Loop-shaping design

A feedback control system configuration

Structure of the cascade controller to be designed

Time domain requirements translation: resume

A three stage procedure

Loop-shaping design

Steady-state controller gain design

Phase lead controller design

Phase lag controller design

Application to dc-motors position control design

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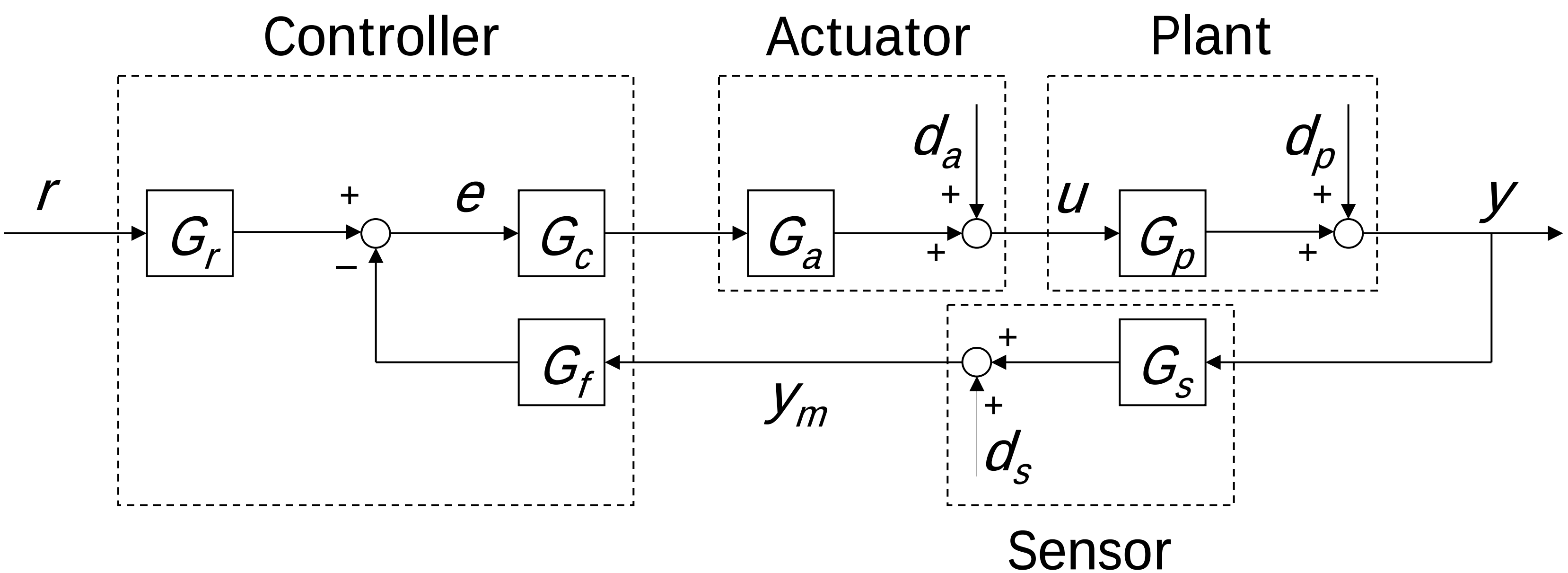
A feedback control system configuration

Output feedback control system configuration

The given parts of the configuration we are dealing with are:

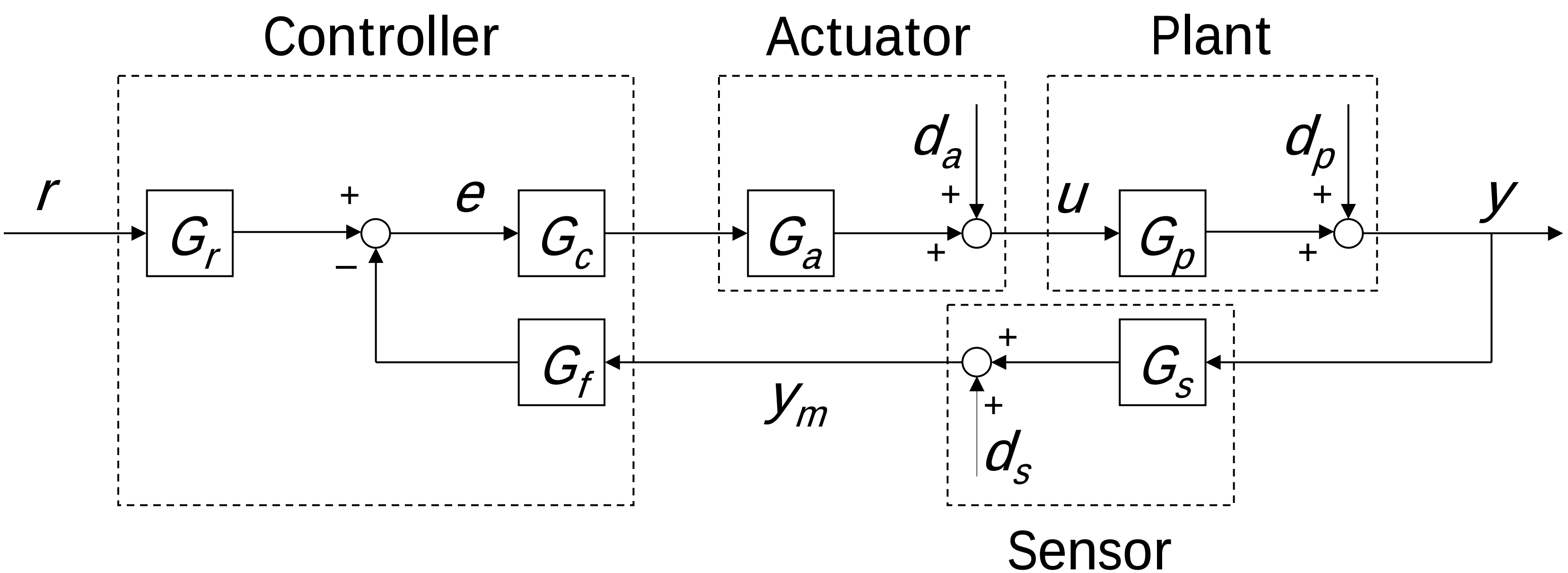
- **plant** G_p with plant disturbance d_p
- **actuator** G_a with actuator disturbance d_a
- **sensor** G_s with sensor noise d_s



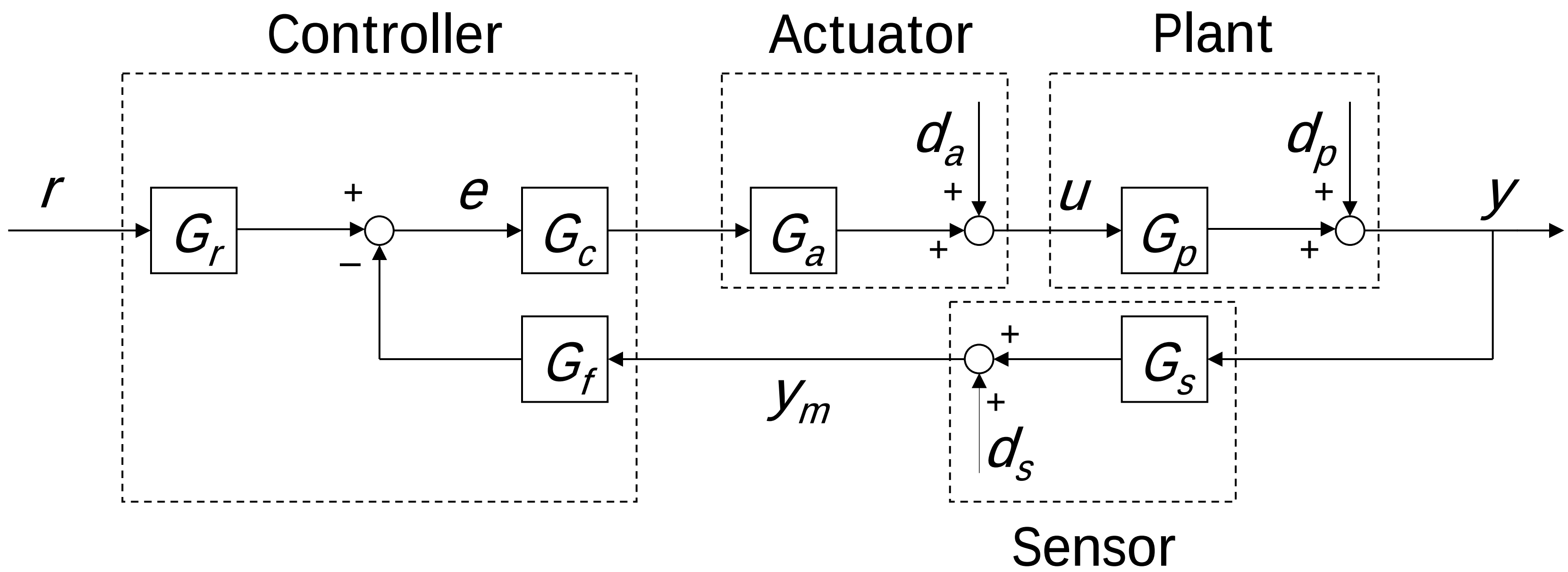
Output feedback control system configuration

The controller to be designed consists of:

- **prefilter** G_r (reference generator)
- **cascade controller** G_c
- **feedback controller** G_f (for 2 DOF or, if constant, for dc gain)



