

Moving between dimensions in EM inversions

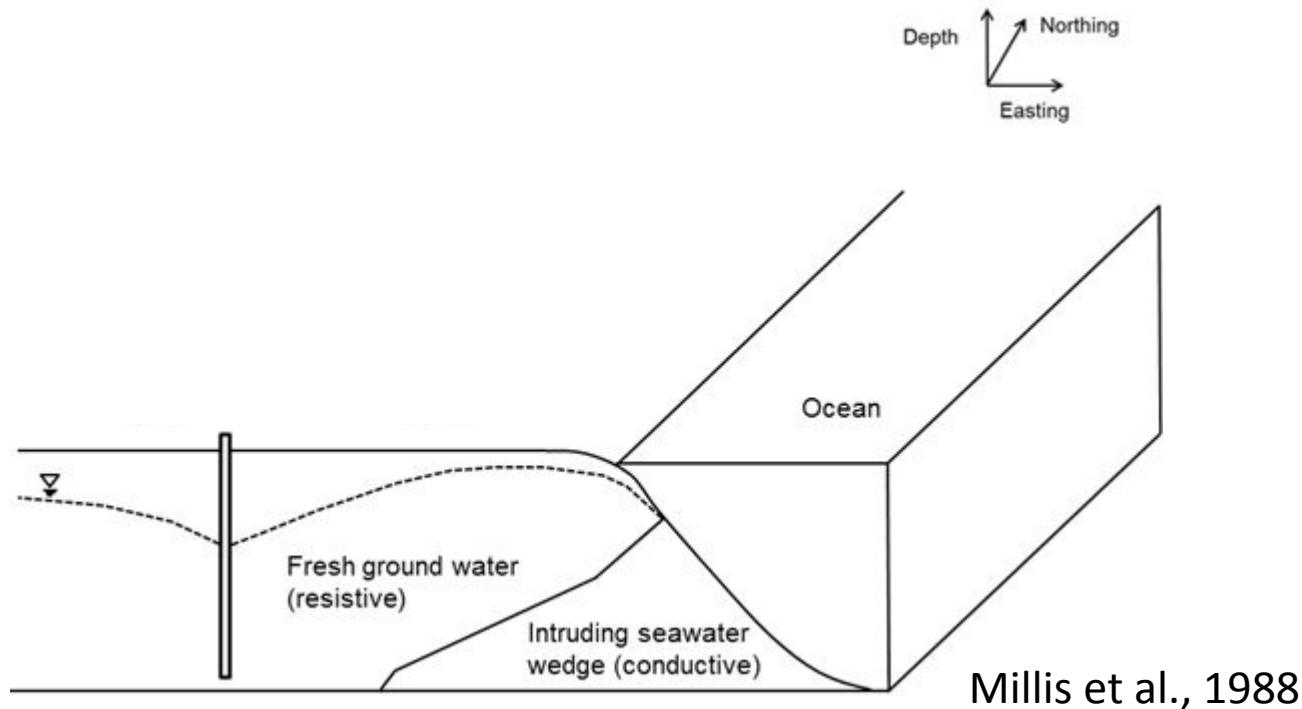


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and Douglas W. Oldenburg
University of British Columbia

Geological problem

- Sea water intrusion

“Where is the **boundary** of fresh and sea water?”



TEM forward problem

- Maxwell's equation in time domain

$$\vec{\nabla} \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

$$\vec{\nabla} \times \mu^{-1} \vec{b} - \vec{j} = \vec{j}_s$$

\vec{e} : Electric field (V/m)

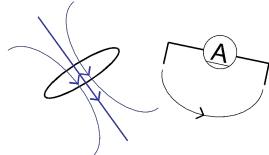
\vec{b} : Magnetic field (Wb/m^2)

\vec{j} : Current density (A/m^2)

\vec{j}_s : Current source (A/m^2)

To solve this problem, we use

- simpegEM



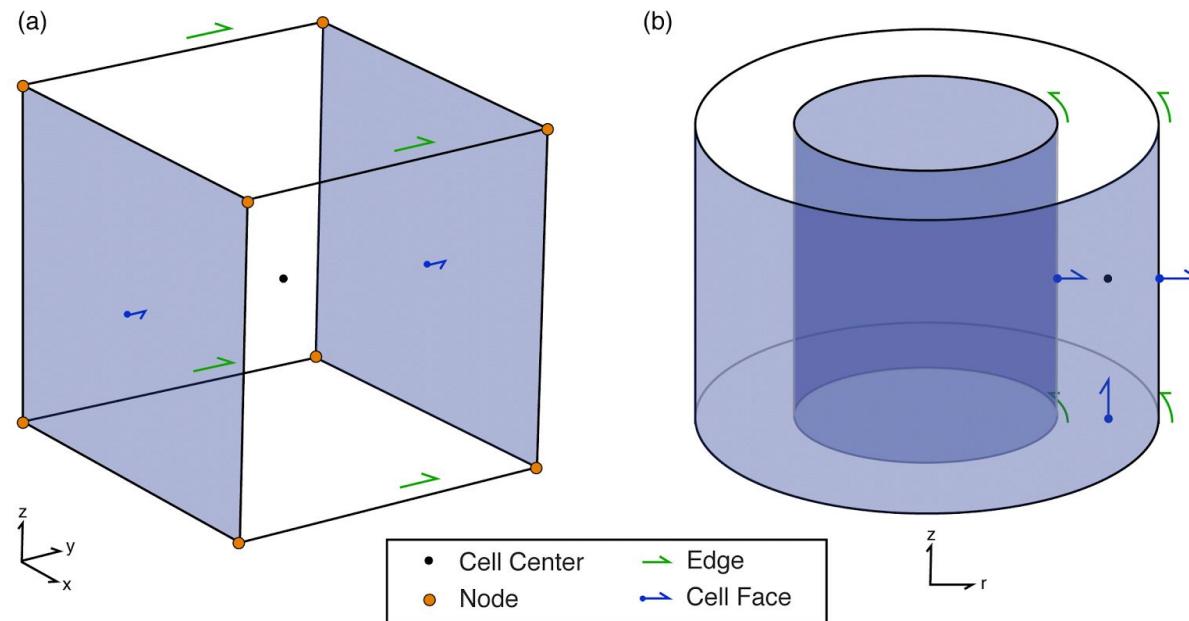
- Frequency and Time
- Tensor and 2D Cylindrical meshes

- SimPEG (Cockett et al., 2015)

- Open source
- Modular
- Well-tested
- Well-documented
- Geophysical inversion frame work

Simulation meshes

- Staggered grids:



\vec{e} -field on Edge

\vec{b} -field on Face

Gradient-based inversion

- Forward problem

$$A(\sigma)u(\sigma) = rhs$$

$$u = A^{-1}rhs$$

$$d = P(u) = F[\sigma]$$

$$J = \frac{\partial F}{\partial m}$$

u : Electromagnetic fields

A : System matrix

$P(\cdot)$: Projection of fields to observations

J : Sensitivity function

- Optimization

$$\phi_d = \|\mathbf{W}_d(\mathbf{d}^{obs} - \mathbf{d}^{pred})\|_2^2$$

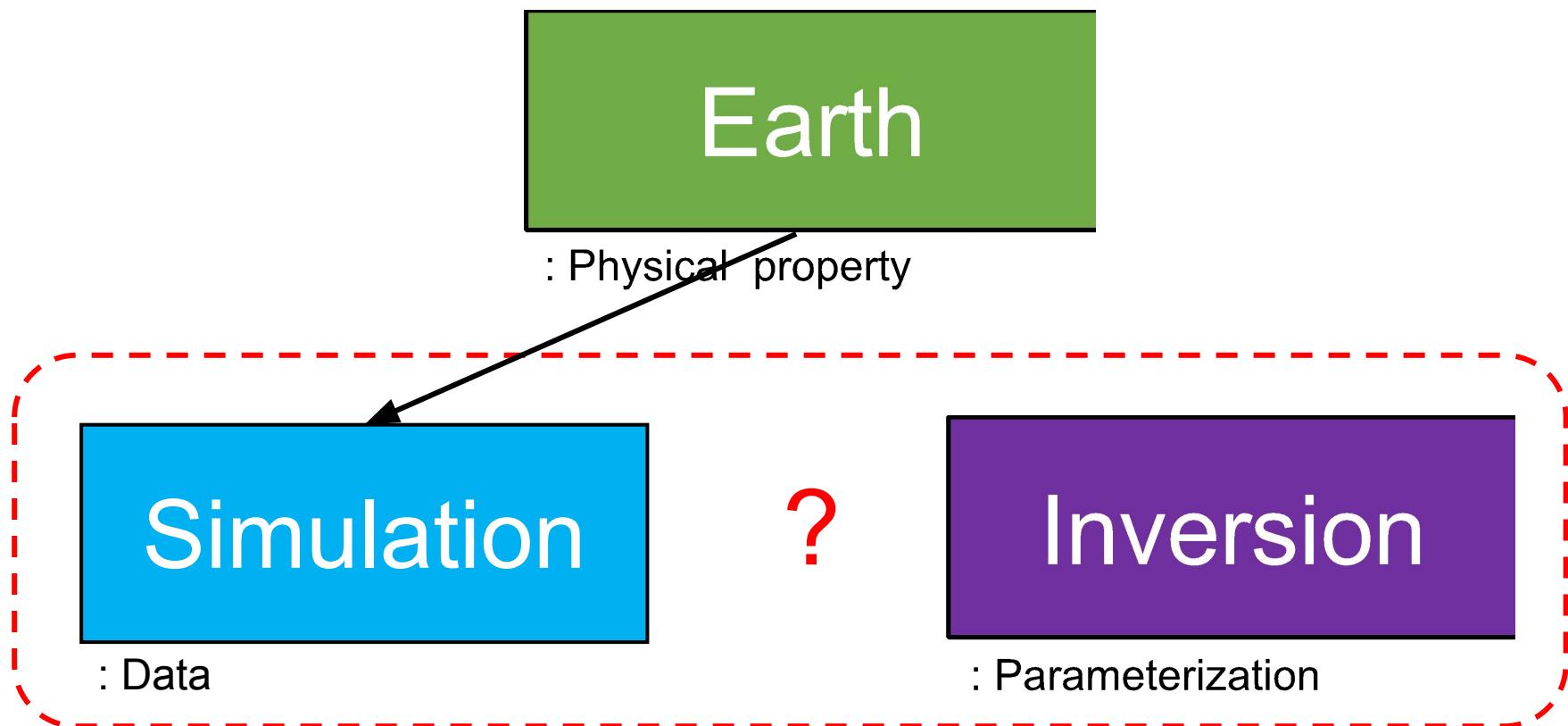
$$\nabla \phi_d = -J^T W_d^T (\mathbf{d}^{pred} - \mathbf{d}^{obs})$$

$$\text{minimize } \phi = \phi_d(\mathbf{m}) + \phi_m(\mathbf{m})$$

$$s.t. \quad 0 \leq \mathbf{m}$$

Inexact Gauss Newton (Nocedal and Wright, 1999):
Always make sure evaluate $J^T vec$ and $J vec$

Model spaces



Model with sensitivity function

- Forward problem

$$d = F[\sigma]$$

σ = conductivity (S/m)

