simpegEM

An open-source resource for simulation and parameter estimation problems in electromagnetic geophysics

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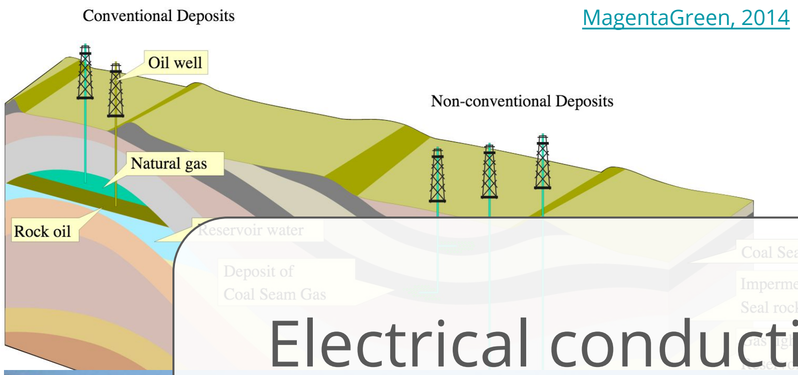
Geophysical Inversion Facility
University of British Columbia

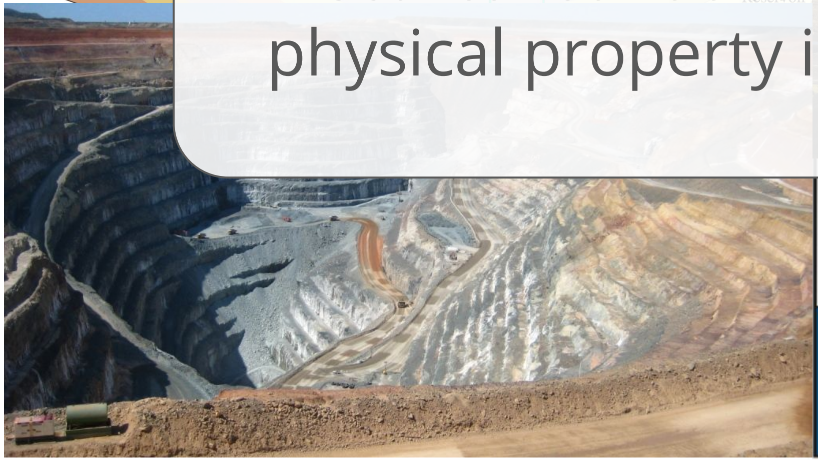
Why Electromagnetics?

MagentaGreen, 2014

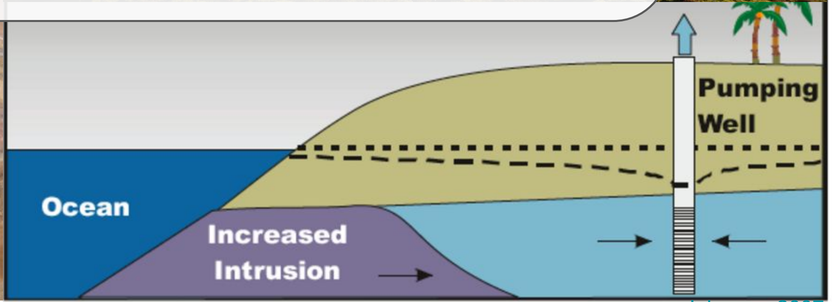


Gretar Ivarsson, 2006

Electrical conductivity is a diagnostic physical property in many settings



Calistemon, 2010



Johnson, 2007

Math!

Maxwell's Equations (quasi-static)

Time

$$\nabla \times \mathbf{e} + \frac{\partial \mathbf{b}}{\partial t} = \mathbf{s_m}$$

$$\nabla \times \mathbf{h} - \mathbf{j} = \mathbf{s_e}$$

Frequency

$$\nabla \times \mathbf{E} + i\omega \mathbf{B} = \mathbf{S_m}$$

$$\nabla \times \mathbf{H} - \mathbf{J} = \mathbf{S_e}$$

Constitutive Relations

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

- Fields
  - $\mathbf{E}$ electric field
  - $\mathbf{H}$ magnetic field
- Fluxes
  - $\mathbf{J}$ current density
  - $\mathbf{B}$ magnetic flux density
- Physical Properties
  - σ electrical conductivity
  - μ magnetic permeability

Steel casing in EM

Physical Properties

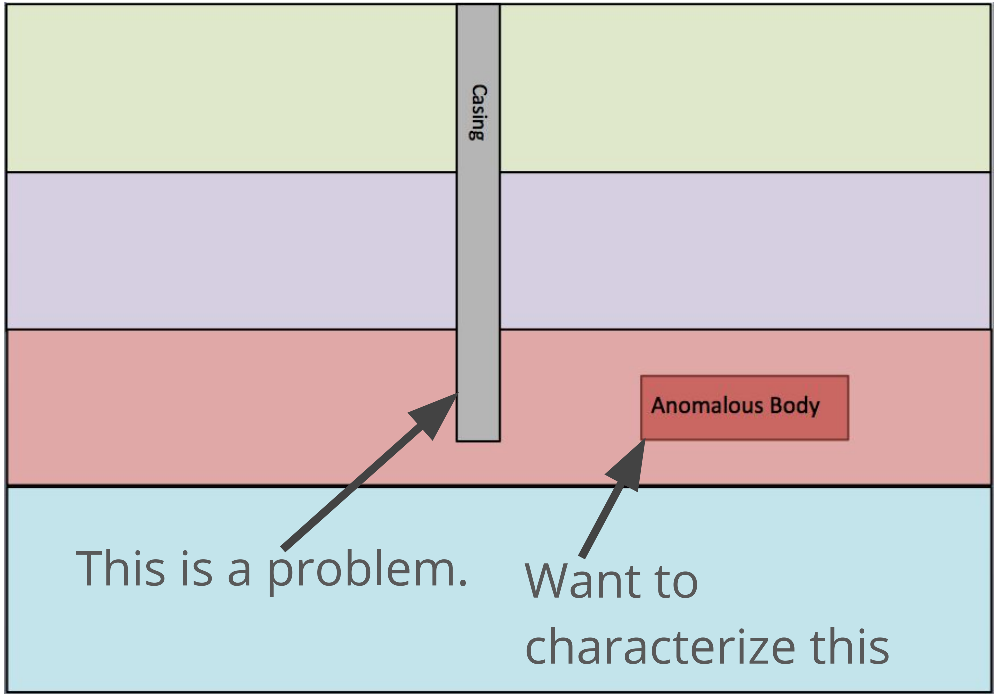
- highly conductive
- significant (variable) magnetic permeability

➡ ***Significant impact on signals***

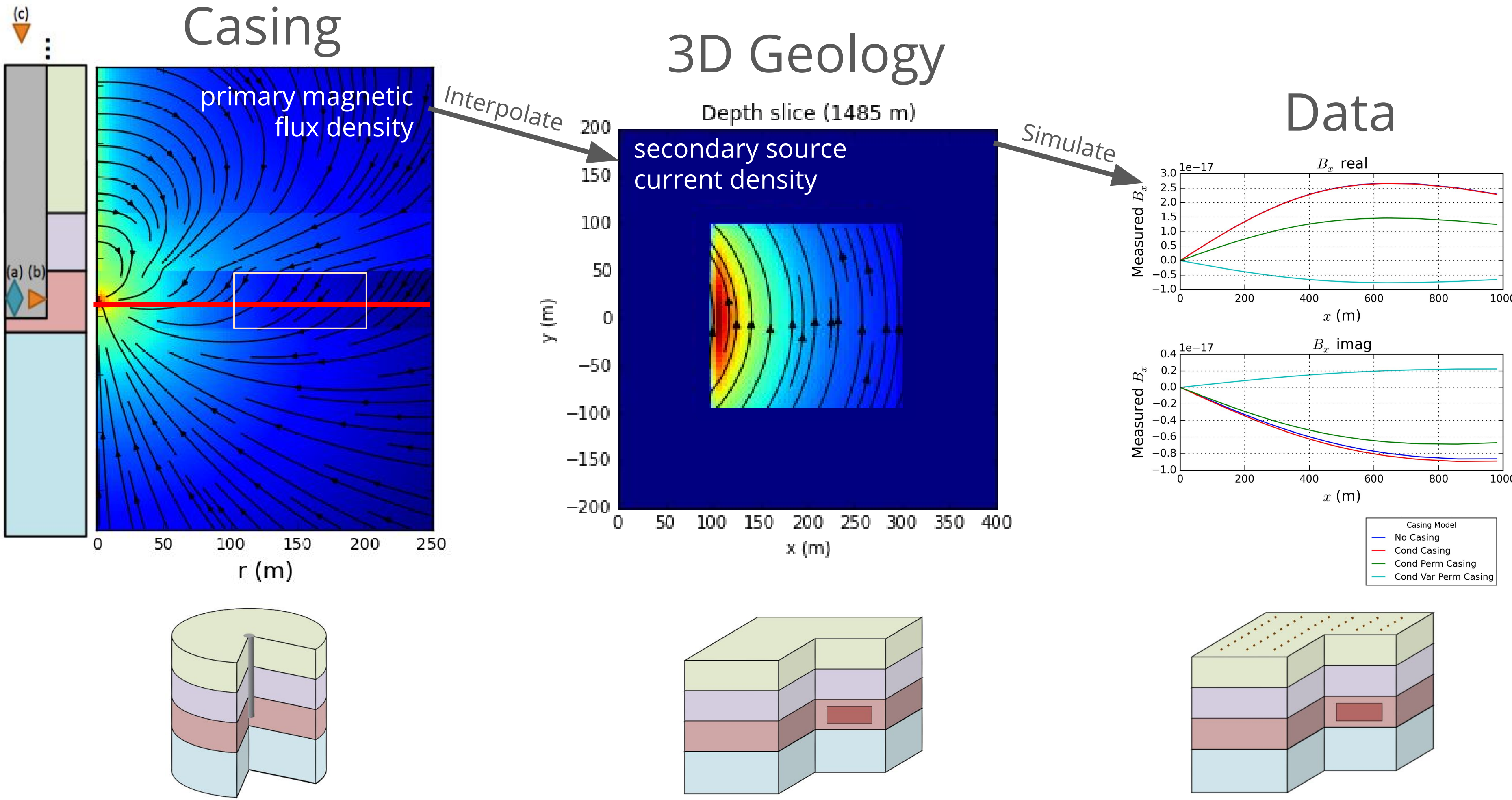
Geometry

- cylindrical
- thin compared to length

➡ ***Numerically challenging***

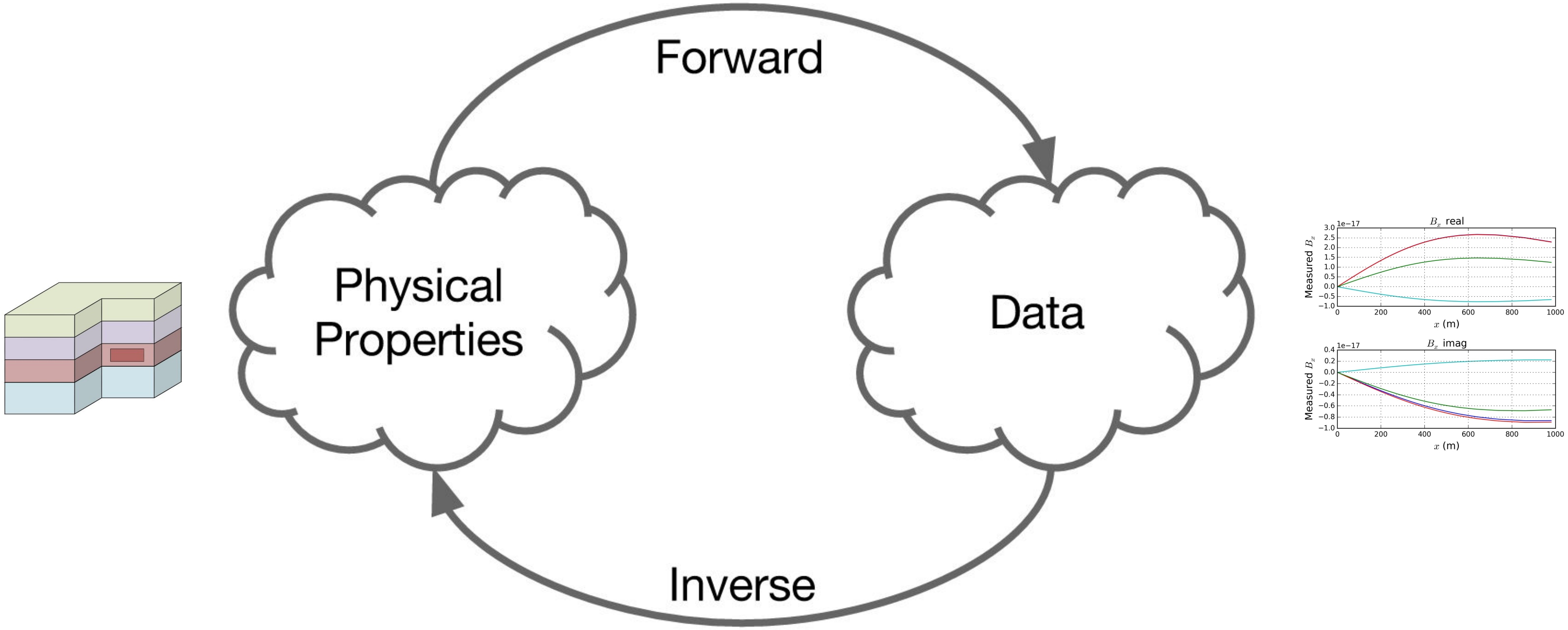


(SEG Abstract: Heagy et al, 2015)



Casing
primary magnetic flux density
(c)
(a) (b)
0 50 100 150 200 250
r (m)
Interpolate
3D Geology
Depth slice (1485 m)
secondary source current density
200 150 100 50 0 −50 −100 −150 −200
y (m)
0 50 100 150 200 250 300 350 400
x (m)
Simulate
Data
B_x real
1e−17
3.0 2.5 2.0 1.5 1.0 0.5 0.0 −0.5 −1.0
Measured B_x
0 200 400 600 800 1000
x (m)
B_x imag
1e−17
0.4 0.2 0.0 −0.2 −0.4 −0.6 −0.8 −1.0
Measured B_x
0 200 400 600 800 1000
x (m)
Casing Model
No Casing
Cond Casing
Cond Perm Casing
Cond Var Perm Casing

Inverse Problem



(How?)