Output feedback control system configuration



Without loss of generality we assume that

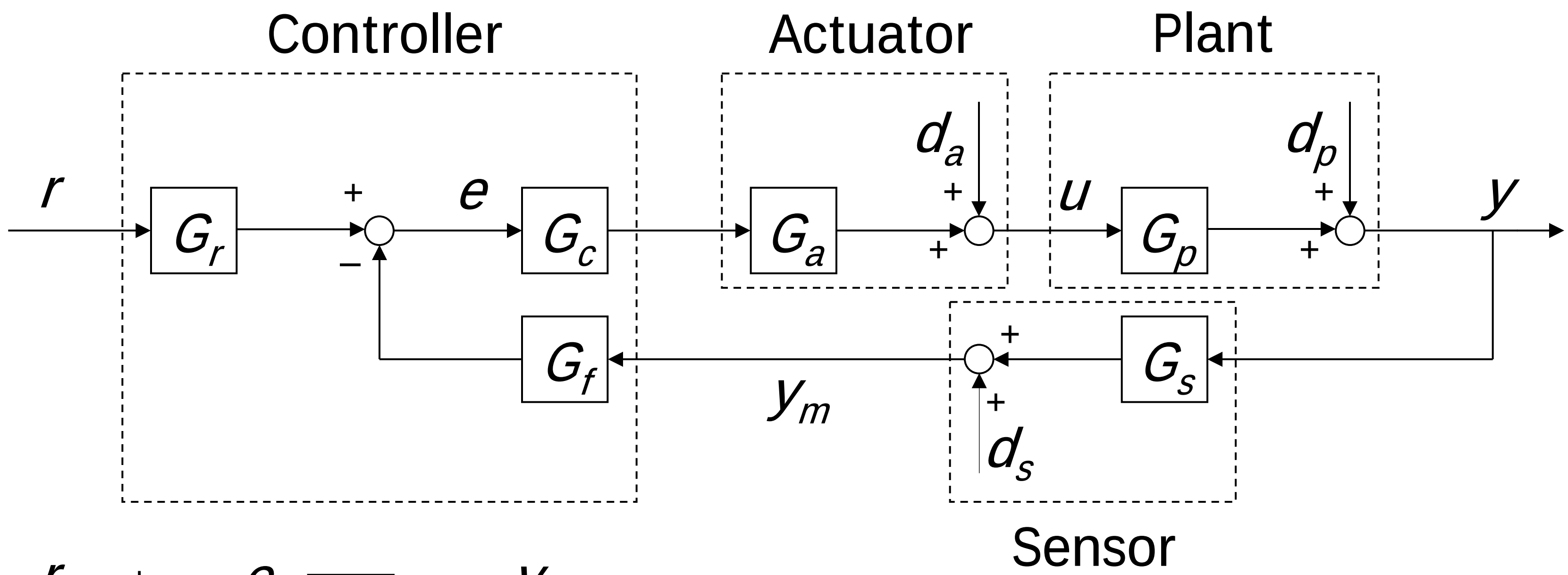
$G_r = 1$

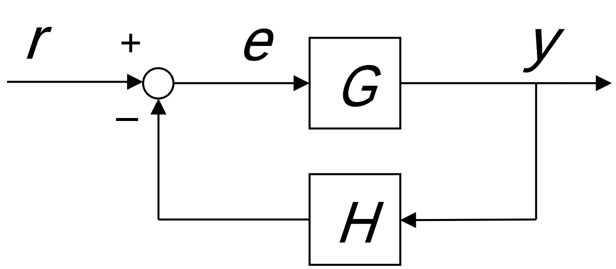
G_a = constant (large bandwidth because of fast dynamics)

G_f = constant (to handle the control system steady-state gain)

G_s = constant (large bandwidth because of fast dynamics)

Output feedback control system configuration





$$G = G_c G_a G_p = \frac{N_G}{D_G} \qquad H = G_s G_f = \frac{N_H}{D_H}$$

$G_r = 1$
$G_a =$ constant
$G_f =$ constant
$G_s =$ constant

Loop-shaping design

Structure of the cascade controller to be designed

Controller structure

- LTI controllers described by the transfer function

$$G_c(s) = \frac{K_c}{s^\nu} \prod_i \left(\frac{1 + \frac{s}{z_{di}}}{1 + \frac{s}{m_{di} z_{di}}} \right) \prod_j \left(\frac{1 + \frac{s}{m_{ij} p_{ij}}}{1 + \frac{s}{p_{ij}}} \right)$$

will be considered from now on.

- The controller has ν poles at $s = 0$ and its generalized steady-state gain is defined as:

$$\lim_{s \to 0} s^\nu G_c(s) = K_c$$

- The plant has p poles at $s = 0$ and its generalized steady-state gain is defined as:

$$\lim_{s \to 0} s^p G_p(s) = K_p$$

Controller structure

The part $\dfrac{K_c}{s^\nu}$ is designed from the steady state requirements

Next we will see how the remaining factors can be designed

$$\prod_i \left(\frac{1+\dfrac{s}{z_{di}}}{1+\dfrac{s}{m_{di} z_{di}}} \right) \prod_j \left(\frac{1+\dfrac{s}{m_j p_j}}{1+\dfrac{s}{p_j}} \right)$$

from the transient time domain requirements

Loop-shaping design

Time domain requirements translation: resume

Time domain requirements translation: resume

- The 2nd order prototype model

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

is used to translate the transient time domain requirements

$$\hat{s} \quad t_r \quad t_{s,\alpha\%}$$

into the relevant indices of the frequency response of the functions *T(s), S(s)* and *L(s)…*

Time domain requirements translation: resume

… in particular:

$$\hat{S} \rightarrow \begin{cases} T_p \rightarrow \text{resonance peak of } |T(j\omega)| \\ S_p \rightarrow \text{resonance peak of } |S(j\omega)| \end{cases}$$

$$\begin{matrix} t_r \\ t_{s,\alpha\%} \end{matrix} \rightarrow \omega_c \rightarrow \text{crossover frequency of } |L(j\omega)|$$

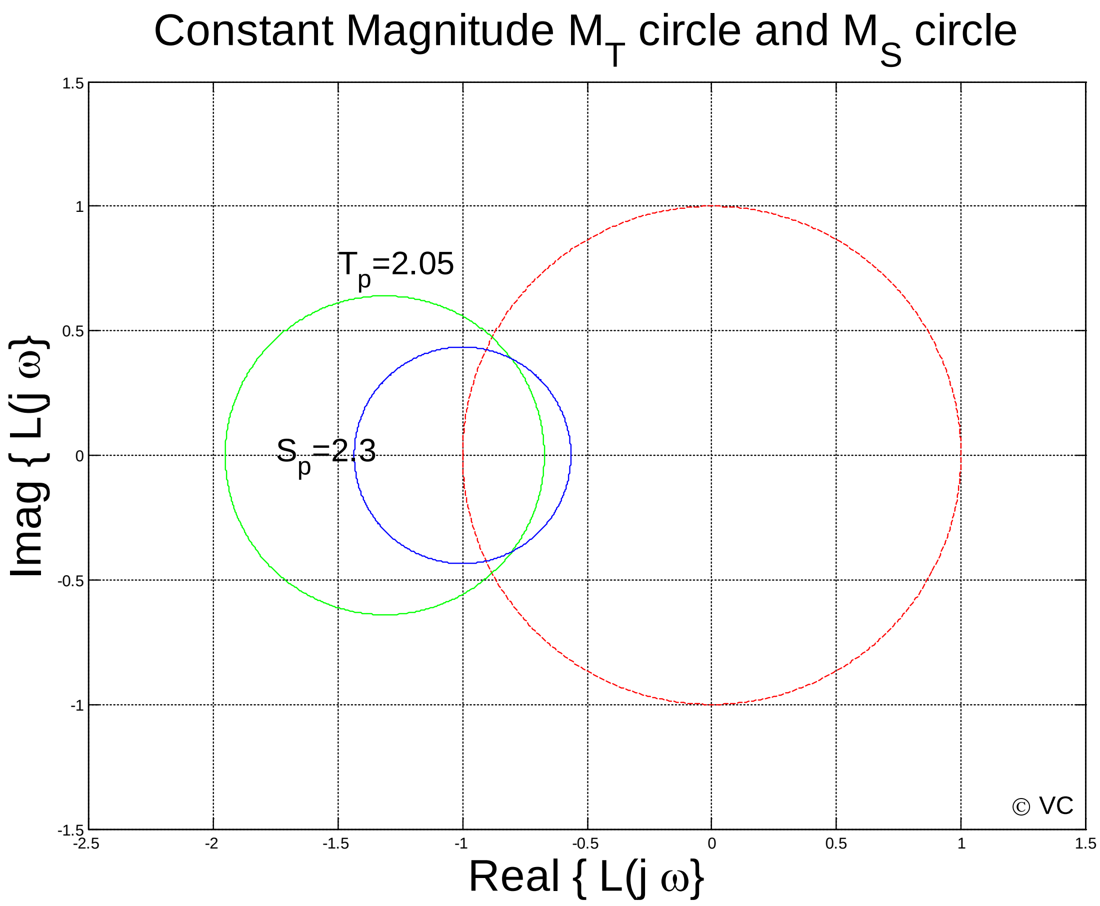
Time domain requirements translation: resume

It is easy to show that S_p and T_p are related to the gain and phase margins.

$$\mathrm{GM} \geq \frac{S_p}{S_p - 1}, \qquad \mathrm{PM} \geq 2\sin^{-1}\left(\frac{1}{2S_p}\right)$$

$$\mathrm{GM} \geq 1 + \frac{1}{T_p}, \qquad \mathrm{PM} \geq 2\sin^{-1}\left(\frac{1}{2T_p}\right)$$