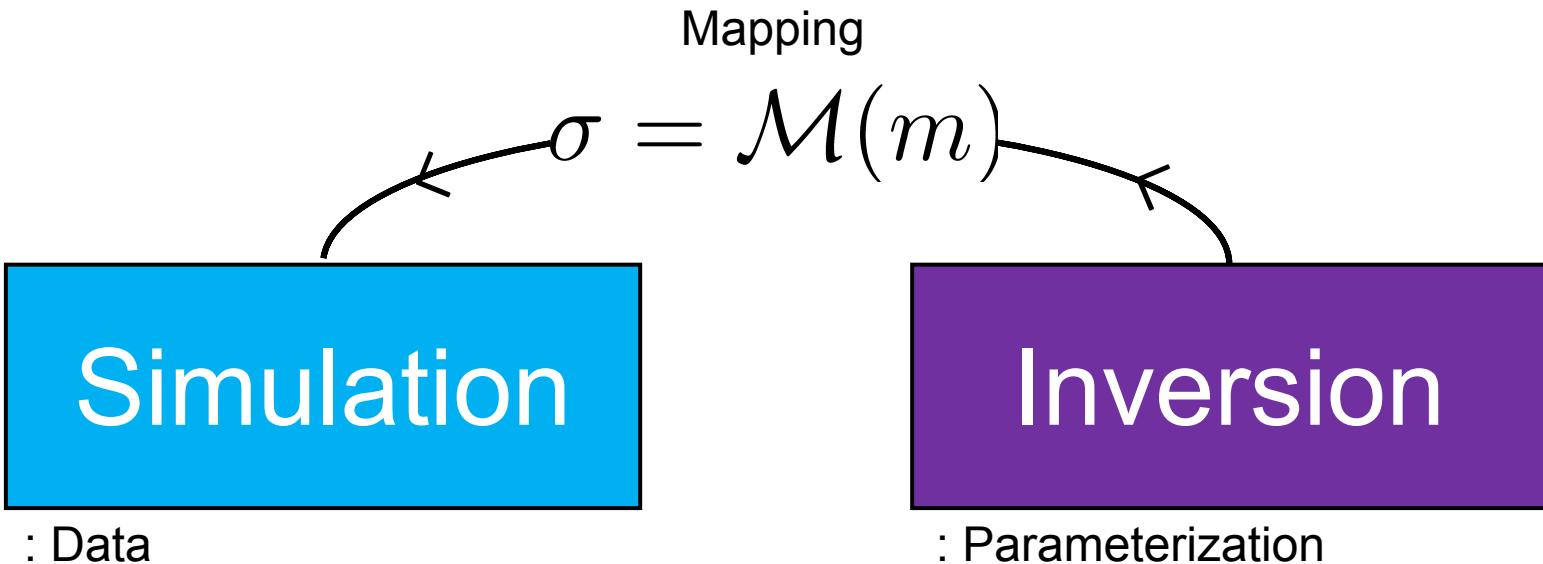
- Sensitivity

Our topic today

$$J = \frac{\partial F}{\partial m} = \frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial m}$$

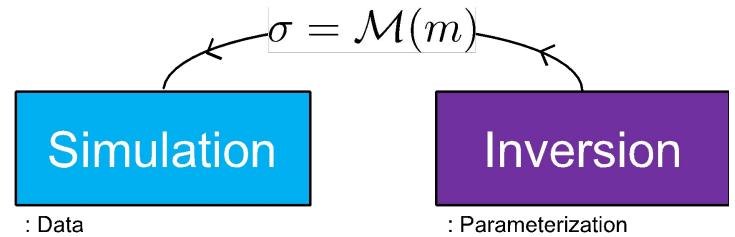
m = inversion model

Mapping function



- From inversion model to simulation model

Exponential map



- Inversion model:

$$m_{log} = \log(\sigma)$$

- Mapping:

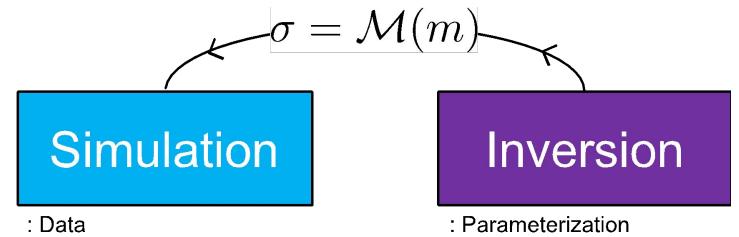
$$\sigma = \mathcal{M}_{log}^{exp}(m_{log}) = \exp(m_{log})$$

- Derivative:

$$\frac{\partial \sigma}{\partial m_{log}} = \exp(m_{log}) = \sigma$$

Surjection map#1

Active cell map



- Inversion model:

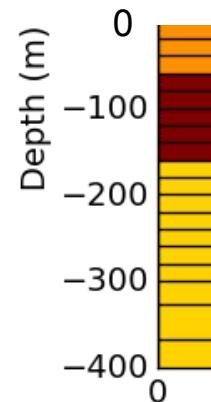
m_{act} : Conductivity of active cells

- Mapping:

$$\begin{aligned}\sigma &= \mathcal{M}_{act}^{surj}(m_{act}) \\ &= P_{act}^{surj} m_{act} + m_{inact}\end{aligned}$$

P_{act}^{sure} : Surjection from active cells to full domain

m_{inact} : Conductivity of inactive cells



- Derivative:

$$\frac{\partial \sigma}{\partial m_{act}} = P_{act}^{surj}$$

Surjection map#2

Vertical 1D map

- Inversion model:

m_{1D} : 1D conductivity

- Mapping:

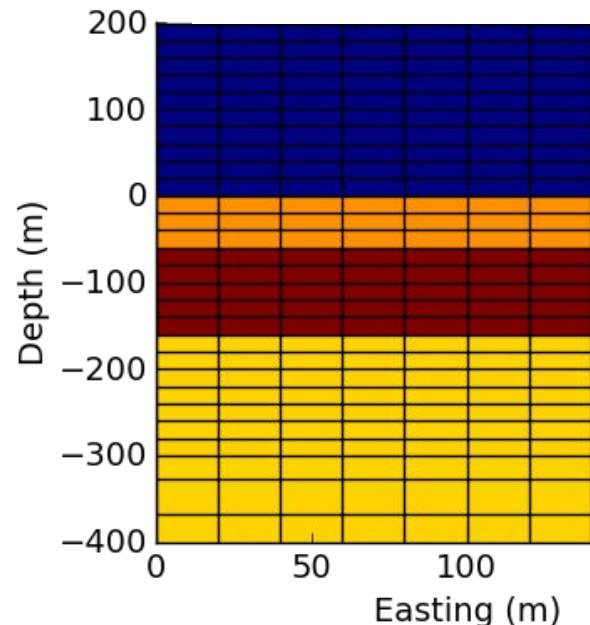
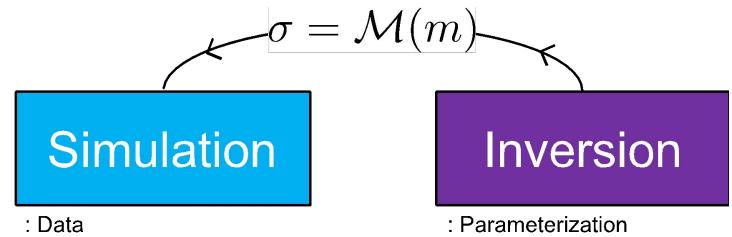
$$\sigma = \mathcal{M}_{1D}^{surj}(m_{1D})$$

$$= P_{1D}^{surj} m_{1D}$$

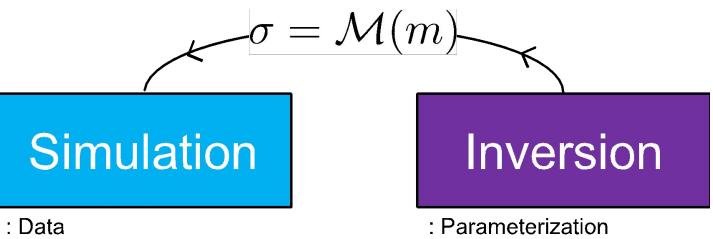
P_{1D}^{surj} : Surjection from 1D conductivity to full domain (2D or 3D)

- Derivative:

$$\frac{\partial \sigma}{\partial m_{1D}} = P_{1D}^{surj}$$



Combo map



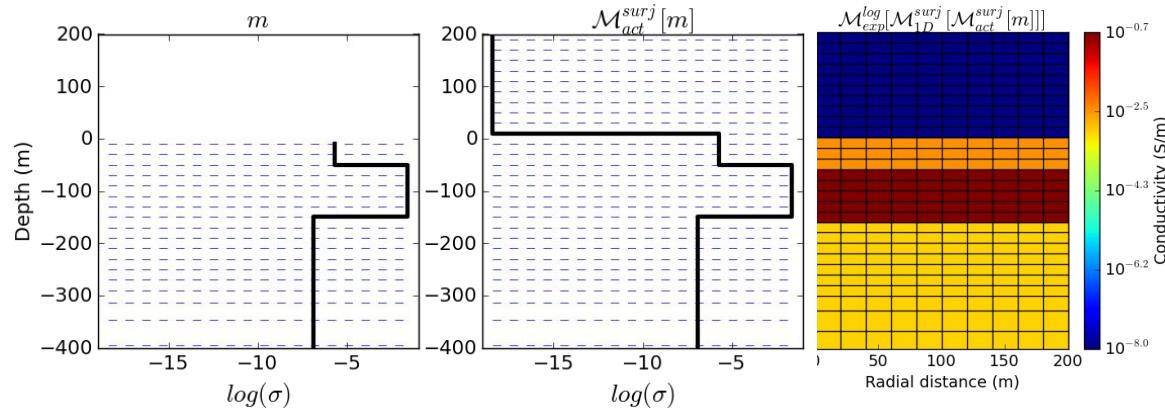
- Inversion model:

m : 1D log conductivity for the subsurface cells

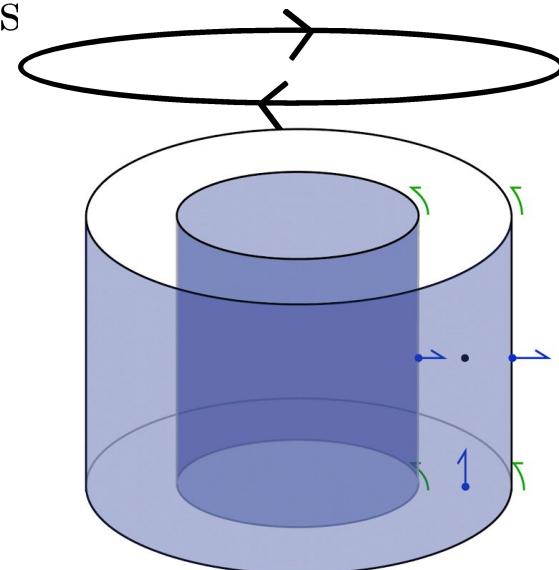
- Combo map:

- Active cell map (\mathcal{M}_{act}^{surj})
- Vertical 1D map (\mathcal{M}_{act}^{surj})
- Exponential map (\mathcal{M}_{exp}^{log})

$$\sigma = \mathcal{M}_{exp}^{log}[\mathcal{M}_{1D}^{surj}[\mathcal{M}_{act}^{surj}(m)]]$$



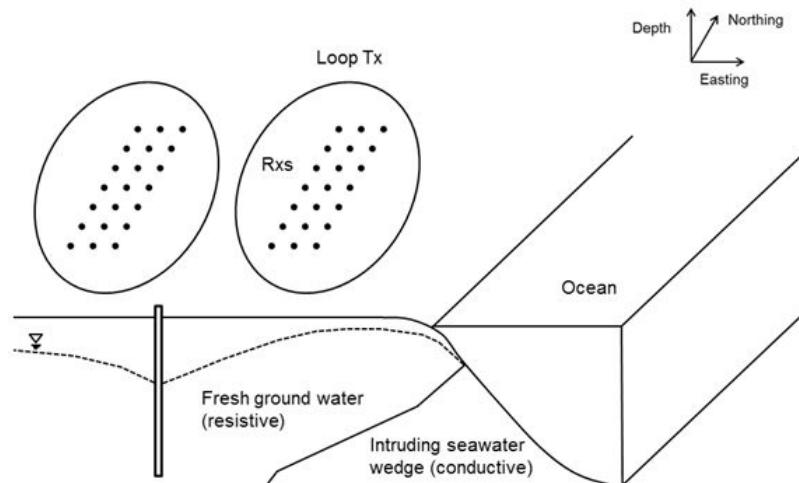
2D cylindrical mesh



Objective

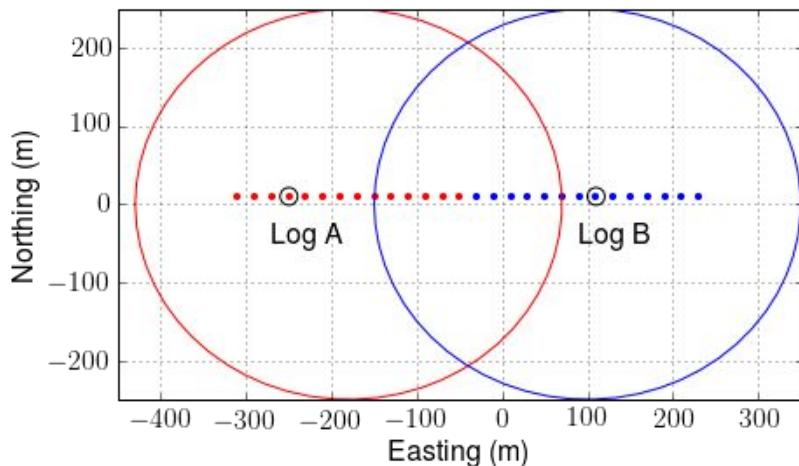
- Use proper inversion models to answer a geological question in EM inversion:

“Where is the **boundary** of fresh and seawater?”

