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Homework 1

Question 1.
The β^--emitter ^{28}Al (half-life 2.30 min) can be produced by the radiative capture of neutrons by ^{27}Al. The 0.0253-eV cross-section for this reaction is 0.23 b. Suppose that a small, 0.01-g aluminum target is placed in a beam of 0.0253-eV neutrons, $\varphi = 3 \times 10^8$ neutrons/cm^2-sec, which strikes the entire target. Calculate:

(a) the neutron density in the beam;
(b) the rate at which ^{28}Al is produced;
(c) the maximum activity (in curies) that can be produced in this experiment.

- (a)

For thermal neutrons, the velocity is approximately 2200 m/s.

$$n = I/n$$

$$n = \frac{3 \times 10^8}{2200 \times 10^2} = 1363\, neutron/cm^3$$

- (b)

From table II.3, $\rho_{\text{Al}} = 2.699\,\text{g/cm}^3$ and $N_{\text{Al}} = 0.06024 \times 10^{24}\,\text{Atoms/cm}^3$

$$\dot{R} = \sigma \varphi N A X = 0.23 \times 3 \times 10^8 \times 0.06024 \times \frac{0.01}{2.699} = 15400\, Interactions/sec$$

- (b)

At saturation activity, production rate = decary rate, $A_{\max} = \dot{R}$

$$A_{\max} = \frac{15400\, Bq}{3.7 \times 10^10} = 4.16 \times 10^{-7}\, Ci$$

Question 2.
Calculate the mean free path of 1-eV neutrons in graphite. The total cross-section of carbon at this energy is 4.8 b.

From table II.3, $N_{\text{C(graphite)}} = 0.08023 \times 10^{-24}$

$$\bar{X} = \frac{1}{\sigma_t N} = \frac{1}{4.8 \times 0.08023} = 2.597\, cm$$

Question 3.
A beam of 2-MeV neutrons is incident on a slab of heavy water (D_2O). The total cross-sections of deuterium and oxygen at this energy are 2.6 b and 1.6 b, respectively.

(a) What is the macroscopic total cross-section of D_2O at 2 MeV?
(b) How thick must the slab be to reduce the intensity of the uncollided beam by a factor of 10?
(c) If an incident neutron has a collision in the slab, what is the relative probability that it collides with deuterium?

- (a)

From table II.3, $N_{D_2O} = 0.03323 \times 10^{-24}$

$$\sigma = 2\sigma_D + \sigma_O = (2 \times 2.6) + (1.6) = 6.8\, b$$

$$\sum_t = \sigma_t N = 6.8 \times 0.03323 = 0.2256\, cm^{-1}$$

- (b)

$$\frac{I_{(\mathrm{x})}}{I_0} = e^{-\sum_t x}$$
$$10 = e^{-0.2256x},\ x = 10.21\,cm$$

- (c)

$$P(D|collision) = \frac{\sum_D}{\sum_t} = \frac{2\sigma_D N}{\sum_t}$$
$$P(D|collision) = \frac{2 \times 2.6 \times 0.03323}{0.2256} = 0.766 = 76.6\%$$

Question 4.
Stainless steel, type 304 having a density of 7.86 g/cm^3, has been used in some reactors. The nominal composition by weight of this material is as follows: carbon, 0.08%; chromium, 19%; nickel, 10%; iron, the remainder. Calculate the macroscopic absorption cross-section of SS-304 at 0.0253 eV.

