



Working towards all the geophysics - but backwards.

Rowan Cockett, Seogi Kang & SimPEG Team
July 12, 2016



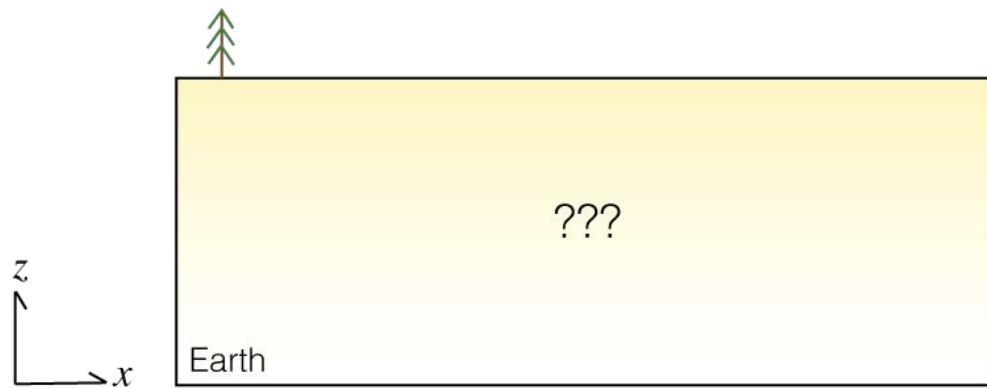
Geophysical Inversion Facility
University of British Columbia

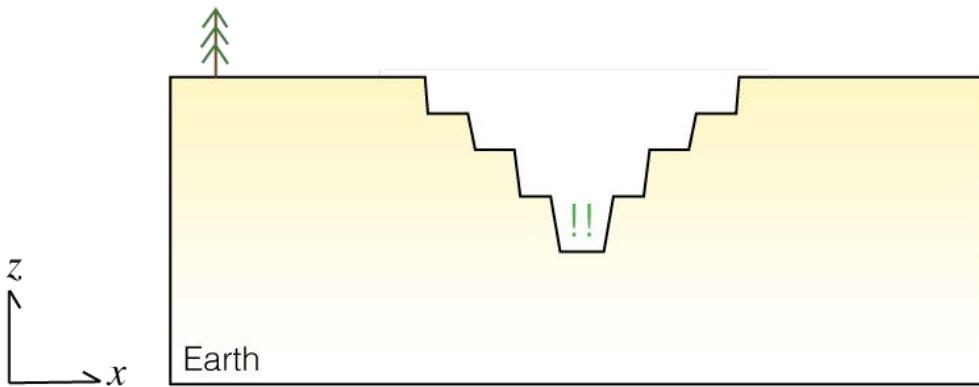
some of

Working towards ~~all~~ the geophysics - but backwards.

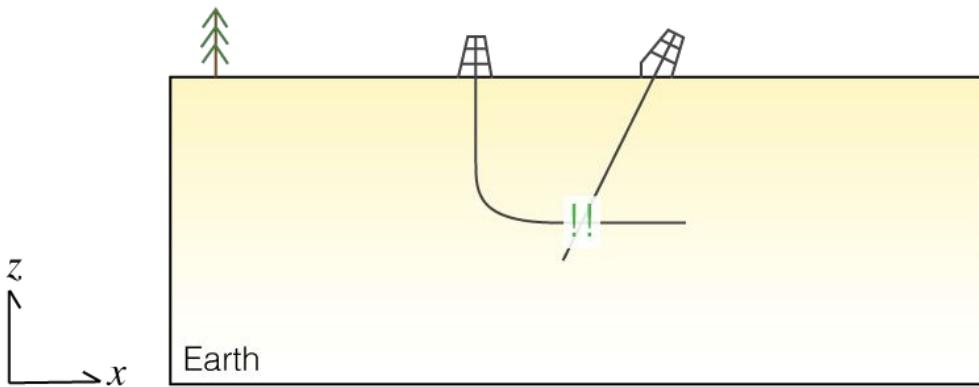
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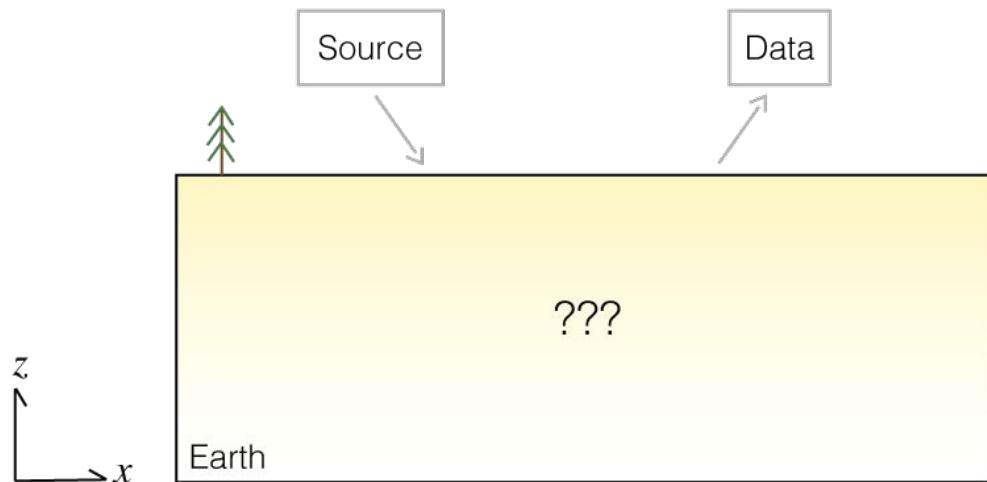




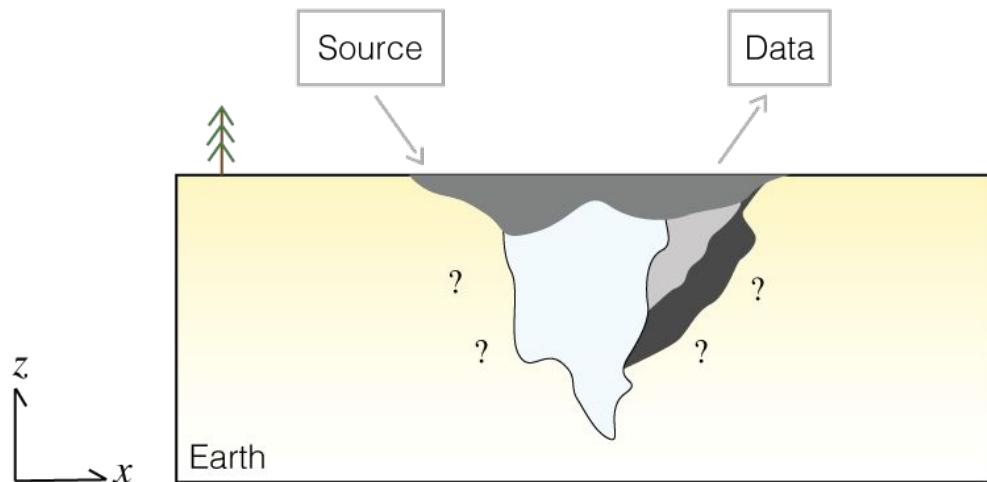
Option 1: Dig it up?



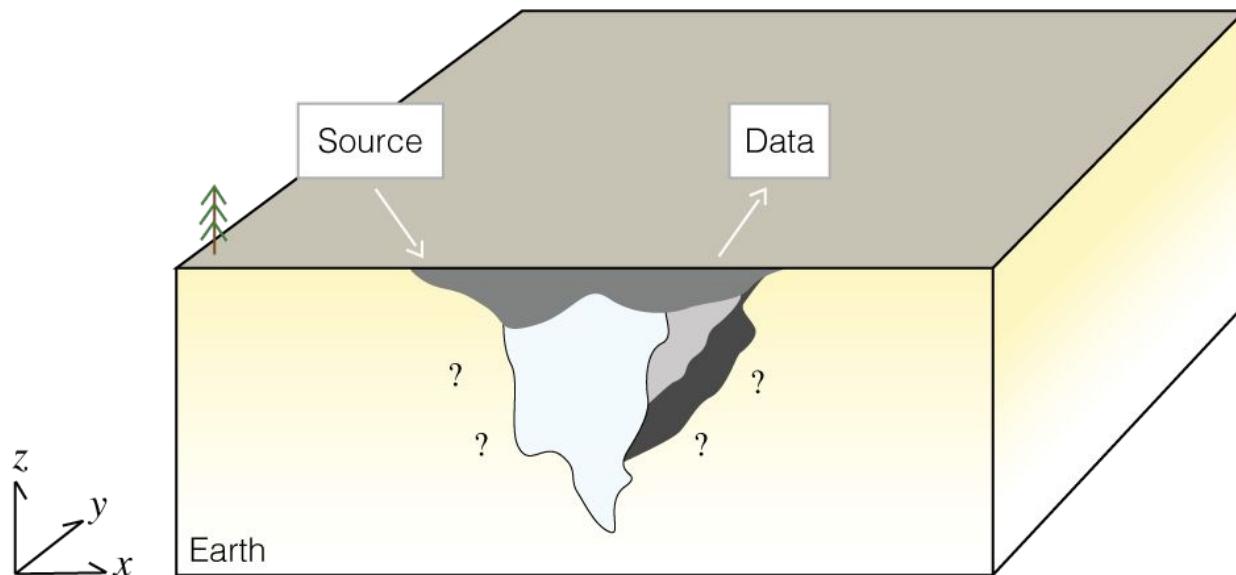
Option 2: Drill it?



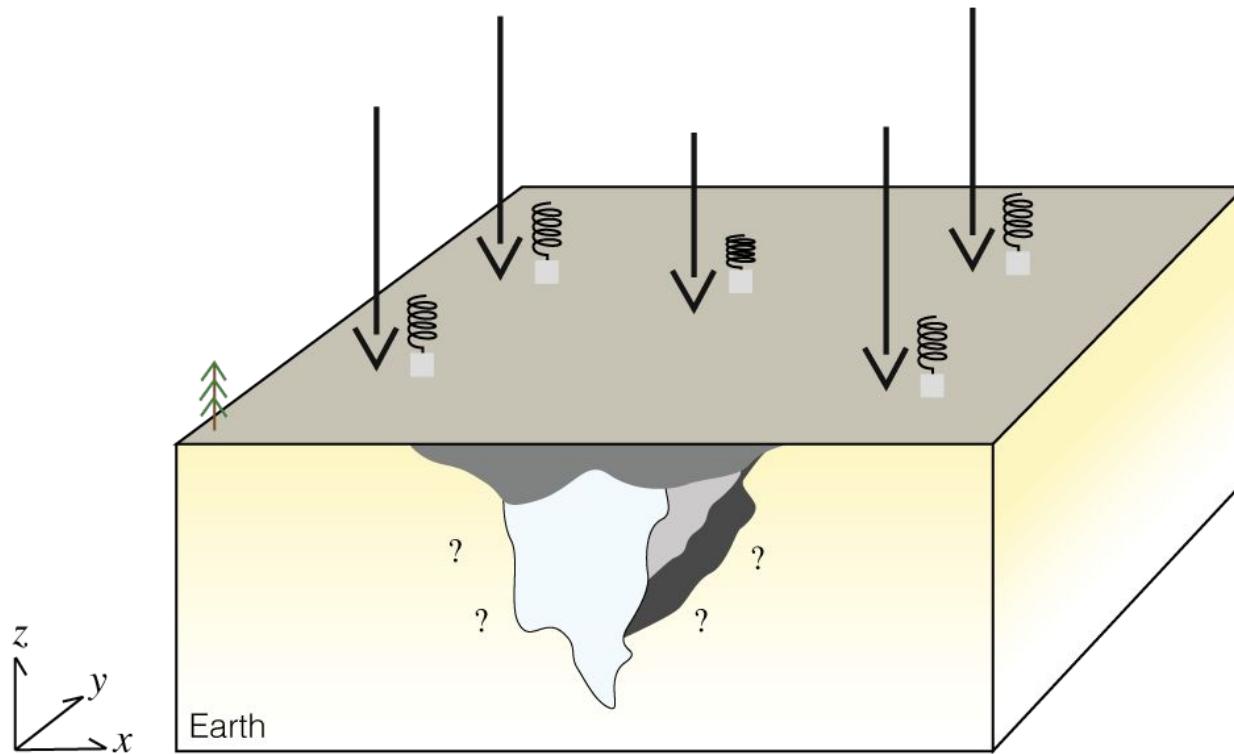
Option 3: Use geophysics!