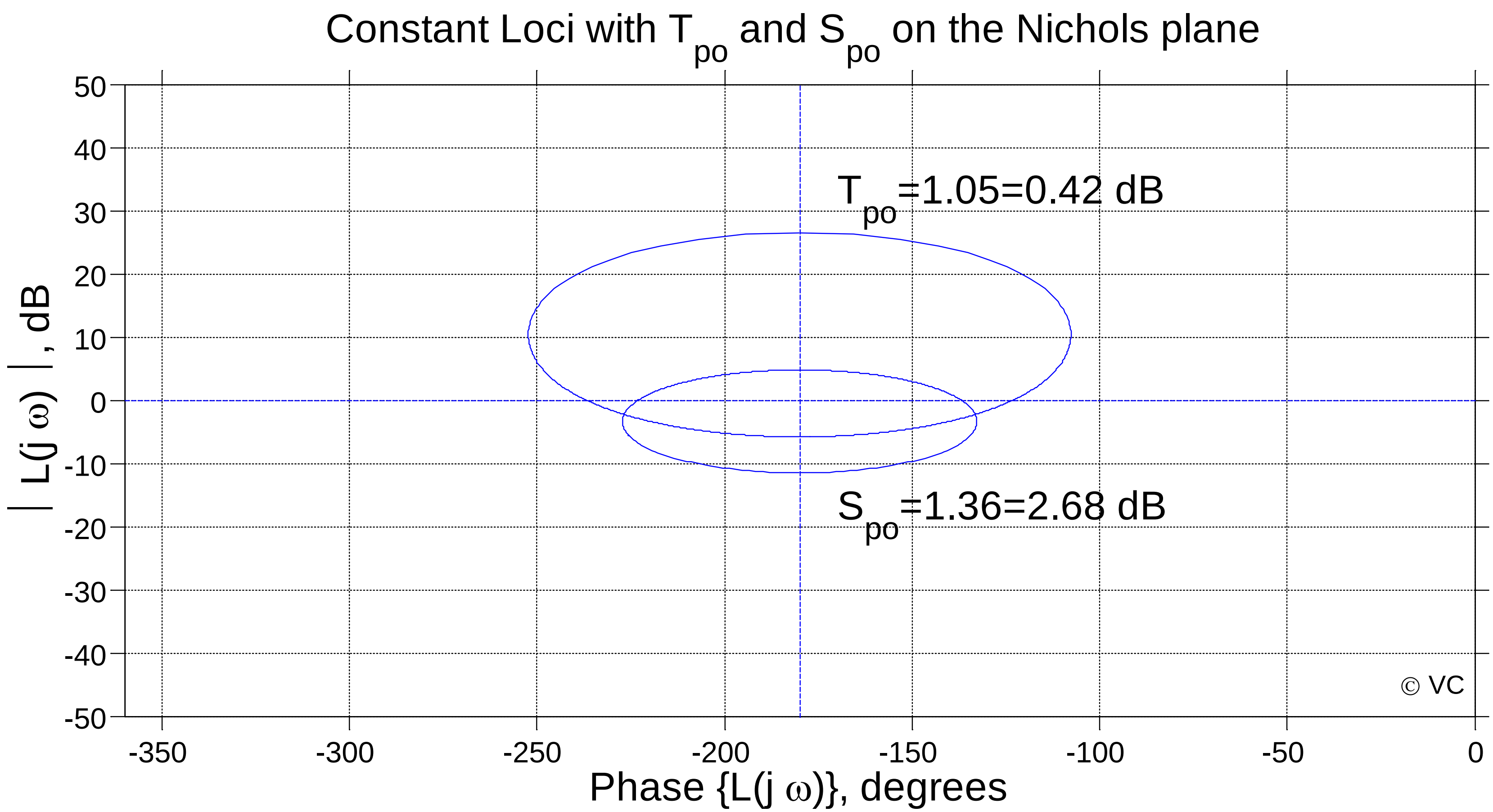
Time domain requirements translation: resume



Time domain requirements translation: resume

- The values of the resonance peaks T_p and S_p of $|T(j\omega)|$ and $|S(j\omega)|$ respectively obtained via the $\hat{s}$ requirement can be used to draw on the Nichols plane the corresponding constant magnitude loci
- Such loci can be considered as constraints which should not be violated by the Nichols plot of the frequency response of the loop function $L(j\omega)$

Time domain requirements translation: resume



Loop-shaping design

A three stage procedure controller design

A three stage procedure controller design

The controller design is accomplished through the following three stage procedure:

Stage 1: Steady-state and transient requirements translation

Stage 2: Controller design through loop-shaping tecniques

Stage 3: Performance analysis of the designed feedback control system, focusing the report on the assigned requirements. If some of the requirements are not fulfilled, go back to stage 2 and modify the designed controller accordingly.

Loop-shaping design

Steady-state controller gain design

Steady-state controller gain design

First, according to the control system type derived from the steady-state requirements, a suitable number of poles at the origin (ν) must be set in the controller.

Thanks to the stability Nyquist criterion, the correct sign of K_c is chosen.

Then, the Nichols plot of the loop function frequency response should be drawn on the phase-magnitude plane (the Nichols plane).

An increase of abs(K_c) shifts the Nichols plot upwards.

A decrease of abs(K_c) shifts the Nichols plot downwards.

Steady-state controller gain design

The abs(K_c) should be greater than the minimum value derived from the steady-state requirements.

The abs(K_c) should be less than the value which makes the Nichols plot of the loop function frequency response intersect the constant loci corresponding to the resonance peaks T_p and S_p .

When we move the abs(K_c), the shape of the loop function frequency response is not modified.

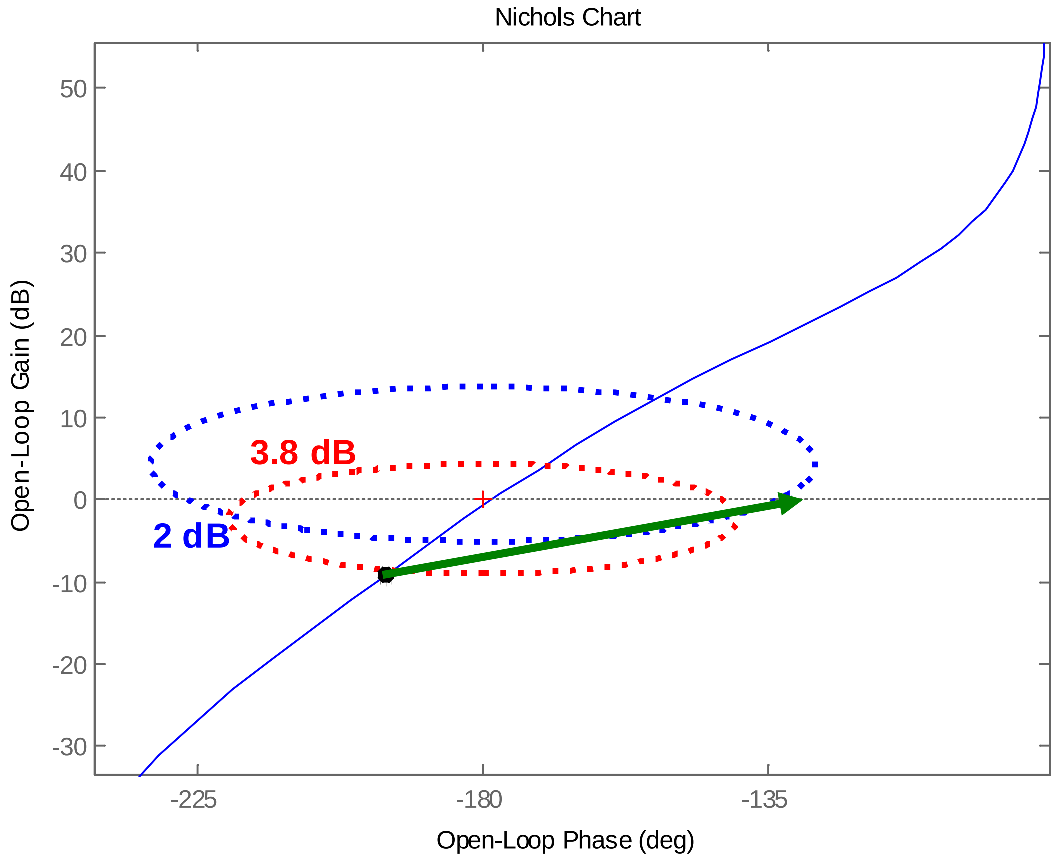
Higher values of abs(K_c) imply higher crossover frequency (faster response), decreased steady state errors to polynomial references and disturbances, decreased stability margin.

Lower values of abs(K_c) imply lower crossover frequency (slow response), increased steady state errors to polynomial references and disturbances, increased stability margin.