Since composite materials (alloys) have densities of their own, the atomic densities of each material should be derived in regards to the alloy density.

$$\Sigma_a = \sum_i N_i \sigma_{a,i} \qquad \text{with} \qquad N_i = \frac{\rho w_i N_A}{A_i}$$

subsituting them will give:

$$\boxed{\Sigma_a = \rho_{alloy} N_A \sum_i \frac{w_i}{A_i} \sigma_{a,i}}$$

Values of w_i, A_i and $\sigma_{a,i}$ is obtained from table II.3

$$\Sigma_a = (\rho_{alloy})(N_A)[(\frac{w_C}{A_C} \times \sigma_C) + (\frac{w_{Cr}}{A_{Cr}} \times \sigma_{Cr}) + (\frac{w_{Ni}}{A_{Ni}} \times \sigma_{Ni}) + (\frac{w_{Fe}}{A_{Fe}} \times \sigma_{Fe})]$$
$$\Sigma_a = (7.86)(0.6022)[(\frac{0.0008}{12} \times 0.0034) + (\frac{0.19}{52} \times 3.1) + (\frac{0.1}{59} \times 4.43) + (\frac{0.7092}{56} \times 2.55)]$$
$$\Sigma_a = 0.242\,cm^{-1}$$

Question 5.
There are no resonances in the total cross-section of ^{12}C from 0.01 eV to cover 1 MeV. If the radiative capture cross-section of this nuclide at 0.0253 eV is 3.4 mb, what is the value of σ_γ at 1 eV?

$$\sigma_\gamma(E) = \sigma_\gamma(E_0) \times \sqrt{\frac{E_0}{E}}$$
$$\sigma_\gamma(1\,eV) = 3.4 \times 10^{-27} \times \sqrt{\frac{0.0253}{1}} = 0.541\,mb$$

Question 6.
The first resonance in the scattering cross-section of the nuclide ^AZ occurs at 1.24 MeV. The separation energies of nuclides ^{A-1}Z, ^AZ, and ^{A+1}Z are 7.00, 7.50, and 8.00 MeV, respectively. Which nucleus and at what energy above the ground state is the level that gives rise to this resonance?

$$E^* = S_n\left({}^{A+1}Z\right) + E$$
$$E^* = 8 + 1.24 = 9.24\,MeV$$

Question 7.
A 2-MeV neutron traveling in water has a head-on collision with an ^{16}O nucleus.

(a) What are the energies of the neutron and nucleus after the collision?
(b) Would you expect the water molecule involved in the collision to remain intact after the event?

- (a)

$$\text{Neutron colliding head-on } \rightarrow \theta_{max} = \pi \rightarrow E^{'} = (\frac{A-1}{A+1})^2 E$$
$$E^{'} = (\frac{16-1}{16+1})^2 \times 2 = 1.557\, MeV$$
$$E_A = 2 - 1.557 = 0.442\, MeV$$

- (b)

No, because the oxygen molecule will recoil with an energy of 4 MeV while the H-O bond in the water molecule is held with an energy of 5 eV.

Question 8.
A 1-MeV neutron strikes a ^{12}C nucleus initially at rest. If the neutron is elastic scattered through an angle of 90°:

(a) What is the energy of the scattered neutron?
(b) What is the energy of the recoiling nucleus?
(c) At what angle does the recoiling nucleus appear?

- (a)

$$E' = \frac{E}{(A+1)^2}\left[\cos\vartheta + \sqrt{A^2 - \sin^2\vartheta}\right]^2$$
$$E' = \frac{1}{(12+1)^2}[\cos(90) + \sqrt{12^2 - \sin^2(90)}]^2 = 0.846\, MeV$$

- (b)

$$E_A = E - E' = 1 - 0.846 = 0.154 MeV$$

- (c)

For a 1 MeV neutron moving at 0.46% speed of light, the lorentz factor is insignificant. Thus, the nonrelativistic model could be applied to derive the angle of recoiled nucleus.