What do we need?

Physics: *Maxwell's Equations*

Time

$$\nabla \times \mathbf{e} + \frac{\partial \mathbf{b}}{\partial t} = \mathbf{s_m}$$

$$\nabla \times \mathbf{h} - \mathbf{j} = \mathbf{s_e}$$

Frequency

$$\nabla \times \mathbf{E} + i\omega \mathbf{B} = \mathbf{S_m}$$

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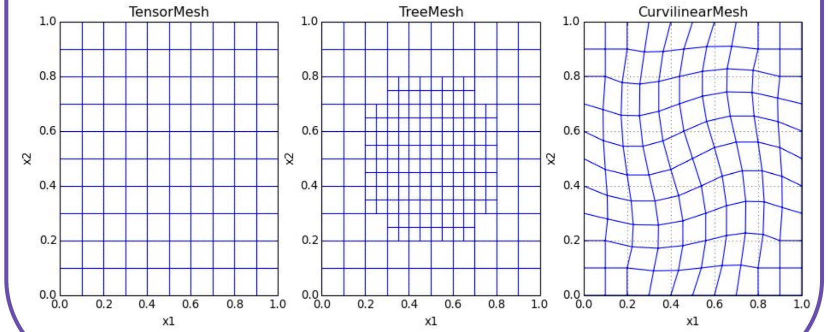
Meshes

2D Cylindrical & 3D



Physical Properties

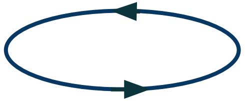
σ electrical conductivity

μ magnetic permeability

anisotropy…

Sources

grounded electric



inductive loop

primary-secondary

…

Data & Sensitivities

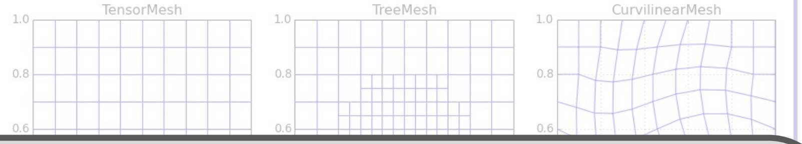
$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$

$$\mathbf{J} = \frac{d\mathcal{F}(\mathbf{m})}{d\mathbf{m}}$$

What do we need?

Meshes

2D Cylindrical & 3D



Physics: *Maxwell's Equations*

Time

Frequency

Also want:

- optimize
- regularize
- invert

σ electrical conductivity

μ magnetic permeability

anisotropy...

grounded electric

inductive loop

primary-secondary

...

$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$

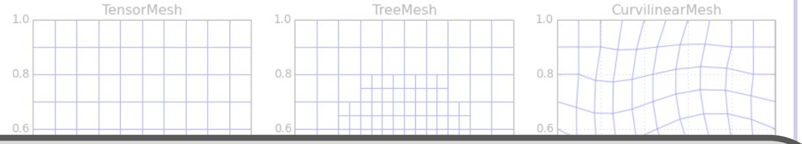
$$\mathbf{J} = \frac{d\mathcal{F}(\mathbf{m})}{d\mathbf{m}}$$

What do we need?

Physics: *Maxwell's Equations*

Meshes

2D Cylindrical & 3D



sensitivity test

```
===================== checkDerivative ===============
iter    h          |ft-f0|    |ft-f0-h*J0*dx|   Order
------------------------------------------------------
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========================= PASS! =====================
You are awesome.
```

and also:

- modular
- extensible

σ electrical conductivity

μ magnetic permeability

anisotropy...

grounded electric

inductive loop

primary-secondary

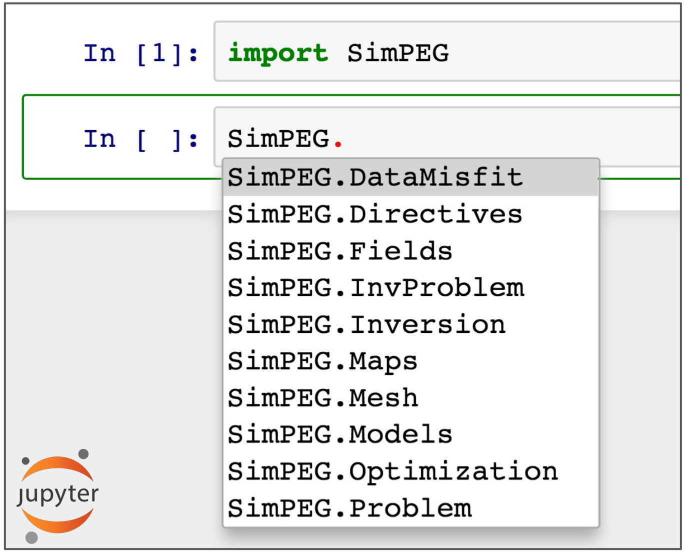
...

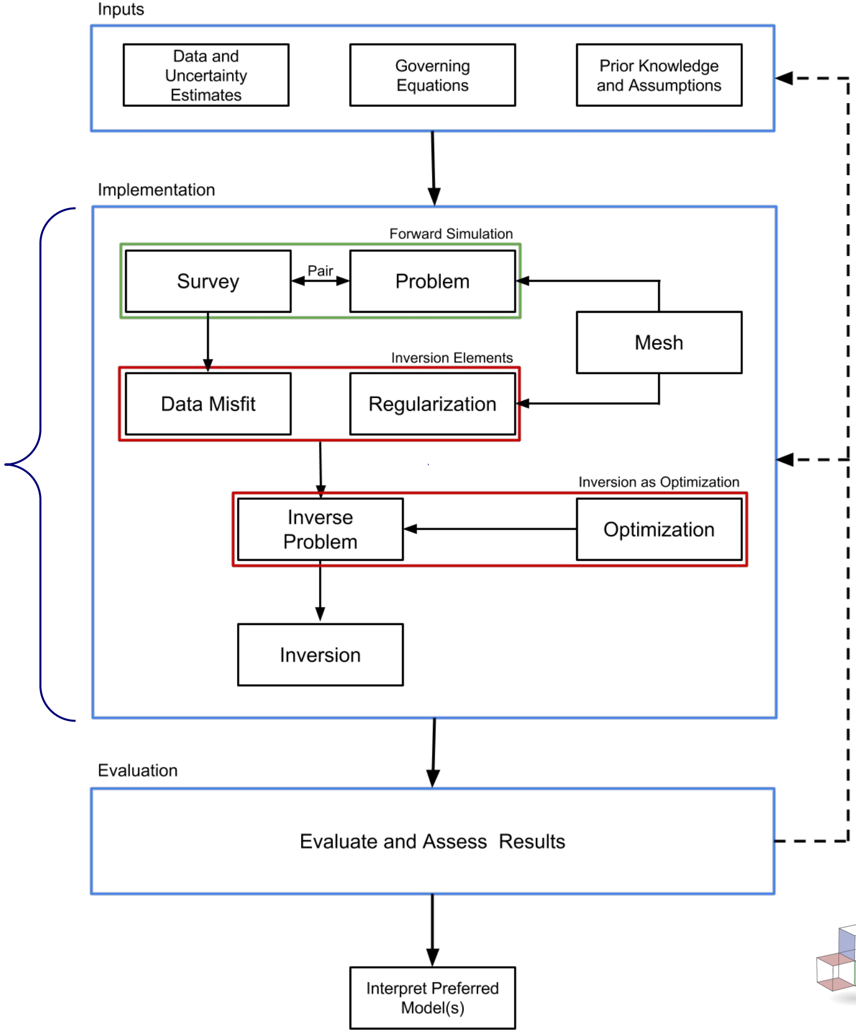
$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$

$$\mathbf{J} = \frac{d\mathcal{F}(\mathbf{m})}{d\mathbf{m}}$$

Open Source!

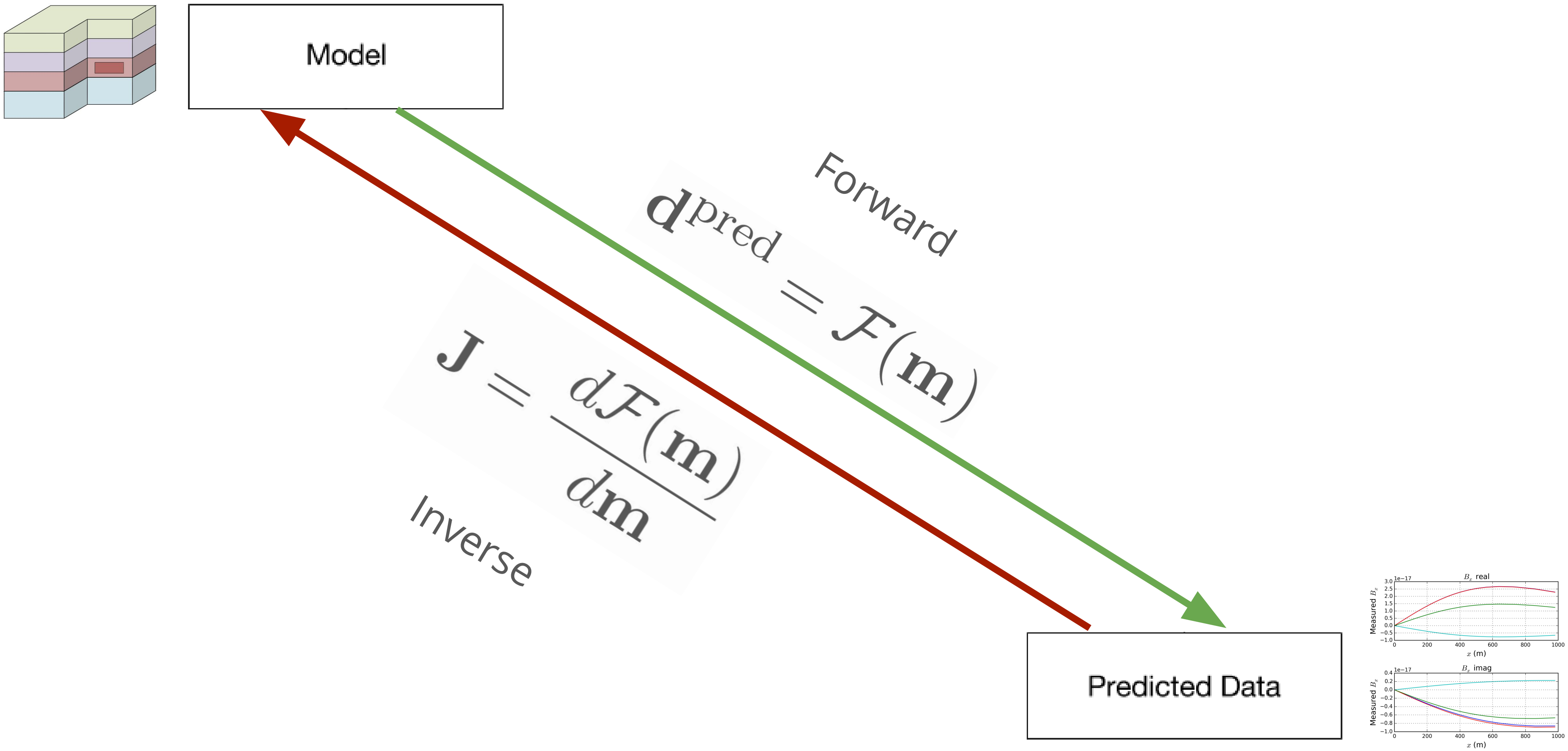
Implemented in Python!



