

An Interactive Shader for Natural Diffraction Gratings

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Abstract

In nature, color production is the result of physical interaction of light with a surface's nano-structure. In his pioneering work, Stam developed limited reflection models based on wave optics, capturing the effect of diffraction on very regular surface structures. We propose an adaption of his BRDF model that can handle complex natural gratings. On top of this, we describe a technique for interactively rendering diffraction effects, as a result of physical interaction of light with biological nano-structures such as snake skin. As input data, our method uses discrete height fields of natural gratings acquired by using atomic force microscopy (AFM). Based on Taylor Series approximation, we leverages precomputation to achieve interactive rendering performance (about 5-15 fps). We demonstrate results of our approach using surface nano-structures of different snake species applied on a measured snake geometry. Lastly, we evaluate the quality of our method by a comparison of the maxima for peak viewing angles using the data produced by our method against the maxima of the grating equation.

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Chapter 1

Results

In this chapter we examine the rendered output results of our implementation of our BRDF models applied to different input patches such as Blaze grating or Elaphe ?? and Xenopeltis ?? snake nano-scaled surface sheds. We are discussing and comparing both, their BRDF maps 1.1 and the corresponding renderings on a snake geometry like shown in section 1.2 for various input parameters. Last we also show a real experimental image showing the effect of diffraction for similar parameters like we have.

1.1 BRDF maps

A BRDF map shows a shader's output for all possible viewing directions for a given, fixed, incident light direction. We assume that each viewing direction is expressed in spherical coordinates (See appendix ??) (θ_v, ϕ_v) and is represented in the map at point

$$(x, y) = (\sin(\theta_v)\cos(\phi_v), \sin(\theta_v)\sin(\phi_v)) \quad (1.1)$$

with its origin at the map-center. The light direction for normal incidence (θ_i, ϕ_i) has been fixed to $(0, 0)$ for our rendered results.

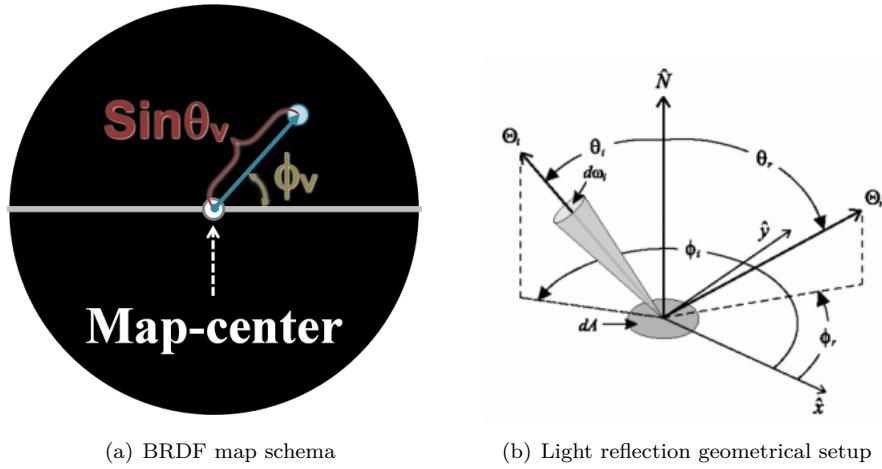


Figure 1.1: BRDF maps¹ for different patches: $\Theta = (\theta_i, \phi_i)$ is the direction of light propagation

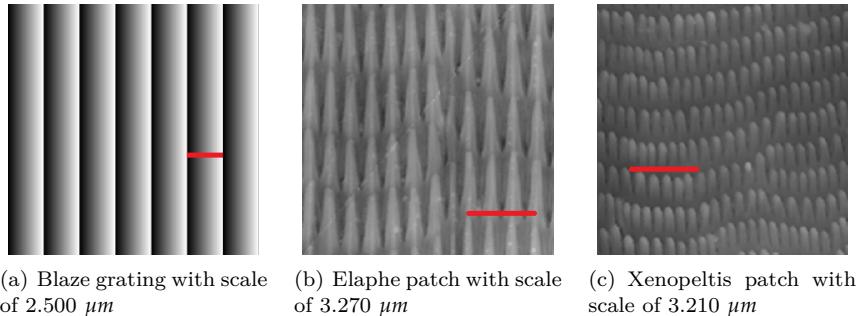


Figure 1.2: Cutouts of our nano-scaled surface gratings used for rendering within our shader with a scale indicator (red line) for each patch. Note that for rendering, we use larger patches.