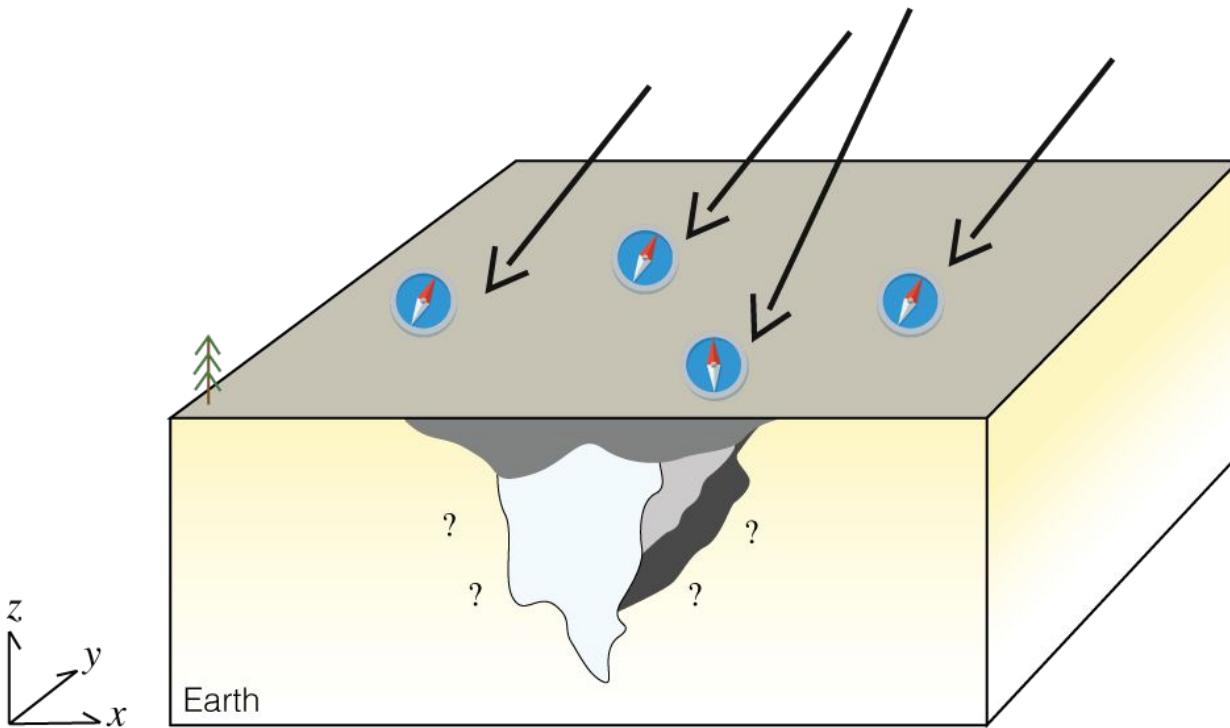
Physical properties

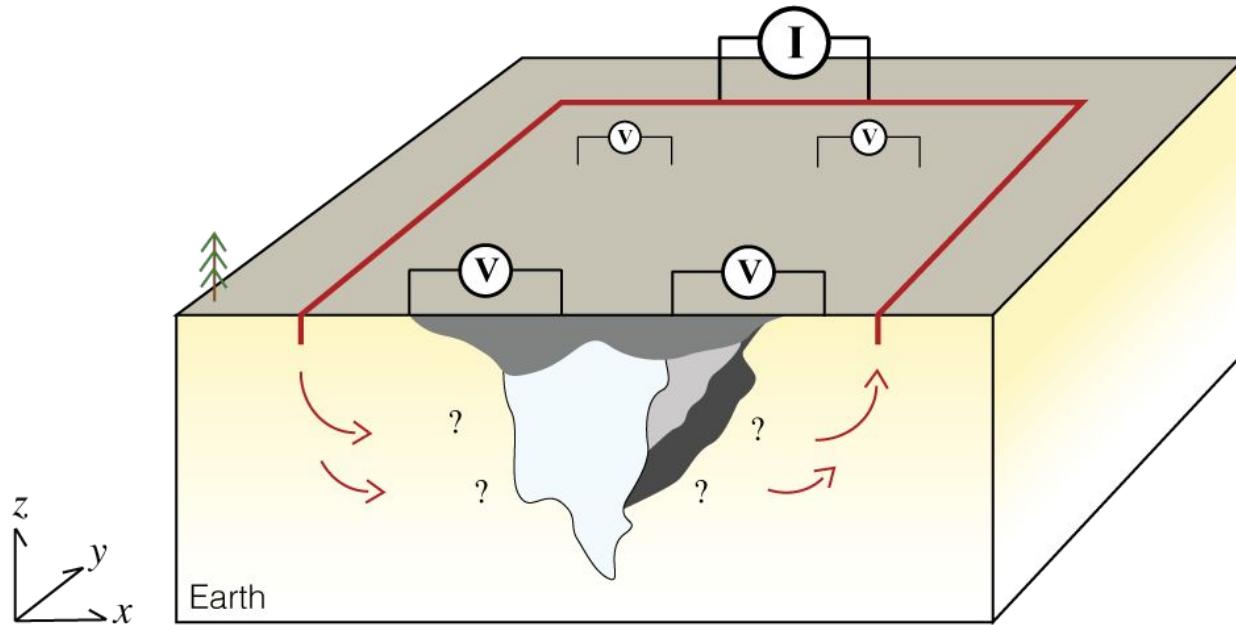


The world is 3D

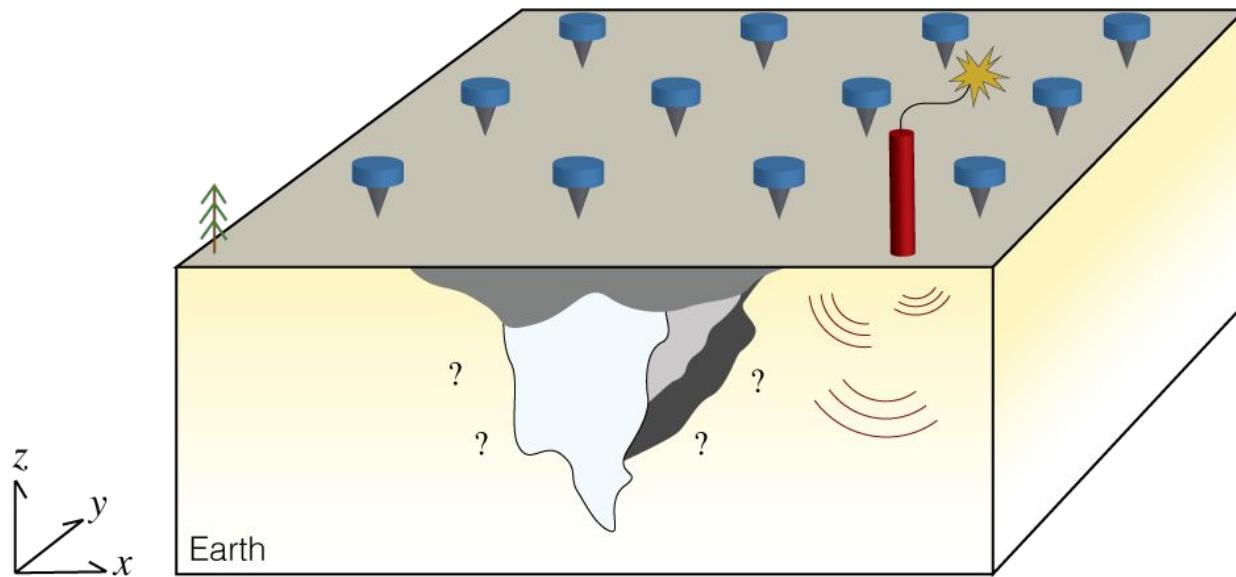


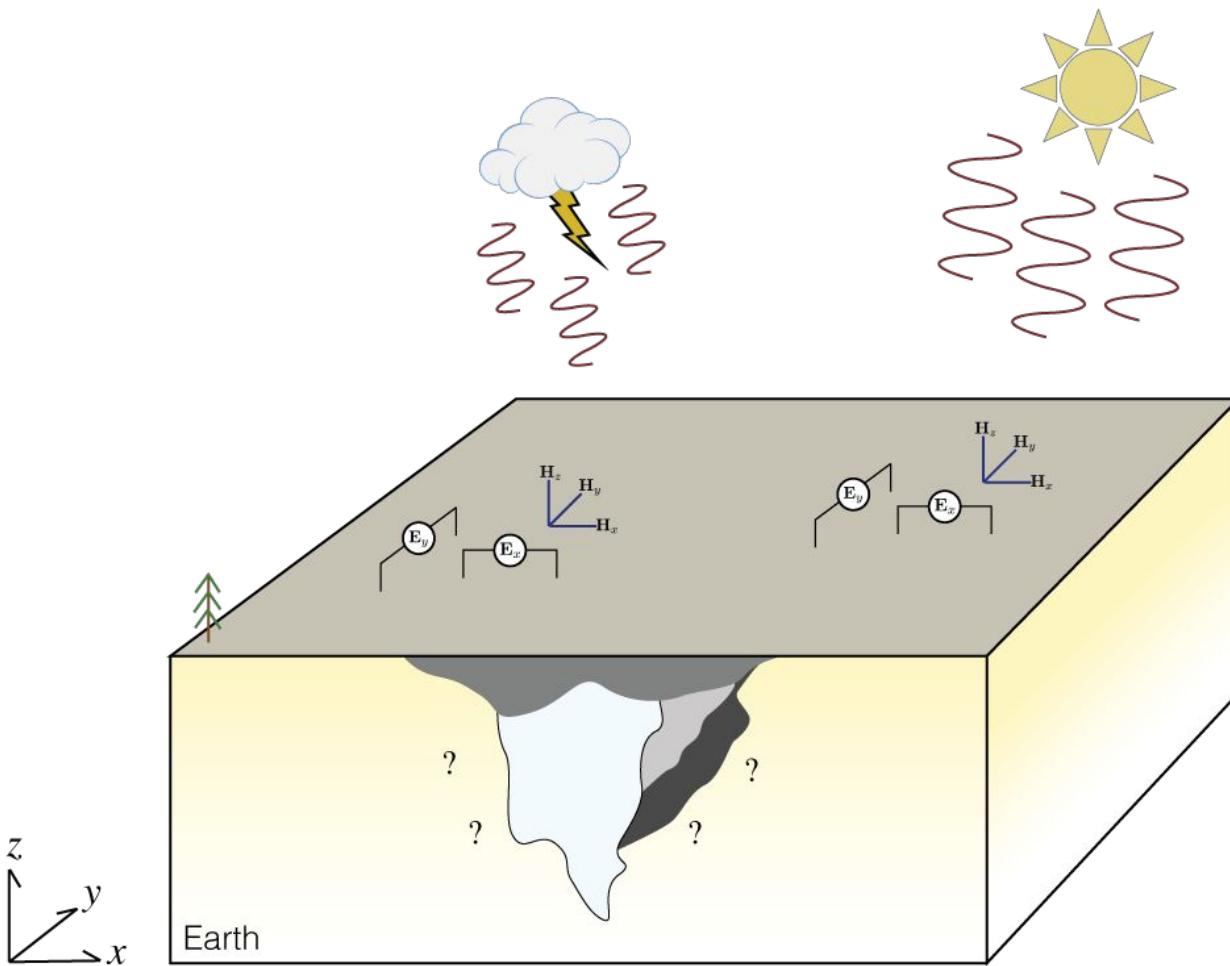
Gravity





Direct current resistivity



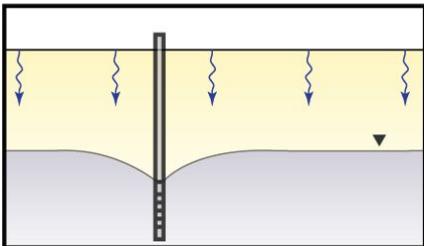


Magnetotellurics

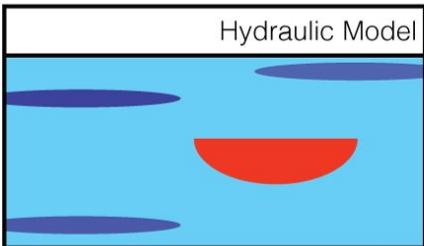


Predict & Decide

Hydrogeology

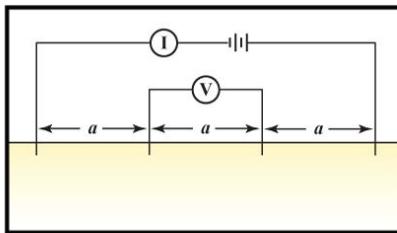


Vadose Zone

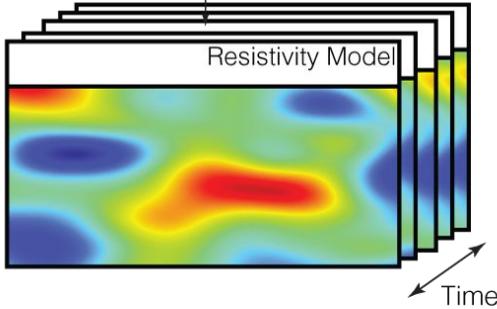


Hydraulic Model

Geophysics



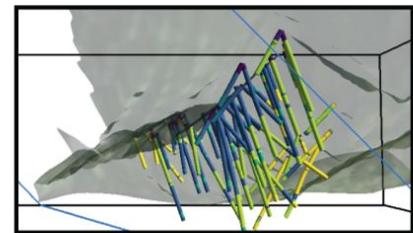
DC Resistivity



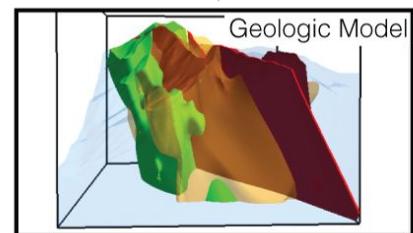
Resistivity Model

Time

Geology



Borehole Data



Geologic Model

Hydrogeology

Geophysics

Geology

Inversion

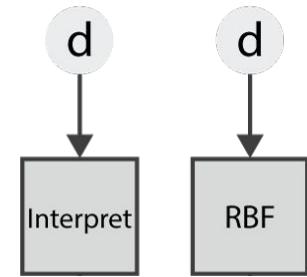
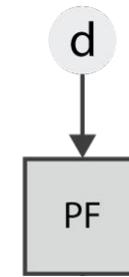
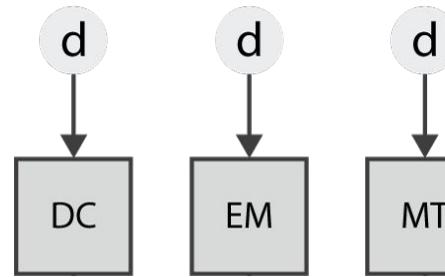
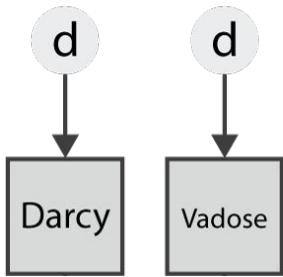