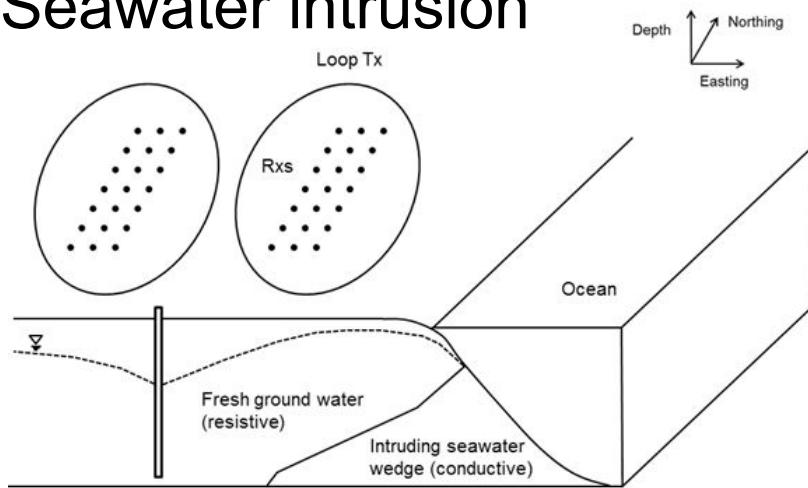


Set up

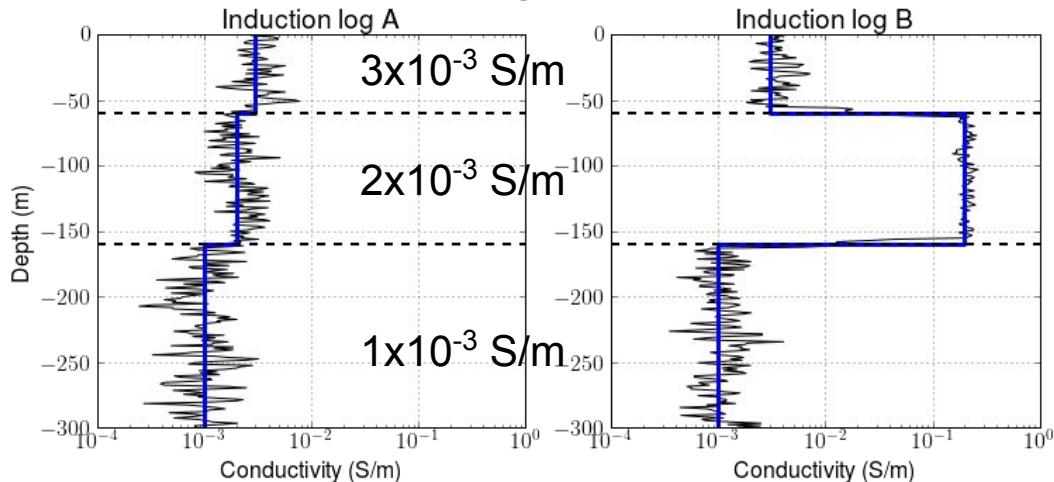
Survey configuration



Seawater intrusion



Two induction logs



- TEM survey

Loop transmitter:

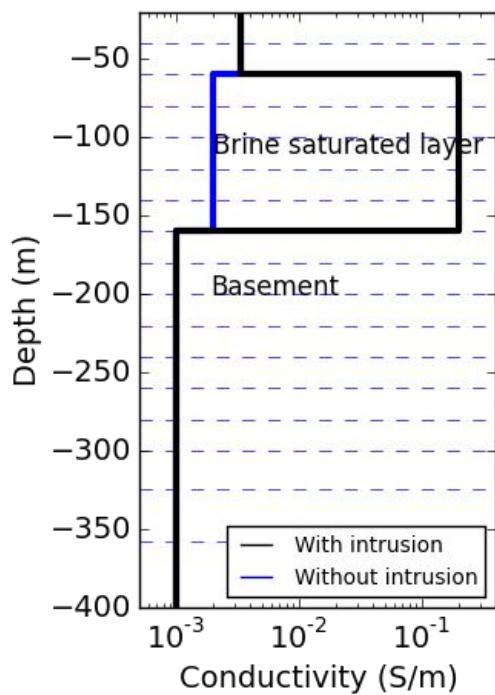
- Radius = 200 m
- Step-off waveform

Flux gate sensors

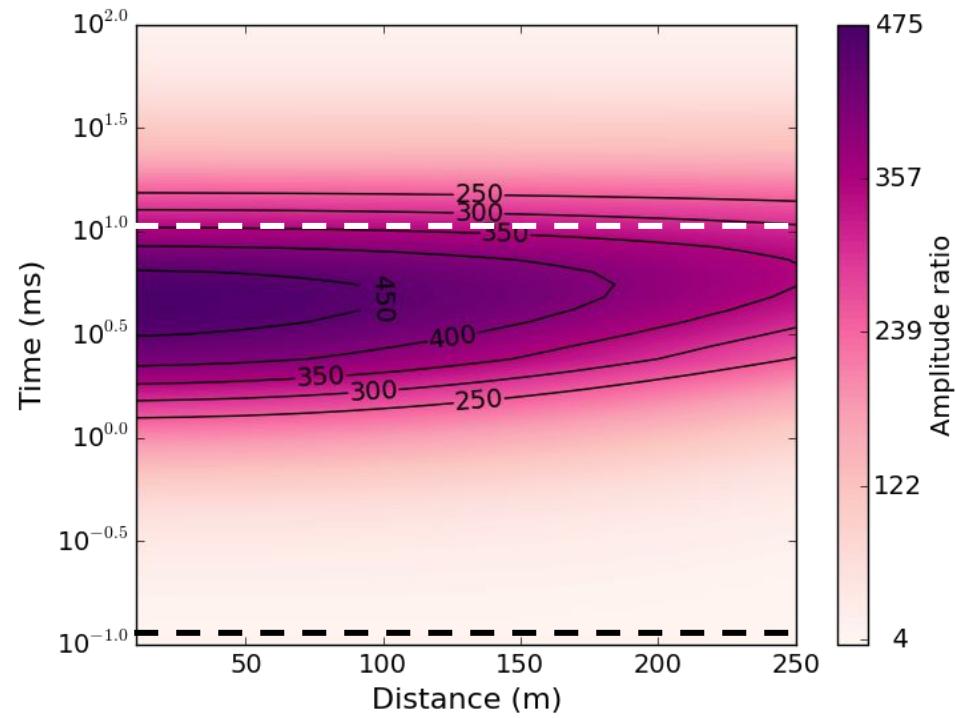
- Measure bz
- 14 / Tx (28 Rxs)

Detectability

1D intrusion model



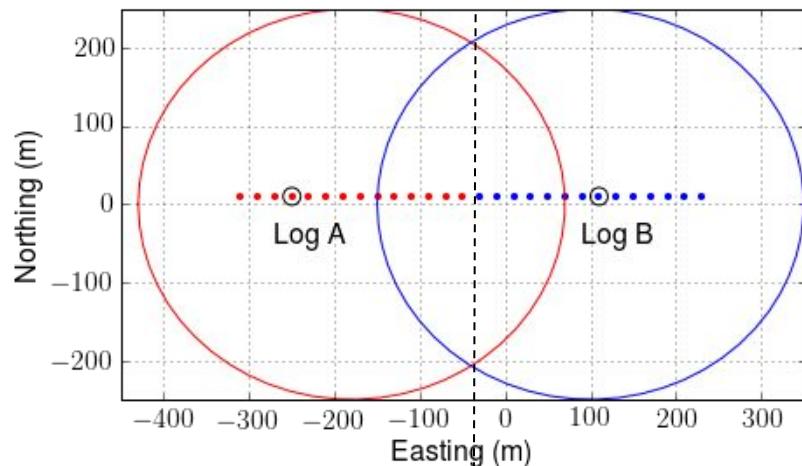
$$\text{Amplitude ratio} = \frac{|\text{With intrusion}|}{|\text{Without intrusion}|}$$



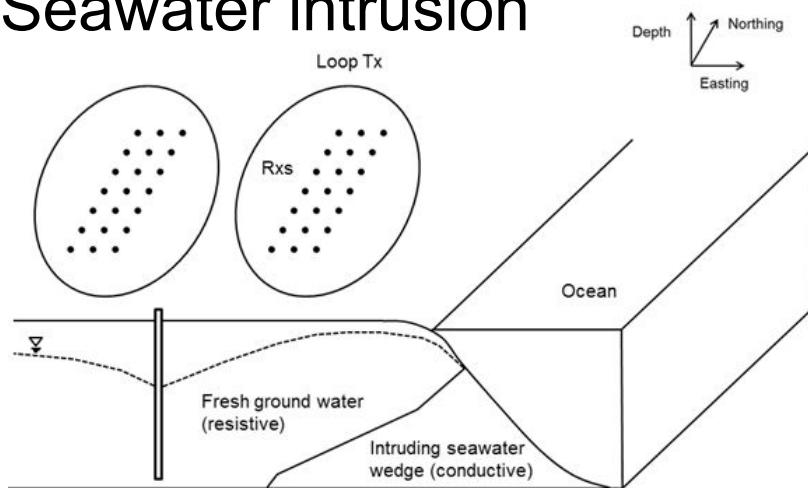
Use: 0.1-10 ms (31 channels)

TEM data from two Tx's

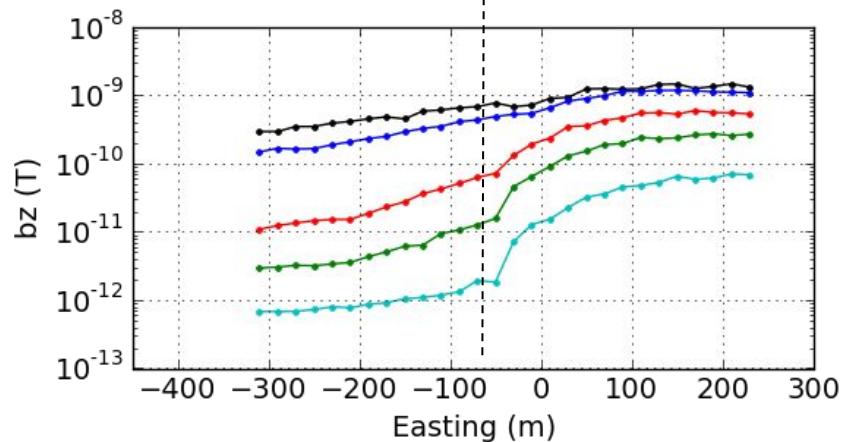
Survey configuration



Seawater intrusion



Observed data



- Obs 0.1 ms
- Obs 0.2 ms
- Obs 1.0 ms
- Obs 2.2 ms
- Obs 4.6 ms

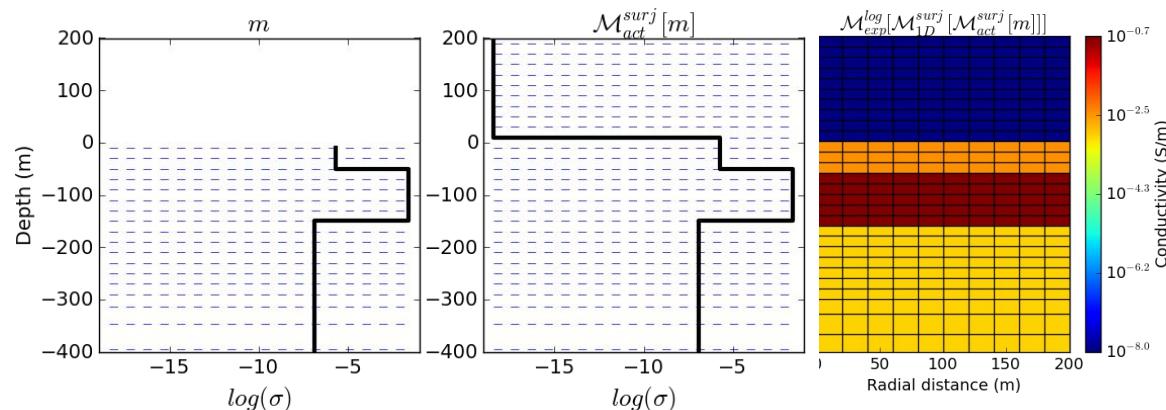
1D inversion

- Inversion model:

m : log conductivity for 1D subsurface cells

- Used mapping:

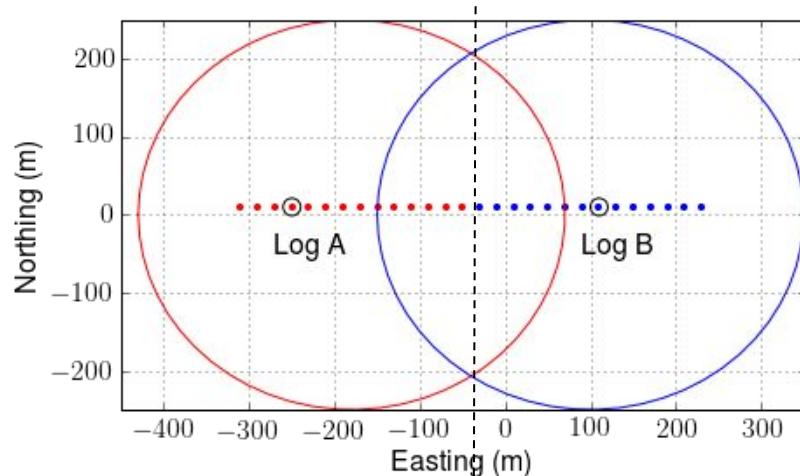
$$\sigma = \mathcal{M}_{log}^{exp}[\mathcal{M}_{1D}^{surj}[\mathcal{M}_{act}^{surj}(m)]]$$



1D inversions for each Tx:

“Overall layering in the region”

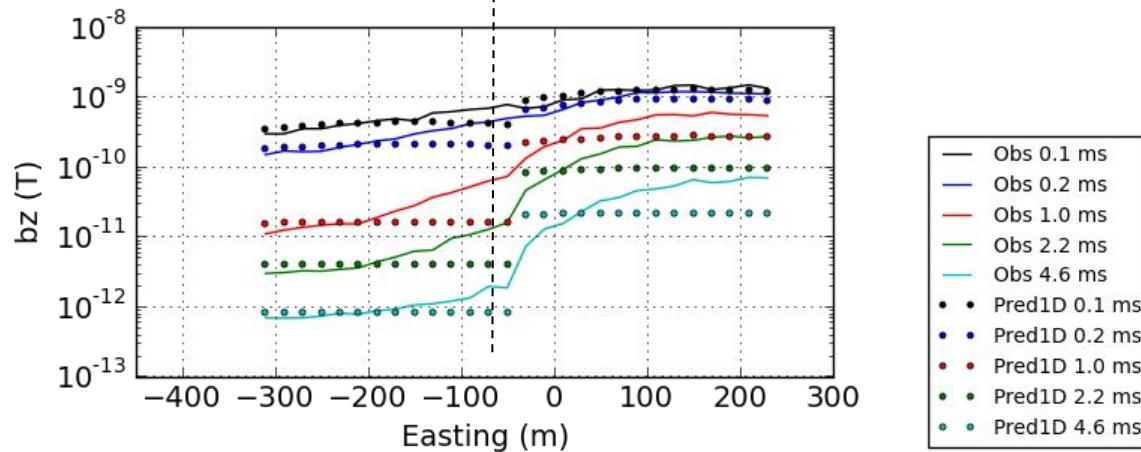
Survey configuration



1D inversions results

Tx1 Tx2

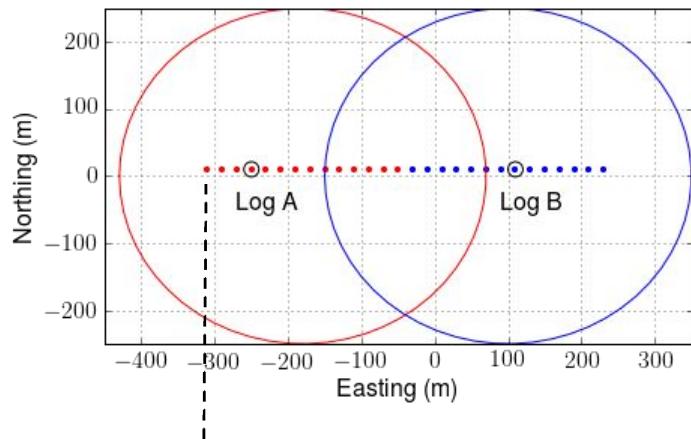
Observed data



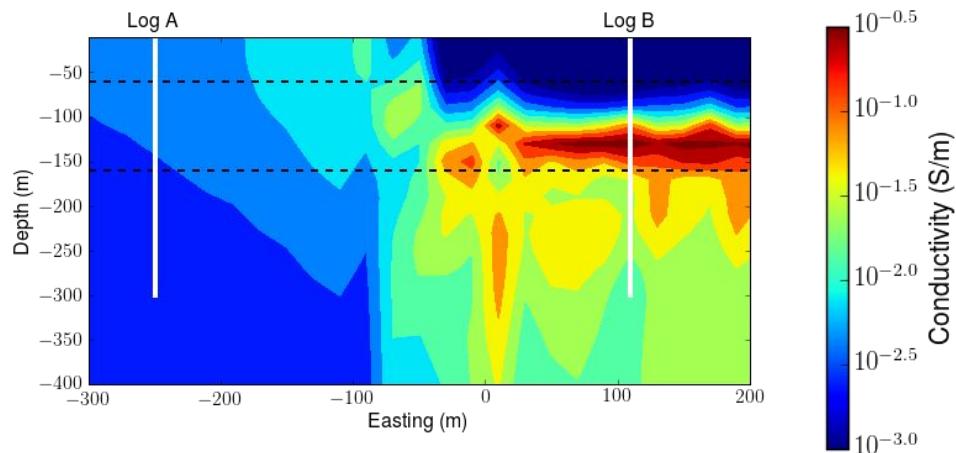
1D stitched inversion:

“Where is horizontal boundary (Easting)?”

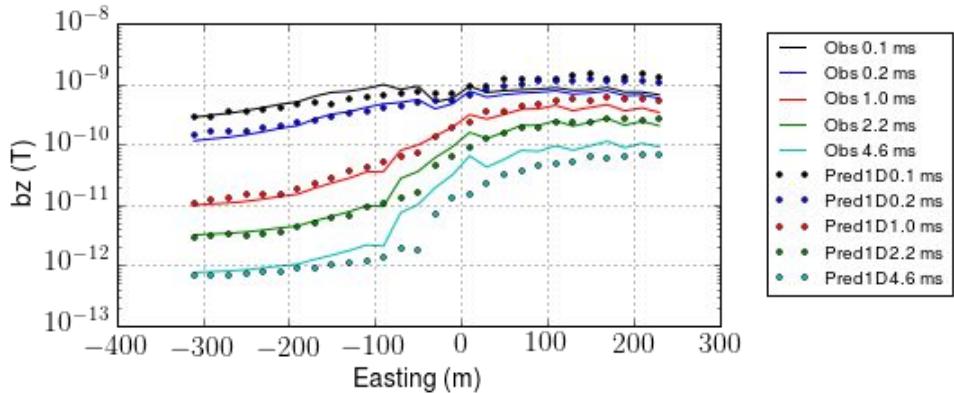
Survey configuration



1D stitched inversion results



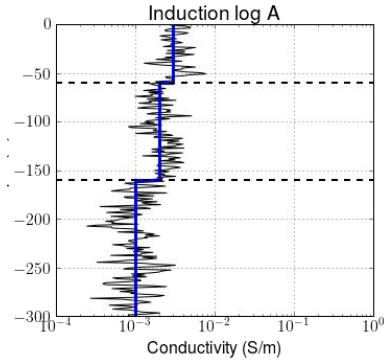
Observed vs. Predicted



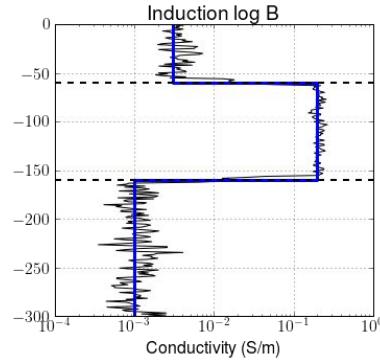
Summary: 1D inversions

1D inversions for each Tx

Tx1

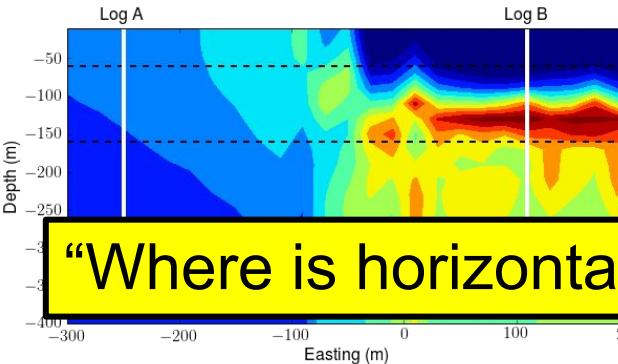
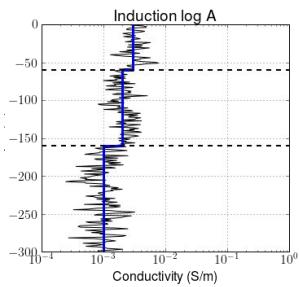


Tx2

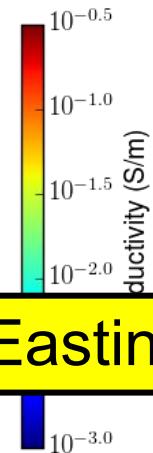


“Overall layering in the region”

1D Stitched inversions

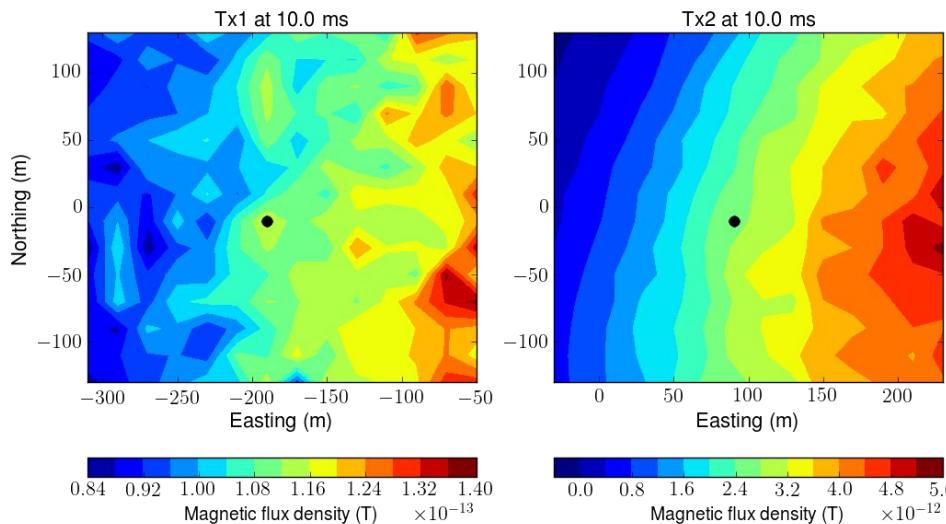
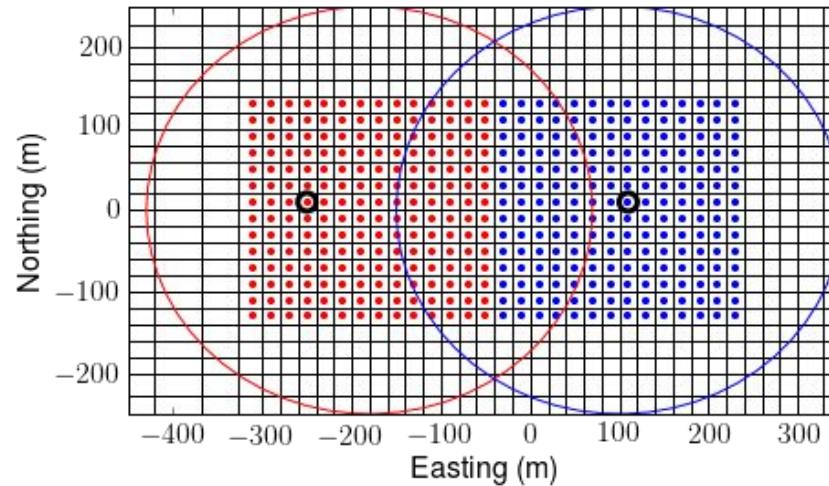


“Where is horizontal boundary (Easting)?”