Figure 1.3 shows the BRDF maps of the full lambda space sampling approach (**FLSS** like introduced in section ??) as described in section ?? applied on different nanoscale surface gratings as shown in figure 1.2. In Subfigure 1.3(a) we see the BRDF map for the Blazed grating, showing high relative brightness for its first order diffraction, i.e. for the Blazed gratings most of the diffracted spectral energy lies in its first order. Notice that the surface of blazed grating is forming a step structure for which the angle between the step normal and the grating normal is denoted by *blaze angle*. Every blazed grating is manufactured in the Littrow² configuration. This means that the blaze angle is chosen such that the diffraction angle and incidence angle are identical. Thus it a blazed grating has maximize efficiency for the wavelength of the used light. Higher diffraction modes are still perceivable (second and higher diffraction orders) but with a much lower relative

¹image source of figure:

- 1.1(a): Taken from D.S.Dhillon's Paper [DSD14b]
- 1.1(b): Taken from <http://math.nist.gov/~FHunt/appearance/brdf.html>

²For further information please see http://en.wikipedia.org/wiki/Blazed_grating.

brightness. The asymmetry of the pattern is due to the asymmetric geometry of the grating 1.2(a).

The finger-like structures contained in the Elaphe surface grating 1.2(b) are quite regularly aligned and hence diffraction occurs along the horizontal axis for the BRDF map as shown in figure 1.3(b). The reason for not seeing any strong diffraction color contribution along other directions in the BRDF map is due to the fact that these ‘nano-fingers’ overlap across layers and thus do not exhibit any well-formed periodicity along finger direction.

For Xenopeltis surface grating 1.2(c), we observe diffraction along many different, almost vertical directions in the BRDF map 1.3(c) since the layers of the finger-like structures do not overlap and are shifted significantly along their length but still exhibit some local consistency. A similar argument holds true for diffraction across locally periodic finger patches with slightly different orientations.

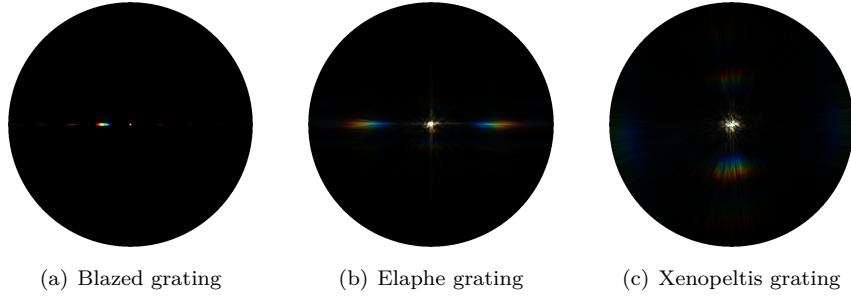


Figure 1.3: BRDF maps for different patches

Figure 1.4 shows BRDF maps of all our BRDF models applied on the Blaze grating. Figure 1.4(a) shows the FLSS shading approach result for our blazed grating and it is used in order to compare with our other rendering approaches.

Figure 1.4(b) shows the BRDF map for the NMM approach, introduces in section ??, which is close to the FLSS approach, verified in section ??, just like in the case of corresponding evaluation in figure ???. Nevertheless there is a small, noticeable difference: For the NMM approach we see a white, circular spot around the map center. Nevertheless, apart from this white spot, the NMM approach resembles the FLSS approach. The reason for this differences is due to the fact that the NMM approach treats the center of a BRDF map as a special case, like described in section ???. Technically, every location around a small ϵ -circumference from the map center gets white color assigned.

Figure 1.4(c) shows the BRDF map for the PQ approach which relies on sinc-interpolation. The PQ BRDF map and the FLSS results are visual alike. Compared to the evaluation plots in figure ???, the BRDF maps even persuade more. Compared to FLSS, one difference we notice is that the first order of diffraction is a little spread for the PQ approach. Without³ applying a sinc-interpolation, this spreading effect would be even strengthened.

Last, let us consider figure 1.4(d) which shows the BRDF map produced by using Nvidia Gem’s implementation [JG04] of Stam’s BRDF model when constraining the y-axis of the BRDF map.

³Note that if we do not perform a sinc-interpolation this would correspond to apply a linear interpolation instead.

This corresponds to a 1d diffraction grating, along the x-axis. This model only uses the spacing d of a given grating. It also always produces highly symmetric results.