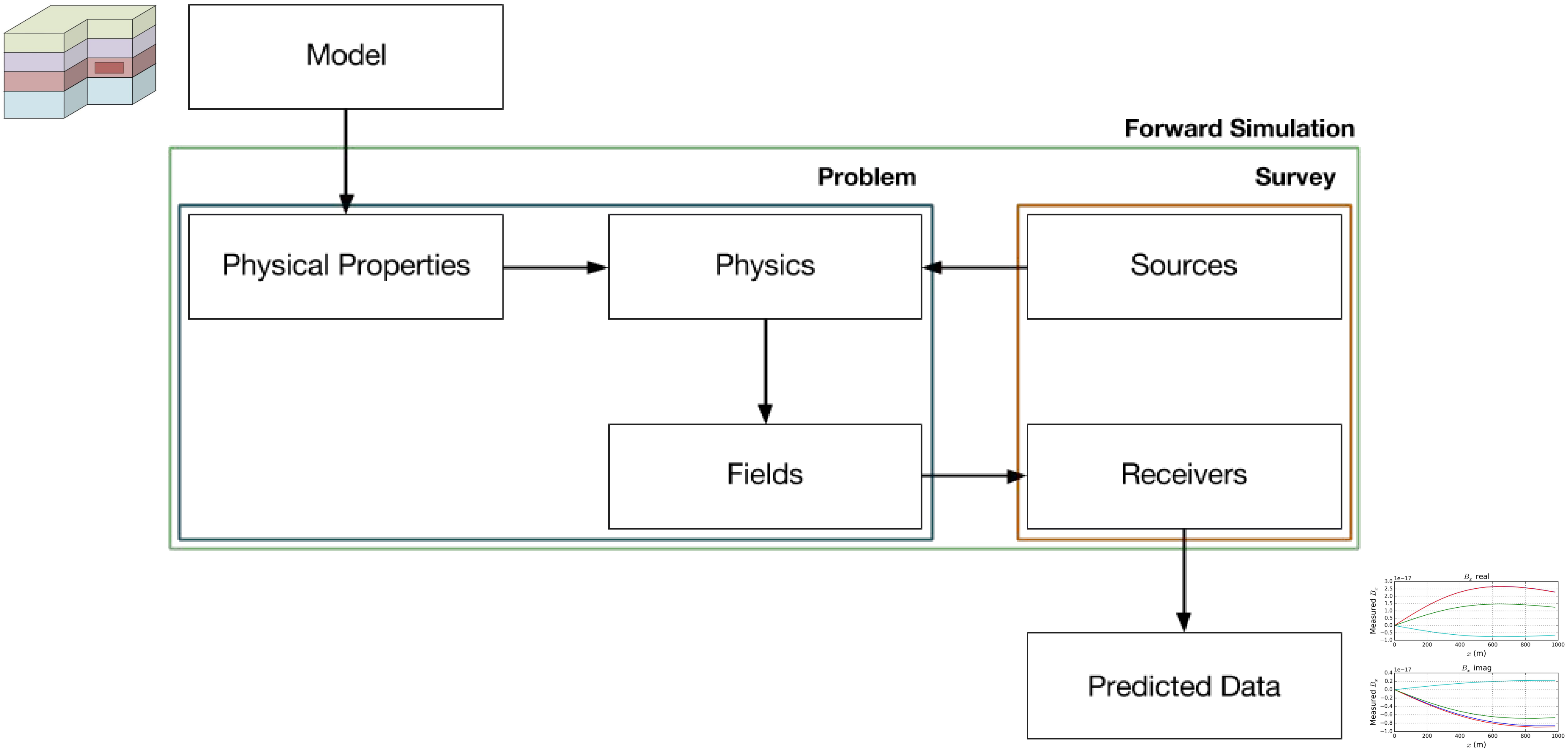


simpeg

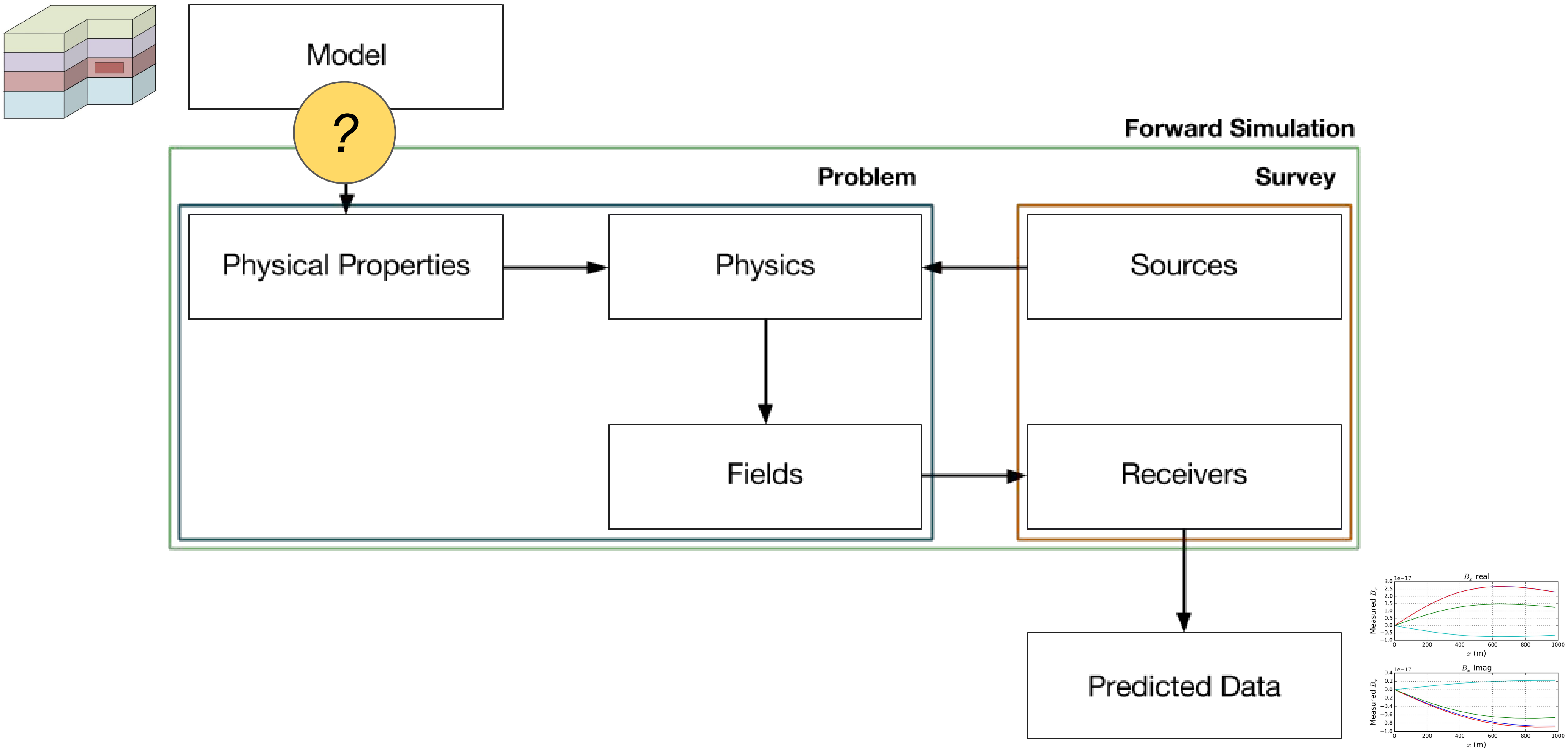
http://simpeg.xyz


Model
Forward
$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$
$\mathbf{J} = \frac{d\mathcal{F}(\mathbf{m})}{d\mathbf{m}}$
Inverse
Predicted Data
B_z real
B_z imag
Measured B_z
x (m)

$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$



$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$

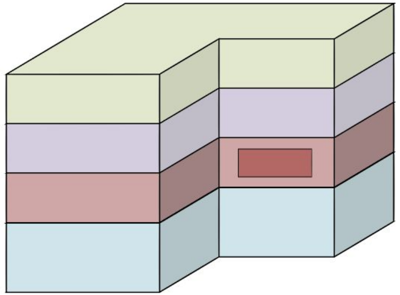


$$\boldsymbol{\sigma} = \mathcal{M}(\mathbf{m})$$

Model & Physical Properties: *What should we invert for?*

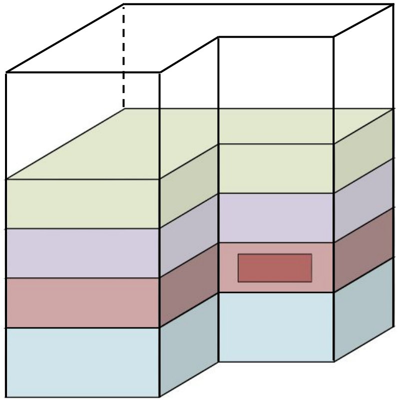
inversion model

$$\mathbf{m} = [\log \sigma]^{\mathrm{Active}}$$



$$\boldsymbol{\sigma} = \mathcal{M}^{exp}(\mathcal{M}^{surj}(\mathbf{m}))$$

physical properties



Derivatives using chain rule:

$$\frac{d\boldsymbol{\sigma}}{d\mathbf{m}} = \frac{d\mathcal{M}^{exp}(\cdot)}{d\cdot}\frac{d\mathcal{M}^{surj}(\mathbf{m})}{d\mathbf{m}}$$

or :

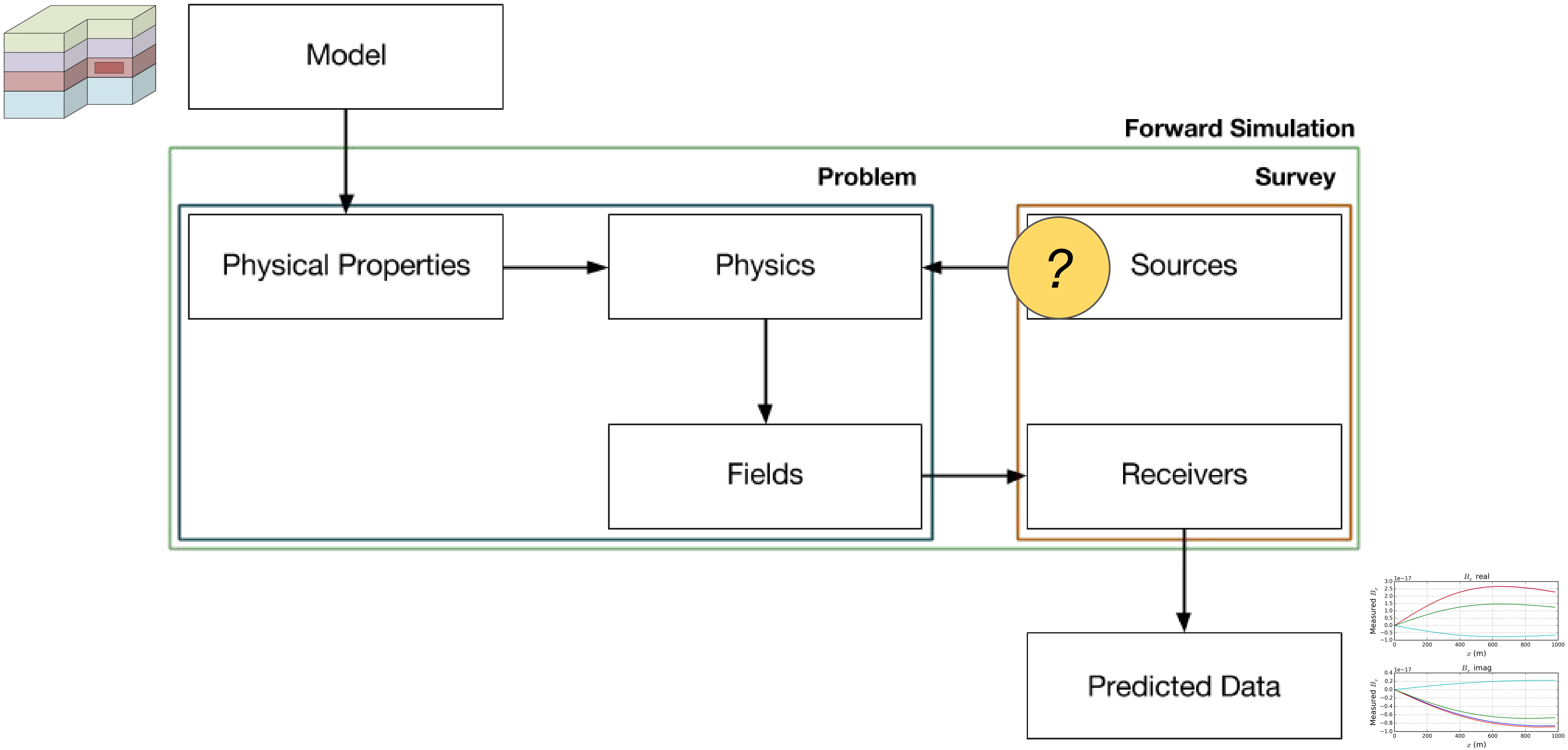
- Active reservoir layer
- Parametric representation
- ...