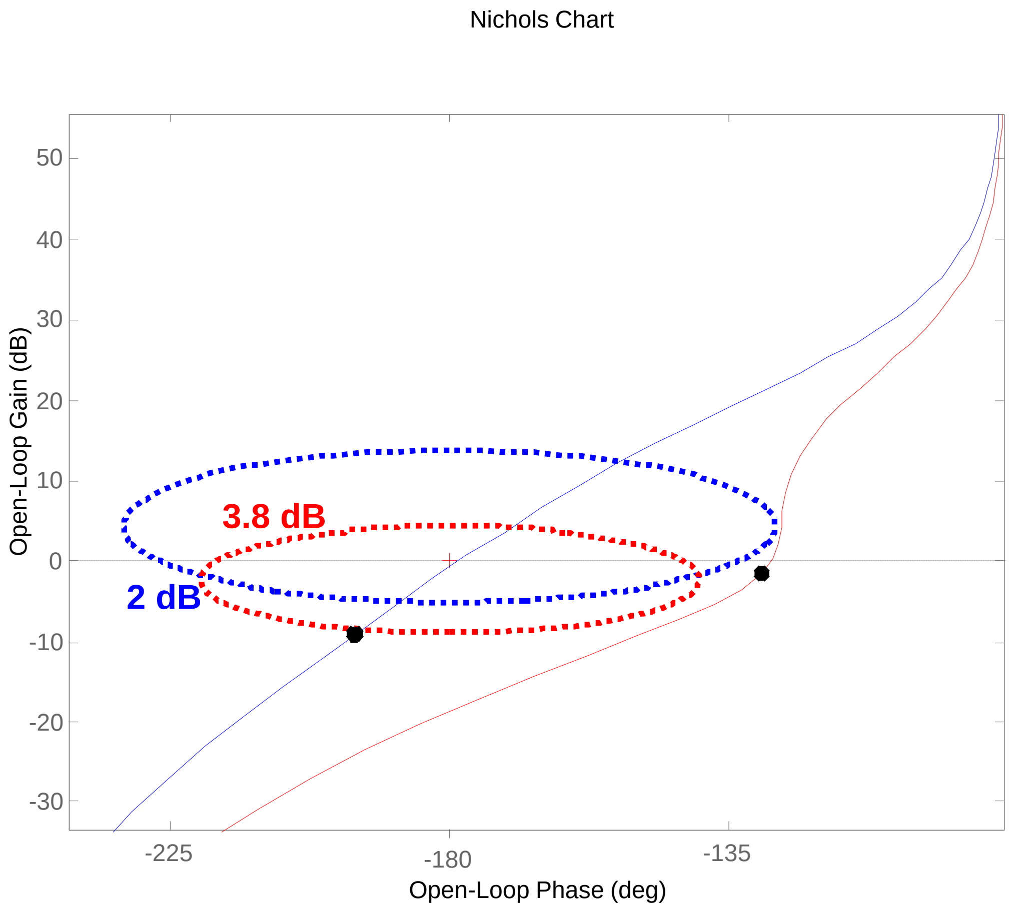
Loop-shaping design

Phase lead controller design

Example 1



- In this case phase lead and magnitude increase actions are required

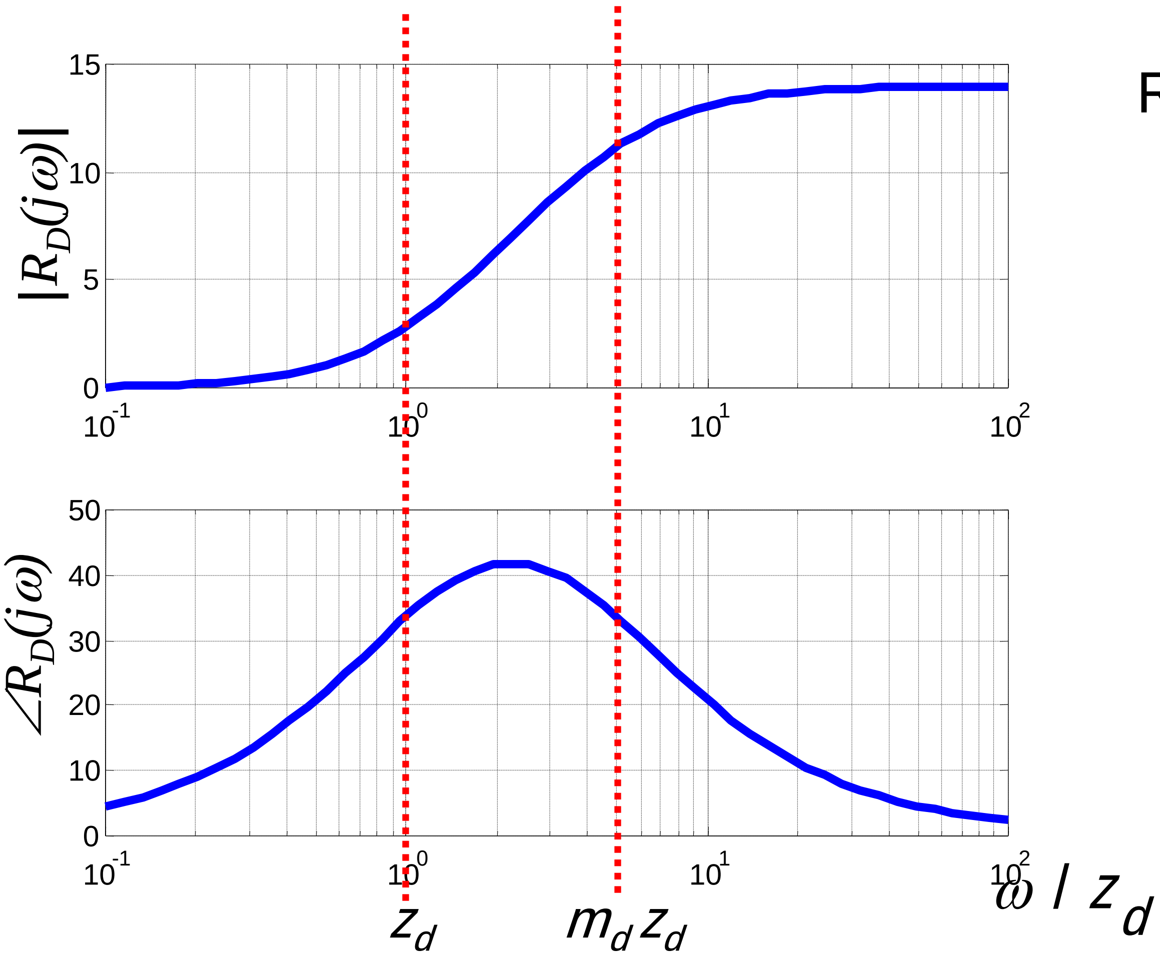

Nichols Chart
Open-Loop Gain (dB)
50
40
30
20
10
0
-10
-20
-30
-225
-180
-135
Open-Loop Phase (deg)
3.8 dB
2 dB

