Parametrized inversion framework for proppant volume in a hydraulically fractured reservoir

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SUMMARY

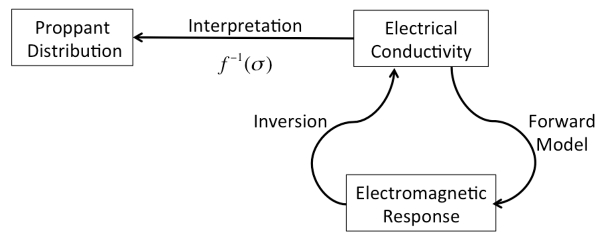
Hydraulic fracturing is an important technique to allow mobilization of hydrocarbons in tight reservoirs. Sand or ceramic proppant is pumped into the fractured reservoir to ensure fractures remain open and permeable after the hydraulic treatment. As such, the distribution of proppant is a controlling factor on where the reservoir is permeable and can be effectively drained. Methods to monitor the fracturing process, such as tiltmeters or microseismic, are not sensitive to proppant distributions in the subsurface after the fracturing treatment is complete (Cipolla and Wright, 2000).
An electrically conductive proppant could create a significant physical property contrast between the propped region of the reservoir and the host rock. Electromagnetic geophysical methods can be used to image this property (Heagy and Oldenburg, 2013). However, traditional geophysical inversions are poorly constrained, requiring *a-priori* information to be incorporated through known electrical properties. We examine a strategy to invert directly for the proppant volume using a parametrization of electrical conductivity in terms of proppant distribution within the reservoir.

INTRODUCTION

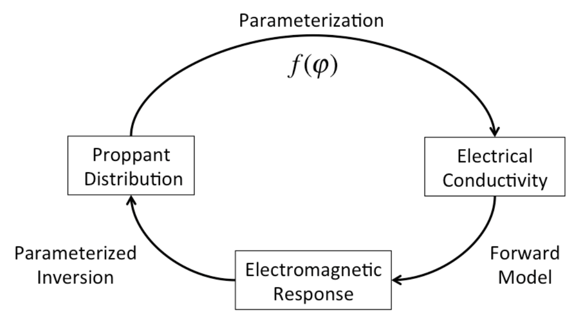
Many of the remaining oil and gas reserves are trapped tightly in low permeability rock formations. These hydrocarbons are difficult to extract because of the limited availability of pathways through which they can flow. Pathways can be opened up using hydraulic fracturing followed by proppant, which stays in the fractures to keep the newly created pathways open (Montgomery et al., 2010).

The distribution of proppant is a determining factor on which regions of the reservoir can be effectively drained. As a result, knowledge of the proppant distribution could provide valuable insight into decisions regarding well placement, well spacing, and fracture completion strategies (Montgomery et al., 2010; King, 2010). Currently no technique provides a satisfactory method of delineating the distribution of proppant within a hydraulically fractured reservoir (Cipolla and Wright, 2000). Electromagnetic (EM) geophysical techniques have been used to distinguish and delineate electrically conductive regions from resistive backgrounds. A large conductivity contrast between the propped region of the reservoir and the host rock is necessary for data sensitivity in EM geophysics. An electrically conductive proppant could be created by using a conductive coating on the particles or using conductive particles. The contrast of this proppant against the background provides a target that can be characterized using EM methods. This requires a proper survey to collect EM data and a methodology to invert these data.

Traditionally, the inverse problem is approached by considering the mapping between the physical property space, electrical conductivity, and data space, the electromagnetic response, as shown in Figure 1(a). The forward modelling provides the machinery to get from model space to data space, and the inversion provides a way to estimate a model from the data. The conductivity model is then used to interpret information about the characteristics of interest, in this case the distribution of proppant within the fractured reservoir. This requires knowledge about the mapping between conductivity and proppant volume.