(SEG Abstract: Kang et al, 2015)

```
In [1]: from SimPEG import EM

In [ ]: EM.Base.BaseEMProblem.
        EM.Base.BaseEMProblem.MeSigma
        EM.Base.BaseEMProblem.MeSigmaDeriv
```

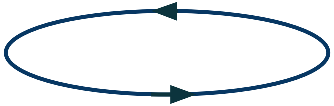
$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$



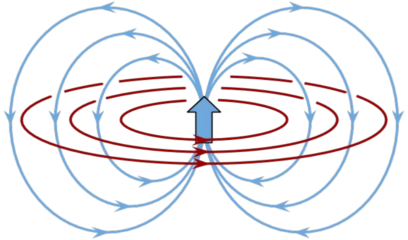
$\mathbf{s_m(m), s_e(m)}$

Sources: *How do we excite the Earth?*

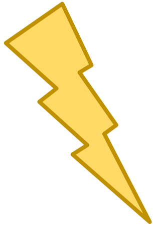
grounded electric



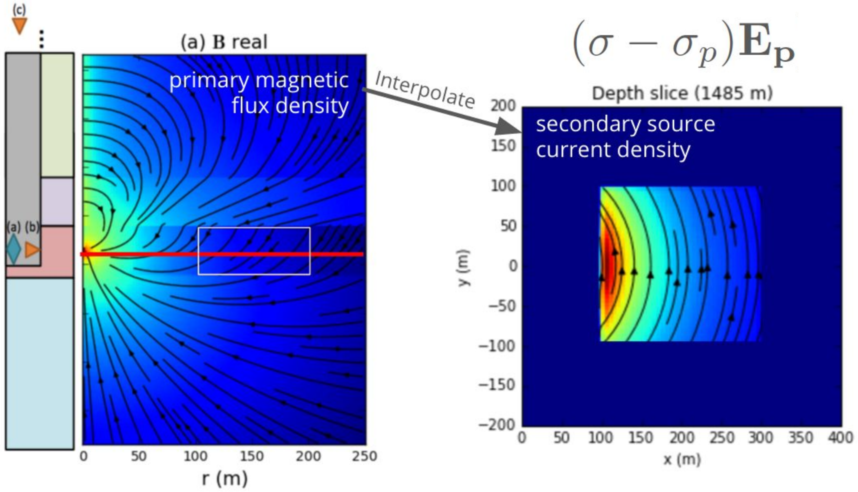
inductive loop



point dipole
(electric or magnetic)



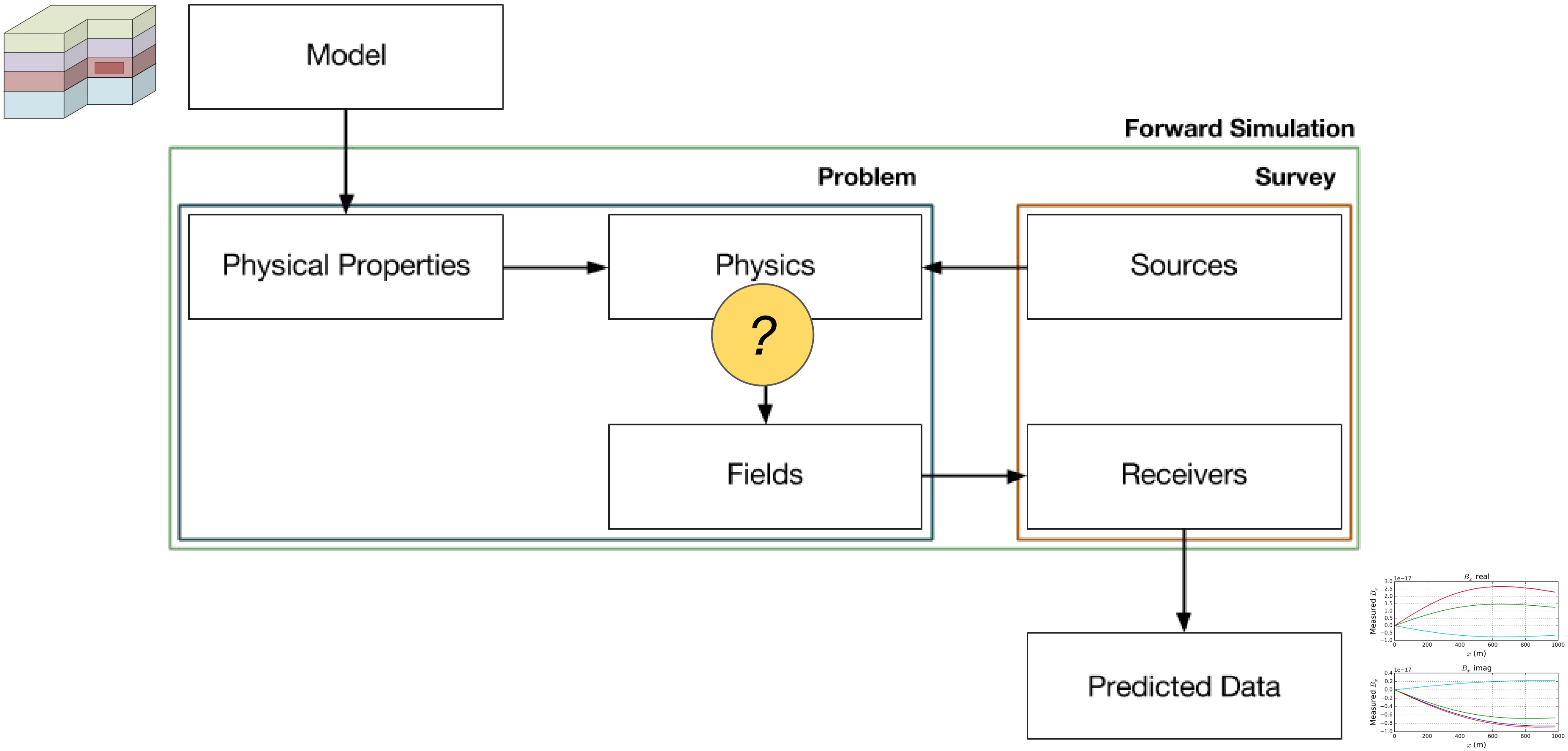
natural source



fields from a primary problem

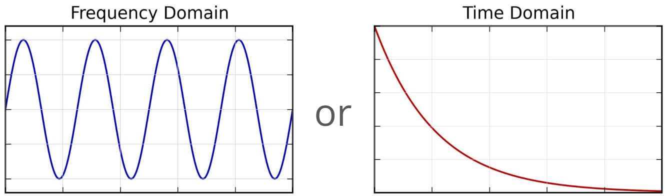
```
In [1]: import SimPEG.EM as EM
In [ ]: EM.FDEM.Src.PrimSecSigma.
        EM.FDEM.Src.PrimSecSigma.eval
        EM.FDEM.Src.PrimSecSigma.evalDeriv
```

$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$



$$\mathbf{A(m)u} = \mathbf{q(s_m, s_e)}$$

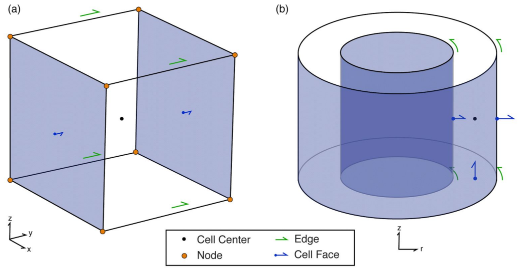
Physics: *How do we solve Maxwell's equations*



or

Solve E-B, H-J ?

$$\nabla \times \mathbf{E} + i\omega \mathbf{B} = \mathbf{S_m}$$
$$\nabla \times \mathbf{H} - \mathbf{J} = \mathbf{S_e}$$



Solve 2nd order system

$$\underbrace{\boxed{\left(\mathbf{C^T M^f_{\mu^{-1}} C} + i\omega \mathbf{M^e_\sigma}\right)}}_{\mathbf{A(m)}} \underbrace{\boxed{\mathbf{e}}}_{\mathbf{u}} = \underbrace{\boxed{\mathbf{C^T M^f_{\mu^{-1}} s_m} - i\omega \mathbf{s_e}}}_{\mathbf{q(s_m, s_e)}}$$

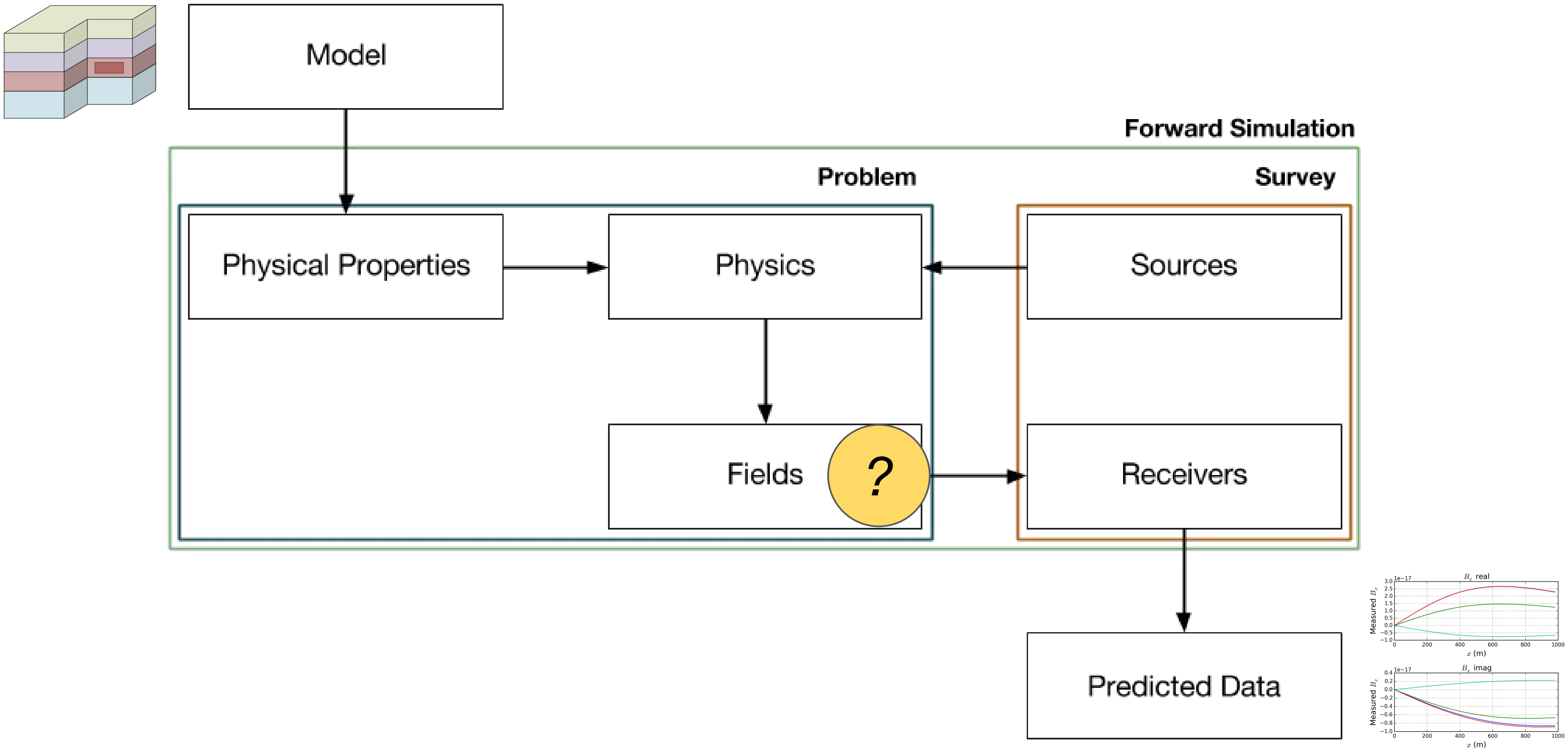
and compute derivative

$$\frac{d\mathbf{u}}{d\mathbf{m}} = \mathbf{A}(\sigma(\mathbf{m}))^{-1}\left(-\frac{d\mathbf{A}(\sigma)\mathbf{u}^{\text{fix}}}{d\mathbf{m}} + \frac{d\mathbf{q}}{d\mathbf{m}}\right)$$

```
In [1]: from SimPEG import EM

In [ ]: EM.FDEM.Problem_e.
        EM.FDEM.Problem_e.getA
        EM.FDEM.Problem_e.getADeriv_m
        EM.FDEM.Problem_e.getRHS
        EM.FDEM.Problem_e.getRHSDeriv_m
```