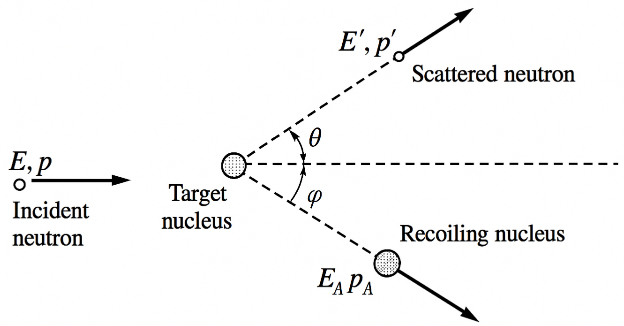

FIGURE 1. Elastic scattering of a neutron by a nucleus.

$$P = P' + P_A \begin{cases} x: & P = P'\cos\theta + P_A\cos\varphi \quad \rightarrow \quad P_A\cos\varphi = P - P'\cos\theta \\ y: & 0 = P'\sin\theta - P_A\sin\varphi \quad \rightarrow \quad P_A\sin\varphi = P'\sin\theta \end{cases}$$

Solving the x and y components by elemination:

$$\frac{P_A\sin\varphi}{P_A\cos\varphi} = \frac{P'\sin\theta}{P - P'\cos\theta}$$

Given that $P = \sqrt{2mE}$, $P' = \sqrt{2mE'}$ and $\theta = 90°$,

$$\tan\varphi = \sqrt{\frac{E'}{E}} \rightarrow \varphi = \tan^{-1}(\sqrt{\frac{E'}{E}})$$

$$\boxed{\varphi = \tan^{-1}(\sqrt{\frac{0.846}{1}}) = 42.6°}$$

Question 9.
Show that the average fractional energy loss in % in elastic scattering for large A is given approximately by:

$$\frac{\overline{\Delta E}}{E} \simeq \frac{200}{A}$$

$$\frac{\overline{\Delta E}}{E} = \frac{1}{2}(1 - (\frac{A-1}{A+1}))^2 \rightarrow f(A) = \frac{1}{2}\left[\frac{(A+1)^2}{(A+1)^2} - \frac{(A-1)^2}{(A+1)^2}\right] = \left[\frac{2A}{(A+1)^2}\right]$$

Since this is an asymptotic function, with leading term 1/A, set $c = \lim_{A\to\infty} Af(A)$ to find it's coefficient

$$\lim_{A\to\infty} Af(A) = \lim_{A\to\infty}\left[\frac{2A^2}{(A+1)^2}\right] \rightarrow \text{Applying L'Hôpital, } \lim_{A\to\infty}\left[\frac{4A}{2A+2}\right] \rightarrow 2$$

$$\lim_{A\to\infty} Af(A) = 2 \rightarrow \left[\lim_{A\to\infty} f(A)\right] \times 100\% = \left[\frac{2}{A}\right] \times 100\%$$

$$\boxed{\frac{\overline{\Delta E}}{E} \times 100\% \simeq \frac{200}{A}\%}$$

Question 10.
The 2,200 meters-per-second flux in an ordinary water reactor is 1.5×10^{13} neutrons/cm^2·sec. At what rate are the thermal neutrons absorbed by the water?

Since water is considered a 1/v nuclei, the absorption density can be found with $F_a = \sum_a(E_0)\Phi_0$

$$F_a = 0.02220 \times 1.5 \times 10^{13} = 3.33 \times 10^{11}\, Neutrons/cm^3 \cdot s$$

Question 11.
A tiny beryllium target located at the center of a three-dimensional Cartesian coordinate system is bombarded by six beams of 0.0253-eV neutrons of intensity 3×10^8 neutrons/cm^2·sec, each incident along a different axis.

(a) What is the 2,200 meters-per-second flux at the target?
(b) How many neutrons are absorbed in the target per cm^3/sec?

- (a)

$$\text{Thermal neutrons } \approx 0.0253\, eV \approx 2200\, m/s$$

$$\Phi = 6 \times 3 \times 10^8 \;= 1.8 \times 10^9 Neutrons/cm^2 \cdot sec$$

- (b)

$$\text{From table II.3, } \sum_a = 0.001137\,cm^{-1}$$
$$F_a = \sum_a(E_0)[\Phi_1 + \Phi_2 + \Phi_3 \ldots] = 0.001137 \times 1.8 \times 10^6 = 2.05 \times 10^6 Neutrons/cm^2 \cdot sec$$

Question 12.