Hydrogeology

Geophysics

Geology

Inversion

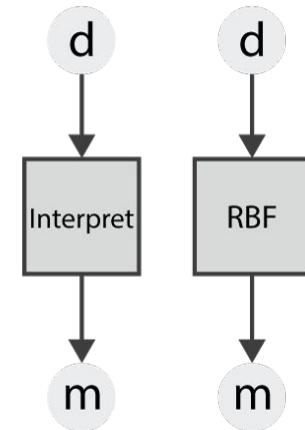
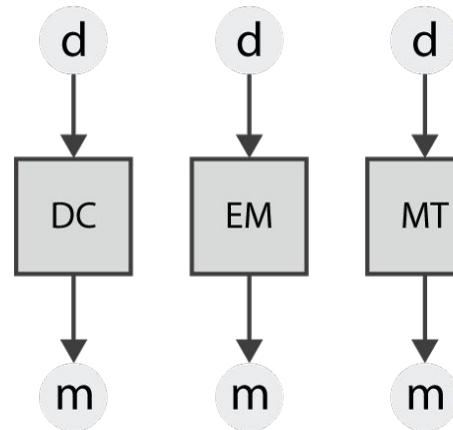
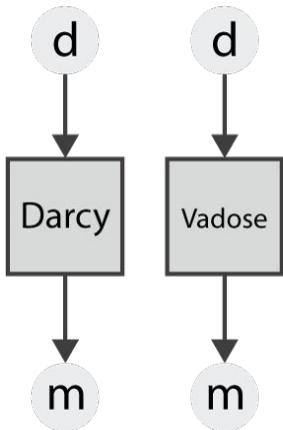


Hydrogeology

Geophysics

Geology

Inversion

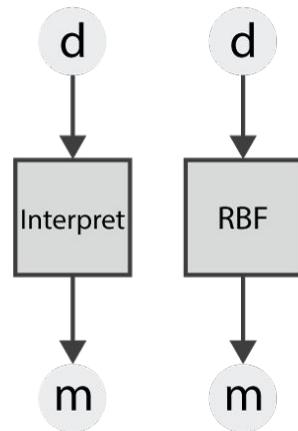
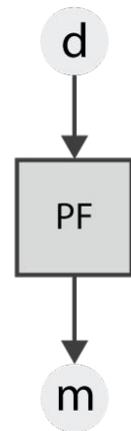
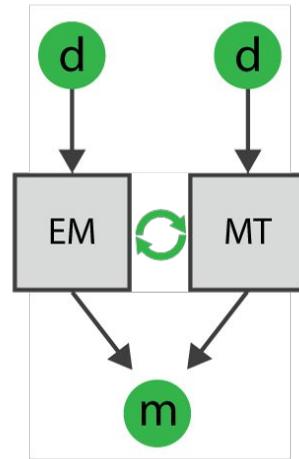
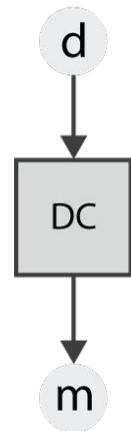
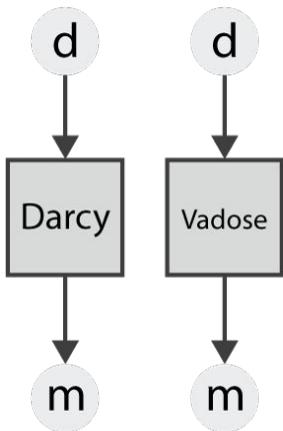


Hydrogeology

Geophysics

Geology

Inversion

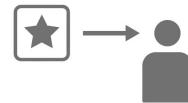
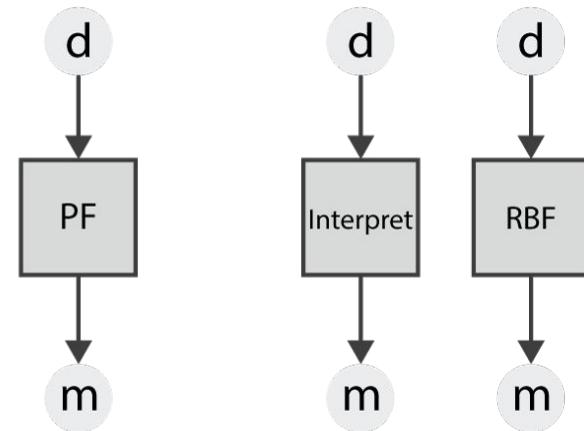
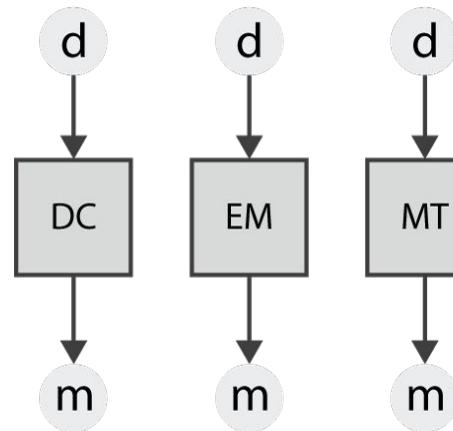
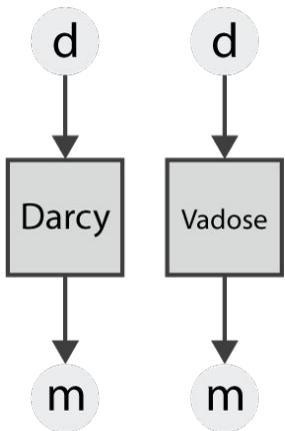


Hydrogeology

Geophysics

Geology

Inversion

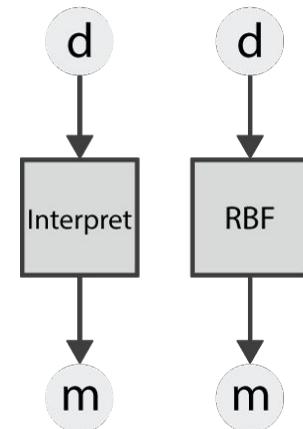
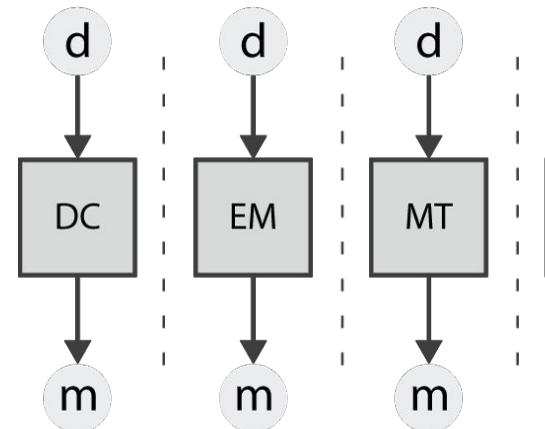
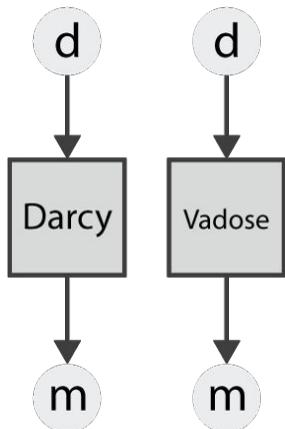


Hydrogeology

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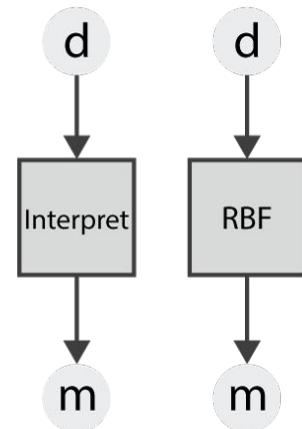
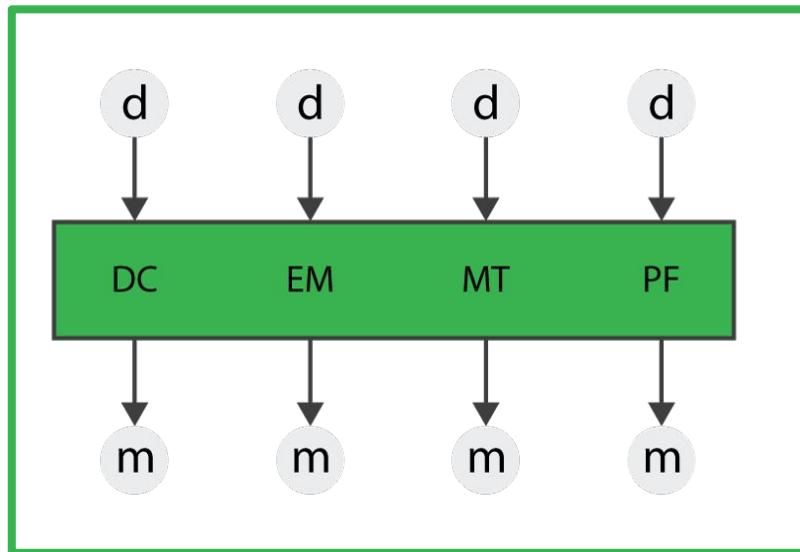
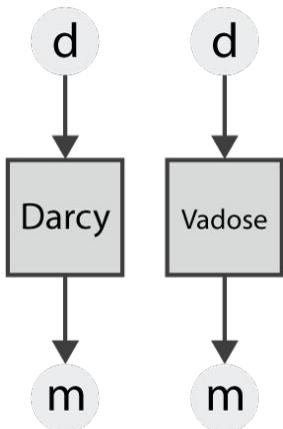


Hydrogeology

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Geology

Inversion

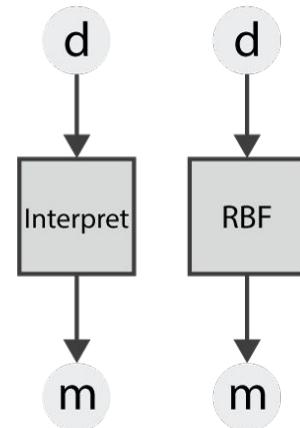
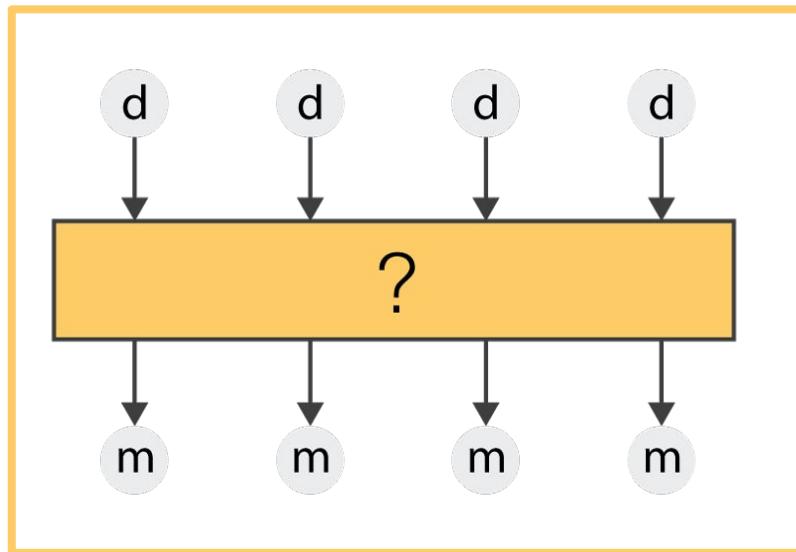
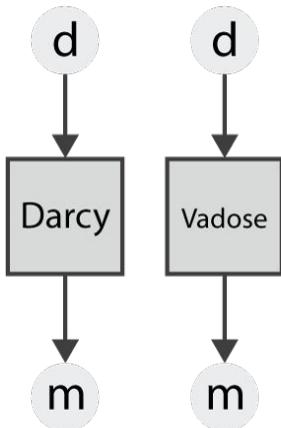