(a)



(b)

Figure 1: (a) Traditional approach to inversion, where the model space, electrical conductivity, is mapped to data space, the electromagnetic response, through a forward model. The inversion then provides a method by which we estimate a model that is consistent with the observed data. The recovered conductivity model is then used to infer information about the reservoir properties of interest, in this case, the distribution of proppant. (b) Parametrized inversion, where we parametrize the model space, electrical conductivity, in terms of the property of interest, the distribution of proppant. By defining such a parametrization, the inversion can provide a means of estimating the properties of interest directly from the data.

We consider an alternative approach, as shown in Figure 1(b), where we incorporate a parametrization of the electrical conductivity as a function of the properties and distribution of proppant within the reservoir. In this manner, we connect the parameter space, which depends on the proppant, to the physical property space, electrical conductivity, through a parametrization. As a result, we provide a mapping from the parameter space to the data space. This can be exploited in the inverse problem, where we are now able to invert directly for the property of interest, namely the distribution of proppant in the reservoir.

We tackle this problem in three stages. First, we characterize the physical properties of a *doped* fractured reservoir. The goal of this characterization is to provide a mapping that connects the parameter space of interest, the proppant distribution, to the physical property space, the bulk electrical conductivity. Next, we choose a geophysical survey which provides data sensitive to the electrical conductivity variations at the scale of the reservoir. Finally, we design an inversion framework which connects the observed data back to a distribution of proppant within the reservoir. We then show the utility of this approach using a simple linear inverse problem.

PARAMETRIZATION

We are interested in characterizing the impact of including electrically conductive proppant in a fractured volume of a reservoir using a relationship of the form

$$\sigma = f(\varphi) \tag{1}$$

where σ is the parametrized electrical conductivity, $\varphi \in [0,1]$ is the volume fraction of proppant in a given region of the reservoir, and f is a parametrization that maps from φ-space to σ-space. This parametrization may be derived empirically from lab measurements or through an upscaling procedure such as effective medium theory (Bruggeman, 1935; Torquato, 2002; Shafiro and Kachanov, 2000; Berryman and Hoversten, 2013). It is important to note that, in general, the parametrization may be such that the inverse $\varphi = f^{-1}(\sigma)$ is not well defined. This inverse mapping is necessary in the traditional approach to inversion, as outlined in Figure 1(a). In such a scenario, mapping from electrical conductivity to proppant distribution requires solving a separate inverse problem with distinct regularization. Requiring the solution of two inverse problems creates a disconnect between regularization on the electrical conductivity and regularization on the proppant distribution. In this case, it may be unclear how regularization on conductivity impacts the characteristics of the proppant model. We find that solving an intermediate inverse problem for electrical conductivity is unsatisfying and potentially problematic.

For this paper, we consider the propped region of the reservoir to be a two-phase material, composed of the host reservoir, with conductivity σ_0, and proppant, with conductivity σ_1. Assuming that the electrical conductivity of the proppant is sufficiently large compared to that of any injected fluid, we can neglect the contribution of the electrical conductivity of the fluid.

For the parametrization, we express the effective conductivity of a propped region of a reservoir using self-consistent effective medium theory (Bruggeman, 1935; Torquato, 2002), namely

$$(1-\varphi)(\sigma-\sigma_0)R^{(0)} + \varphi(\sigma-\sigma_1)R^{(1)} = 0. \tag{2}$$

When we consider the effective conductivity of the fractured rock to be isotropic the factor $R^{(i)}$ is given by

$$R^{(i)} = \left[1 + \frac{1}{3}\frac{\sigma_i - \sigma}{\sigma}\right]^{-1}. \tag{3}$$

Note that if we were to consider an anisotropic conductivity, the R would instead be a matrix (see for example Shafiro and Kachanov (2000); Berryman and Hoversten (2013); Torquato (2002)).