More data for 3D inversion

“Distribution of intruded seawater”

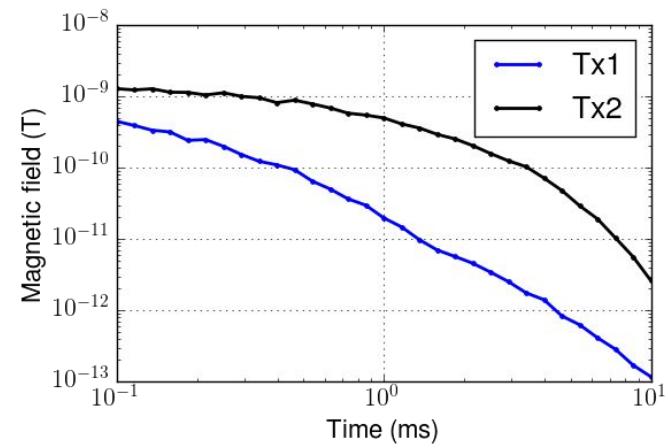


Loop transmitter:

- Radius = 200 m
- Step-off waveform

Magnetic field sensors:

- Measure bz
- 196 Rxs / Tx
- 31 channels (0.1-10 ms)



3D inversion:

“Distribution of intruded seawater”

- Inversion model

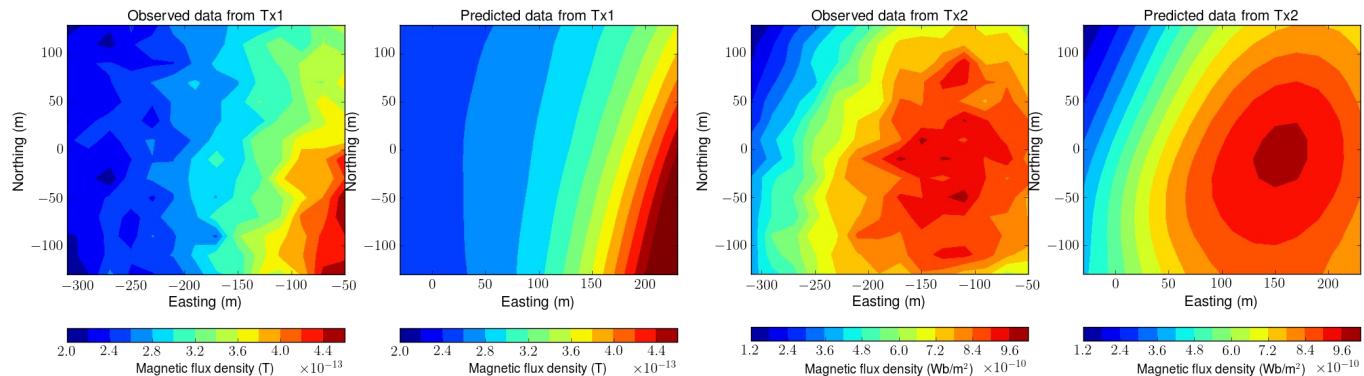
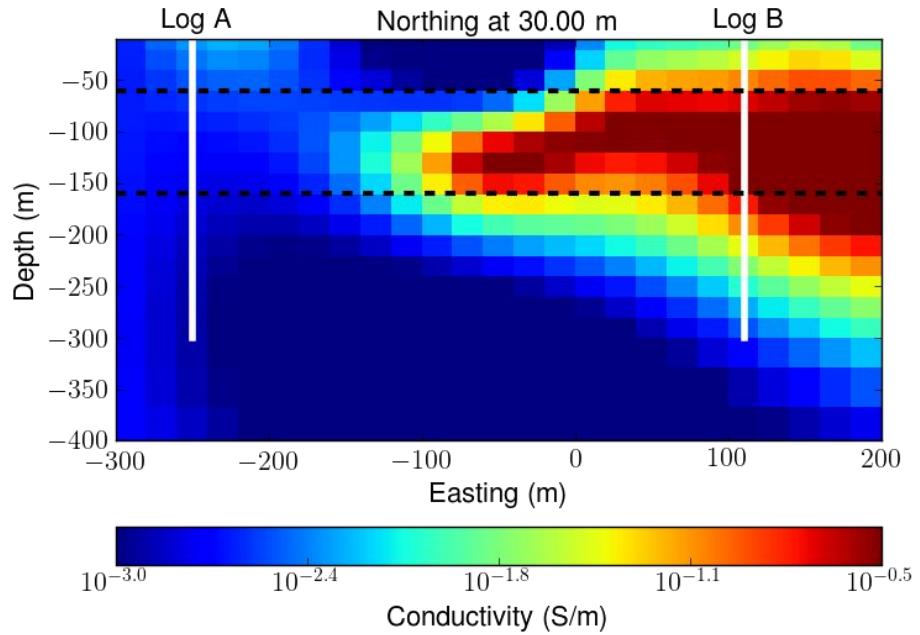
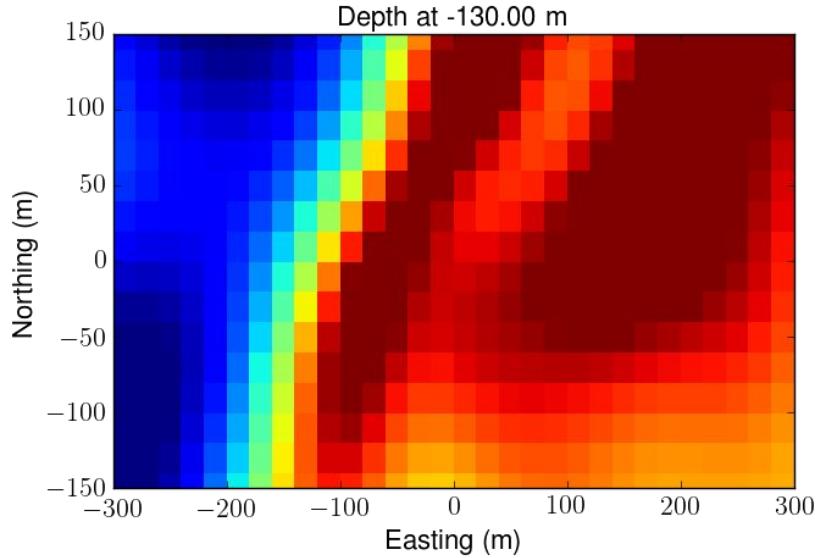
m : Log conductivity of subsurface cells (3D)

- Used mapping:

$$\sigma = \mathcal{M}_{log}^{exp}[\mathcal{M}_{act}^{surj}(m)]$$

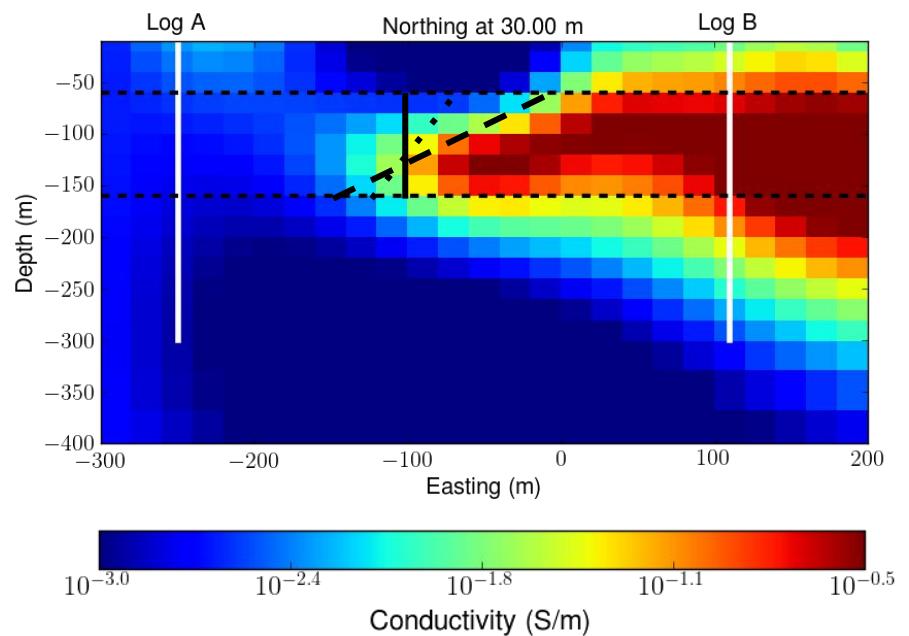
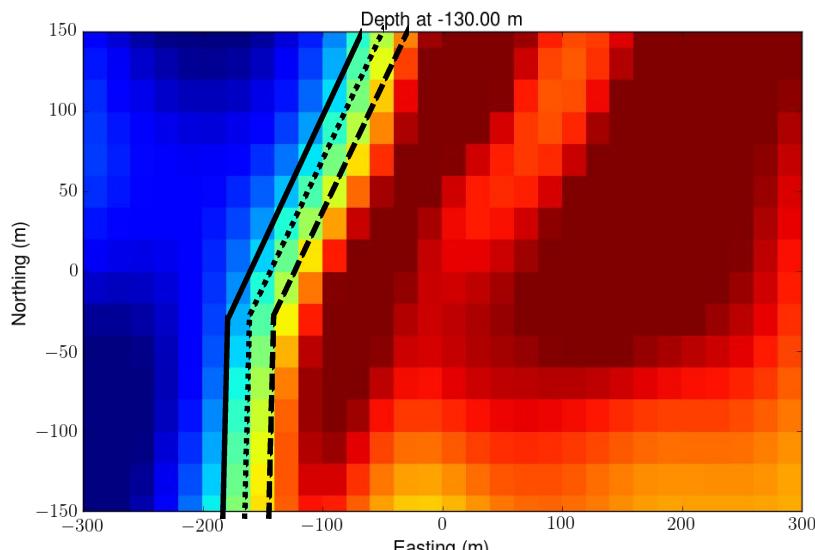
3D inversion:

“Distribution of intruded seawater”



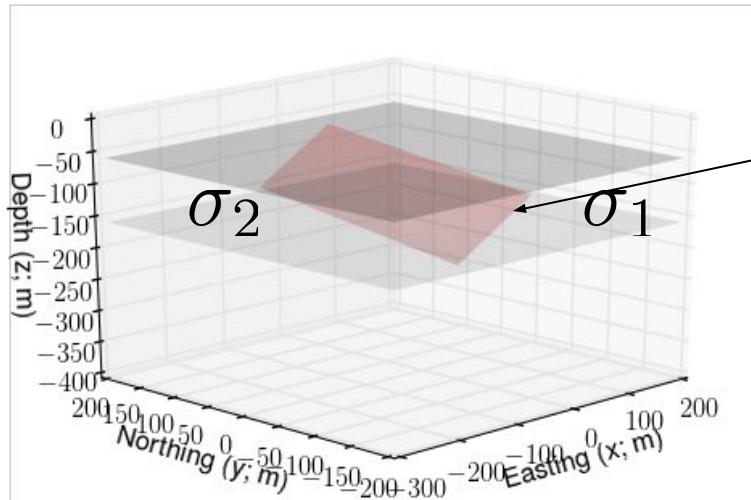
Our question:

“Where is the **boundary** of fresh and sea water?”