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Geology

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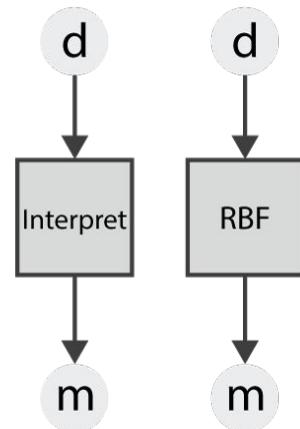
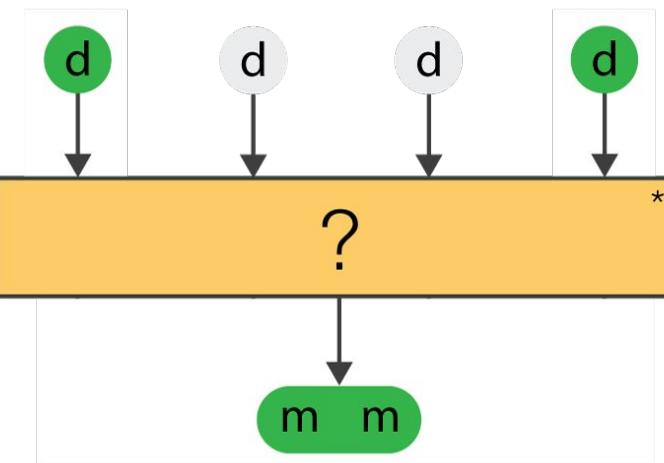
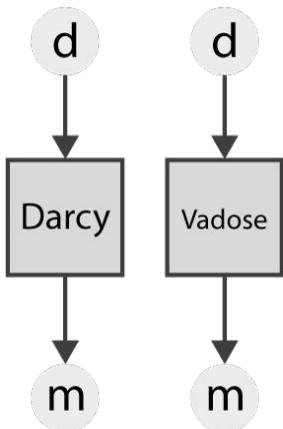


Hydrogeology

Geophysics

Geology

Inversion



Inputs

Data and
Uncertainty
Estimates

Governing
Equations

Prior Knowledge
and Assumptions

Inversion Implementation

Forward Simulation

- Discretization of the earth
- Simulation of fields/fluxes
- Survey geometry

Inversion

- Data Misfit
- Regularization
- Statement of inverse problem
- Numerical optimization

Evaluation

Evaluate and Assess Results

Interpret Preferred
Model(s)

Science! (in SimPEG)

Data

- T_x
- R_x
- $S_{req/times}$
- $\text{type } (E_x, E_y, \dots)$
- dobs
- `projectFields(u=None)`
- `setProblems probData=self`
- `dPred(m)`
- residual
- residualWeighted
- `plotData` → `probSectors`
- `filter` → only some data → stochastic.
- `User Data`
- `call Data: sdiag(mdict=)`

Mesh

- $h / H / \text{refine}$
- X_0
- operators

Model

- eg. by Land Model
- m [array?]
- transforms
- transformDeriv.

$$\Delta m = H^{-1} g$$

Problem

- mesh
- J (-approx)
- J^T (-approx)
- J dense
- create Synthetic Data
- use fields
- o Data needed for Tx locs.
- diag $J^T J$ (-approx)
- trace $J^T J$ (-approx)

ObjFunc

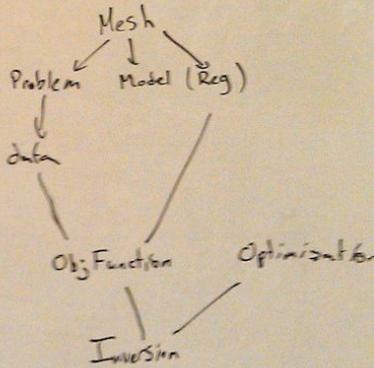
- Inversion Reg Optimization.
- * functions
 - $\rightarrow P$
 - $\rightarrow \text{reg}$
 - $\rightarrow \text{obj funct.}$
 - $\rightarrow \text{model}$
- $\rightarrow \text{dofIndJInv}$

$$J \Delta m \approx \underline{\underline{\delta d}}$$

$$\| J \Delta m - \underline{\underline{\delta d}} \|_F^2$$

$$\Delta m = (J^T J)^{-1} J^T \underline{\underline{\delta d}}$$

$$\left[\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right] \quad \left[\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right]$$



- * How do we explain the data?
 What problem explains this data?
 How do we optimize a model to fit this data?

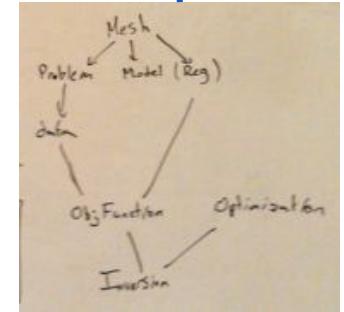
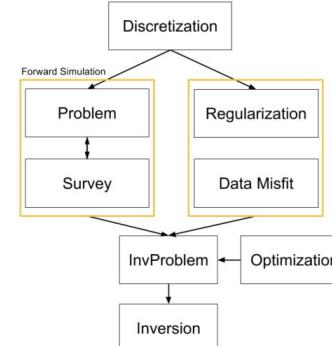
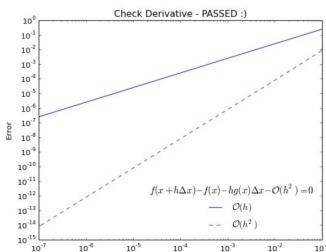
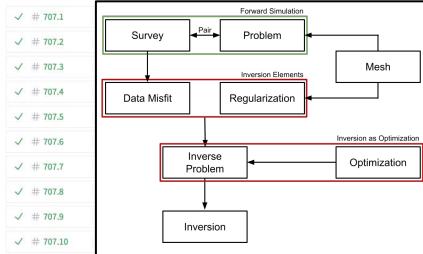
We create models and physics that explain data!

```
In [1]: import SimPEG
```

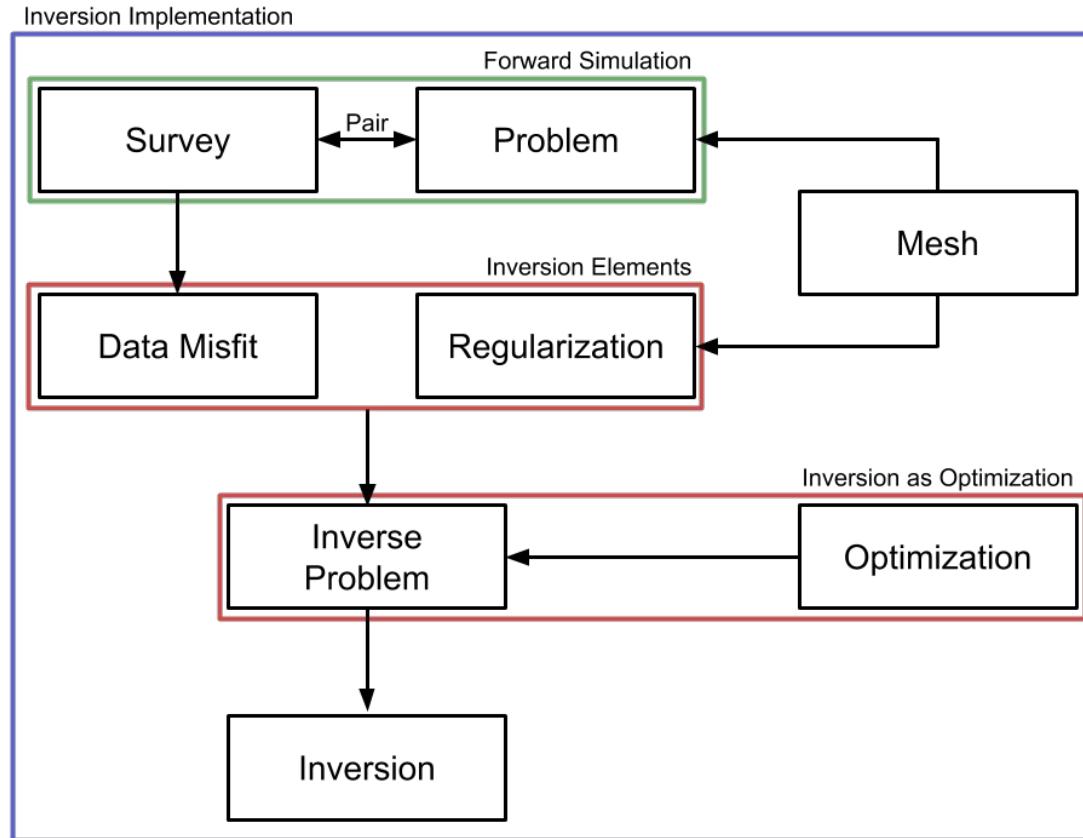
```
In [ ]: SimPEG.|
```

```
SimPEG.DataMisfit
SimPEG.Directives
SimPEG.InvProblem
SimPEG.Inversion
SimPEG.Maps
SimPEG.Mesh
SimPEG.Models
SimPEG.Optimization
SimPEG.Problem
SimPEG.Regularization
```

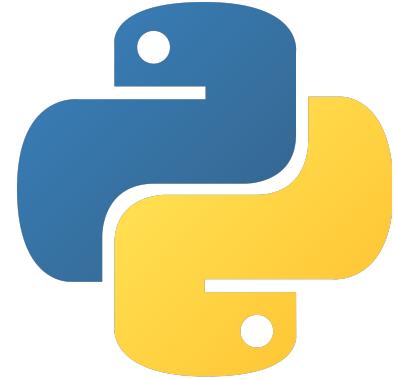
```
✓ # 707.1
✓ # 707.2
✓ # 707.3
✓ # 707.4
✓ # 707.5
✓ # 707.6
✓ # 707.7
✓ # 707.8
✓ # 707.9
✓ # 707.10
Run 430 tests in 245.984s
OK
```



Implementation



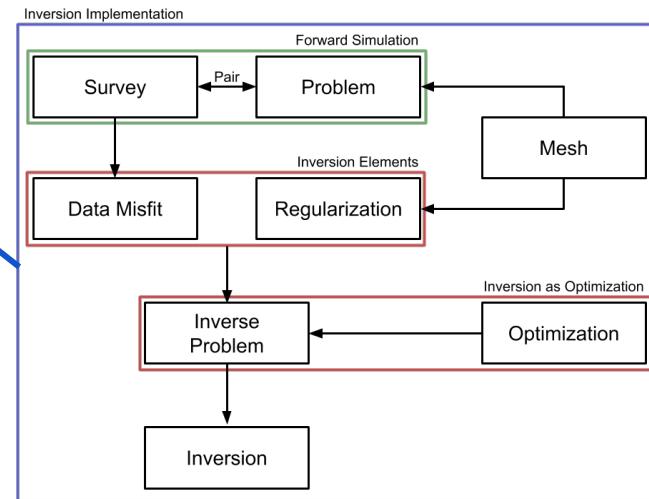
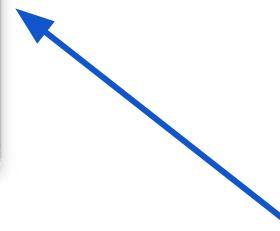
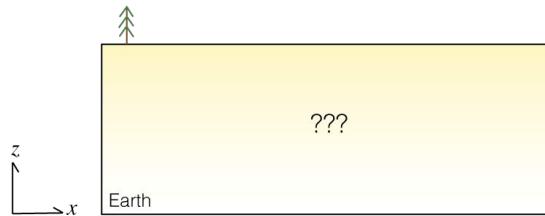
Interactive Geophysics!



```
In [1]: import SimPEG
```

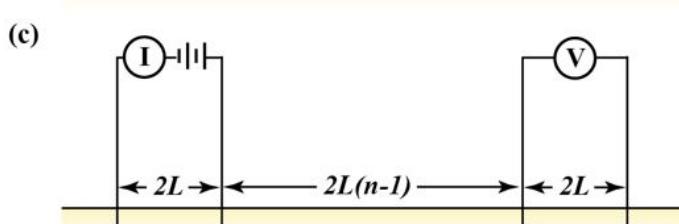
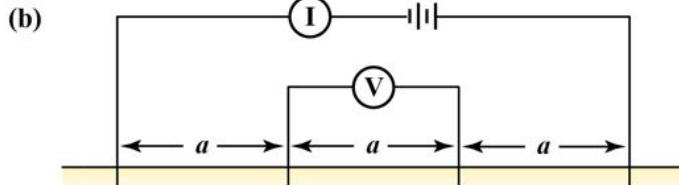
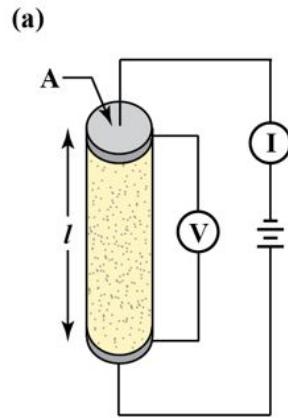
```
In [ ]: SimPEG.|
```

```
SimPEG.DataMisfit  
SimPEG.Directives  
SimPEG.InvProblem  
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SimPEG.Maps  
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SimPEG.Models  
SimPEG.Optimization  
SimPEG.Problem  
SimPEG.Regularization
```