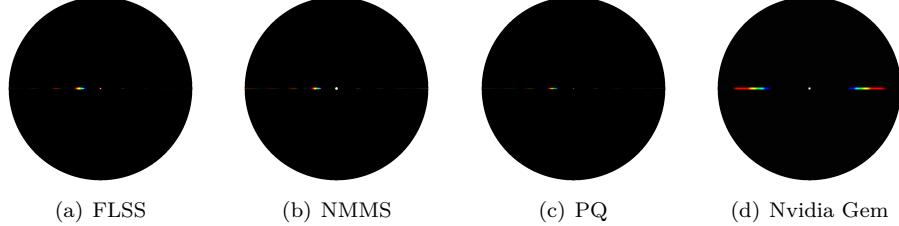


Figure 1.4: BRDF maps for Blazed grating comparing our different rendering approaches

Figure 1.5 and figure 1.6 show the BRDF maps for different wavelength step sizes used in the fragment shader for the FLSS approach applied on the blazed grating and the Elpape snake shed, respectively. Within our fragment shaders the outermost loop iterates over the range [380nm, 780nm] for a given step size λ_{step} to integrate over the wavelength spectrum. Having bigger step sizes implies having fewer λ -samples which will reduce the overall runtime of a shader but, it will also introduce artifacts and therefore lower the overall shading quality. For Elpape surface grating, artifacts are perceivable when $\lambda_{step} \leq 10\text{nm}$. Results produced by using 5nm step sizes do not differ from those produced by using $\lambda_{step} = 1\text{nm}$. This allows us to set λ_{step} at 5nm. For a Blazed grating we may choose even bigger step sizes without losing any rendering quality (see figure 1.5).