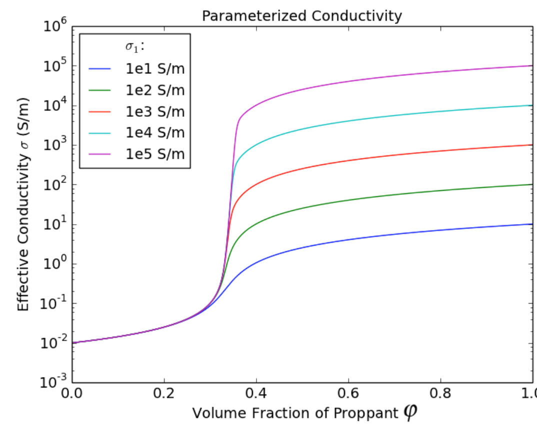


Figure 2: Parametrization of electrical conductivity as a function of proppant volume, according to equation 2. The background conductivity is $\sigma_0 = 10^{-2}$ S/m, and five different proppant conductivities (σ_1) are shown.

In order for this parametrization to be a function of φ alone, we require that both the electrical conductivity of the host reservoir, as well as that of the proppant to be known *a-priori*. Reasonable estimates for these conductivities should be available. The proppant's electrical conductivity can be measured before pumping begins. Additionally, a model of the conductivity of the reservoir, σ_0, can be estimated using well logs, or, ideally, estimated by performing an EM survey of the reservoir prior to the fracture treatment. Note that in general, σ_0 may be a distributed parameter, which varies with position in space. Using this parametrization, we next define the mapping from electrical conductivity to the electromagnetic response.

ELECTROMAGNETIC RESPONSE

The electromagnetic response of a medium is governed by Maxwell's equations. Here, we consider the frequency-domain problem, under the quasi-static approximation, in which case, the responses are given by

$$\begin{aligned} \nabla \times \mathbf{E} + i\omega\mathbf{B} &= 0, \\ \nabla \times \mu^{-1}\mathbf{B} - \sigma(\varphi)\mathbf{E} &= \mathbf{J_s}. \end{aligned} \tag{4}$$

$\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic flux density, ω is the angular frequency, μ is the magnetic permeability, $\sigma(\varphi)$ is the parametrized electrical conductivity, and $\mathbf{J_s}$ is the source term (cf. Ward and Hohmann (1988)). Equation 4 can be reduced to a single second order equation. By eliminating the magnetic field using

$$\mathbf{B} = \frac{-1}{i\omega} \nabla \times \mathbf{E}, \tag{5}$$

we define our simulation, in terms of the electric field only, as

$$\mathscr{F}(\mathbf{E}, \sigma(\varphi)) = \nabla \times \mu^{-1} \nabla \times \mathbf{E} + i\omega\sigma(\varphi)\mathbf{E} + i\omega\mathbf{J_s} = 0. \tag{6}$$

To perform the modeling, we discretize Maxwell's equations (4) using a mimetic finite volume approach (cf. Haber et al. (2000a)). By solving for the electric field $\mathbf{E}$ for a given proppant distribution and conductivity model $\sigma(\varphi)$, we define our predicted data, d^{pred}, as a projection of the fields to the receiver locations. This then provides the mapping from electrical conductivity space to data space.

INVERSION FRAMEWORK

We develop the inverse problem using a discretize then optimize approach, where we aim to minimize a regularization on the parameter φ subject to constraints on the data misfit and the parametrization.

Data Misfit

The result of an EM survey is data in the form of fields or fluxes. The quality of a particular model, in terms of its ability to reproduce these data, is assessed through a data misfit term. Assuming the errors in the measured fields and fluxes are Gaussian and uncorrelated, we define the EM data misfit as

$$\phi_{EM} = \frac{1}{2} \sum_{j=1}^{N} \left(\frac{d_j^{obs} - d_j^{\text{pred}}(\varphi)}{\varepsilon_j} \right)^2 \tag{7}$$

where d^{obs} is the observed EM data, d^{pred} is the predicted EM data, N is the number of data, and ε_j is the standard deviation of the jth datum (cf. Oldenburg and Li (2005)).

The EM data misfit term does not account for all of the data available to us in this problem. Since we have control over the amount of proppant pumped into the reservoir, the total proppant volume, V_{prop}, is an additional datum we can consider in the inversion. We define the volume misfit term as

$$\phi_V = \frac{1}{2} \left(\frac{\int \varphi dV - V_{\text{prop}}}{\varepsilon_V} \right)^2, \tag{8}$$

where ε_V is the uncertainty in our total volume estimate.