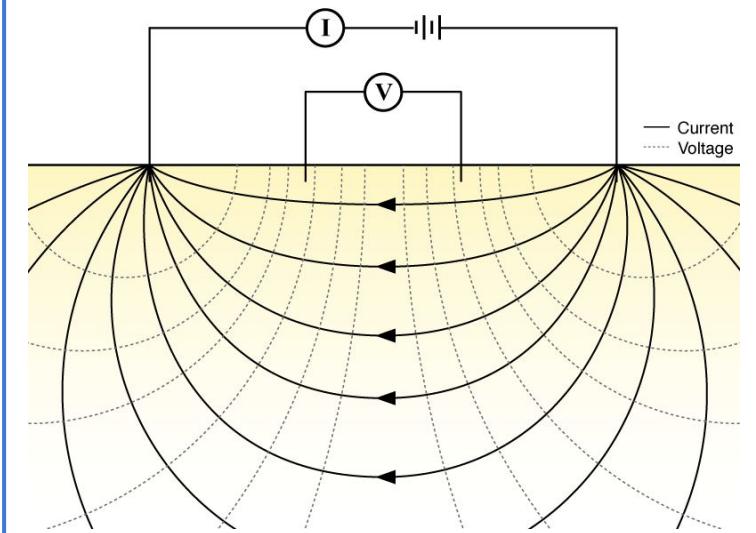


Survey & Problem

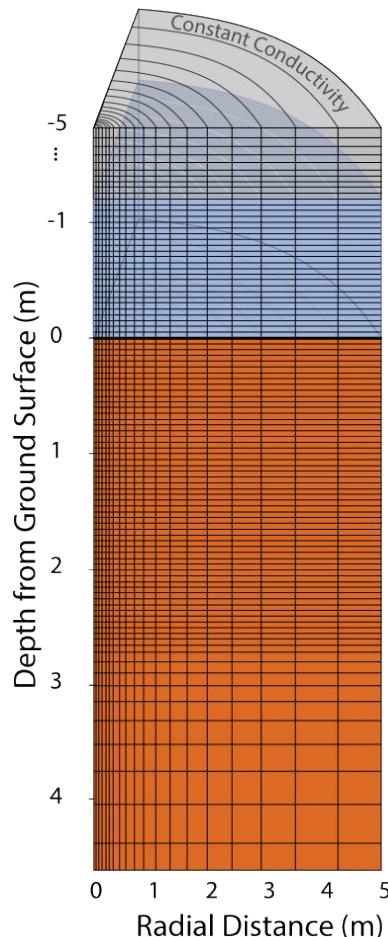
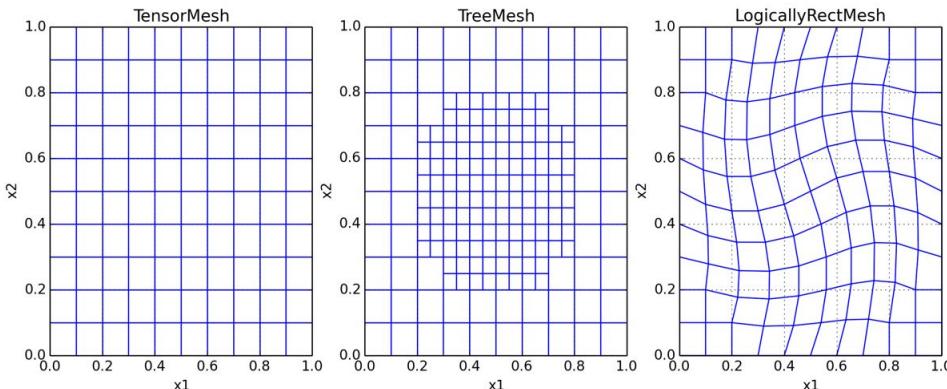
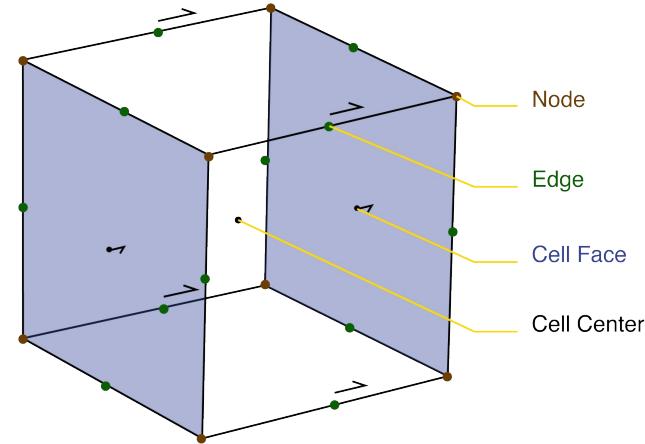
Data collection and geometry



Physics

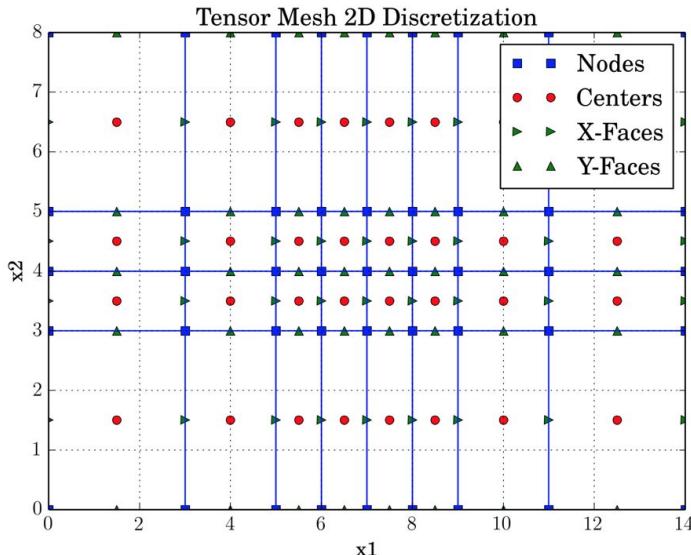


Finite Volume



Creating a Mesh

```
hx = [3,2,1,1,1,1,2,3]
hy = [3,1,1,3]
M = Mesh.TensorMesh([hx, hy])
M.plotGrid(faces=True, nodes=True, centers=True)
```



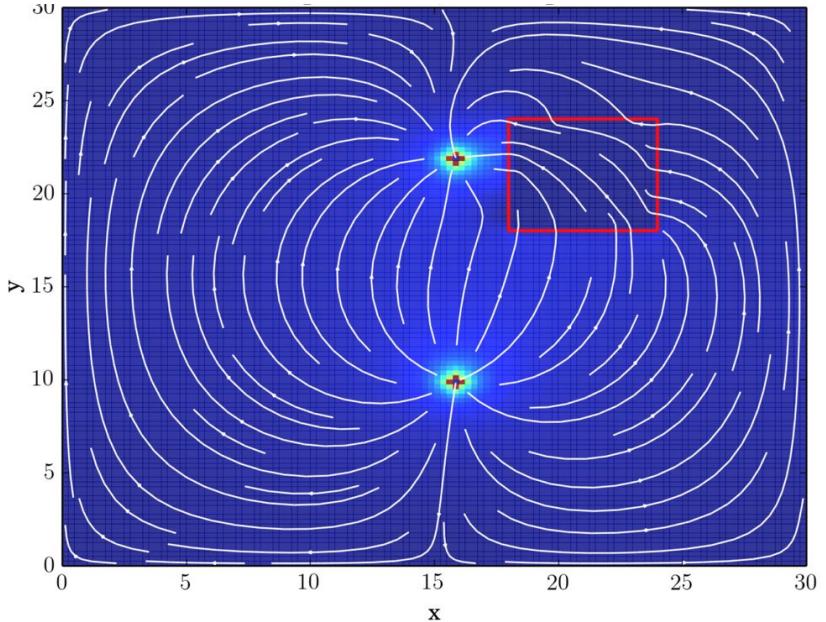
Property or Function

dim, x0
nC, nN, nF, nE
vol, area, edge
gridN, gridCC, ...
faceDiv, edgeCurl, cellGrad
aveF2CC, aveN2CC, etc.
getEdgeInnerProduct()
getInterpolationMat(loc)

DC Resistivity

$$\nabla \cdot (-\sigma \nabla \phi) = \mathbf{I}(\delta(\mathbf{r} - \mathbf{r}_{s^+}) - \delta(\mathbf{r} - \mathbf{r}_{s^-}))$$

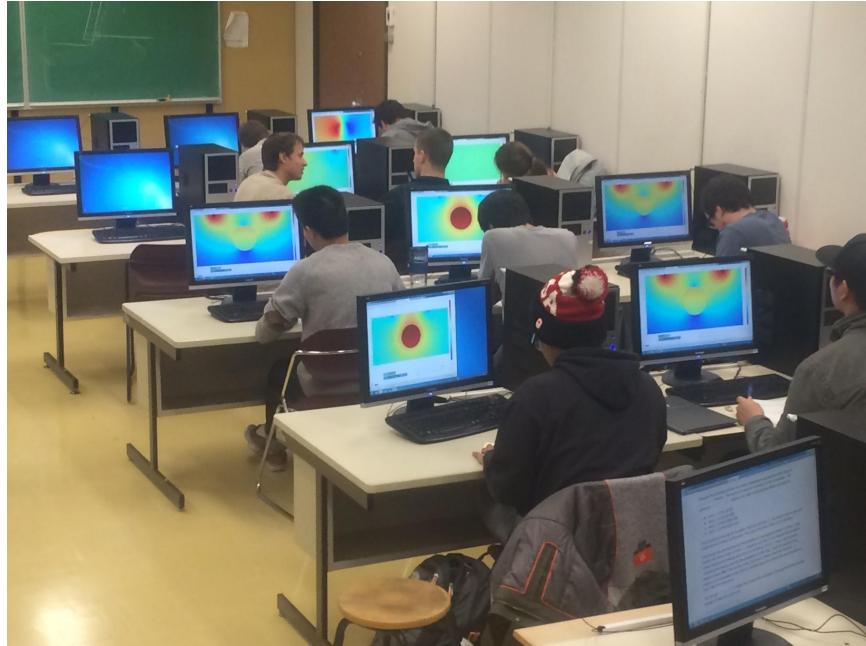
```
D = M.faceDiv
G = M.cellGrad
# Harmonically average sigma
MsigI = sdInv(sdIag(M.aveF2CC.T*(1/sig)))
A = D*MsigI*G
A[0,0] *= 1/M.vol[0] # Remove the null space
Ainv = Solver(A) # Create a default Solver
phi = Ainv * ( - q )
```



DC Resistivity

$$\nabla \cdot (-\sigma \nabla \phi) = \mathbf{I}(\delta(\mathbf{r} - \mathbf{r}_{s^+}) - \delta(\mathbf{r} - \mathbf{r}_{s^-}))$$

```
D = M.faceDiv
G = M.cellGrad
# Harmonically average sigma
MsigI = sdInv(sdiag(M.aveF2CC.T*(1/sig)))
A = D*MsigI*G
A[0,0] *= 1/M.vol[0] # Remove the null space
Ainv = Solver(A) # Create a default Solver
phi = Ainv * ( - q )
```



Time Domain Electromagnetics

32 lines of code

$$\nabla \times \vec{e} + \frac{\partial \vec{b}}{\partial t} = 0,$$

$$\nabla \times \frac{1}{\mu_0} \vec{b} - \sigma \vec{e} = \vec{j}_s.$$

$$\mathbf{C}\vec{e}^{(t+1)} + \frac{\vec{b}^{(t+1)} - \vec{b}^{(t)}}{\Delta t} = 0$$