The introduced example motivates the use of the

lead network

$$R_d(s) = \frac{1+\dfrac{s}{z_d}}{1+\dfrac{s}{m_d z_d}}, \quad m_d > 1$$

The lead network: frequency response

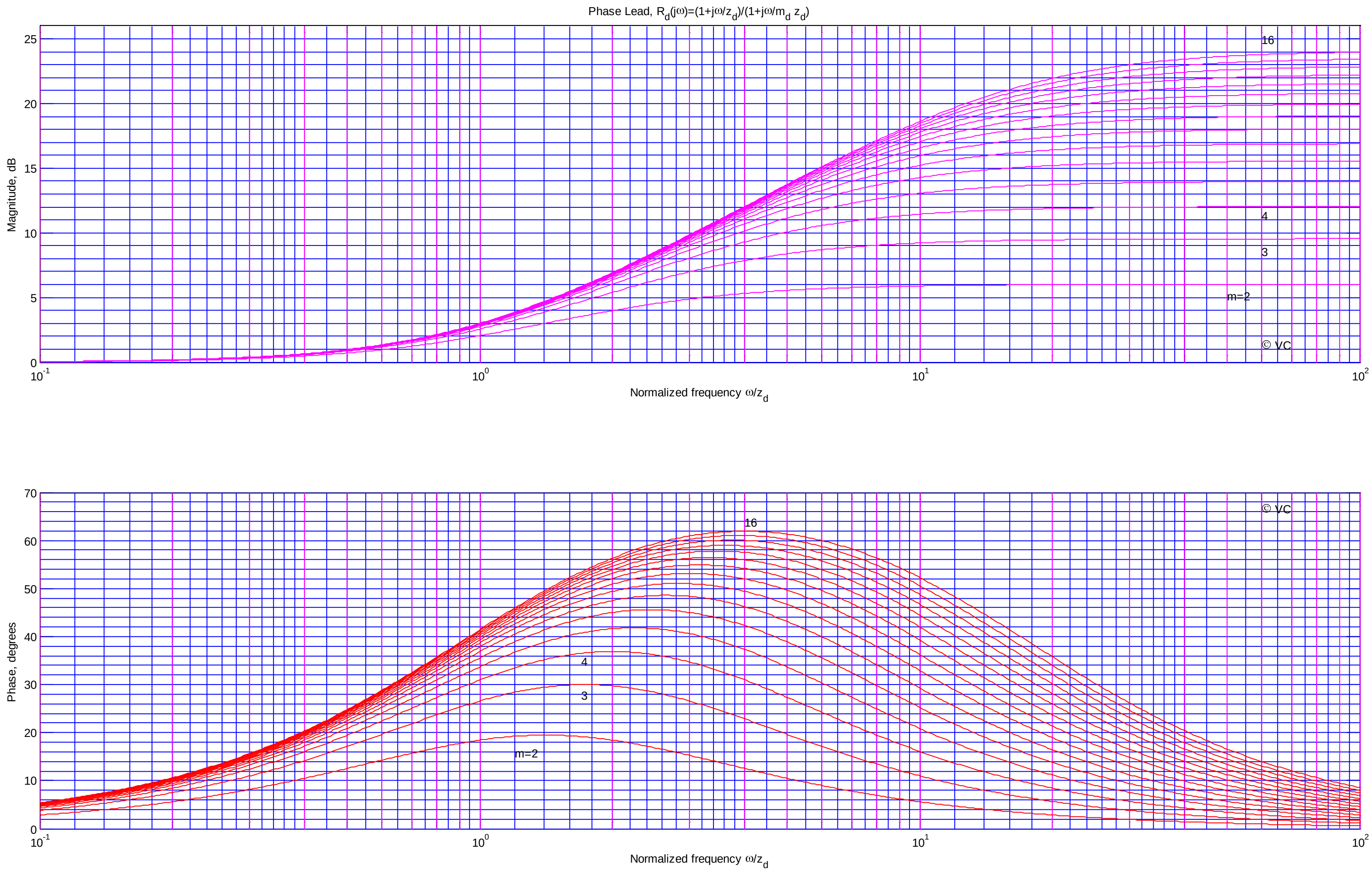


$$R_d(s) = \frac{1 + \dfrac{s}{z_d}}{1 + \dfrac{s}{m_d z_d}}, \quad m_d > 1$$

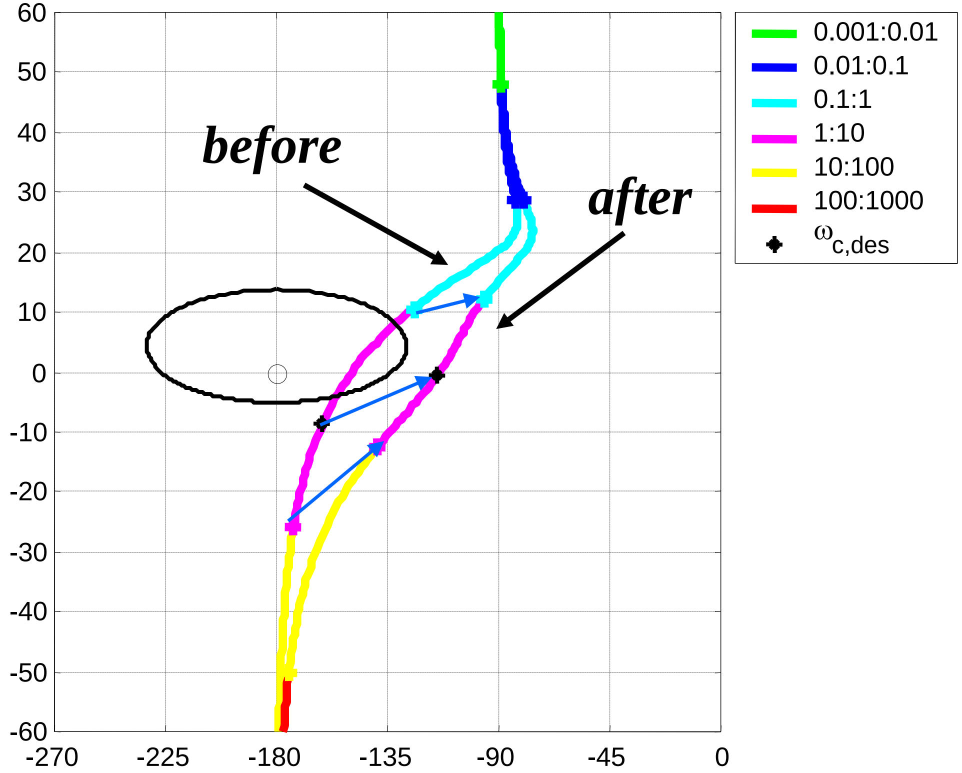
Magnitude increase

Phase lead

Phase lead frequency response



The lead network: effects

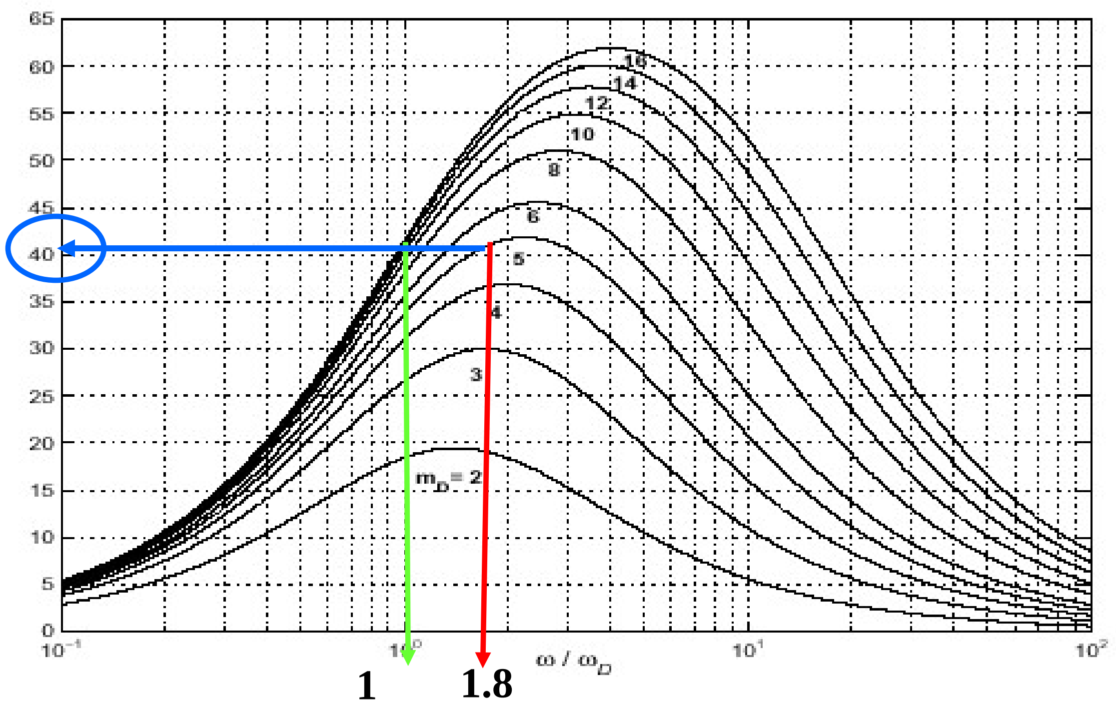


Effects on the Nichols plane:

- phase lead and magnitude increase gives rise to an oblique shift of the loop Nichols plot over the frequencies of interest.

- Suppose that a 40° phase lead is required at $\omega_{c,des} = 7\ rad/s$

several choices can be made:



- $m_D = 5$; $\omega/z_D = 1.8$

$\rightarrow\ (\omega/z_D)|_{\omega=\omega c} = 1.8$

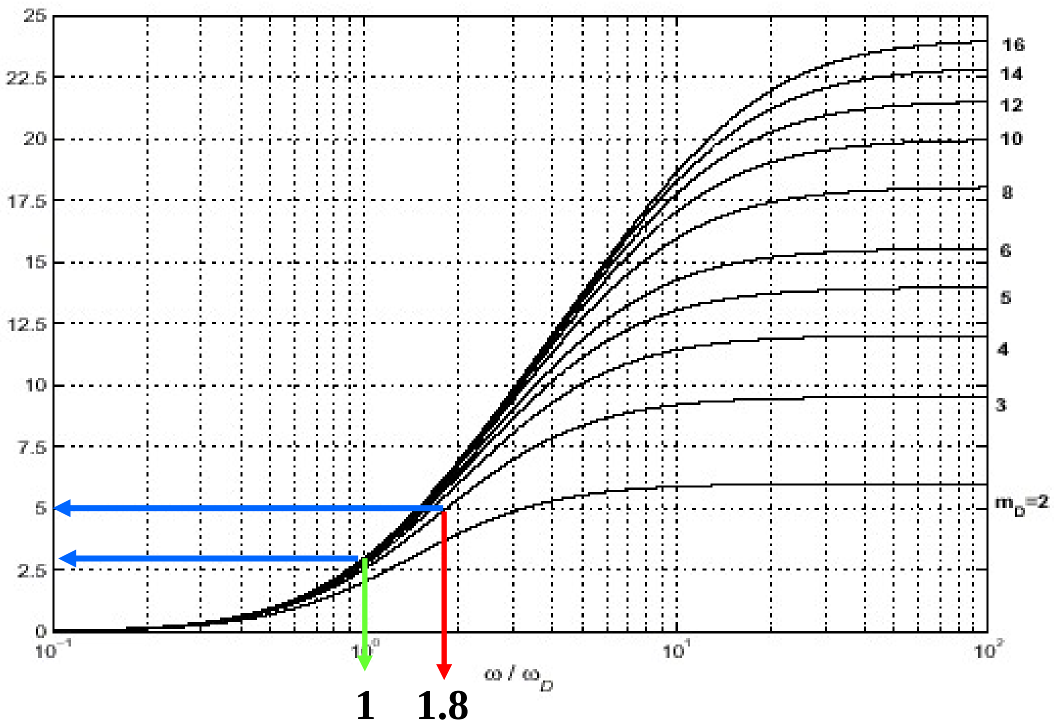
$z_D = \omega_c / 1.8 = 3.89$ rad/s

- $m_D = 10$; $\omega/z_D = 1$

$\rightarrow\ (\omega/z_D)|_{\omega=\omega c} = 1$

$z_D = \omega_c / 1 = 7$ rad/s

- The corresponding magnitude increases are:

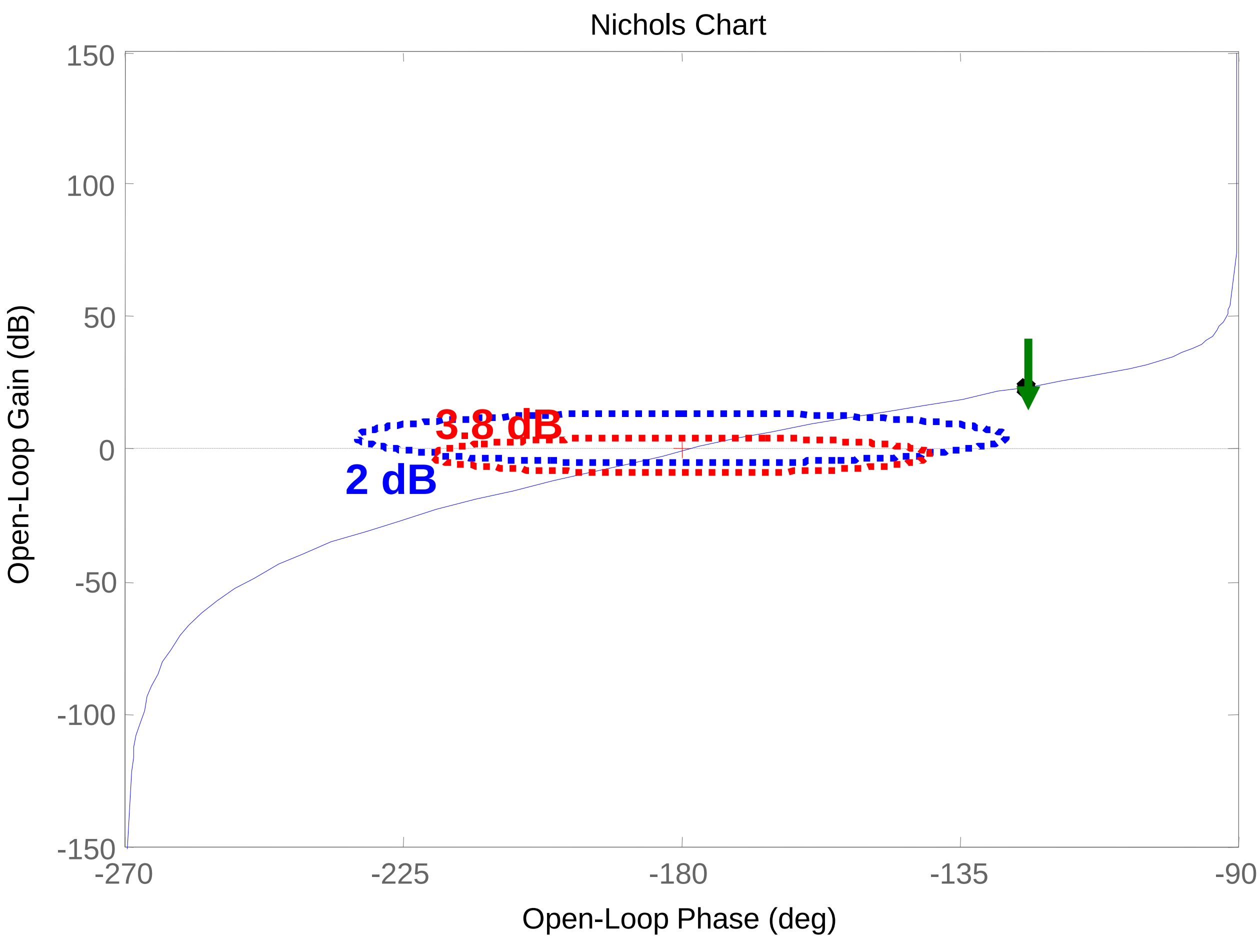


- 5 dB
- 3 dB

Loop-shaping design

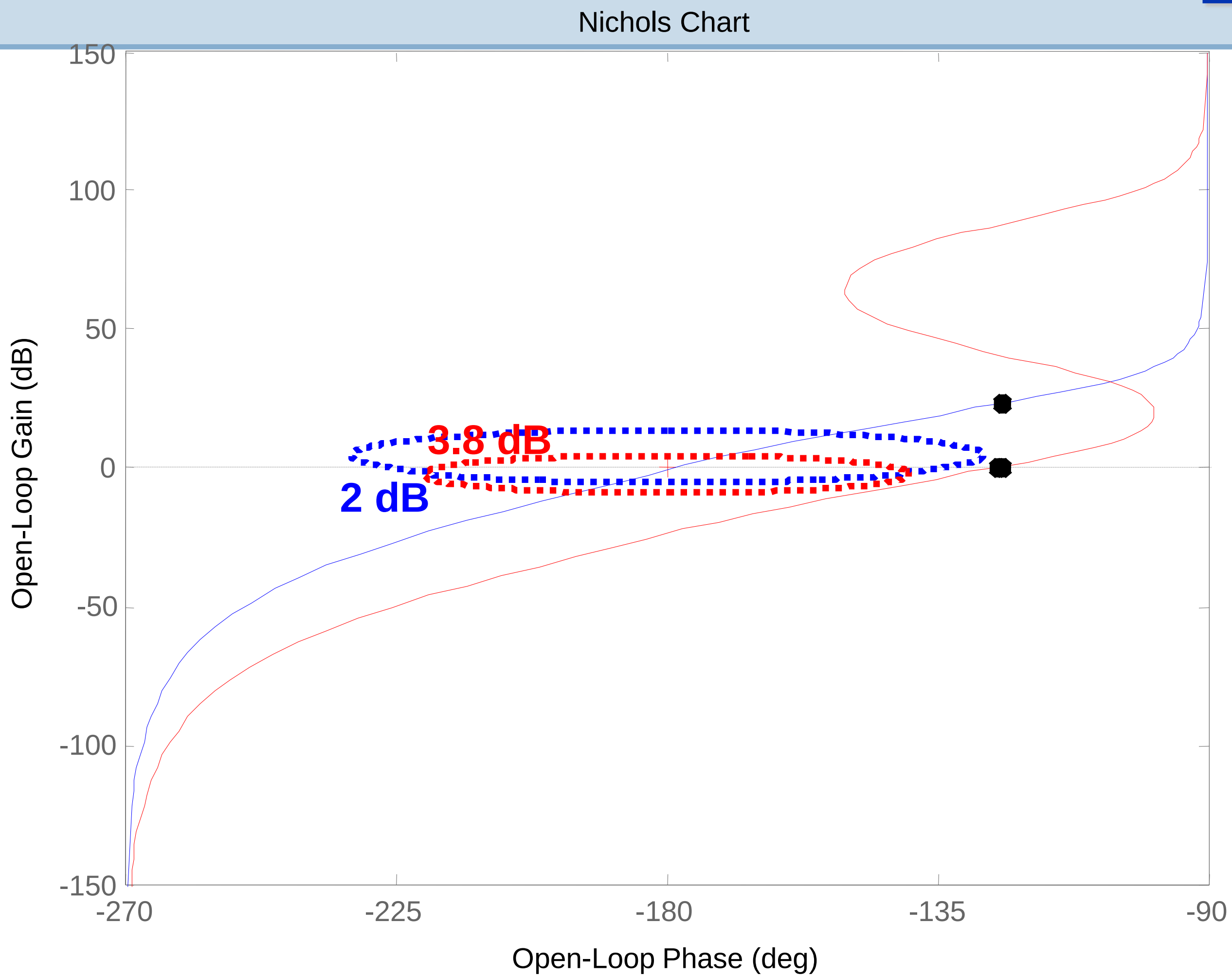
Phase lag controller design

Example 2



A magnitude attenuation action is required

Example 2



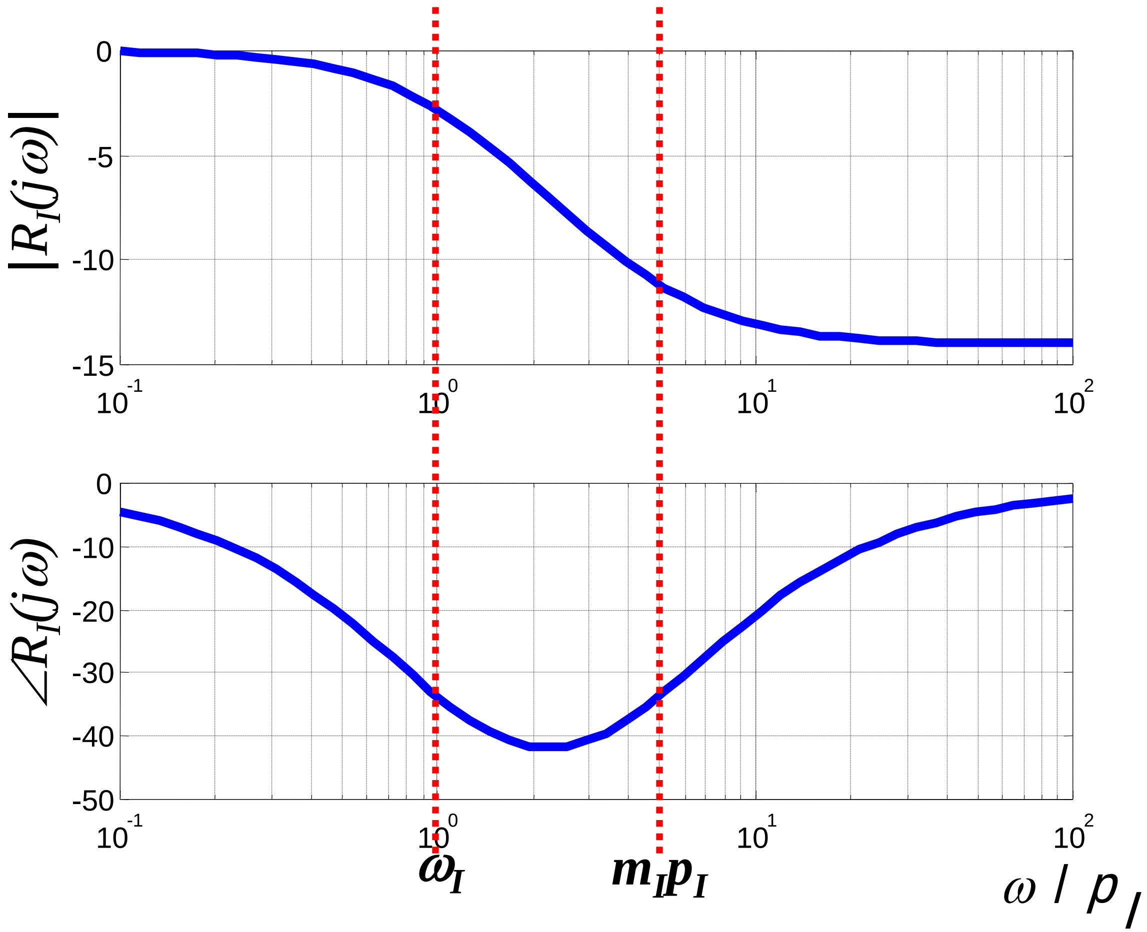
Nichols Chart
150
100
50
0
-50
-100
-150
-270
-225
-180
-135
-90
Open-Loop Gain (dB)
Open-Loop Phase (deg)
3.8 dB
2 dB