The data misfit term is defined by combining both the EM data misfit and volume misfit terms (equations 7 and 8):

$$\phi_d = \phi_{EM} + \phi_V. \tag{9}$$

Regularization

Defining regularization in the parameter space allows a significant amount of *a-priori* information to be easily incorporated into the inversion. We start by considering the regularization of the form

$$\begin{aligned} \phi_p = \frac{\alpha_s}{2} \int (w_d(x,y,z)\varphi)^2 dV + \frac{\alpha_x}{2} \int \left(\frac{\partial \varphi}{\partial x} \right)^2 dV \\ + \frac{\alpha_y}{2} \int \left(\frac{\partial \varphi}{\partial y} \right)^2 dV + \frac{\alpha_z}{2} \int \left(\frac{\partial \varphi}{\partial z} \right)^2 dV, \end{aligned} \tag{10}$$

where α_s, α_x, α_y, α_z weight the relative smallness and first-order smoothness in the x, y and z directions, respectively (cf. Oldenburg and Li (2005)). w_d is a distance weighting function, which may be chosen to penalize the placement of proppant far from the injection point. Alternatively, it can be specified based on other available data, such as microseismic. The relative weights on the partial derivatives of φ can be chosen to align with the regional stress directions or chosen to incorporate *a-priori* knowledge about fracture orientation, such as microseismic, tiltmeter or well-log data (Cipolla and Wright, 2000).

Defining the Inverse Problem

Having constructed the parametrization of electrical conductivity in terms of the distribution of proppant, the forward model, and the regularization we wish to impose, we combine these elements to define the inverse problem. The aim of the parametrized inversion is to find a proppant distribution model, φ^* such that

$$\begin{aligned} \varphi^* = \underset{\varphi}{\text{minimize}} \ \phi_d + \beta\phi_m \\ \text{subject to} \ \ 0 \leq \varphi \leq 1, \end{aligned} \tag{11}$$

and $\phi_d \leq \phi_d^*$, where ϕ_d^* is a target data misfit chosen, for example, by the discrepancy principal. Linear bound constraints are applied on φ as it is a volume fraction of proppant. β is a trade-off parameter that weights the model objective function ϕ_m. β may be selected using a statistical approach such as generalized cross-validation. Alternatively, a continuation approach may be taken, where a cooling schedule is applied on β (Oldenburg and Li, 2005; Nocedal and Wright, 1999).

Computing the Sensitivity

To apply any gradient based optimization method, we require the sensitivity of the fields with respect to the model, φ. The sensitivity can be found by considering the derivative of $\mathscr{F}$, as given in equation 12, with respect to φ, namely

$$\frac{\partial \mathbf{E}}{\partial \varphi} = \left(\frac{\partial \mathscr{F}}{\partial \mathbf{E}} \right)^{-1} \frac{\partial \mathscr{F}}{\partial \sigma} \frac{\partial \sigma}{\partial \varphi} \tag{12}$$

Note that the first two terms on the right hand side of (12) define the sensitivity of the electric field with respect to electrical conductivity; these terms capture the physics of the problem. The parametrization acts to change the space in which we search for a model, this is incorporated by the third term of (12). A common technique is to parameterize electrical conductivity in log-space, which leads to a similar equation. Once the sensitivities have been defined, the inverse problem can be solved (cf. Haber et al. (2000b)).

EXAMPLE

To demonstrate our inversion framework and parameterization, we consider a representative linear inverse problem, namely, the 2D ray-tracing tomography problem. Tomography provides a simple physical problem to develop an understanding of the impact of searching for a model in the parameter space of interest, φ-space.

We use the model of a block ($\varphi = 0.4$) in a whole-space ($\varphi = 0$) as shown in Figure 3(a). We use $\sigma_0 = 10^{-2}$ S/m, $\sigma_1 = 10$

S/m, and the parametrization of $\sigma(\varphi)$ given by equation 2. The unit-square domain is discretized uniformly using a 30×30 mesh. There are 10 sources and 10 receivers, giving a total of $N = 100$ data; the geometry is shown in Figure 3(a).