$$\mathbf{C}^\top \mathbf{M}_{\mu^{-1}}^f \vec{b}^{(t+1)} - \mathbf{M}_\sigma^e \vec{e}^{(t+1)} = \mathbf{M}^e \vec{j}_s^{(t+1)}$$

```
from SimPEG import *
import simpegEM as EM
from pymatsolver import MumpsSolver
from scipy.constants import mu_0

# Create the computational mesh
cs, nc, npad = 20., 20, 5 # cell sz, num cells/padding
h = [(cs,npad,-1.3),(cs,nc),(cs,npad,1.3)]
mesh = Mesh.TensorMesh([h,h,h], 'CCC')

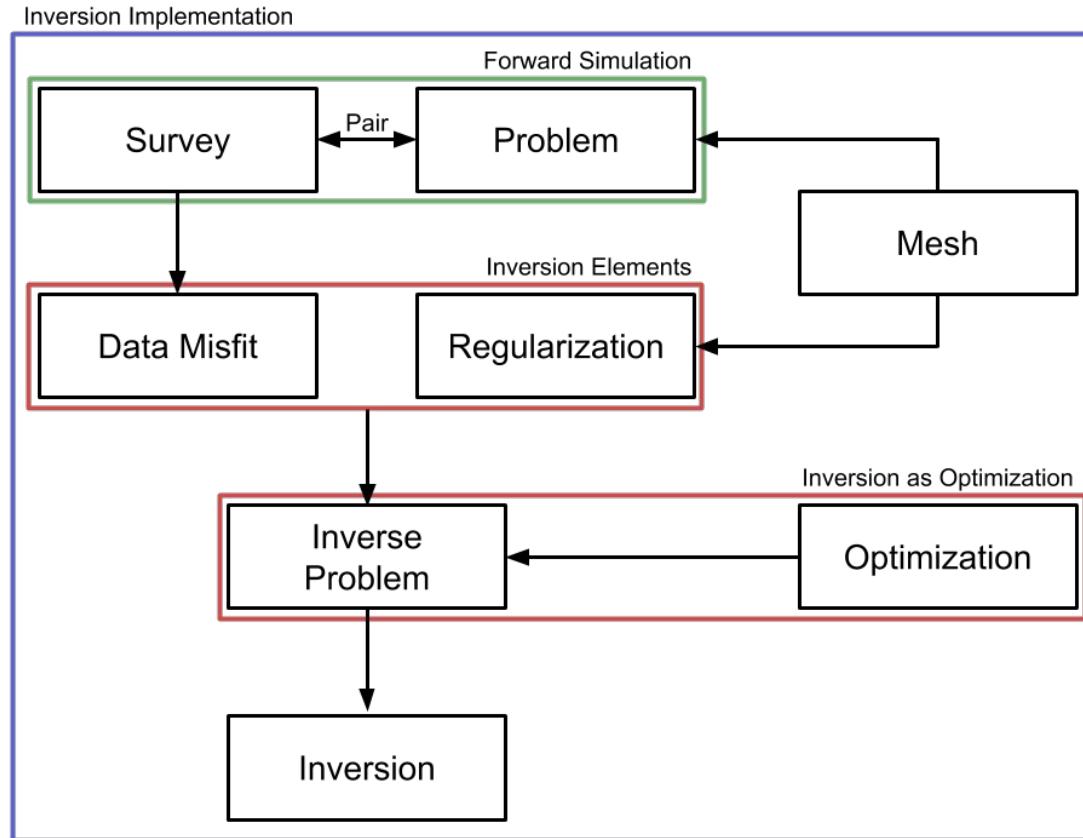
# Create a half-space conductivity
sigma = np.ones(mesh.nC)*1e-8
sigma[mesh.gridCC[:,2] < 0] = 1e-3

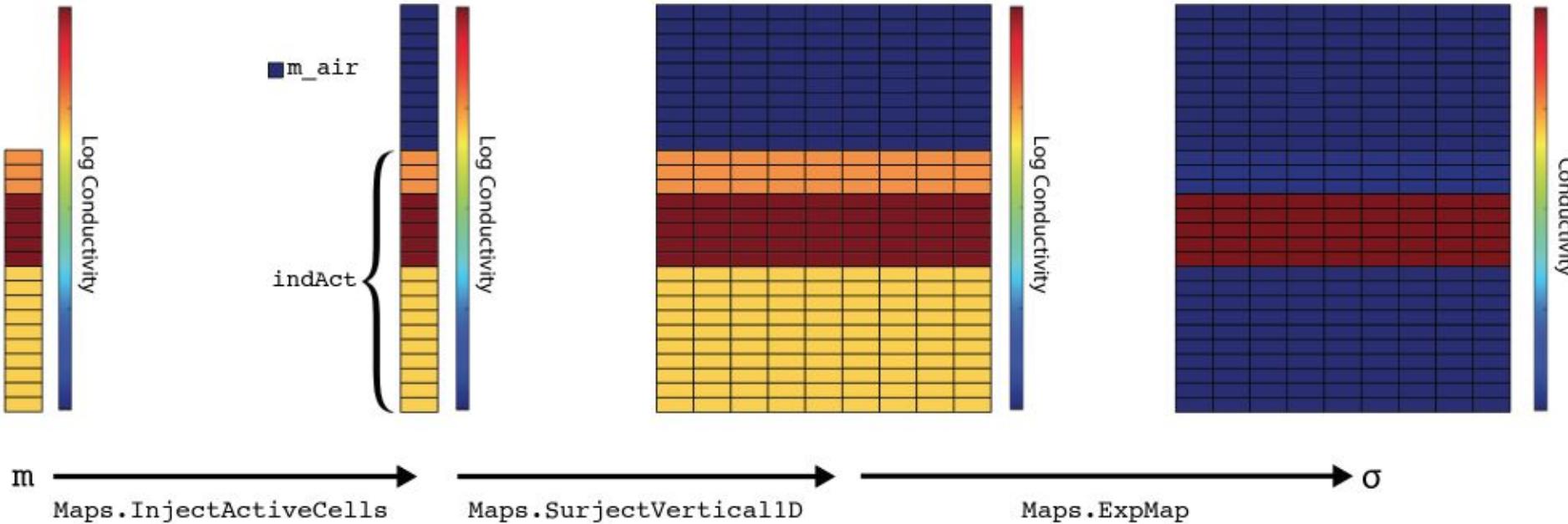
# Create a source in the center of our domain (0,0,0)
As = EM.Sources.MagneticDipoleVectorPotential(
    np.zeros(3), mesh, ['Ex','Ey','Ez'])
C = mesh.edgeCurl
b0 = C*As

# Create inner products
MesigI = mesh.getEdgeInnerProduct(sigma, invMat=True)
Mfmui = mesh.getFaceInnerProduct(1./mu_0)
Me    = mesh.getEdgeInnerProduct()

# Maxwell's Equation (eliminate e, j_s=0)
dt = 1e-7 # Choose a time-step
A = Mfmui*C*MesigI*C.T*Mfmui + 1.0/dt*Mfmui
Ainv = MumpsSolver(A) # Factor the matrix!
# Solve for b using Backward Euler!
B = [b0] + range(299)
for i in range(len(B)-1):
    B[i+1] = Ainv * ( 1.0/dt*Mfmui*B[i] )
```

Implementation





```
In [ ]: from SimPEG import Mesh, Maps, np
mesh = Mesh.CylMesh([20, 20]) # SimPEG cylindrically symmetric mesh
m_air = np.log(1e-8) # value of the model in the air cells
indAct = mesh.vectorCCz < 0. # define active cells to be subsurface only
mapping = ( Maps.ExpMap(mesh) *
            Maps.SurjectVertical1D(mesh) *
            Maps.InjectActiveCells(mesh, indAct, m_air, nC=mesh.nCz) )
```

Inversion Elements

Data Misfit

$$\phi_d(\mathbf{m}) = \frac{1}{2} \|\mathbf{W}_d(F[\mathbf{m}] - \mathbf{d}_{\text{obs}})\|_2^2$$

Regularization

$$\phi_m(\mathbf{m}) = \frac{1}{2} \|\mathbf{W}_m(\mathbf{m} - \mathbf{m}_{\text{ref}})\|_2^2$$

$$\phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m})$$

Inverse Problem

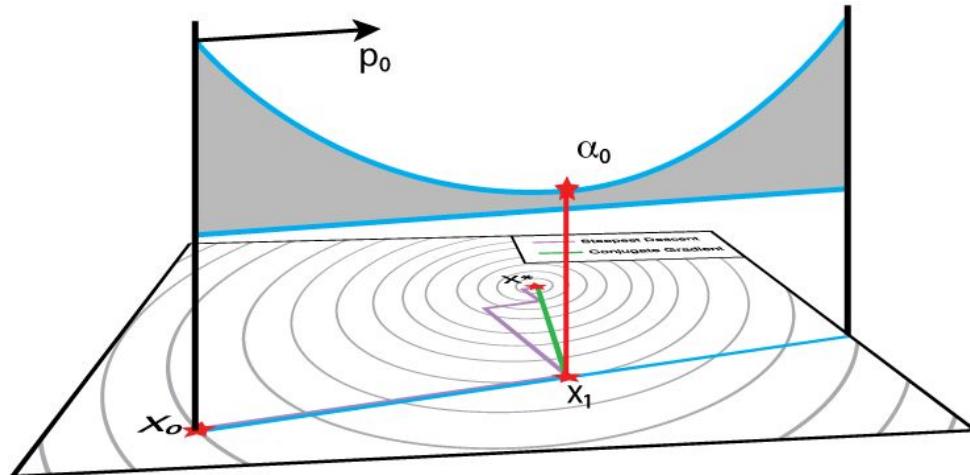
Optimization

```
In [1]: from SimPEG import Optimization
```

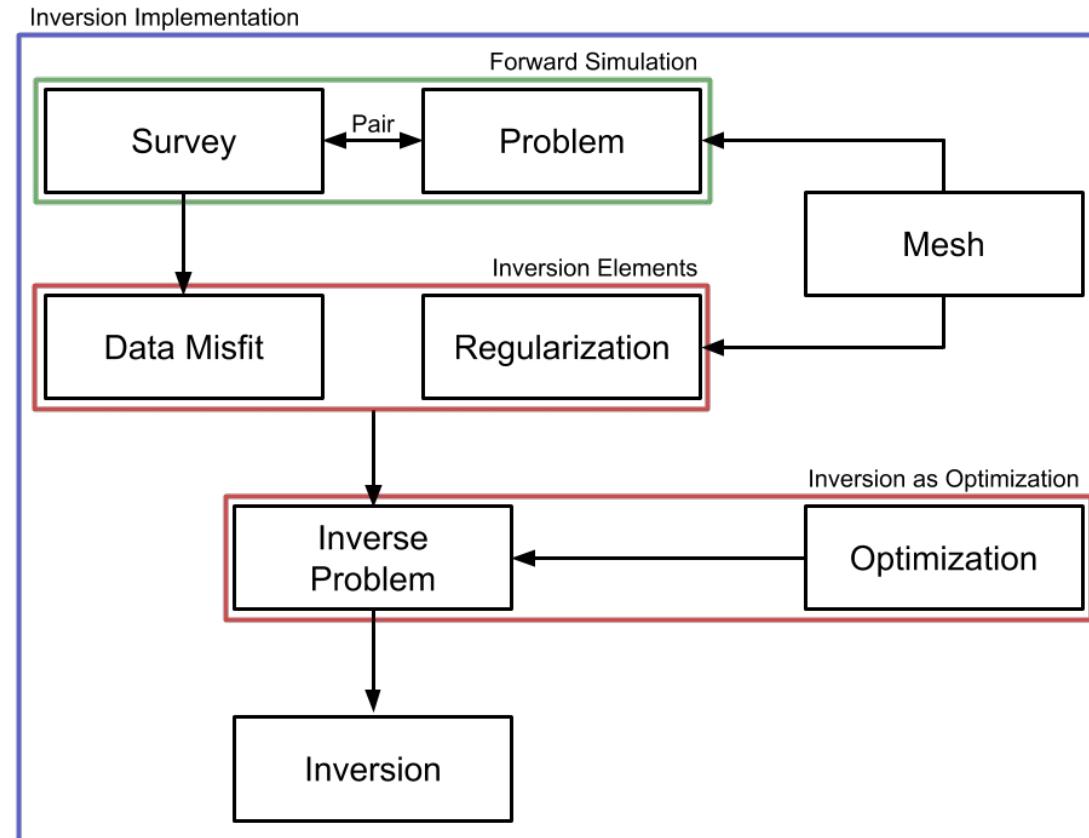
```
In [ ]: Optimization.  
Optimization.BFGS  
Optimization.GaussNewton  
Optimization.InexactGaussNewton  
Optimization.IterationPrinters  
Optimization.Minimize  
Optimization.NewtonRoot  
Optimization.ProjectedGNCG  
Optimization.ProjectedGradient  
Optimization.Remember  
Optimization.Solver
```

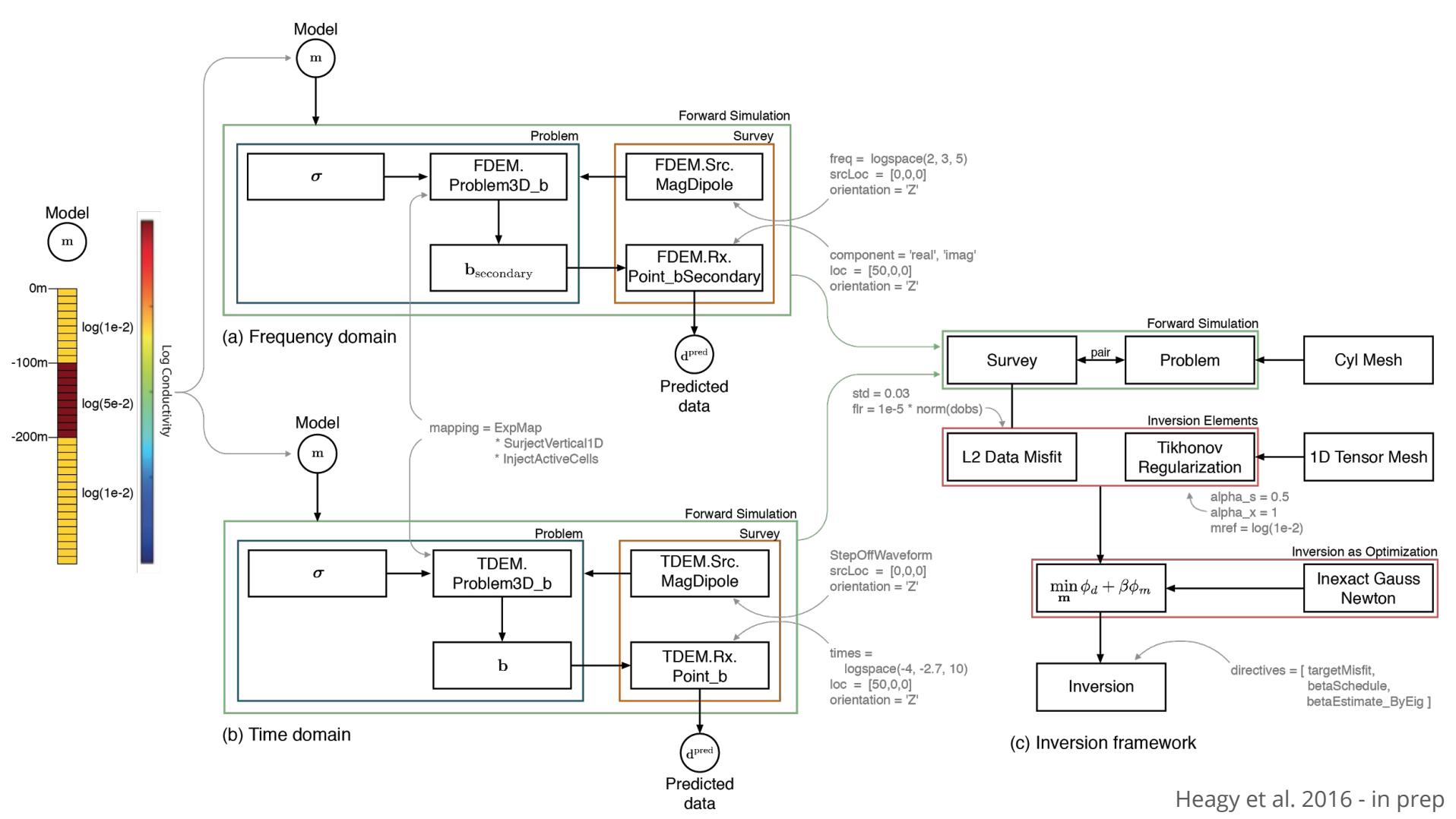
$$\underset{\mathbf{m}}{\text{minimize}} \quad \phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m})$$