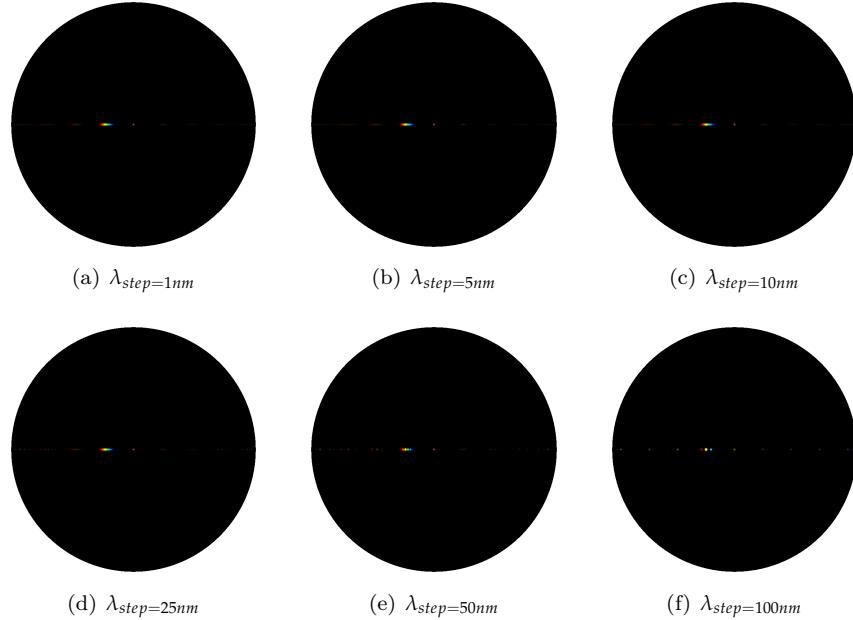


Figure 1.5: Blazed grating at $2.5\mu\text{m}$: Different λ step sizes

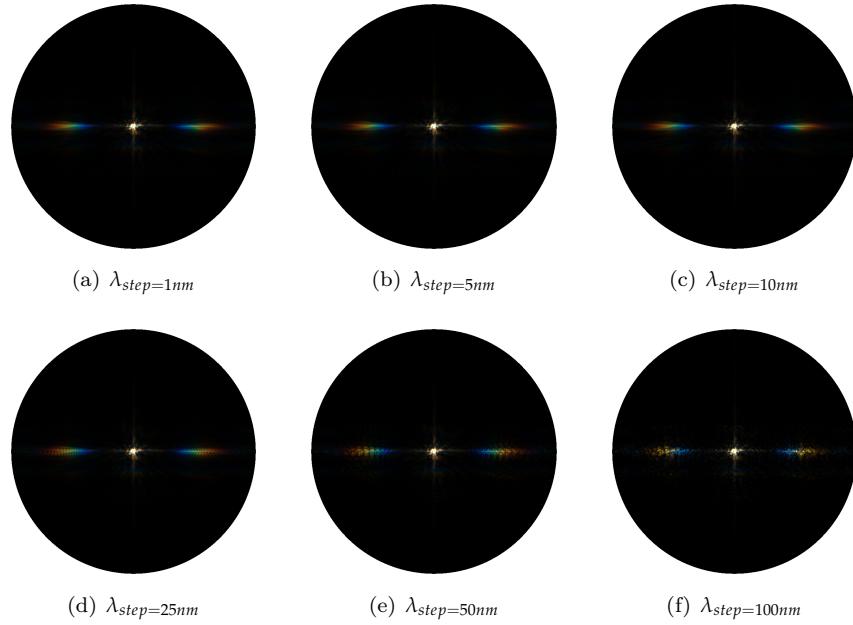


Figure 1.6: Elaphe grating at $65\mu m$: Different λ step sizes

The figures 1.7, 1.8, 1.9 show a comparison of the BRDF maps produced by the FLSS approach (on the left) and the PQ shading approach (on the right) applied on all our patches. For Blazed grating, as already mentioned, we notice that both approaches, FLSS and PQ, resemble each other. We also notice that for PQ map, the first order diffraction color contribution is spread. For the Elaphe and Xenopeltis grating we notice similar shaped BRDF patterns, even when the angle of light varies, but nevertheless, they also contain some artifacts.

In general, a Blazed Grating is manufactured in a way that a large fraction of the incident light is diffracted preferentially into the first order. Therefore, most of the energy in its BRDF map lies in the first order of diffraction at its blaze angle. This implies that largest portion of the color contribution, visible on the corresponding BRDF map, lies at that angle. In figure 1.7, in contrast to the results produced by the FLSS approach, we see color fringes at the first order modes in the BRDF map produced by our PQ approach. This implies that the PQ approach does not produce reliable results which also affirms our evaluation plots shown in figure ???. A similar argumentation hold true for the PQ approach, when we do not apply we a sinc-interpolation like shown in figure 1.7(c).

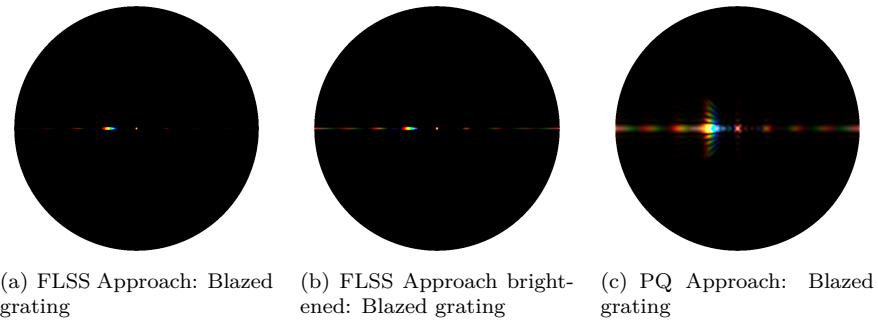


Figure 1.7: A comparison between the PQ- and the FLSS approach applied on an Blazed grating.

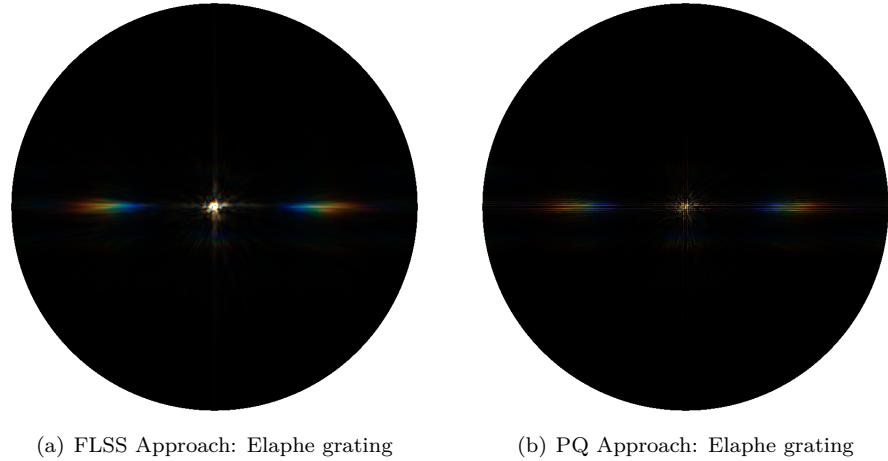


Figure 1.8: A comparison between the PQ- and the FLSS approach applied on an Elaphe grating.