Gaussian noise (2%) was added and the data inverted in both σ-space and φ-space (Figure 3(b) and Figure 3(c), respectively). In each case, the same relative weights on the smallness (α_s) and smoothness (α_x, α_y) were applied. Equivalent bounds were enforced on φ and σ, and equivalent reference and starting models were used ($\varphi = 0$). The volume datum was included in the φ-inversion, but not in the σ-inversion. Each inversion was run until $\phi_d \leq N$, thus satisfying the discrepancy principle.

As can be seen in Figure 3(b), the inverse mapping creates an interpretation of φ that is very sensitive to minor variations in σ-space. This results from the choice of regularization applied in σ-space, which does not translate to an equivalent regularization in φ-space as the mapping between these two parameters (2) is non-linear.

The parametrized inversion, as outlined in this paper, is performed in φ-space. We show the recovered φ-model in Figure 3(c). By applying the regularization directly on φ, both the spatial extent and the maximum value of the recovered model is much closer to the true model. This simple model is a powerful demonstration of how building the inverse problem around the parameter of interest allows not only incorporation of additional data, but also application of regularization in a physically meaningful and tangible manner.

CONCLUSION

In hydraulic fracturing, the proppant distribution is an important variable in management decisions. The methods currently applied are not adequate in their ability to detect the distribution of proppant following the fracture treatment. Electromagnetic (EM) methods provide a promising approach as they are sensitive to contrasts in electrical conductivity, which can be introduced through the proppant. Traditional inversions are approached with a focus on physical properties, such as electrical conductivity. This disconnects the problem from the question we are ultimately aiming to answer. By framing the inverse problem in terms of the parameter of interest, we carry this motivation throughout the entire inversion process. We have demonstrated the utility of this approach using a simple linear inverse problem. Such an approach could provide a powerful framework for incorporating management decision variables into the geophysical inversion process.

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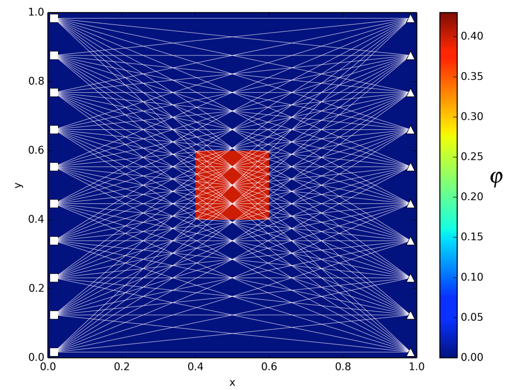

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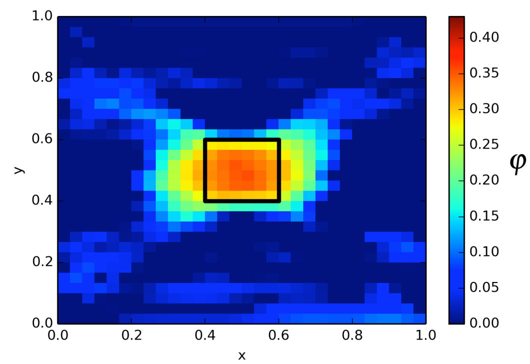

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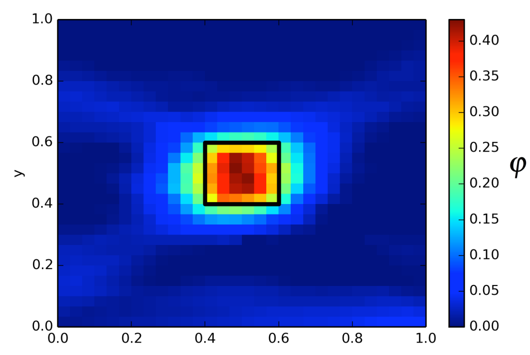

(c)

Figure 3: (a) True model, (b) model recovered by inverting for conductivity and preforming the inverse mapping to φ, (c) model recovered by inverting for φ directly. A black outline of the true model is shown on each recovered model.

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