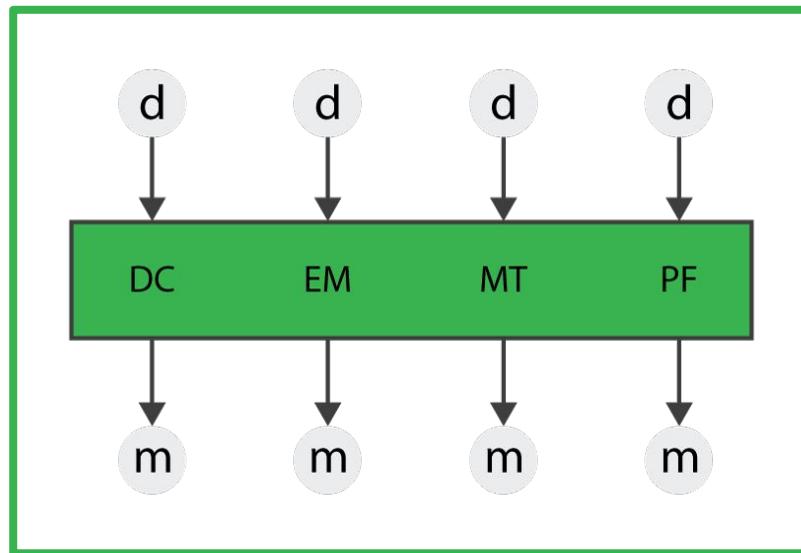
$$\text{s.t.} \quad \phi_d \leq \phi_d^*, \quad \mathbf{m}_i^L \leq \mathbf{m}_i \leq \mathbf{m}_i^H$$



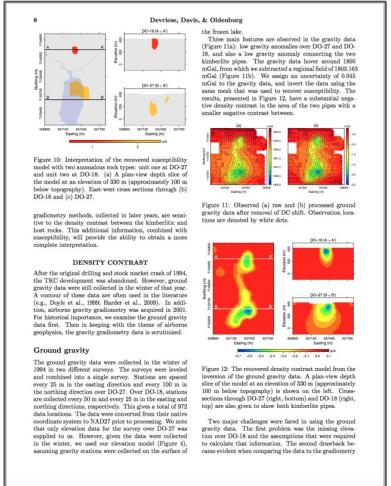
Bringing it together.







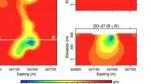
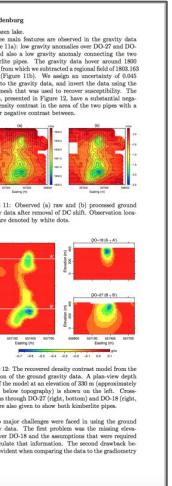
Where are the diamonds?



gradiometry methods, collected in later years, are sensitive to the density contrast between the kimberlite and host rocks. By comparing the observed density and magnetic susceptibility, will provide the ability to obtain a more complete interpretation.

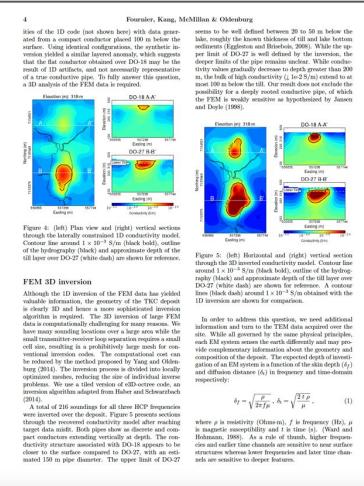
DEPTH CONTRAST
Although the original drilling and stock market crash in 1994, the TKC development was abandoned. However, ground gravity surveys were conducted in the TKC region. A contour of these data are often used in the literature (e.g. Doyle et al., 1995; Hunter et al., 2010). In addition to the TKC region, we also collected ground gravity data. For historical purposes, we examine the ground gravity data from the TKC region. The TKC region has a complex geology, the ground gravity data are more complex than the TKC region.

Ground gravity
The total ground gravity data were collected in the winter of 1994. The data were collected in two separate surveys and combined into a single survey. Stations are spaced approximately 100 m apart and the data are collected along the north-south direction over DO-27. Over DO-18, stations are spaced every 20 m and every 20 m along the east-west direction. The total number of stations is a total of 872 data locations. The data were converted from their native coordinate system to a UTM coordinate system. Since the data is in the water, we used our elevation model (Figure 4), sampling gravity stations were collected on the surface of



DEPTH 3D inversion
Although the 1D inversion of the FEM data has yielded valuable information, the TKC region is a complex geological environment and hence a more sophisticated inversion algorithm is required. The 3D inversion of large FEM data sets is a challenging task. The TKC region has many sounding locations over a large area with the most frequent sounding locations being near the crater center site, resulting in a prohibitively large needs for computation time. Therefore, we developed a 3D inversion scheme to reduce the method proposed by Yang and Oldenburg (2003). The 3D inversion scheme uses a set of optimized meshes, reducing the size of individual inversion problems. We used the same frequency-domain inversion algorithm adopted from Haber and Scheichl (2009).

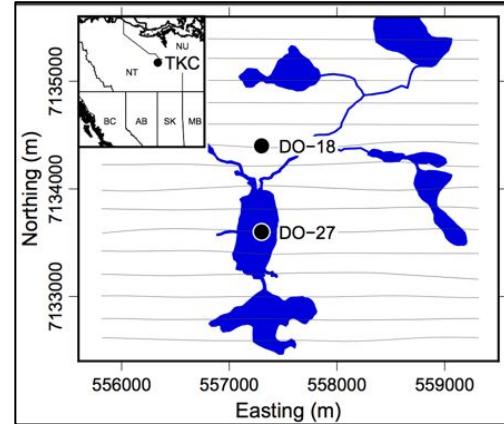
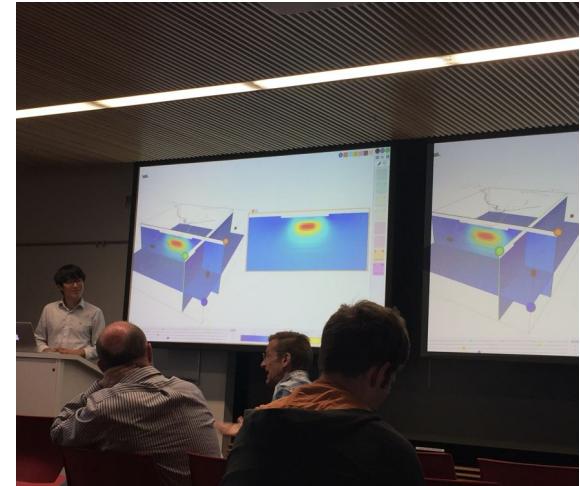
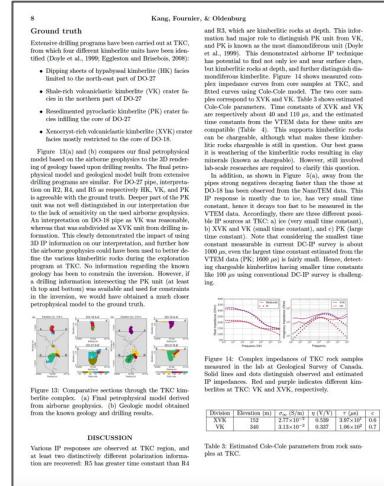
Two major challenges were faced in using the ground gravity data for the 3D inversion. The main challenge was the lack of a suitable model to represent the TKC region and combine it into a single survey. Stations are spaced very far apart and the data are collected along the north-south direction over DO-27. Over DO-18, stations are spaced every 20 m and every 20 m along the east-west direction. The total number of stations is a total of 872 data locations. The data were converted from their native coordinate system to a UTM coordinate system. Since the data is in the water, we used our elevation model (Figure 4), sampling gravity stations were collected on the surface of



In order to solve for the unknowns, we used additional information from the TSEND data acquired over the site. While all gathered from the same physical principles, the TSEND data and ground gravity data can provide complementary information about the porosity and density of the subsurface. The TSEND data can be used to reduce the noise introduced by the method proposed by Yang and Oldenburg (2003). The TSEND data can also be used to validate the 3D inversion results. The validation of an EM survey is a function of the skin depth (δ) and diffusion distance (d) in frequency and time-domain respectively:

$$d = \sqrt{\frac{f}{2\pi\mu}}, \quad \delta = \sqrt{\frac{f^2\mu}{\rho}} \quad (1)$$

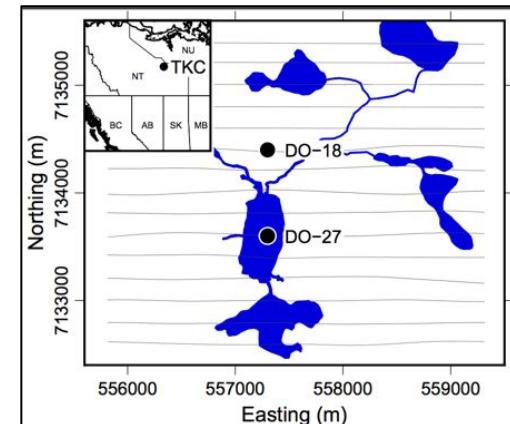
where f is frequency (Hz), μ is magnetic susceptibility and ρ is density (kg/m³). What and how much information can be obtained from the TSEND data? As discussed earlier, the TSEND data frequencies and earlier time channels are sensitive to near surface features whereas lower frequencies and later time channels are sensitive to deeper features.



Where are the diamonds?

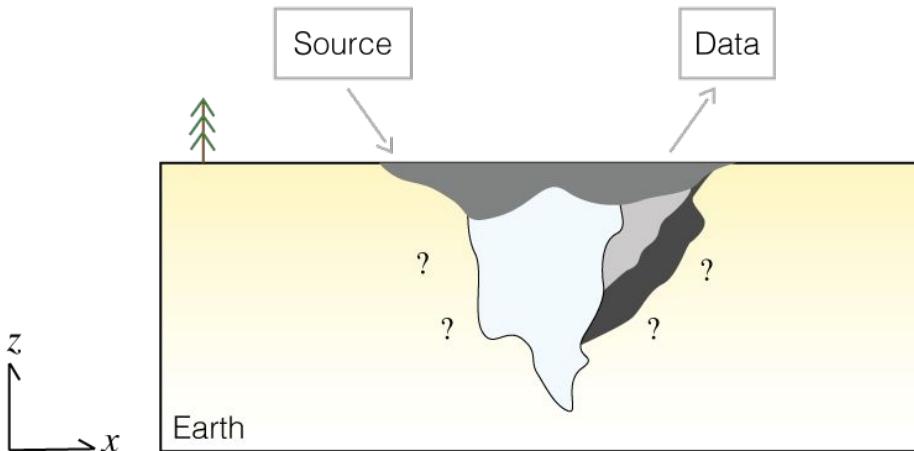
Kimberlite Pipe

- Different geologic units
- Specific units is diamond-rich



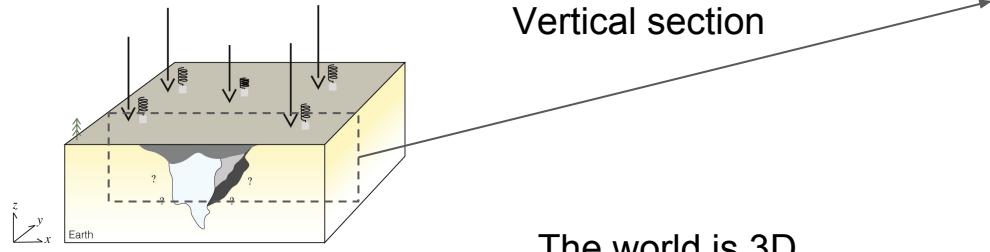
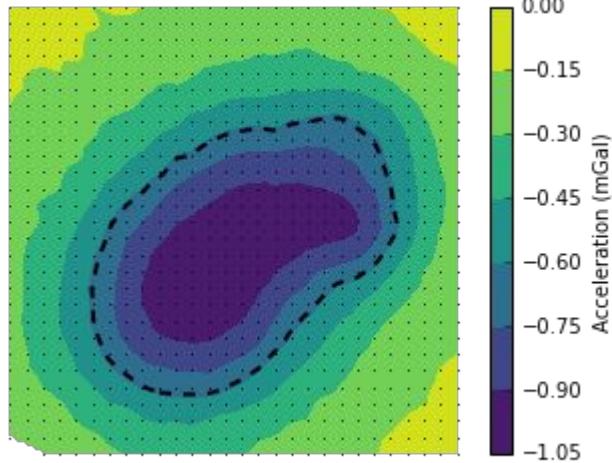
Physical Properties

- Low density
- Moderate susceptibility
- High conductivity



Physical properties

Gravity

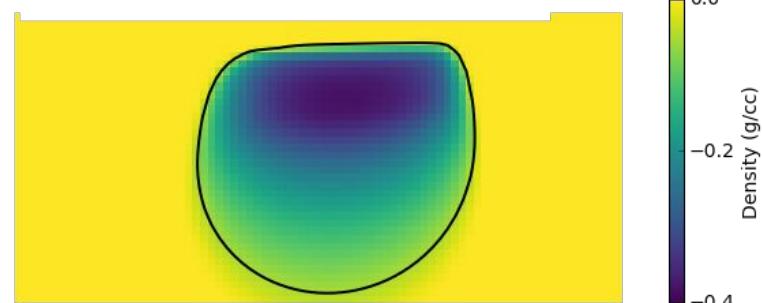


The world is 3D

Rock model?