The introduced example motivates the use of the

lag network

$$R_i(s) = \frac{1 + \frac{s}{m_i p_i}}{1 + \frac{s}{p_i}}, m_i > 1$$

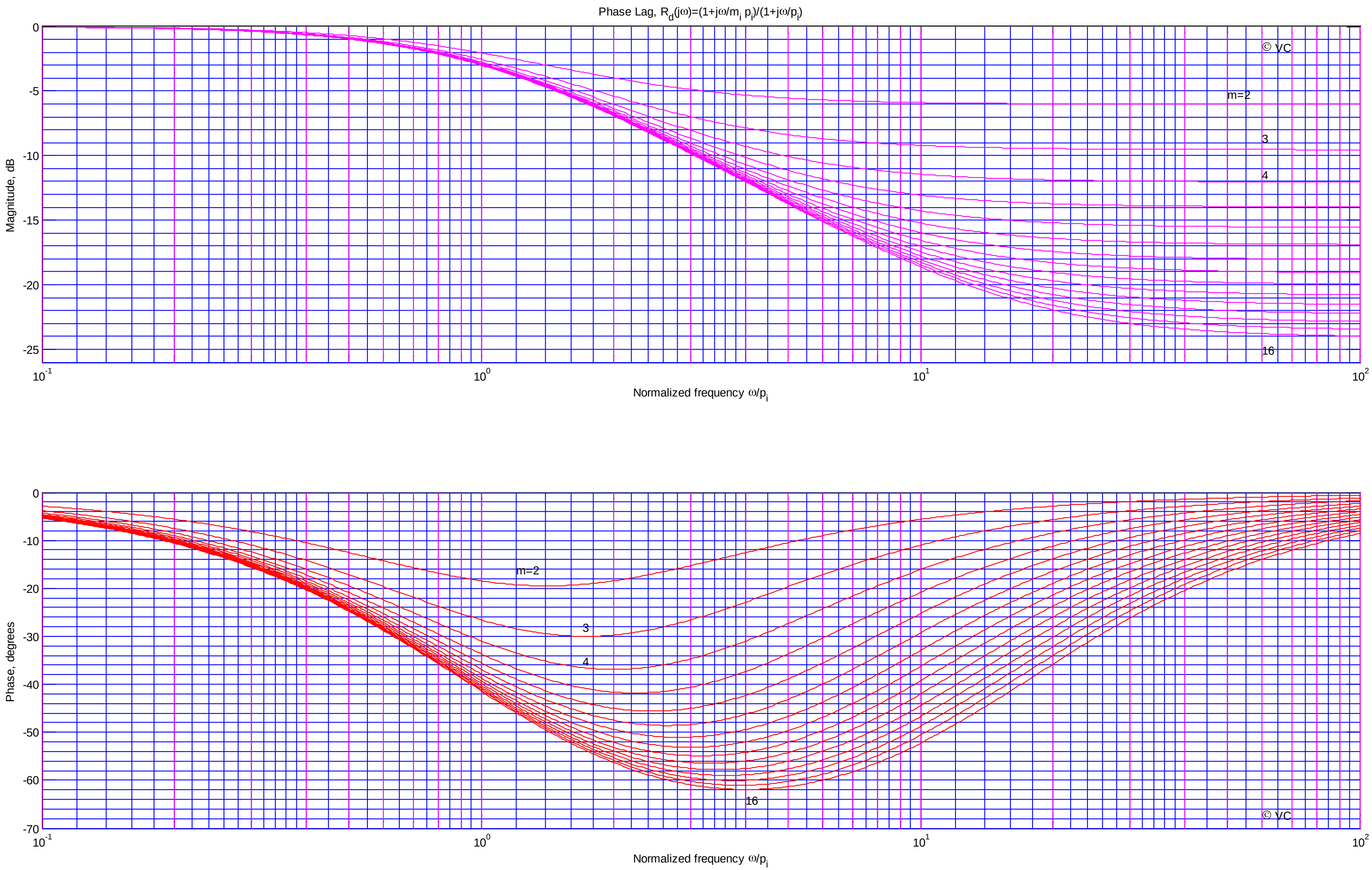
The lag network: frequency response

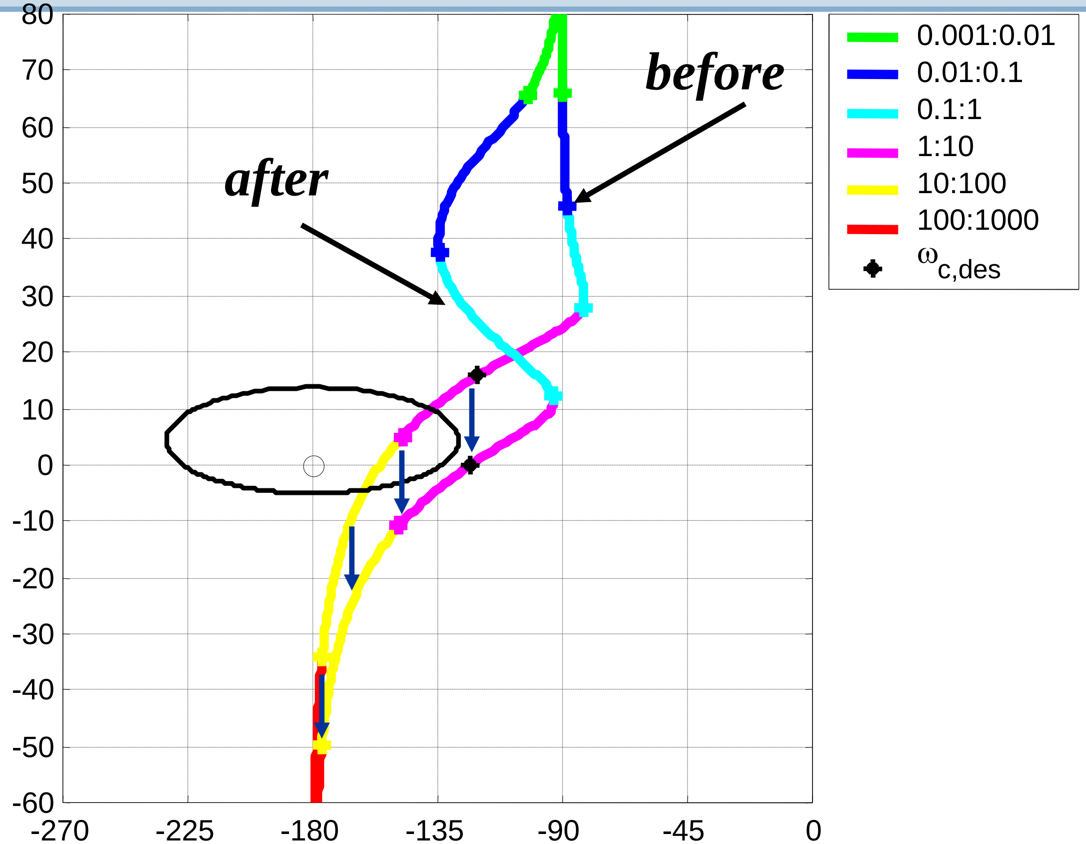


$$R_i(s) = \frac{1 + \dfrac{s}{m_i p_i}}{1 + \dfrac{s}{p_i}}, \; m_i > 1$$

- Magnitude attenuation

- Phase lag


Phase Lag, $R_d(j\omega)=(1+j\omega/m_i\ p_i)/(1+j\omega/p_i)$
Magnitude, dB
Normalized frequency ω/p_i
m=2
3
4
16
© VC
Phase, degrees
Normalized frequency ω/p_i
m=2
3
4
16
© VC

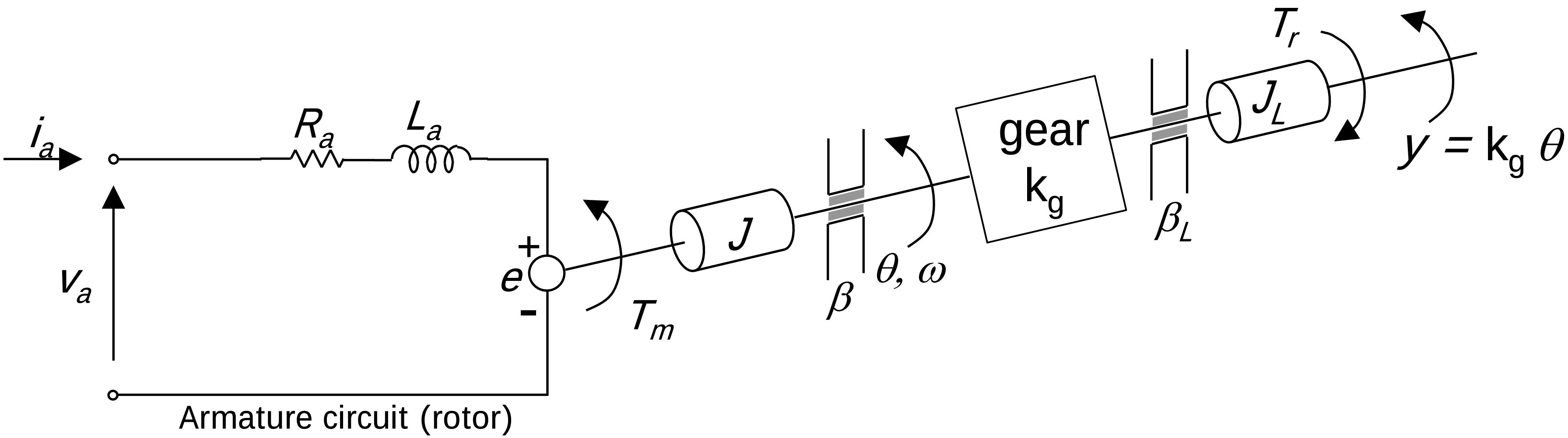


Effects on the Nichols plane:

- the magnitude attenuation effect gives rise to a vertical shift of the loop Nichols plot over the frequency of interest.

