

1. When applying the stencil to the 2D matrix, each point executes 9 floating-point operations (5 multiplications and 4 additions). So for horizontal slabs of size $n \times (n/p)$, the computation of each process is

$$9\gamma \frac{n^2}{p}.$$

In this case, the communication between neighbors is only the adjacent rows between processes, which can be communicated using sendrecv in $O(n)$ time. Since each process except for the first and last has two neighbors, the communication cost can be approximated as

$$2(\alpha + \beta n).$$

Therefore the speedup is

$$\frac{9n^2\gamma}{\frac{n^2}{p}9\gamma + 2\alpha + 2\beta n} = \frac{p}{1 + \frac{2p(\alpha + \beta n)}{9n^2\gamma}}.$$

The efficiency thus simplifies to

$$\frac{1}{1 + \frac{2p\alpha}{9n^2\gamma} + \frac{2p\beta}{9n\gamma}}.$$

For a fixed memory $M = n^2/p$, the efficiency becomes

$$\frac{1}{1 + \frac{2\alpha}{9\gamma M} + \frac{2\sqrt{p}\beta}{9\sqrt{M}\gamma}}.$$

Since

$$E \sim \frac{1}{\sqrt{p}},$$

this approach does not scale weakly.

2. When the domain is divided into patches of size $(n/\sqrt{p}) \times (n/\sqrt{p})$, the computation of each process remains the same, but the communication cost becomes

$$4\left(\alpha + \beta \frac{n}{\sqrt{p}}\right)$$

since each process does a sendrecv with its 4 neighbors.

The speedup is then

$$\frac{9n^2\gamma}{\frac{n^2}{p}9\gamma + 4\alpha + 4\beta\frac{n}{\sqrt{p}}}.$$

The efficiency is

$$\frac{1}{1 + \frac{4p\alpha}{9n^2\gamma} + \frac{4\beta\sqrt{p}}{9n\gamma}}.$$

For the fixed memory $M = n^2/p$, it becomes

$$\frac{1}{1 + \frac{4\alpha}{9M\gamma} + \frac{4\beta}{9\sqrt{M}\gamma}}.$$

Since this is a constant, this approach scales weakly.

Plugging in the given numbers, the time spent on computation of each processor is 9 seconds, and the communication time is $\approx 1.305 \times 10^{-4}$ seconds.