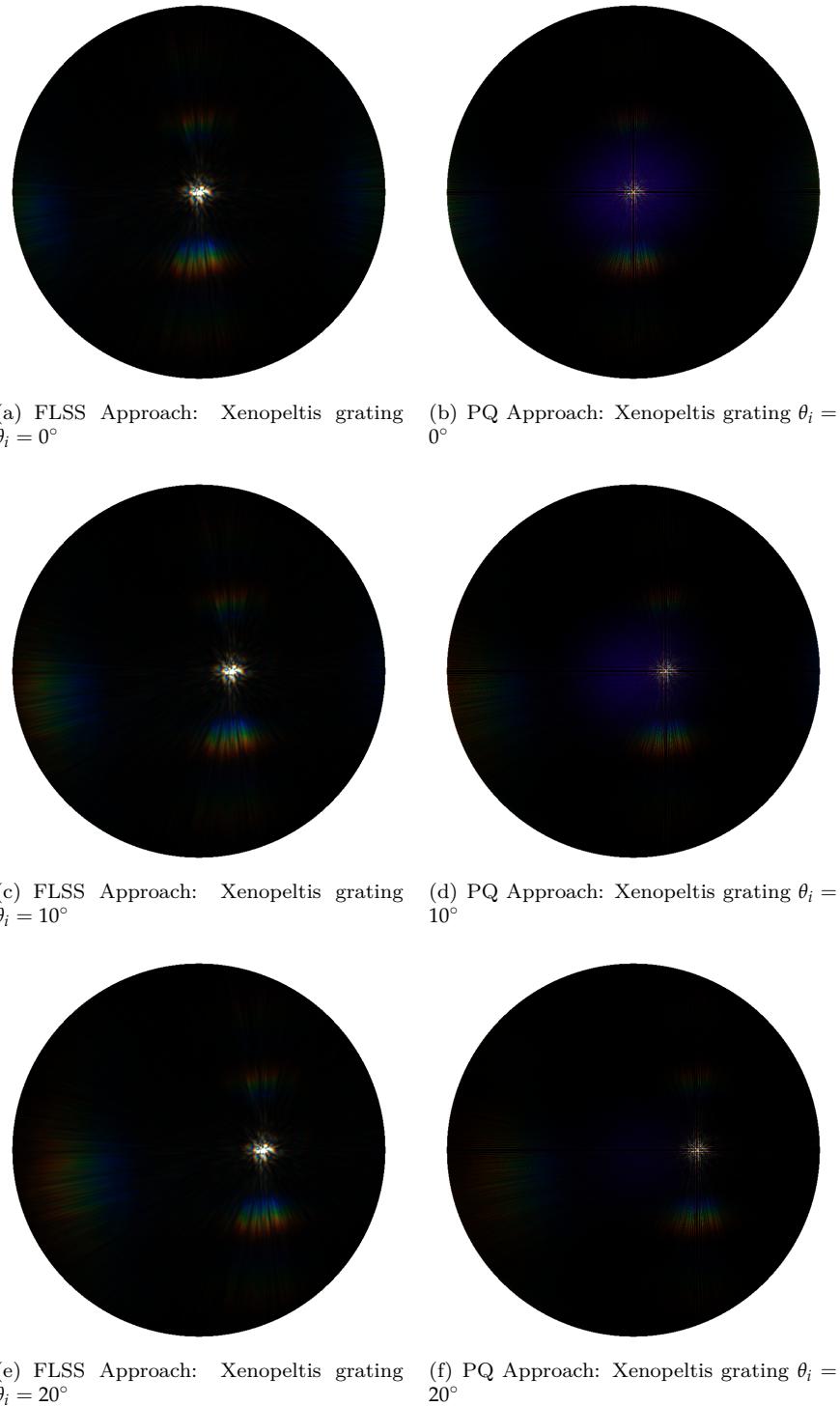


Figure 1.9: A comparison between the PQ- and the FLSS approach applied on an Xenopeltis grating.

Figure 1.10 shows BRDF maps for the full lambda sampling approach applied to the Blazed grating, while varying the value for the spatial variance σ_s . This akin to changing the coherence length for the incident light. The lower the coherence length, the fewer interacting grating periods produce blurred diffraction bands for different λ which overlap to produce poorly resolved colors.