$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$



$$\mathbf{e}, \mathbf{b} = \mathbf{F}^{\mathrm{e,b}}(\mathbf{m}, \mathbf{u}, \mathbf{s_m}, \mathbf{s_e})$$

Fields: *How do we calculate the EM fields and fluxes?*

compute fields everywhere:

$$\mathbf{b} = -\frac{1}{i\omega}(\mathbf{C}\boxed{\mathbf{u}} - \boxed{\mathbf{s_m}}) + \boxed{\mathbf{b_p}}$$

what we solved for

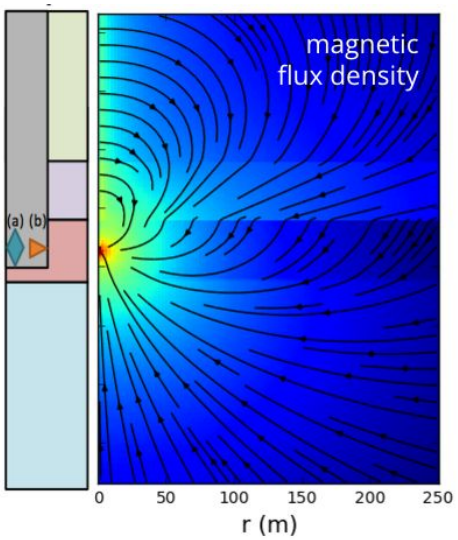
from source

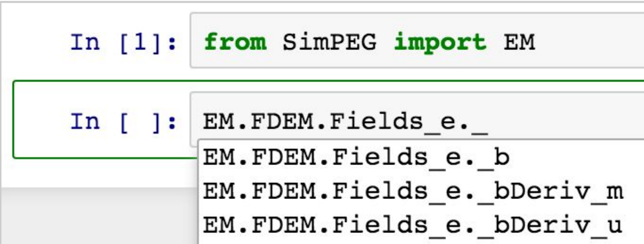
derivative

$$\frac{d\ \mathbf{e}, \mathbf{b}}{d\mathbf{m}} = \frac{\partial \mathbf{F}^{\mathrm{e,b}}}{\partial \mathbf{m}} + \frac{\partial \mathbf{F}^{\mathrm{e,b}}}{\partial \mathbf{u}} \boxed{\frac{d\mathbf{u}}{d\mathbf{m}}} + \frac{\partial \mathbf{F}^{\mathrm{e,b}}}{\partial \mathbf{s_m}} \boxed{\frac{d\mathbf{s_m}}{d\mathbf{m}}} + \frac{\partial \mathbf{F}^{\mathrm{e,b}}}{\partial \mathbf{s_e}} \boxed{\frac{d\mathbf{s_e}}{d\mathbf{m}}}$$

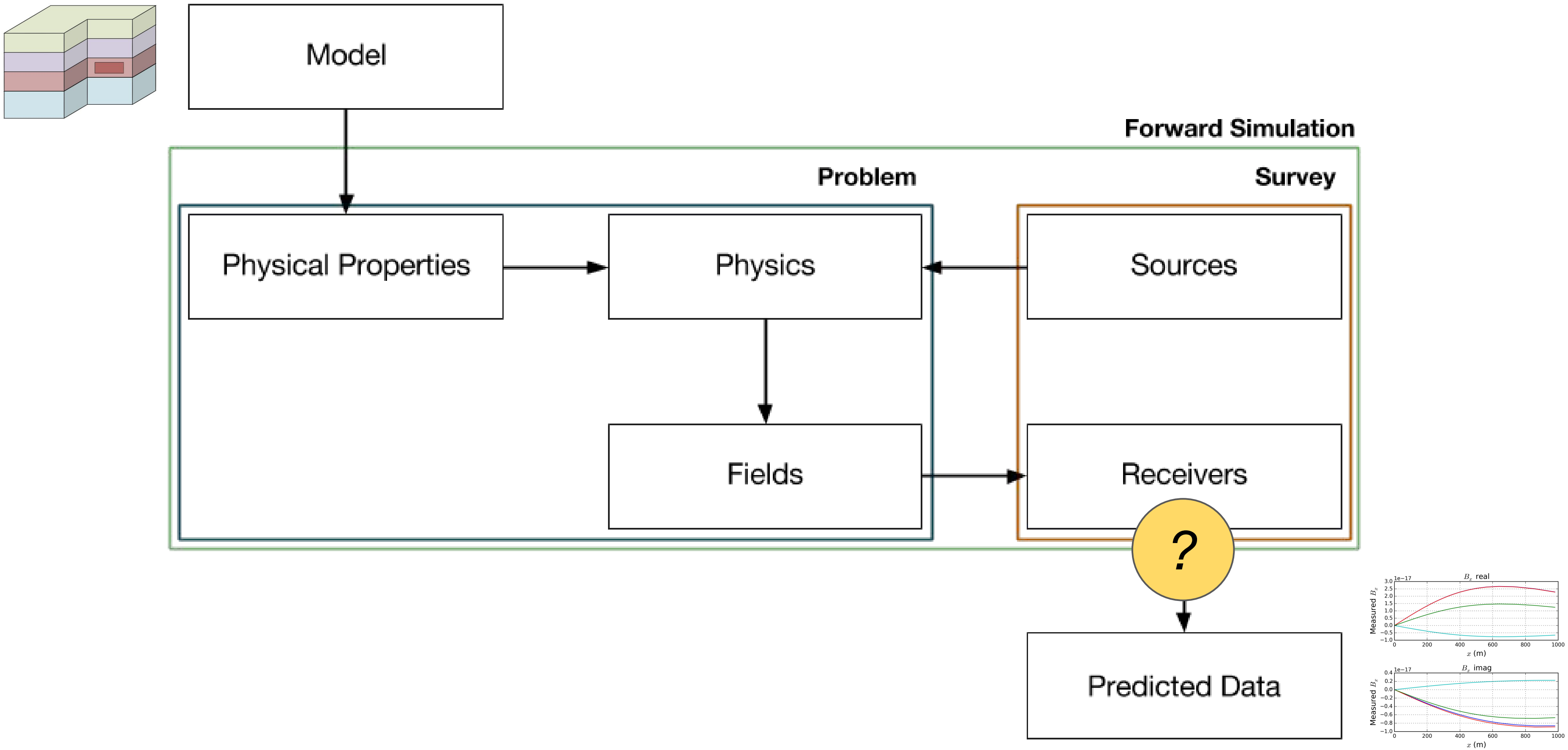
from physics

from source



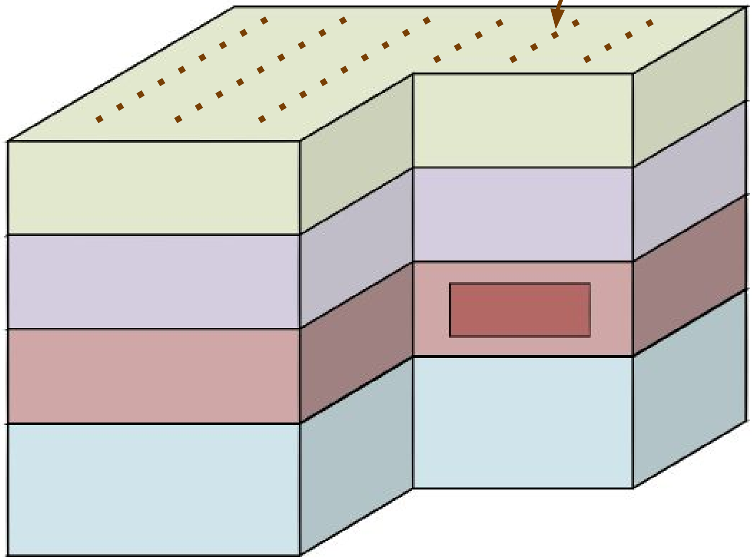


$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$



$$\mathbf{d}^{\text{pred}} = \mathbf{P}(\mathbf{e}, \mathbf{b})$$

Receivers: *What, and where, do we measure?*



Data:

$\mathbf{e_x}^{\text{real}}$

$\mathbf{e_x}^{\text{imag}}$

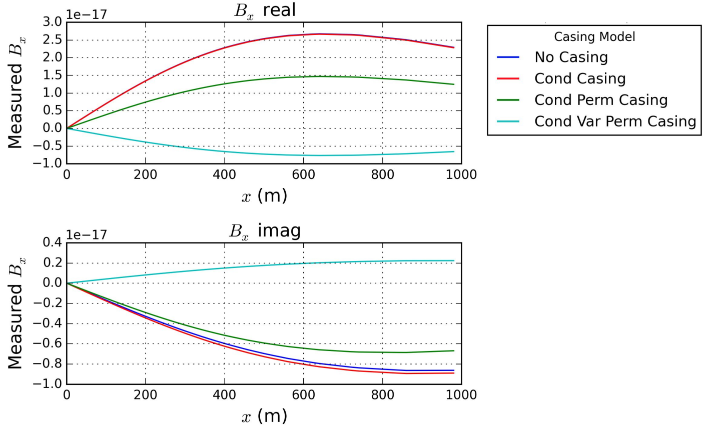
$\mathbf{e_y}^{\text{real}}$

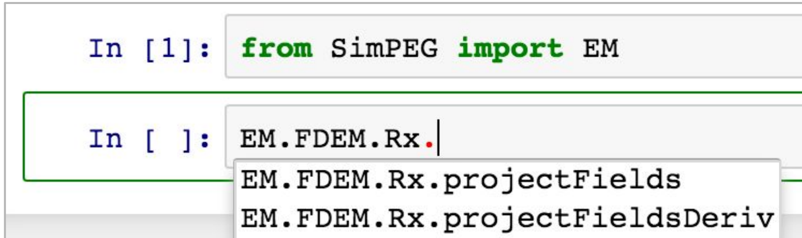
$\mathbf{e_y}^{\text{imag}}$

$\mathbf{b_z}^{\text{real}}$

$\mathbf{b_z}^{\text{imag}}$

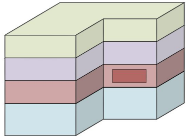
...