The control rods for a certain reactor are made of an alloy of cadmium (5 w/o), indium (15 w/o) and silver (80 w/o). Calculate the rate at which thermal neutrons are absorbed per gram of this material at a temperature of 400 °C in a 2,200 meters-per-second flux of 5×10^{13} neutrons/cm^2·sec.

[Note: Silver is a 1/ν absorber.]

$$F_{a,t} = \Phi_0 \sum_i ((\sum_a(E_0))_i (g_a(T))_i w_i \quad \text{with} \quad g_a(T) = 1 \quad \text{for 1/v materials}$$
$$F_{a,t} = \Phi_0[(\sum_a(E_0))_{Cd}(g_a(T))_{Cd} w_{Cd} + (\sum_a(E_0))_{In}(g_a(T))_{In} w_{In} + (\sum_a(E_0))_{Ag}(g_a(T))_{Ag} w_{Ag}]$$
$$F_{a,t} = 5 \times 10^{13}[(113.56)(2.5589)(0.05) + (7.419)(1.1011)(0.15) + (3.725)(1)(0.8)]$$
$$F_{a,t} = 9.367 \times 10^{14}\, Neutron/cm^3.s$$

Question 13. Find roots of $x^2 - 8x = 9$.

We proceed by factoring,

$$
\begin{aligned}
x^2 - 8x - 9 &= 9 - 9 && \text{Subtract 9 on both sides.}\\
x^2 - x + 9x - 9 &= 0 && \text{Breaking the middle term.}\\
(x-1)(x+9) &= 0 && \text{Pulling out common } (x-1).\\
x &\in \{1, -9\} && f(x)g(x) = 0 \Rightarrow f(x) = 0 \vee g(x) = 0.
\end{aligned}
$$

Question 14. Figure 2 shows two cipher wheels. The left one is from Jeffrey Hoffstein, et al. [1] (pg. 3). Write a Python 3 program that uses it to encrypt: `FOUR SCORE AND SEVEN YEARS AGO`.

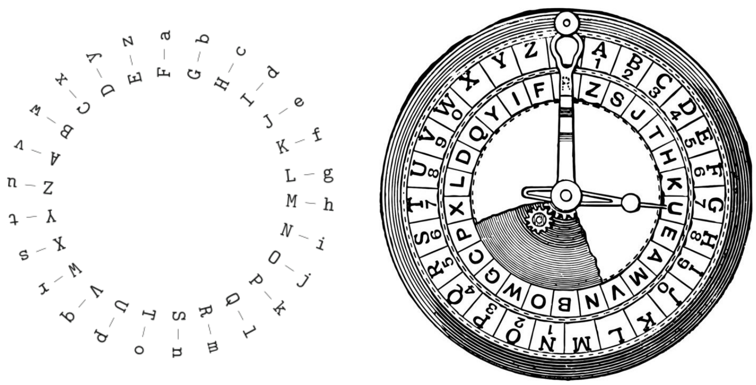

FIGURE 2. Cipher wheels.

The Python program is given in listing 1 and the encryption is given in table 1.

```python
def encrypt(plain):
    cipher = ''
    for c in plain:
        cipher = cipher+c if c==' ' else cipher+chr(((ord(c)-60) % 26)+65)
    return cipher
print(encrypt("FOUR SCORE AND SEVEN YEARS AGO"))
```

LISTING 1. Python 3 implementing figure 2 left wheel.

Plain Text	FOUR	SCORE	AND	SEVEN	YEARS	AGO
Cipher Text	KTZW	XHTWJ	FSI	XJAJS	DJFWX	FLT

Table 1. Caesar cipher

REFERENCES

[1] Jeffrey Hoffstein, Jill Pipher, Joseph H Silverman, and Joseph H Silverman. *An introduction to mathematical cryptography*, volume 1. Springer, 2008.

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