Parameterize interface?

- Interface



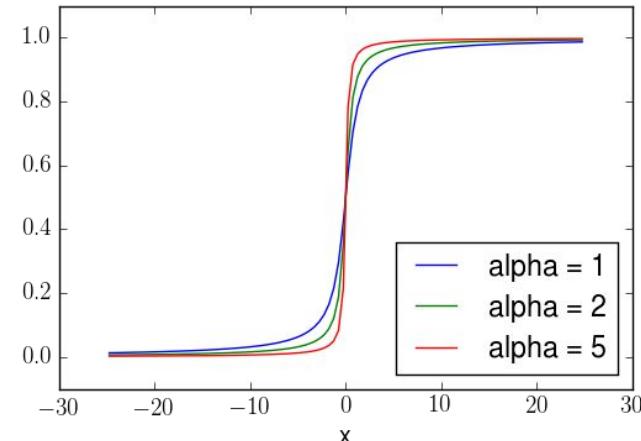
$$f(y, z; c)$$

$$g(x, y, z; c) = f(y, z; c) - x$$

c : Coefficients of arbitrary geometric function

- Analytic Heaviside step function:

$$H(x, z) = \frac{\tan^{-1}(\alpha g(x, y, z))}{\pi} + 0.5$$



Kim et al., 1999
Newman and Commer, 2005

Geometric map

Li et al., 2008

Pidlisecky et al., 2011

McMillan et al., 2015

- Inversion model

m : m_1, m_2 , geometric coefficients (c)

$$m_1 = \log(\sigma_1)$$

$$m_2 = \log(\sigma_2)$$

- Mapping:

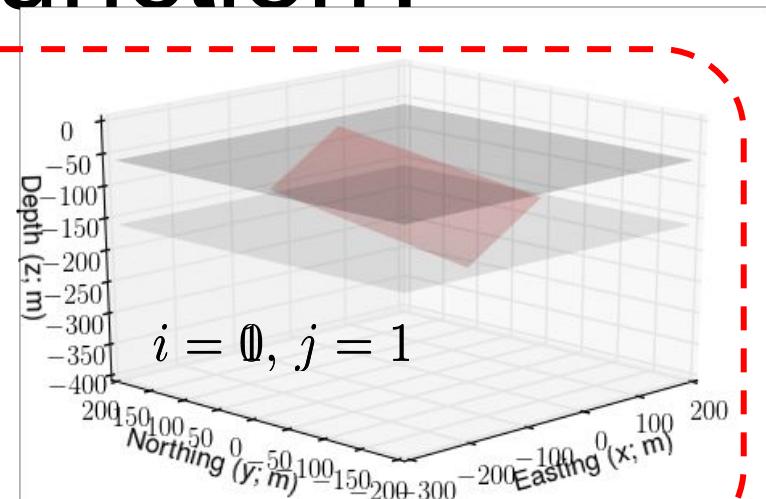
$$\sigma = \mathcal{M}_{geom}^{surj}(m) = m_1 + (m_2 - m_1)H(x, y, z; c)$$

Which geometric function?

- 2D polynomial:

$$f(y, z; c) = c_{ij} y^i z^j$$

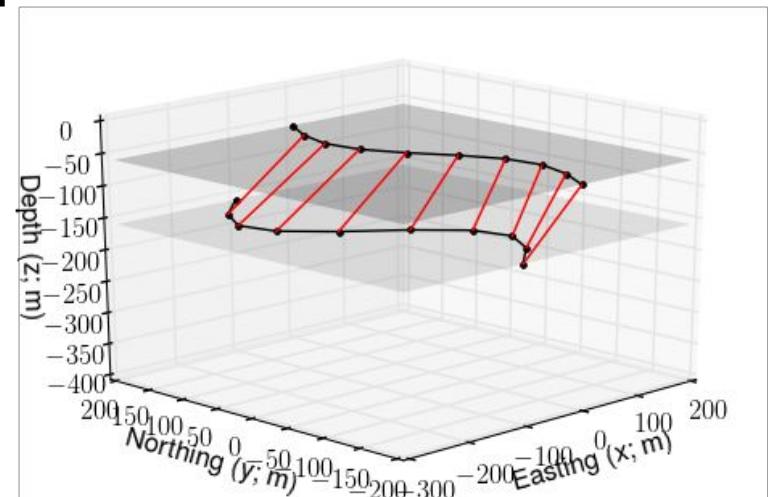
$$\text{order} = (i + 1)(j + 1)$$



- Two linearly interpolated Splines:

$$f_{1D}(y, z = z_{top}; c_1)$$

$$f_{1D}(y, z = z_{bottom}; c_2)$$



Setting up inversion model using: “Geometric map” (2D polynomial)

- Geometric map
 - 2D polynomial: $f(y, z; c) = c_{ij}y^i z^j$
 - 1st order for each y and z directions

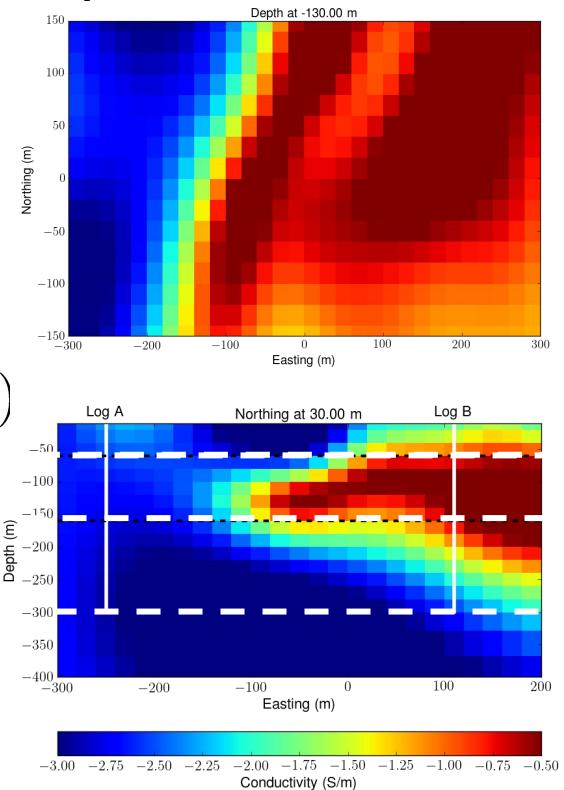
$$\sigma = \mathcal{M}_{geom}^{surj}(m) = \sigma_1 + (\sigma_2 - \sigma_1)H(x, y, z; c)$$

- Active cell map: only aquifer

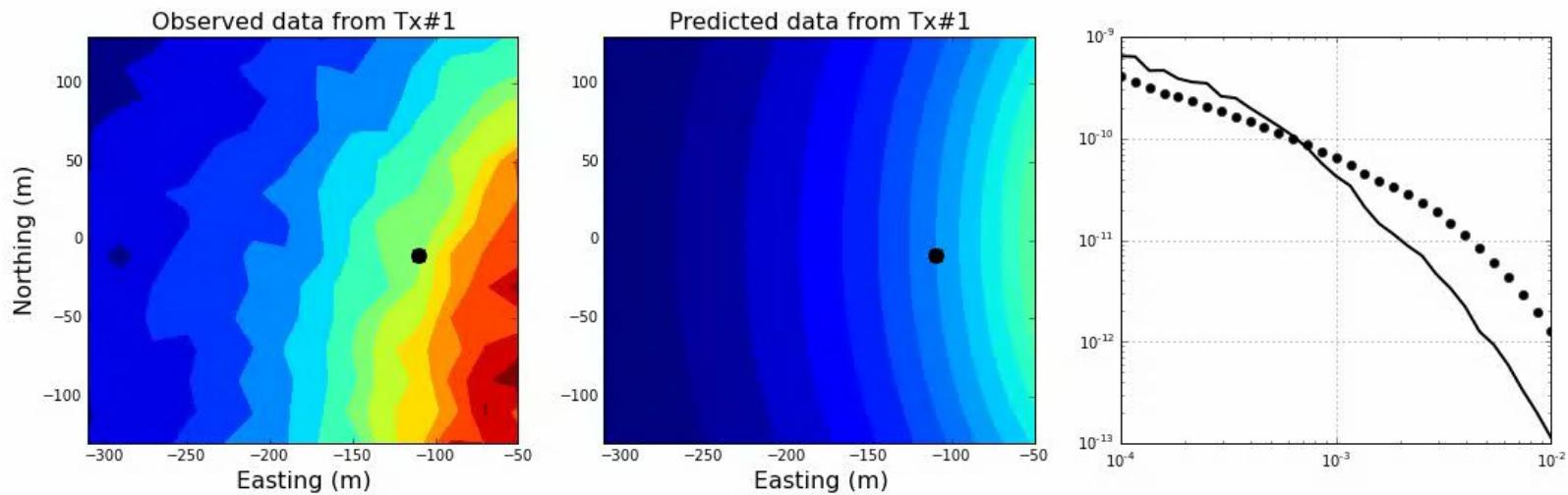
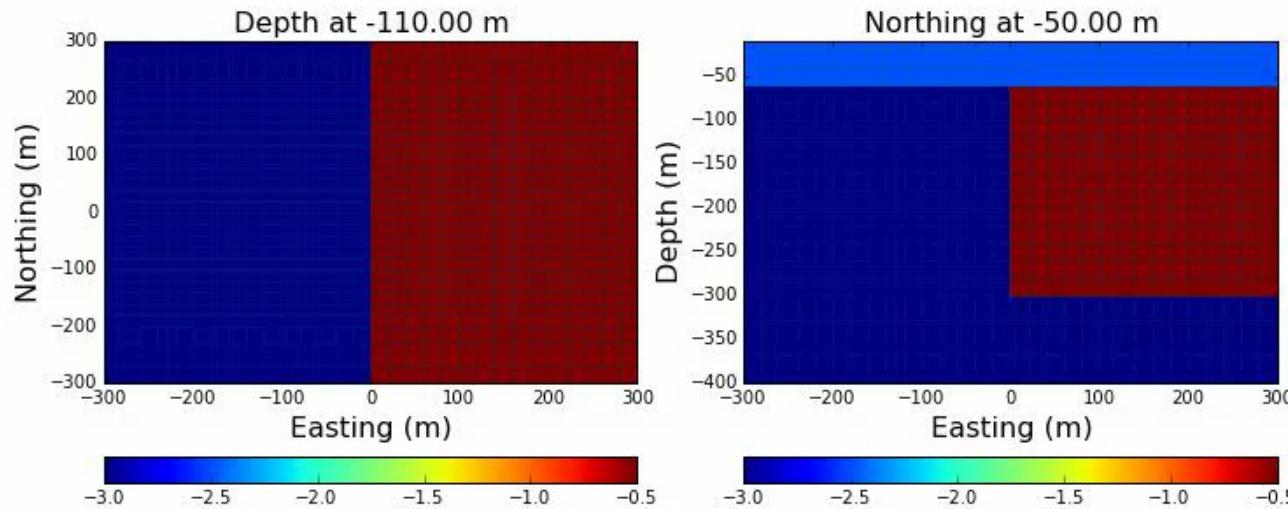
- Exponential map

- Used mapping:

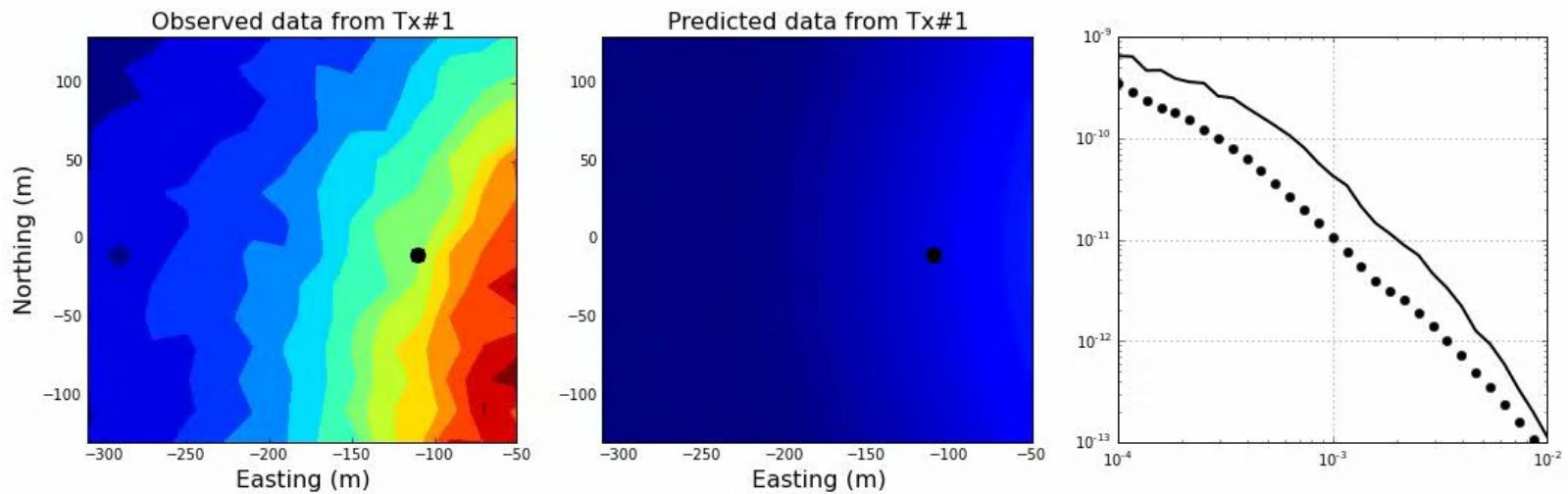
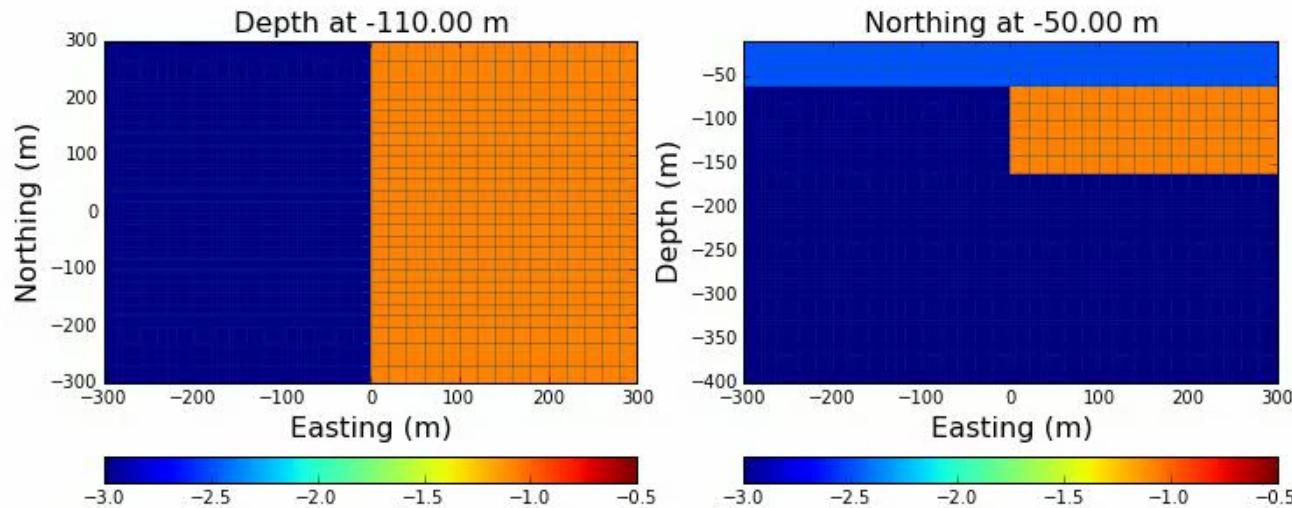
$$\sigma = \mathcal{M}_{act}^{surj}[\mathcal{M}_{geom}^{surj}(m)]$$



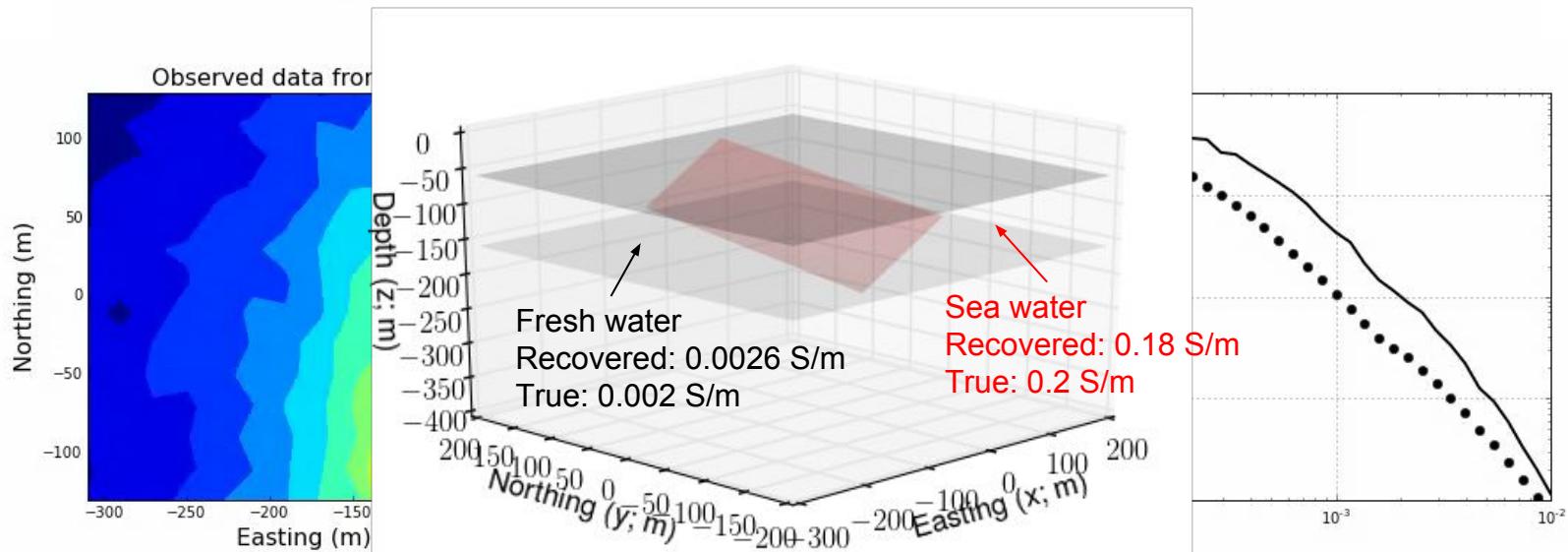
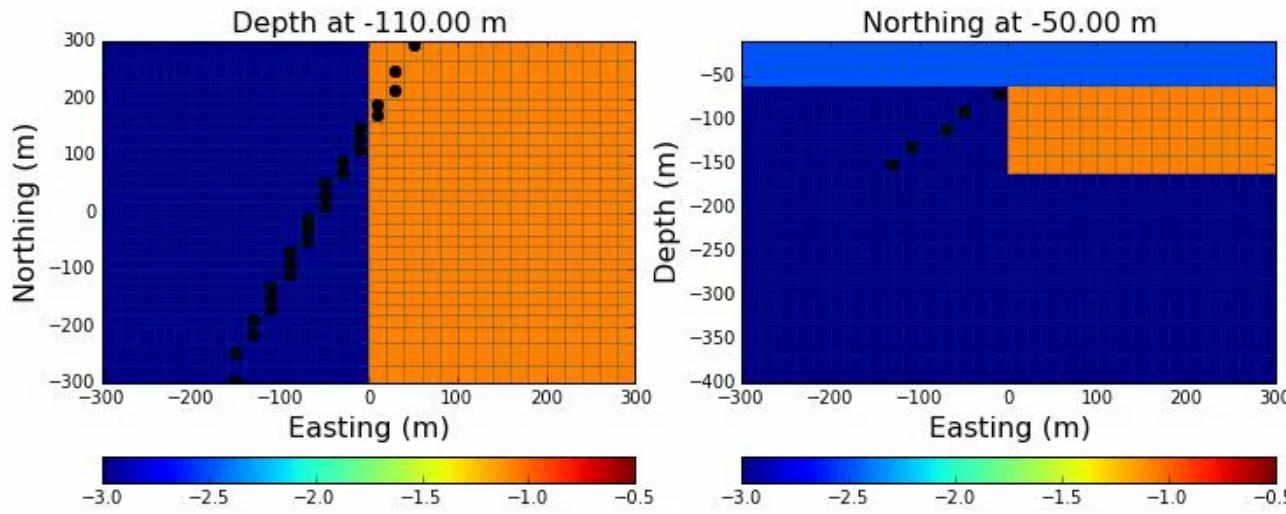
Inverting interface (-300 m)



Inverting interface (-160 m)



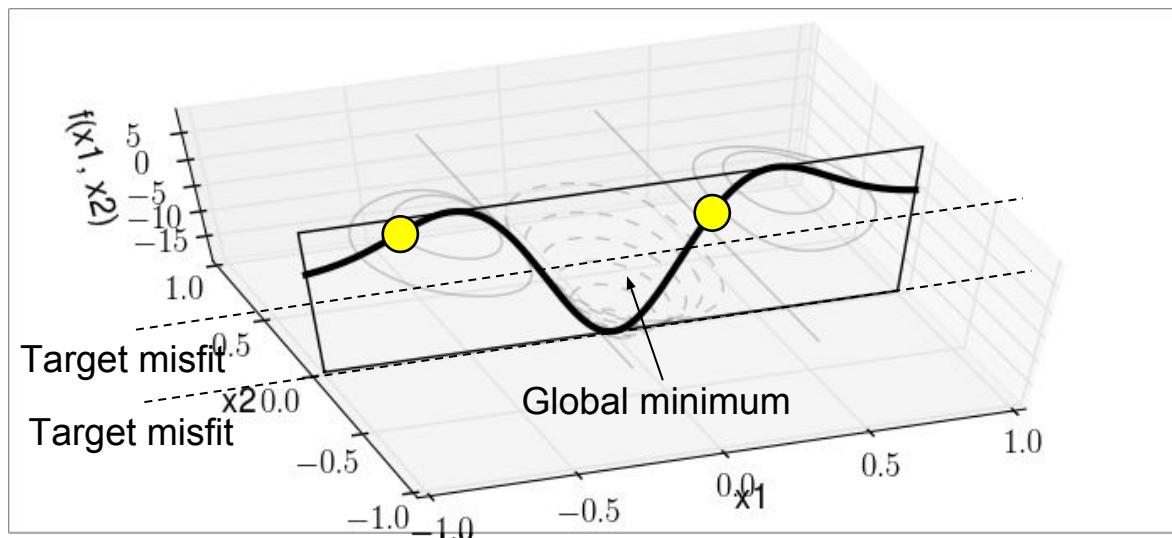
Compare with true boundary



Lowering the dimensionality?

- Is it always good?

	Higher Dimensions	Lower Dimensions
Non-uniqueness	High	Low
Fitting	Easy	Hard
Importance of initial guess (relative)	Low	High



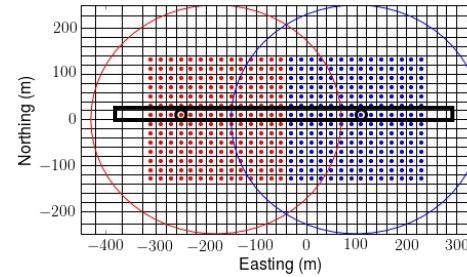
Lowering dimension
Level set: $x_2 = 0$

Summary

- Sub-questions:

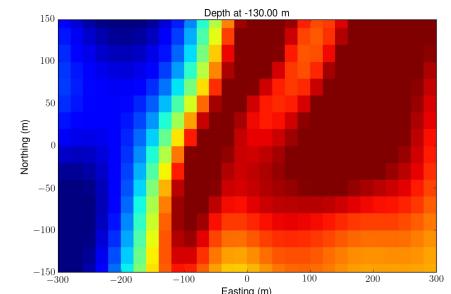
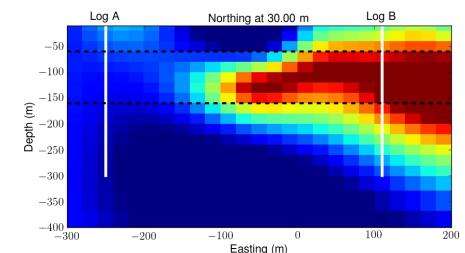
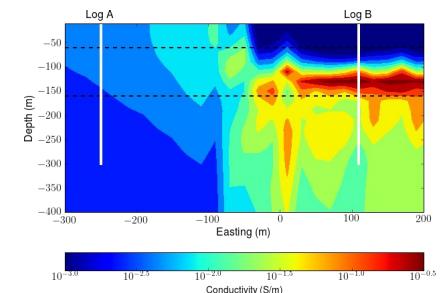
- “Overall layering in this region?”
- “Where is horizontal boundary?”
- “Distribution of the intruded seawater?”