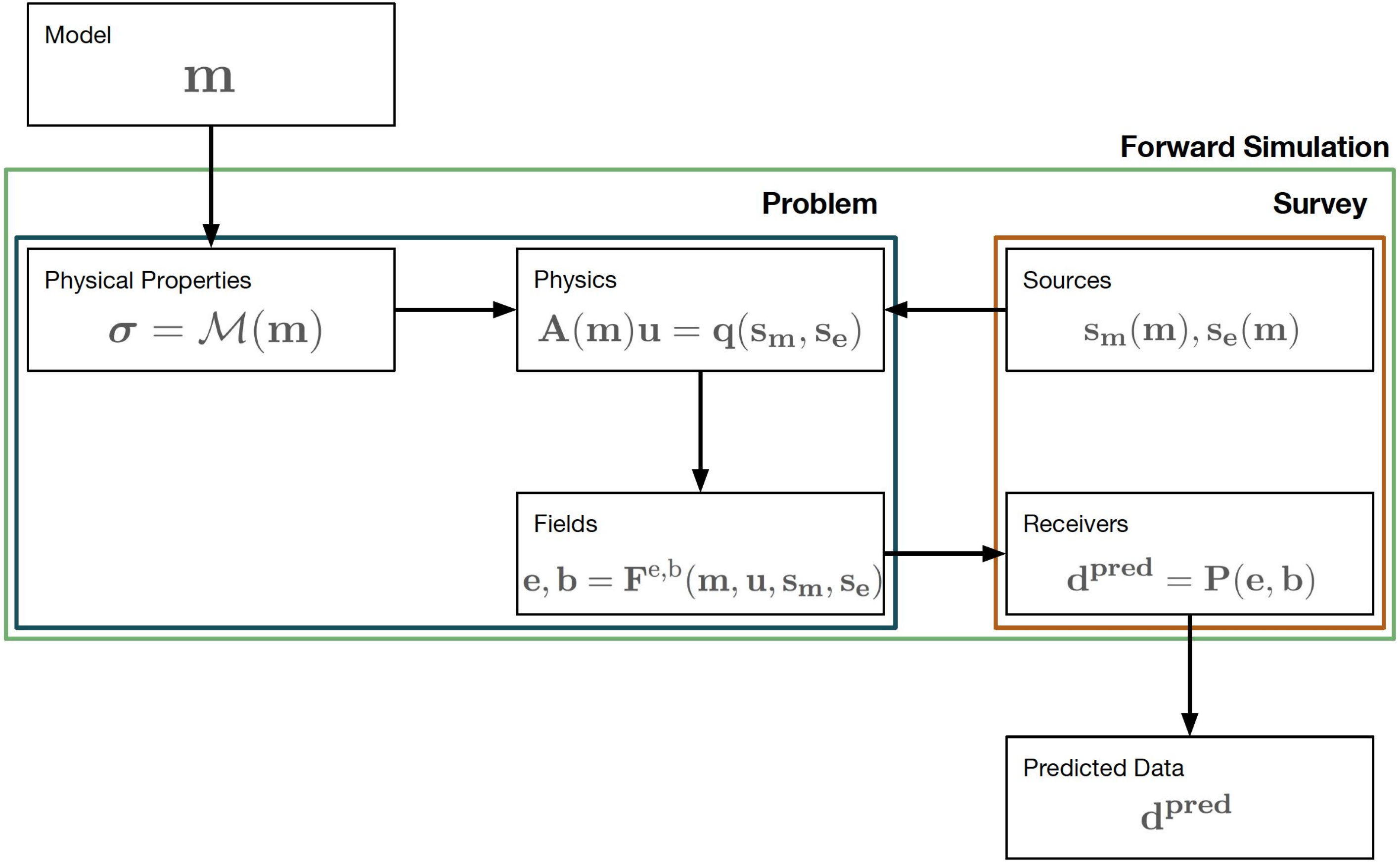


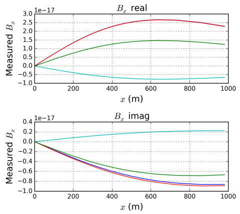


Plug in the pieces

$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$

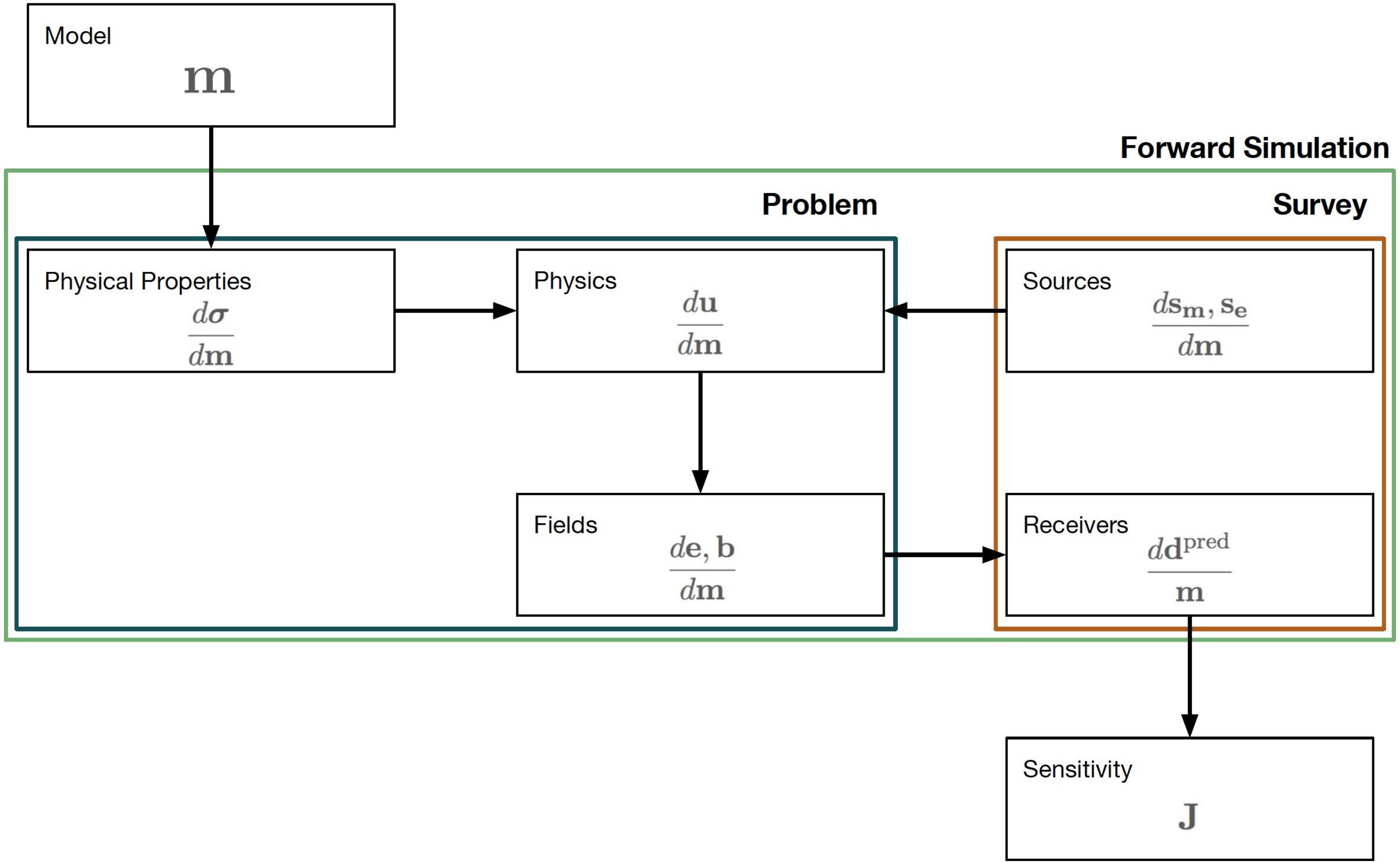






Sensitivities

$$\mathbf{J} = \frac{d\mathcal{F}(\mathbf{m})}{d\mathbf{m}}$$



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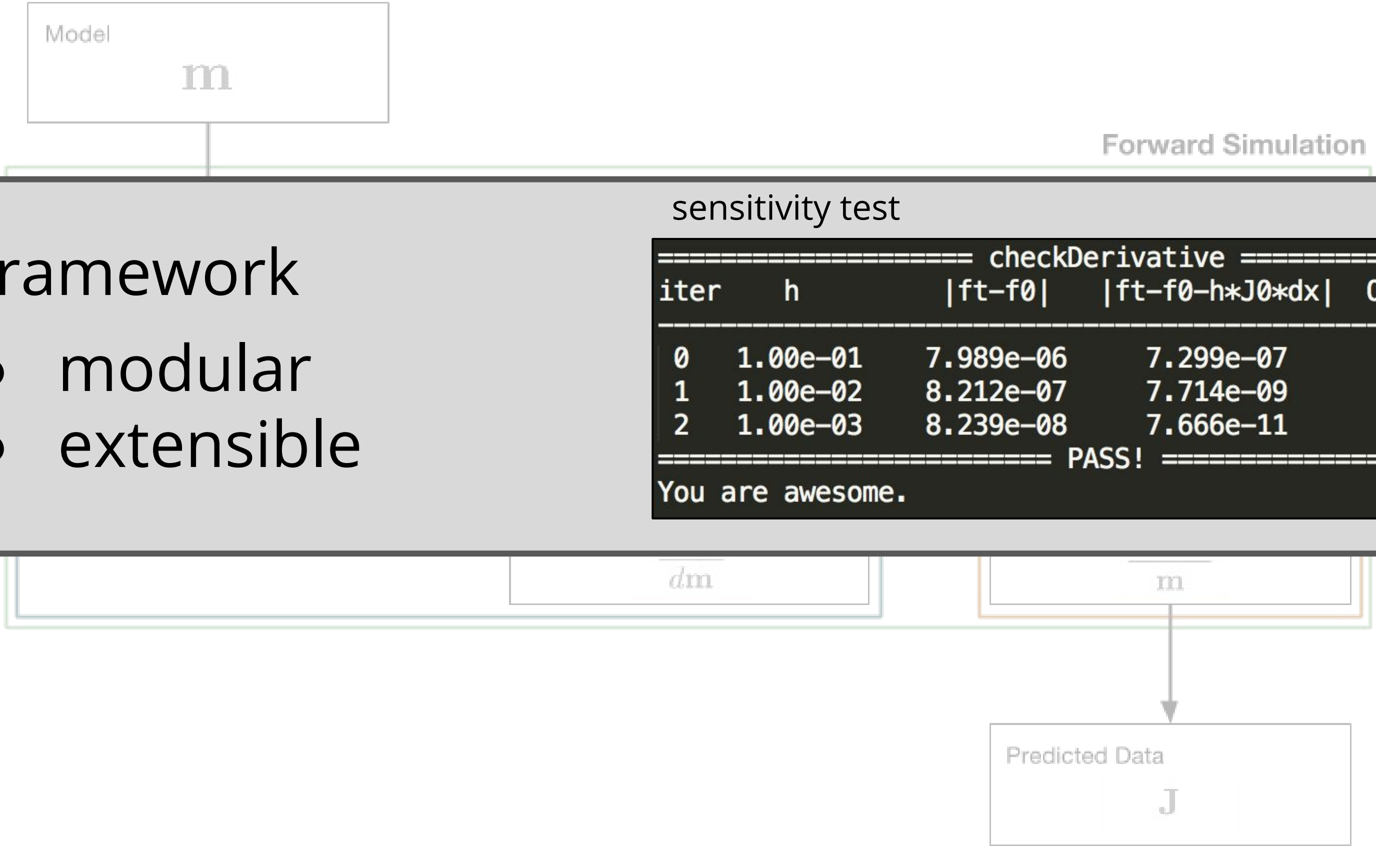


Framework

- modular
- extensible

sensitivity test

```
==================== checkDerivative ===============
iter    h           |ft-f0|    |ft-f0-h*J0*dx|   Order
-----------------------------------------------------
 0   1.00e-01    7.989e-06      7.299e-07        nan
 1   1.00e-02    8.212e-07      7.714e-09        1.976
 2   1.00e-03    8.239e-08      7.666e-11        2.003
========================= PASS! ====================
You are awesome.
```

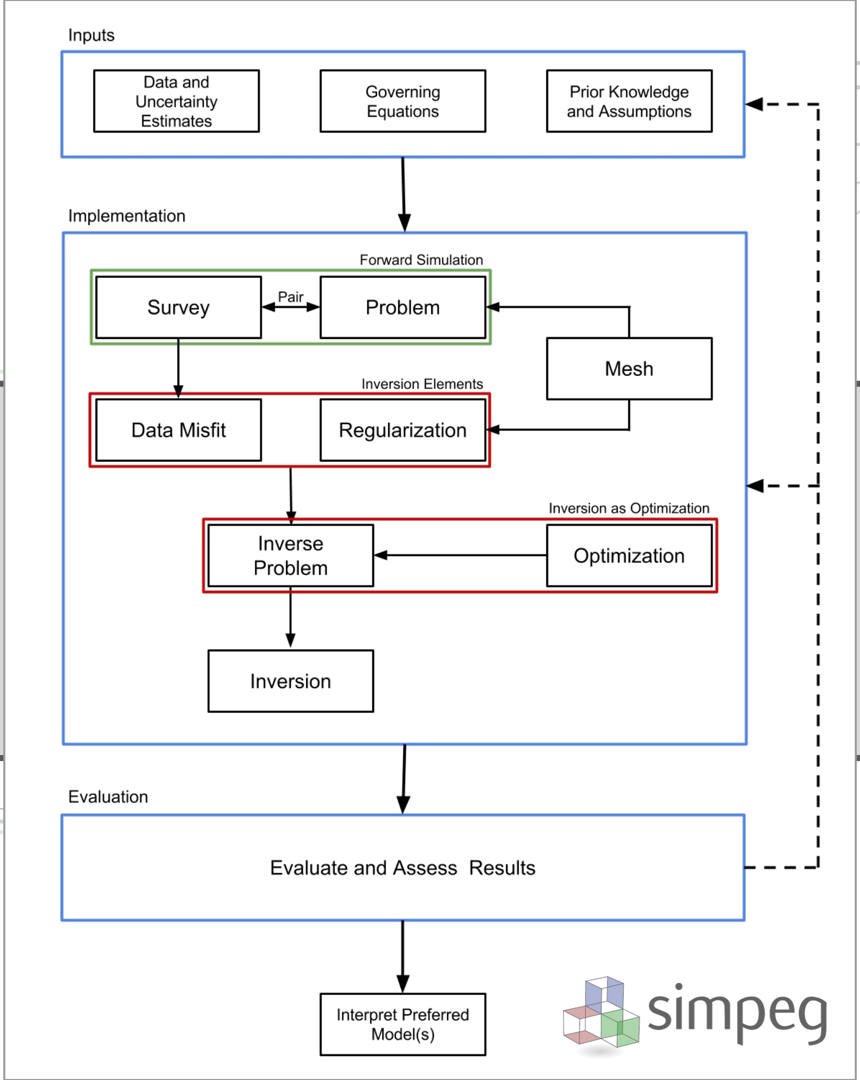
Sensitivities

Model

$\mathbf{m}$

Framework

- modular
- extensible



http://simpeg.xyz

Built in the open

MIT GitHub

pypi v0.1.3 | downloads 832/month | license MIT | build passing | coverage 77%

Part of an Ecosystem

Package	State
SimPEG	✔
simpegEM	↻
simpegMT	↻
simpegFLOW	⚗
simpegDC	⚗
simpegPF	⚗
simpegSEIS	🔧
simpegGPR	🔧

Documentation

SimPEG

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- Projects Using SimPEG
- Installation
- SimPEG Meshes
- Differential Operators

Read the Docs v: latest

Docs » SimPEG Documentation Edit on GitHub

SimPEG Documentation

simpeg

SimPEG: Simulation and Parameter Estimation in Geophysics

simpeg DOCS CODE JOURNAL CONTACT

Simulation and Parameter Estimation in Geophysics

An open source python package for simulation and gradient based parameter estimation in geophysical applications.