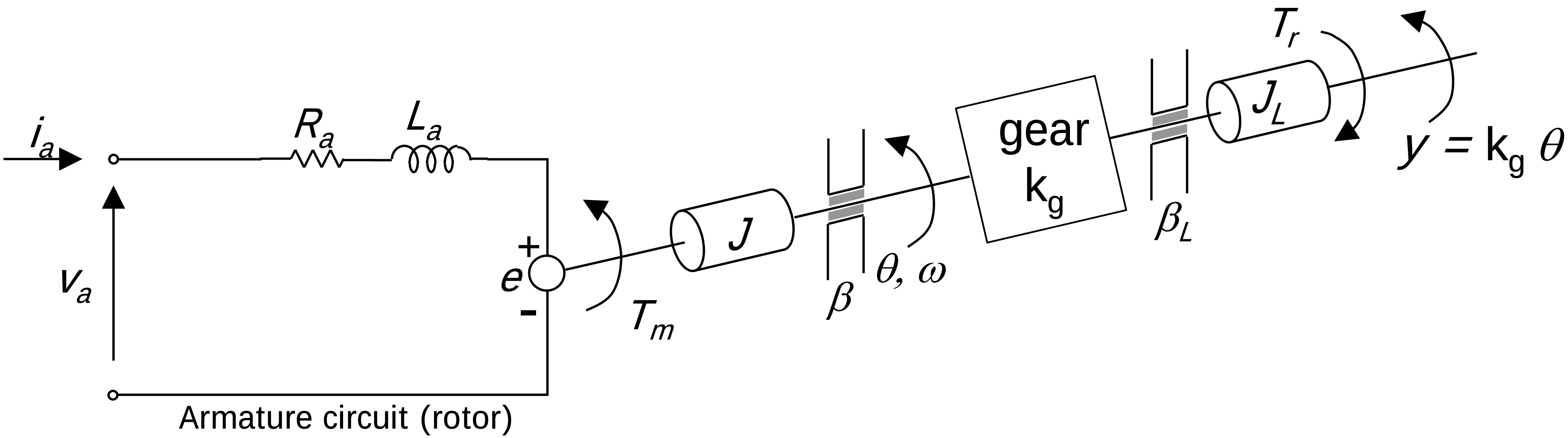
Loop-shaping design

Application to dc-motors position control design



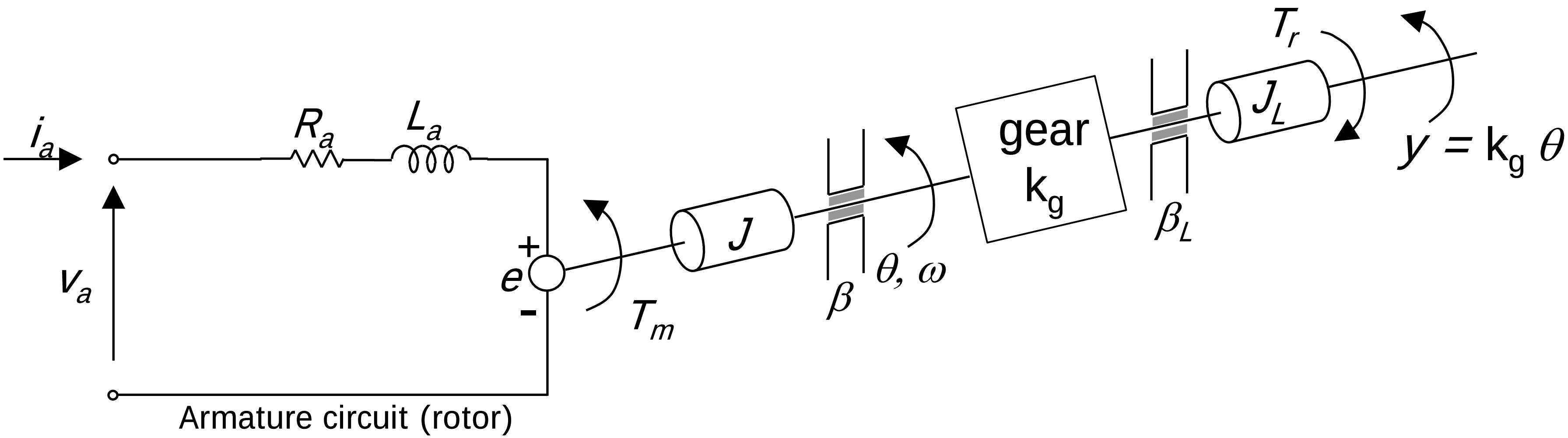
Suppose we are given the plant transfer function:

$$G_p(s) = \frac{Y(s)}{V_a(s)} = \frac{1.61}{s\left(1+\frac{s}{54.67}\right)\left(1+\frac{s}{5.49}\right)}$$



... and the disturbance transfer function

$$G_d(s) = \frac{Y(s)}{T_r(s)} = \frac{3.23\left(1+\frac{s}{60}\right)}{s\left(1+\frac{s}{54.67}\right)\left(1+\frac{s}{5.49}\right)}$$

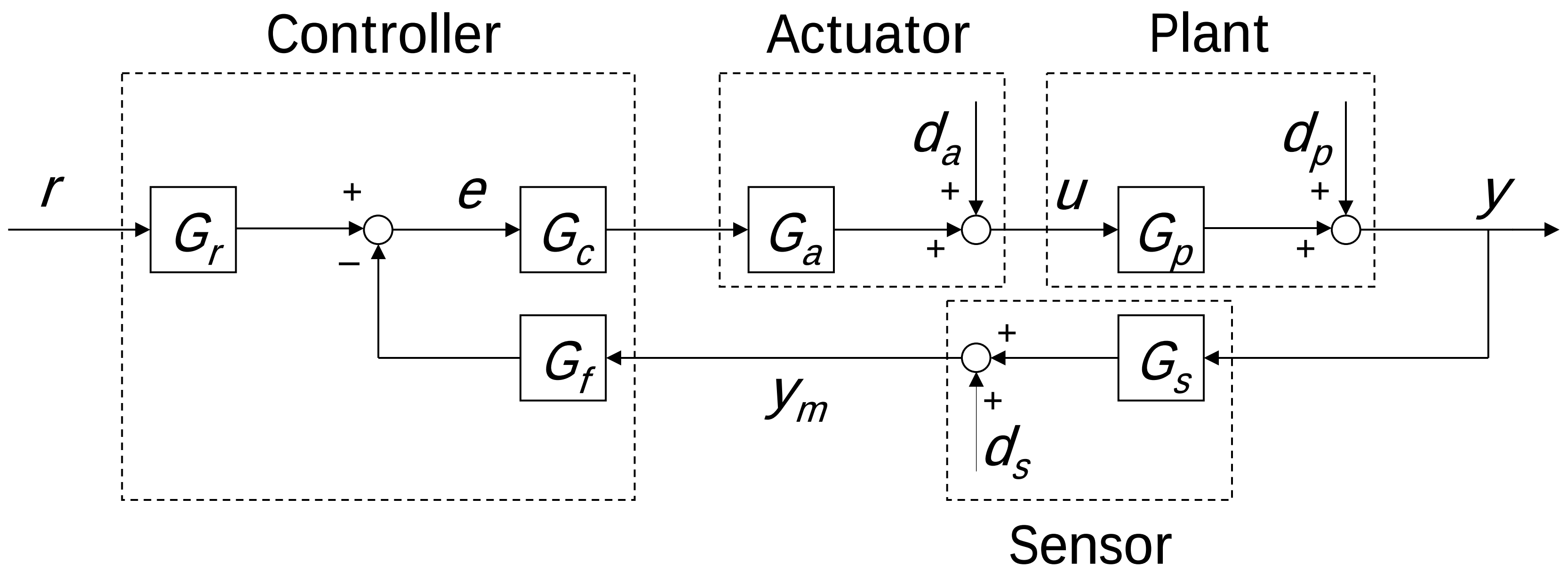


Design a control system such that:

$$\hat{s} \le 10\%$$

$$\left|e_r^\infty\right| \le \rho_r = 2 \cdot 10^{-2} \quad \text{when} \quad r(t) = R_0\, t; \; \left|R_0\right| \le \frac{\pi}{10}$$

Feedback control systems to be designed



$G_r = 1$
$G_a = 1$
$K_d = 1$
$G_s = 1$