- “Where is the **boundary** of fresh and sea water?”



Fresh water
 Recovered: 0.0026 S/m
 True: 0.002 S/m

Sea water
 Recovered: 0.18 S/m
 True: 0.2 S/m

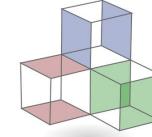


Summary

- Inversion is not a magic box
- We need to ask proper questions using our inversion tools
- Inversion tools need flexibility to handle those questions

Thank you

Acknowledgements

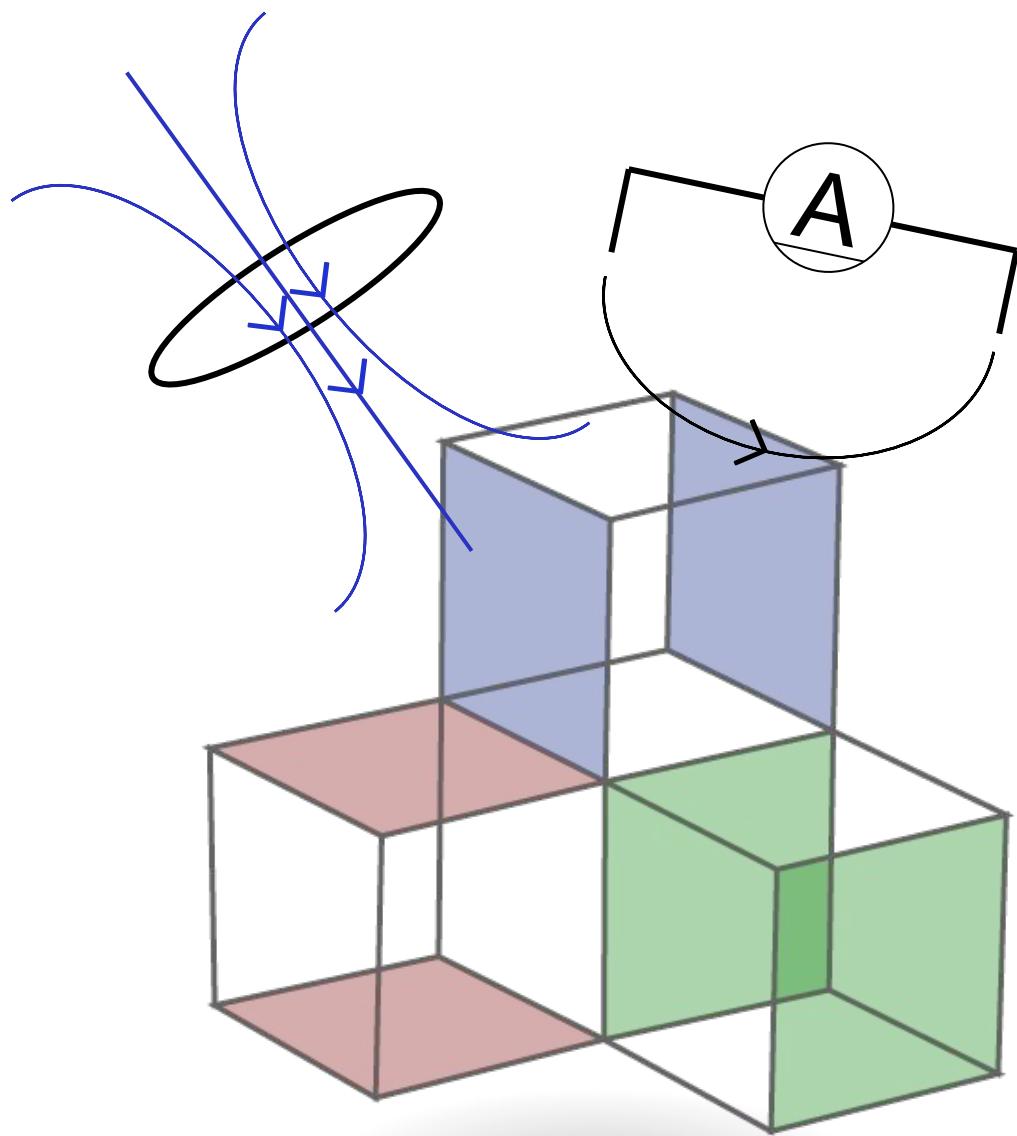


Simulation and Parameter Estimation in
Geophysics

- Thanks to:

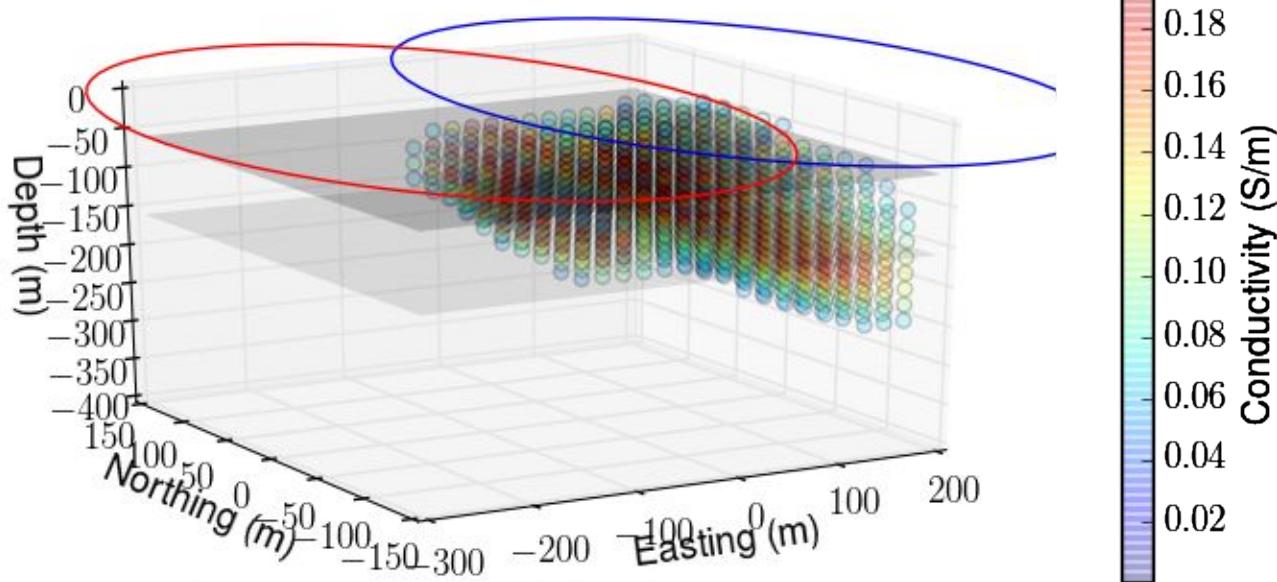
Check out: www.simpeg.xyz

- Other SimPEG developers for developing this geophysical inversion package
- Michael McMillan for constructive discussion on parametric inversion
- Klara for constructive discussion about seawater intrusion problem
- GIFT members, especially Kris Davis for GIFTtools (building seawater model)



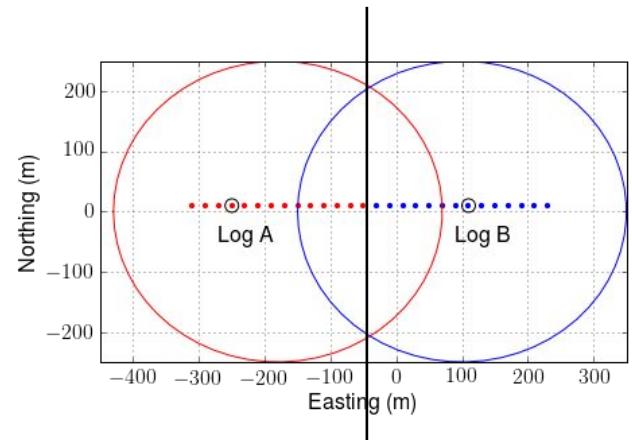
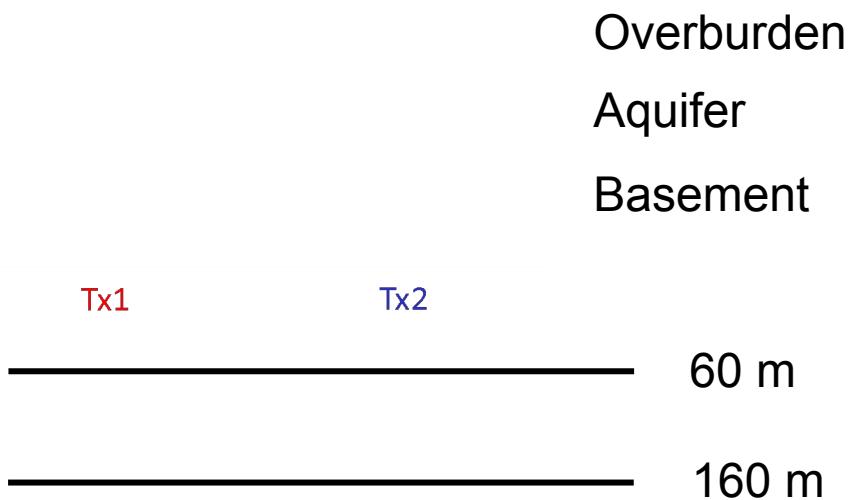
3D distribution of seawater

Cut off > 0.05 S/m



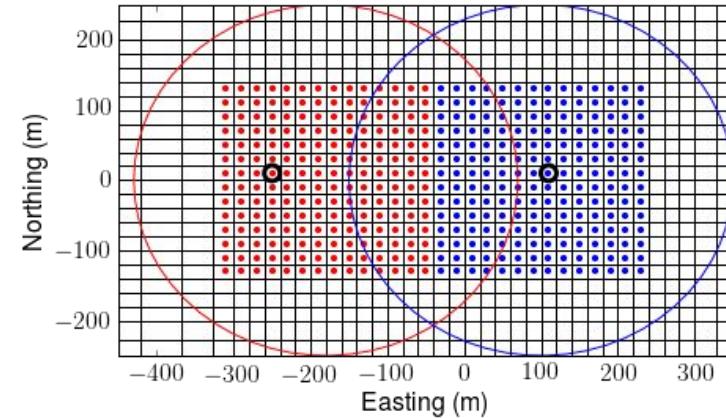
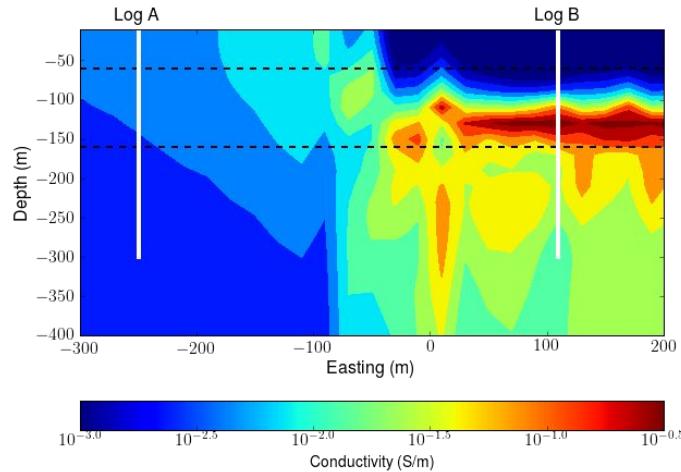
Let's think

“Overall layering in the region?”



Let's think

“Where is horizontal boundary (Easting)?”



“Distribution of intruded seawater?”