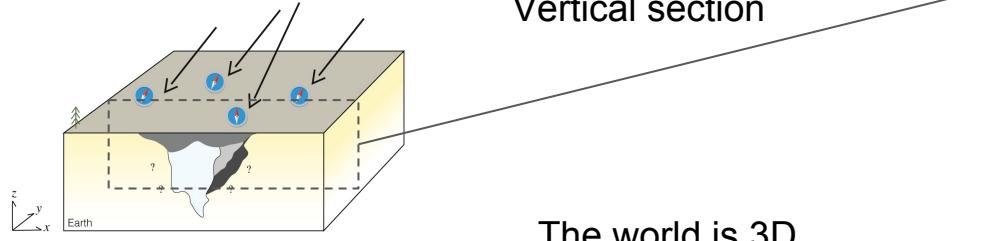
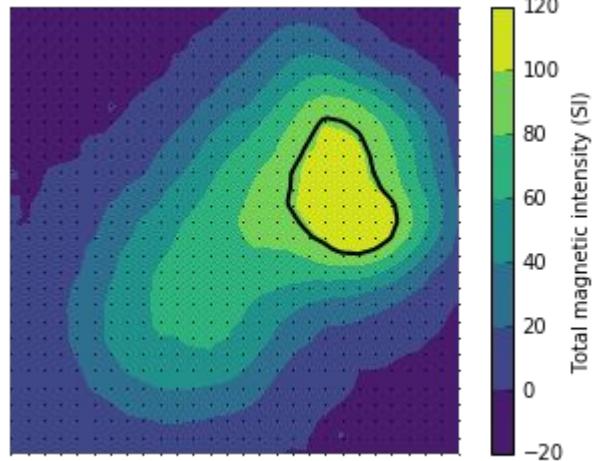


Recovered 3D Density



0.0
-0.2
-0.4

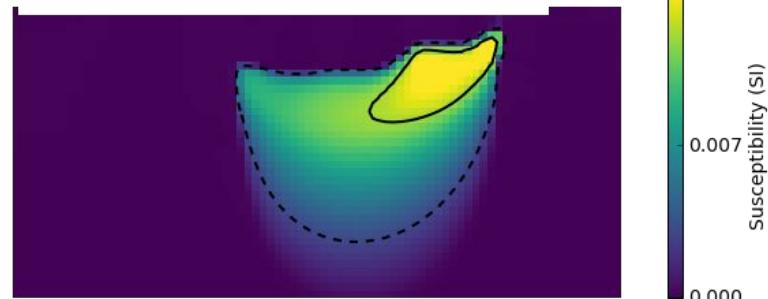
Magnetics



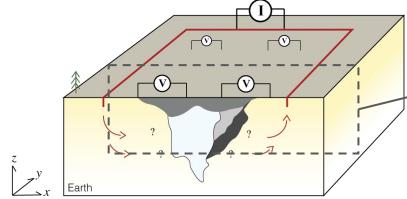
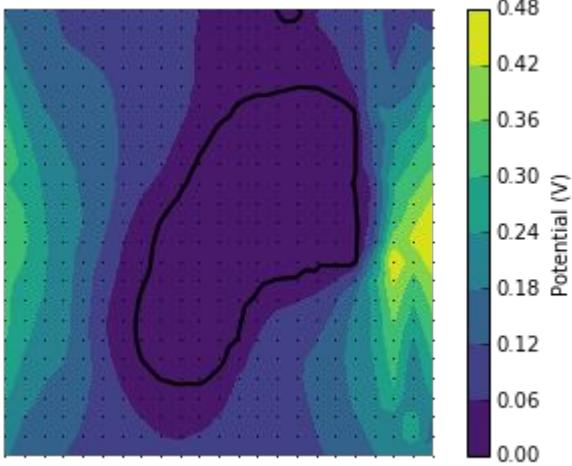
Rock model?



Recovered 3D Susceptibility



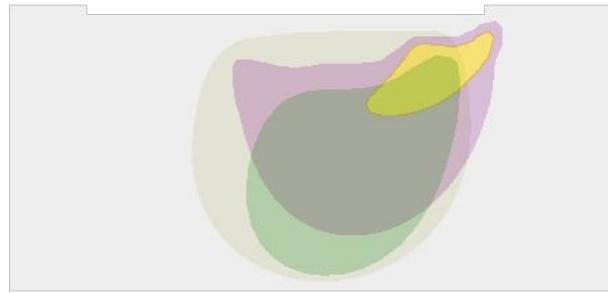
Direct Current (DC)



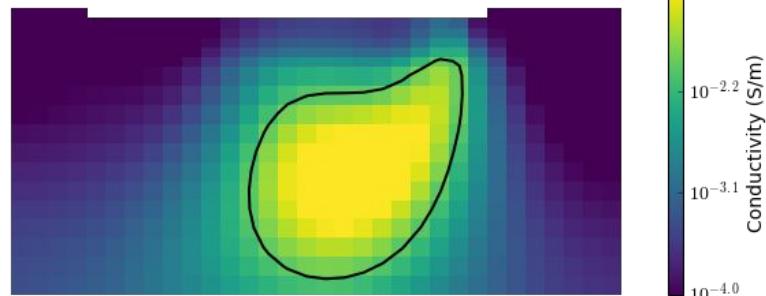
Vertical section

The world is 3D

Rock model?

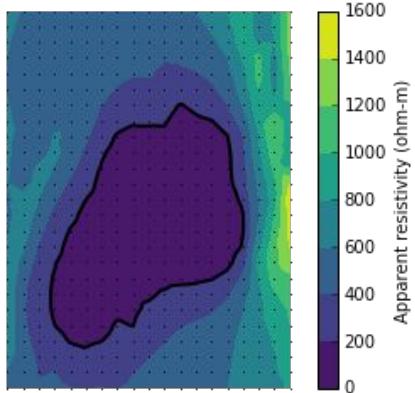


Recovered 3D Conductivity

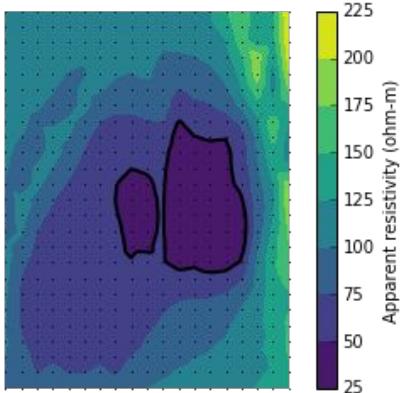


Magnetotellurics (MT)

1000 Hz



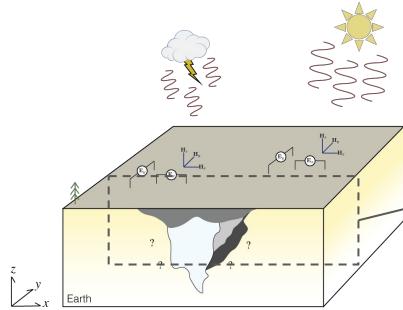
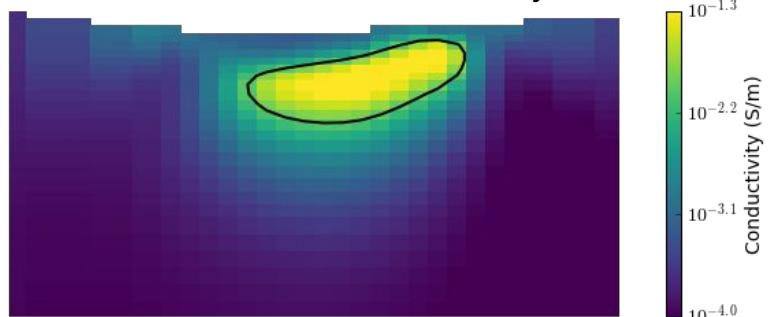
21544 Hz



Rock model?



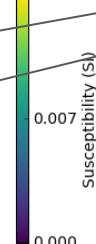
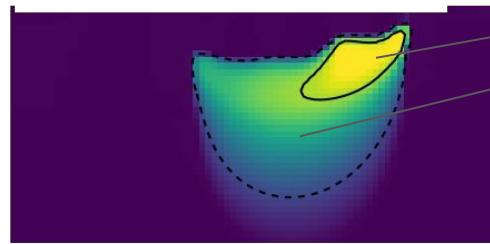
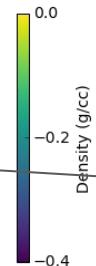
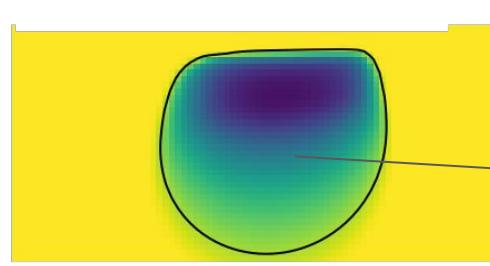
Recovered 3D Conductivity



Vertical section

The world is 3D

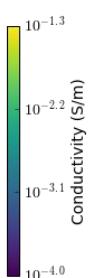
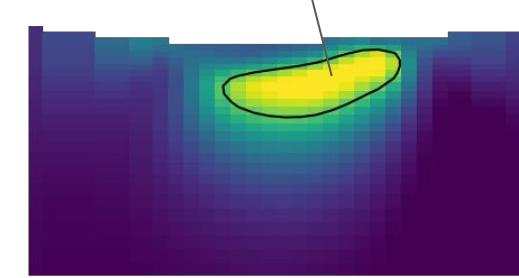
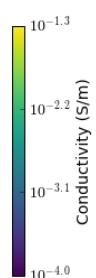
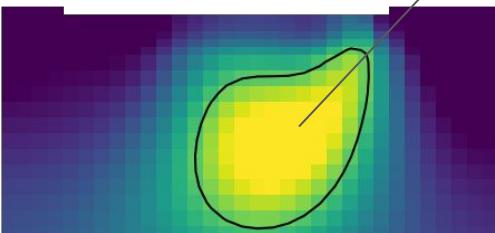
Integration or Interpretation?



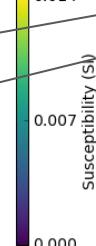
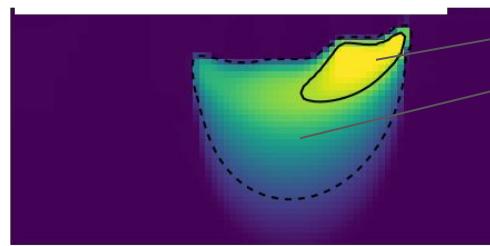
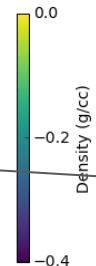
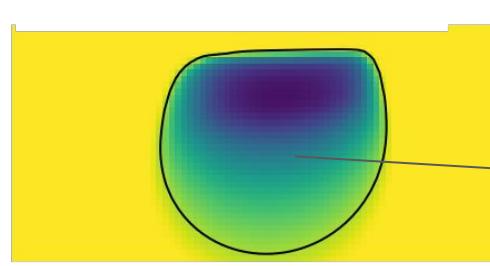
Rock model?

Diamond-rich kimberlite

- Low density
- Moderate susceptibility
- High conductivity

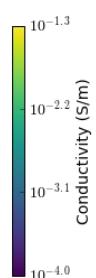
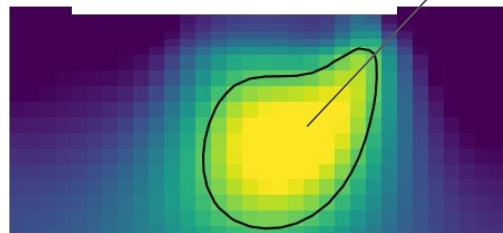


Integration or Interpretation?

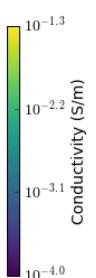
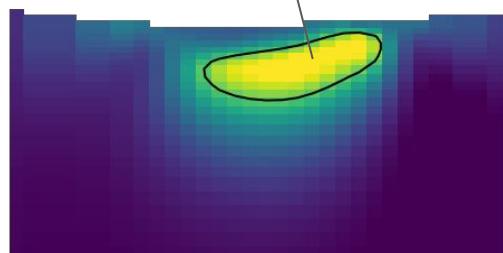


Diamond-rich kimberlite

- Low density
- Moderate susceptibility
- High conductivity



Rock model?



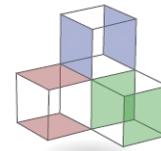
From geophysics to a geologic model

Geologic model



Rock model from geophysics





simpeg