Installation

The easiest way to install SimPEG is from PyPI, using pip:

```
> pip install SimPEG
```

Read more detailed installations instructions in the documentation.

Get the source code at Github.

The team:

Rowan | Seogi | Lindsey | Guðni | Adam | Doug | Contributors

Thank You!

Questions?

simpeg.xyz

github.com/simpeg

lindsey@simpeg.xyz

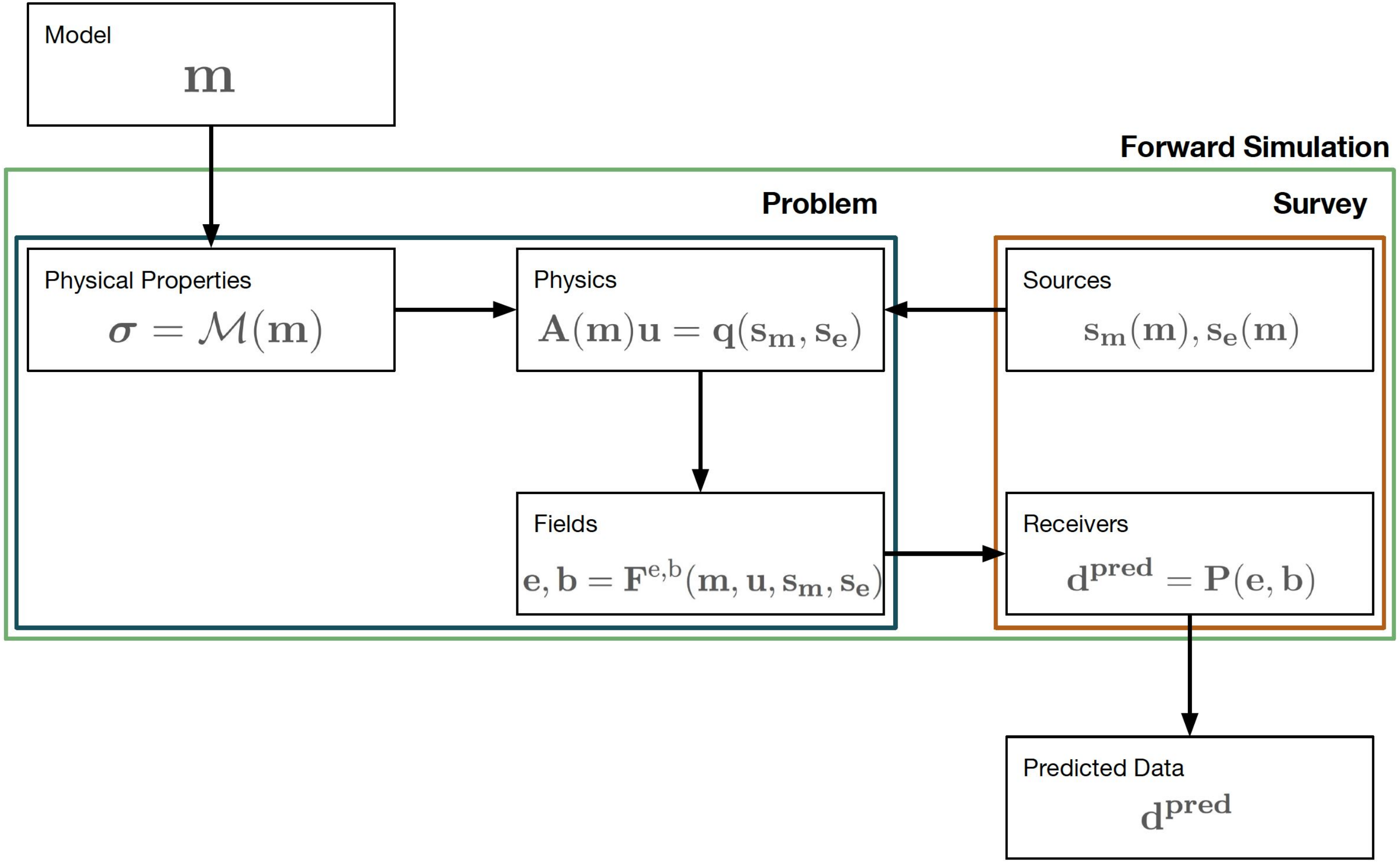
Other Presentations!

- Wed 4:15 - SimPEG - Rowan Cockett
- Wed 4:20 - Time Domain IP - Seogi Kang
- Fri 8-12 - Simulations & Education - Lindsey Heagy

SCRAPS

Forward Simulation

$$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$$



Want more?

simpeg.xyz

github.com/simpeg

lindsey@simpeg.xyz

Practices

pypi v0.1.3 | downloads 832/month | license MIT | build passing | coverage 77%

Testing

```
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---------------------------------------------------------
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========================= PASS! =========================
You are awesome.
```

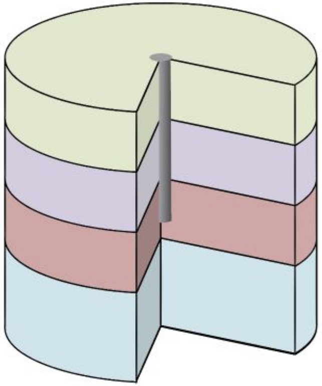
Steel casing in EM

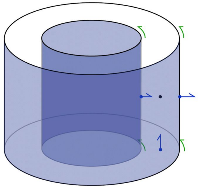
Primary

Estimate: $\sigma_p \; \mu_p$

$$\nabla \times \mathbf{E_p} + i\omega \mathbf{B_p} = 0$$
$$\nabla \times \mu_p^{-1} \mathbf{B_p} - \sigma_p \mathbf{E_p} = \mathbf{q}$$

Solve for: $\mathbf{E_p} \; \mathbf{B_p}$





Interpolate to compute source

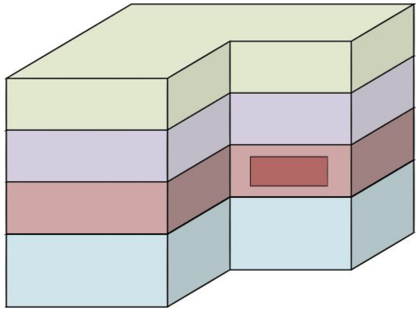
Secondary

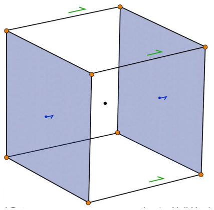
Invert for 3D conductivity : σ

$$\nabla \times \mathbf{E_s} + i\omega \mathbf{B_s} = 0$$
$$\nabla \times \mu^{-1} \mathbf{B_s} - \sigma \mathbf{E_s} = \tilde{\mathbf{q}}$$

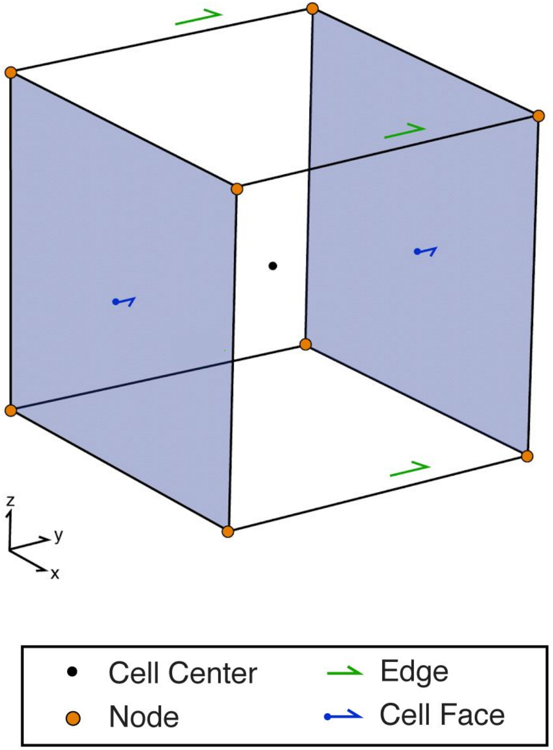
$$\tilde{\mathbf{q}} = (\boxed{\sigma} - \sigma_p)\mathbf{E_p}$$

Model dependence on RHS

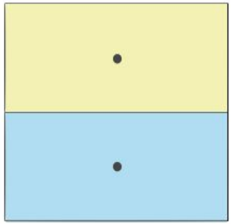




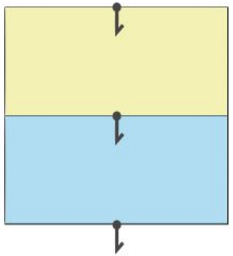
Finite Volume Forward Modelling



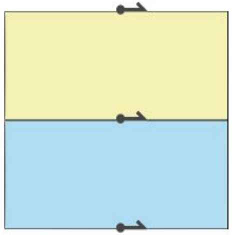
- Physical Properties
  - σ electrical conductivity
  - μ magnetic permeability



- Fluxes
  - $\mathbf{J}$ current density
  - $\mathbf{B}$ magnetic flux density



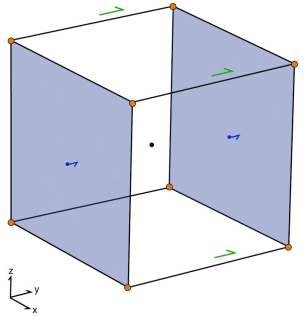
- Fields
  - $\mathbf{E}$ electric field
  - $\mathbf{H}$ magnetic field



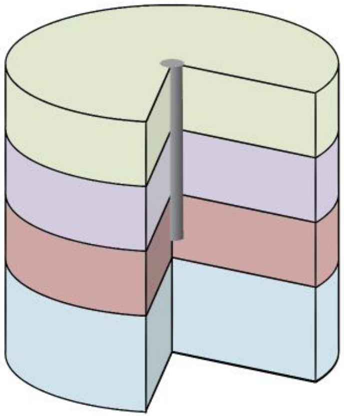
Steel casing in EM

Motivation: How do we characterize 3D conductivity distributions in settings with steel cased wells?

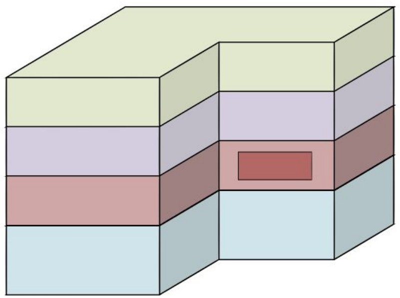
$$\nabla \times \mathbf{E} + i\omega \mathbf{B} = 0$$
$$\nabla \times \mathbf{H} - \mathbf{J} = \mathbf{q}$$



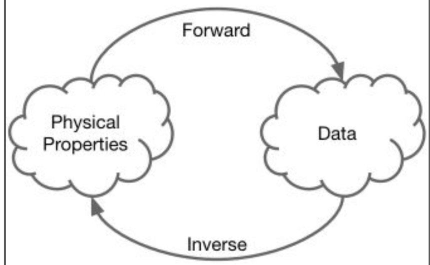
Modelling Maxwell's equations



Modelling the Casing



Modelling 3D geology

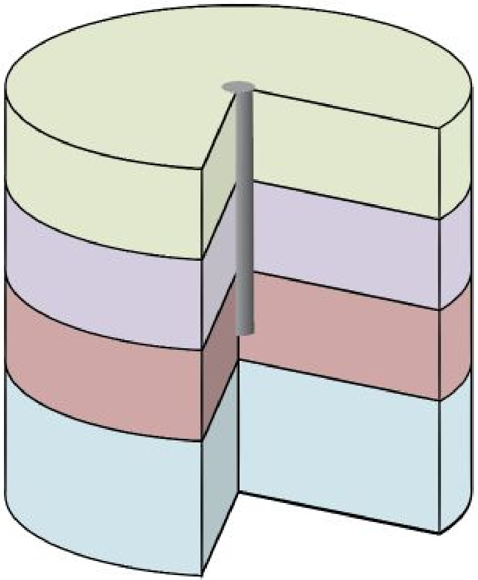


Approaching the inverse problem

Modelling with 3D geology: Primary Secondary

Primary: $\nabla \times \mathbf{E_p} + i\omega \mathbf{B_p} = 0$

$$\nabla \times \mu_p^{-1} \mathbf{B_p} - \sigma_p \mathbf{E_p} = \mathbf{q}$$



Casing & Source, Layered Earth

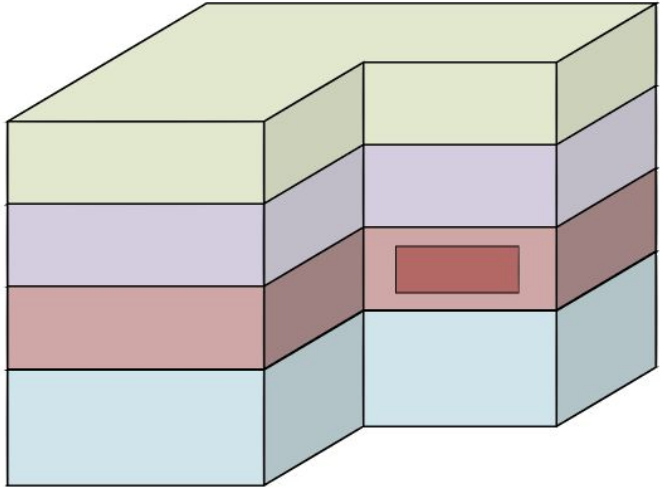
Interpolate

Secondary:

