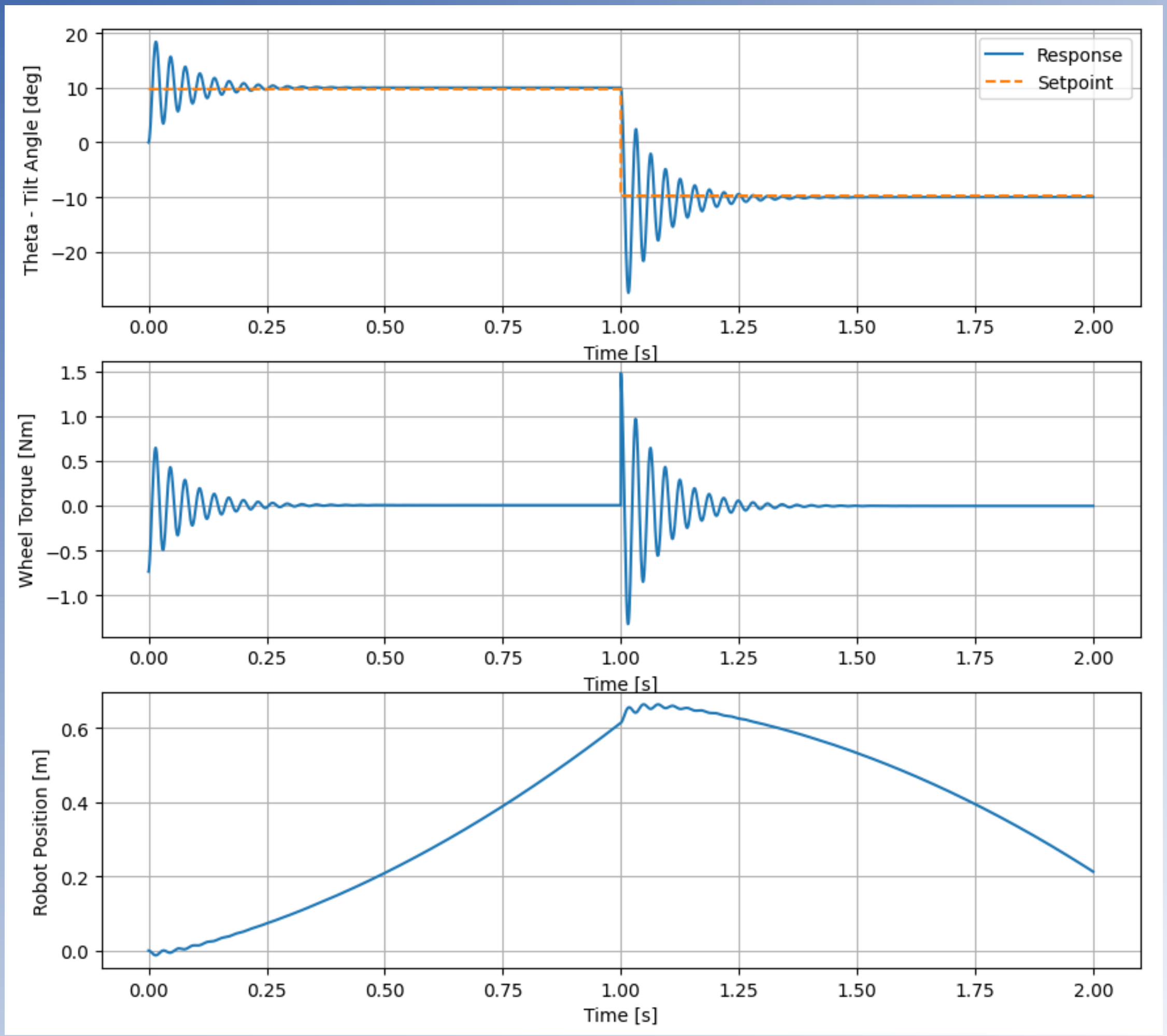
Simulation



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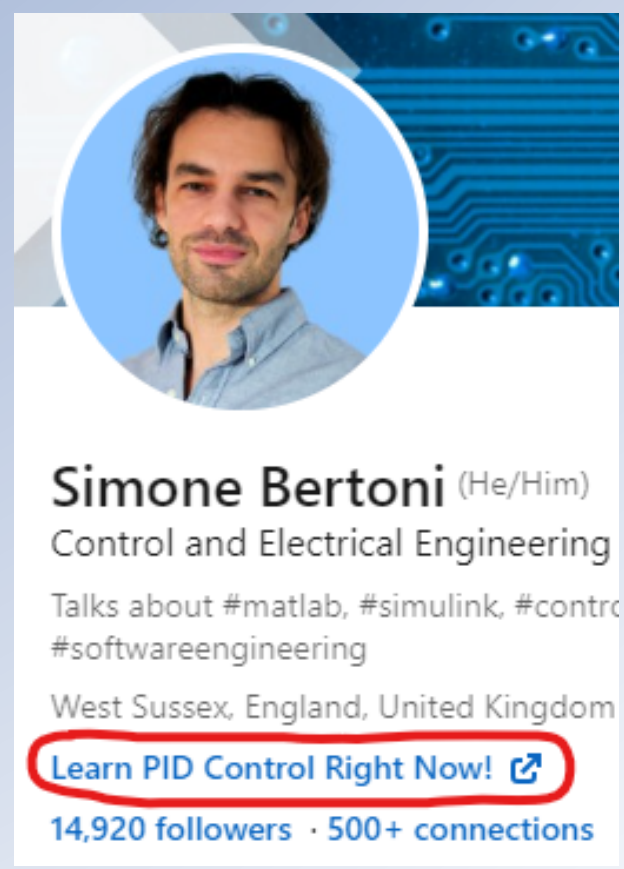
PID Control

Interested in PID Control? Check out my digital course:

https://simonebertoni.thinkific.com/

Find the link here!



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