$$\nabla \times \mathbf{E_s} + i\omega \mathbf{B_s} = 0$$

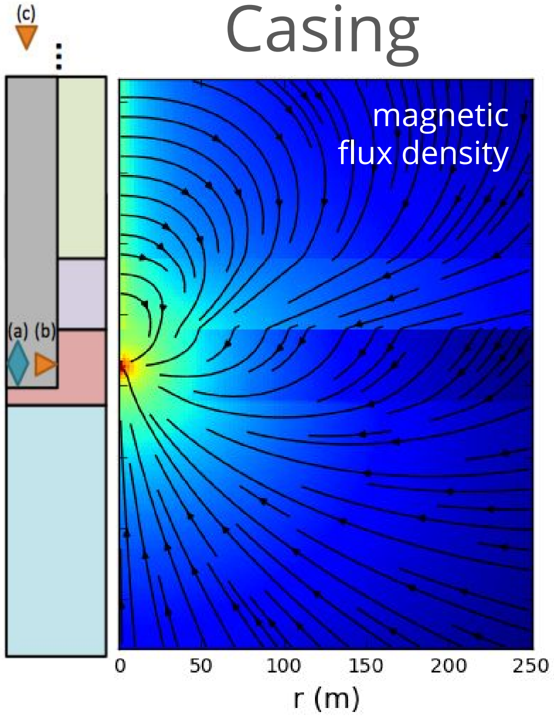
$$\nabla \times \mu^{-1} \mathbf{B_s} - \sigma \mathbf{E_s} = \tilde{\mathbf{q}}$$

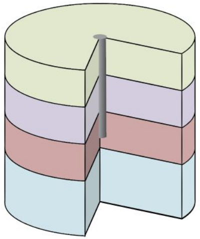
$$\tilde{\mathbf{q}} = -\left(\nabla \times (\mu^{-1} - \mu_p^{-1})\mathbf{B_p} - (\sigma - \sigma_p)\mathbf{E_p}\right)$$



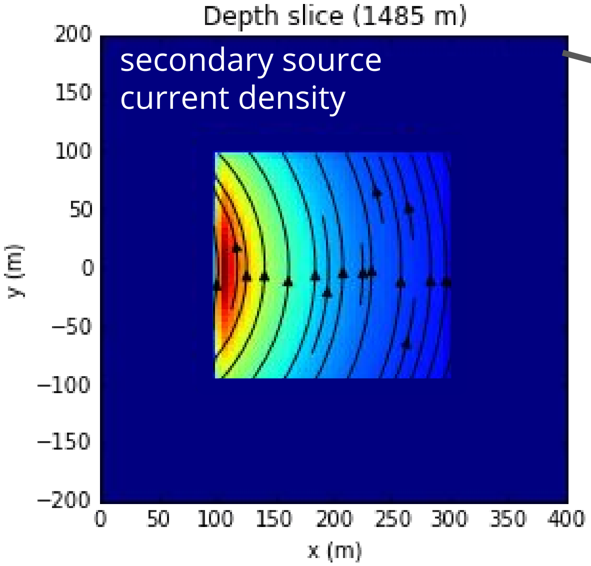
Fields from casing, 3D Earth

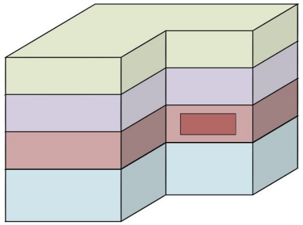
Casing



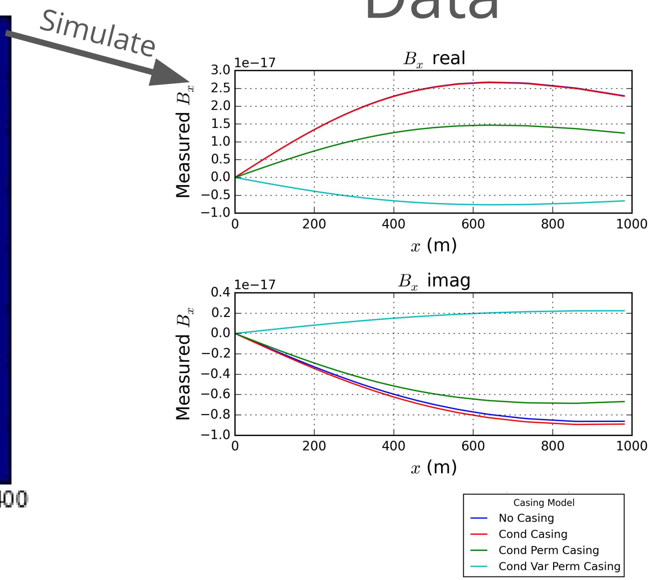


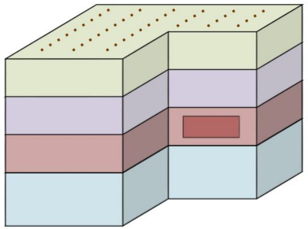
3D Geology

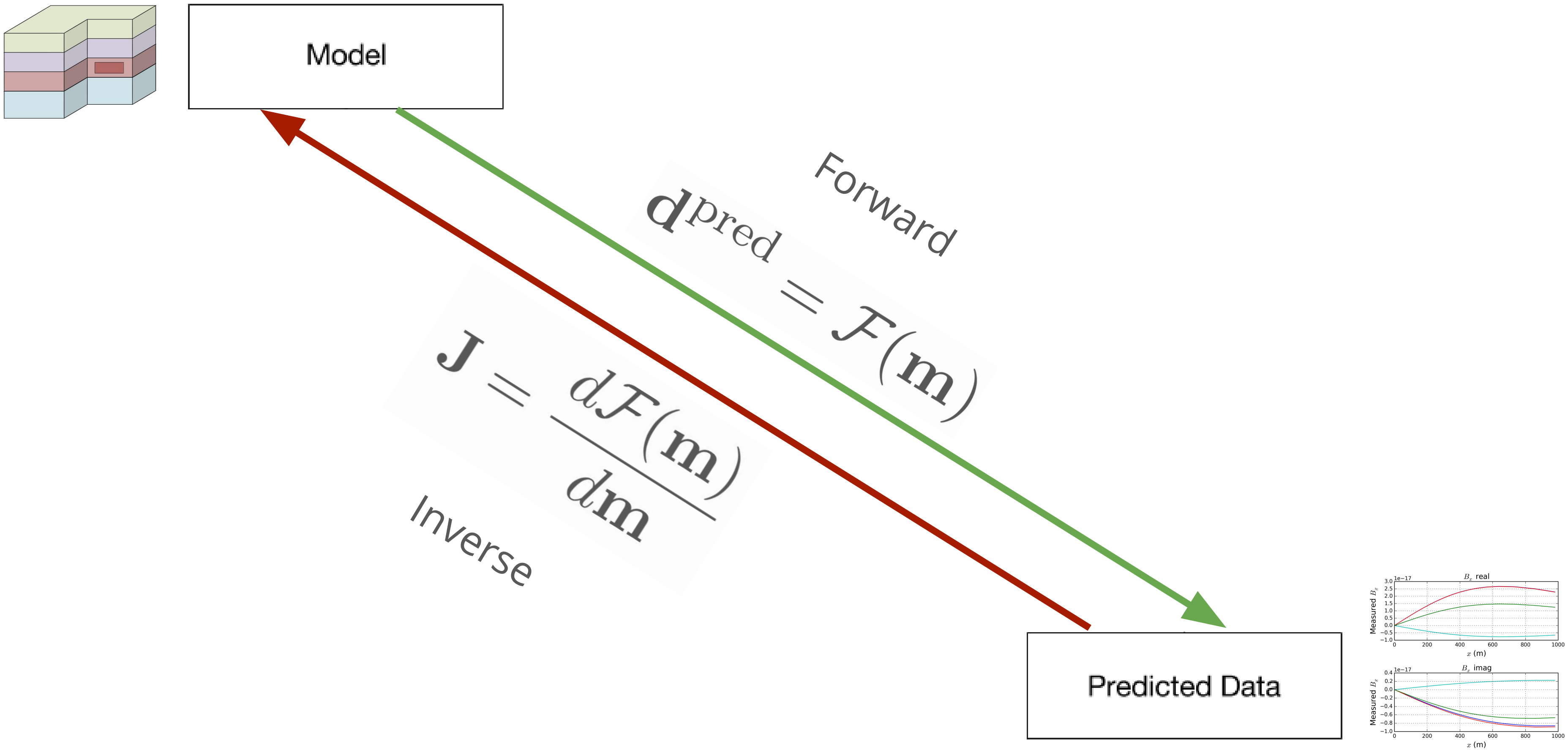


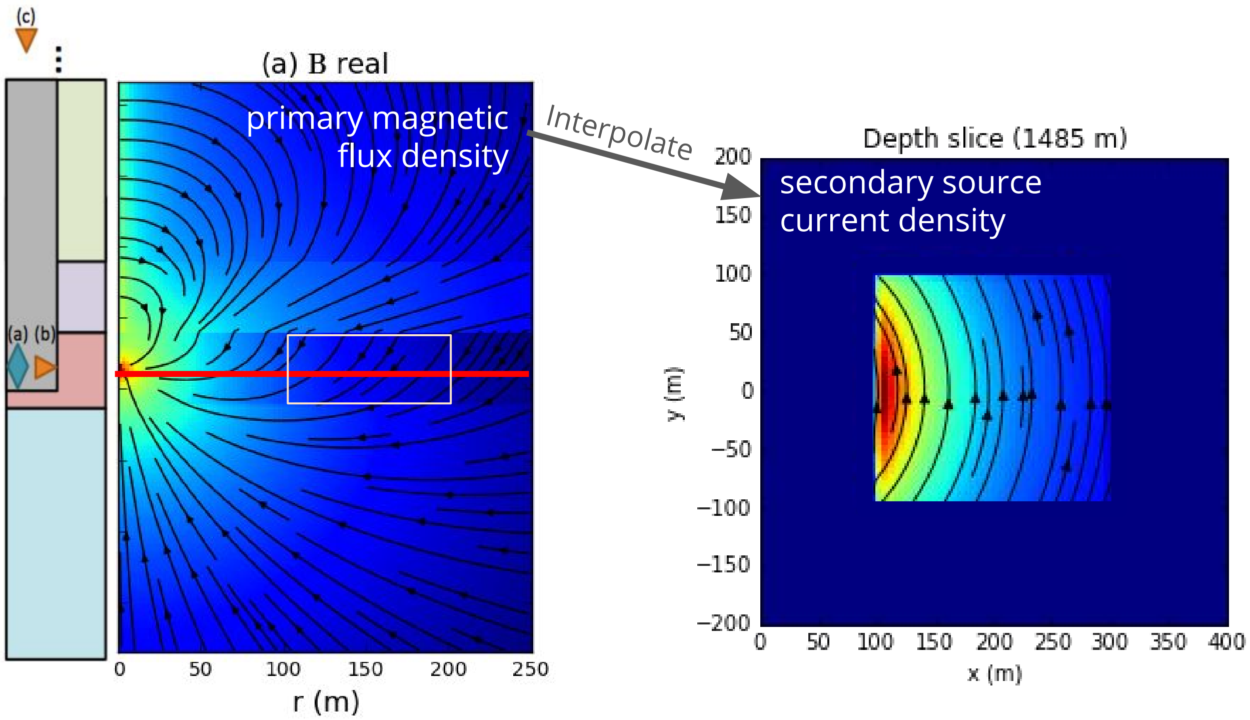


Data






Model
Forward
$\mathbf{d}^{\text{pred}} = \mathcal{F}(\mathbf{m})$
$\mathbf{J} = \frac{d\mathcal{F}(\mathbf{m})}{d\mathbf{m}}$
Inverse
Predicted Data
B_z real
B_z imag
Measured B_z
x (m)


(c)
(a) (b)
(a) B real
primary magnetic
flux density
0
50
100
150
200
250
r (m)
Interpolate
Depth slice (1485 m)
secondary source
current density
200
150
100
50
0
−50
−100
−150
−200
y (m)
0
50
100
150
200
250
300
350
400
x (m)