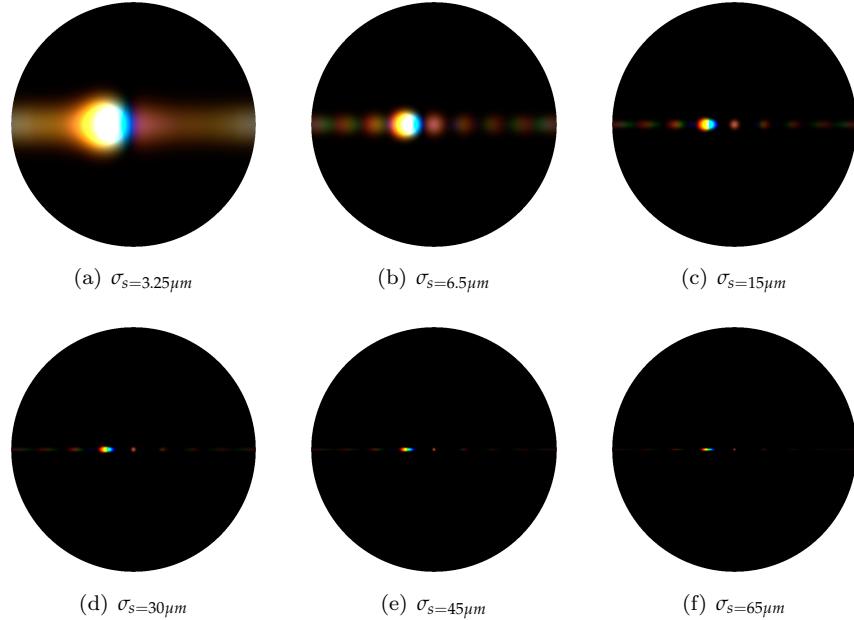


Figure 1.10: Blazed grating with periodicity of $2.5\mu m$: Different σ_s

Figures 1.11 and 1.12 show the BRDF maps the reference-, FLSS approach using different values for N in the taylor series approximation. For both input patches we clearly visually observe the convergence of the taylor series for higher values of N . We visually observe convergence of the Taylor series for all our patches a very large value of N^4 .

Like discussed in section ?? there exists a certain value of N for which our approach converges. For all our shading approaches, applied on our gratings, we visually observed a convergence of their BRDF maps when using $N \geq 39$ DFT terms. Furthermore, for a Blazed grating it satisfies to use only $N \geq 7$ - and for an Elaphe grating only $N \geq 9$ DFT terms. Notice, that these numbers of required DFT terms were empirically determined by trial and error strategy.

However, by making use of taylor error term estimates, like introduced in the appendix section ??, we can derive an upper bound for N . Since this computation is dependent on many aspects, such as on the grating spacing, the pixel-width correspondance, the used lambda space for sampling, it is usually simpler to determine empirically actual values for N .

In algorithm ?? we compute the DFT terms of a provided height field h raised to the power of the imaginary number i times an integer, i.e. we evaluate the expression $DFT(h)^n \cdot i^n$. Since we multiply our height field by i^n and then apply the DFT operator, basically, there exist four

⁴Using N equal to 39 lead to visual convergence for all our used gratings.

possible convergence images, each having its own convergence radius.

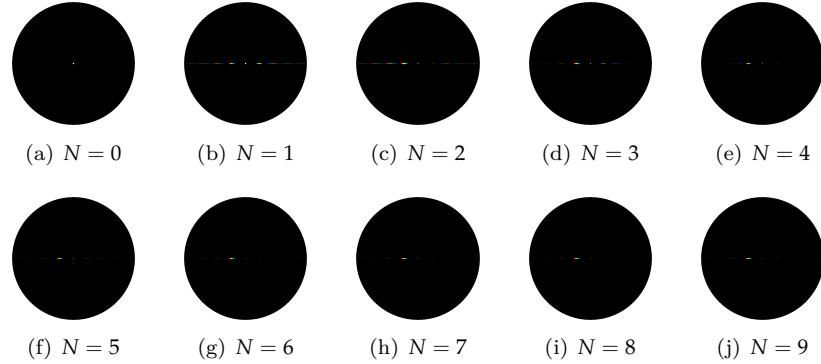


Figure 1.11: Blazed grating at $2.5\mu\text{m}$: N Taylor Iterations

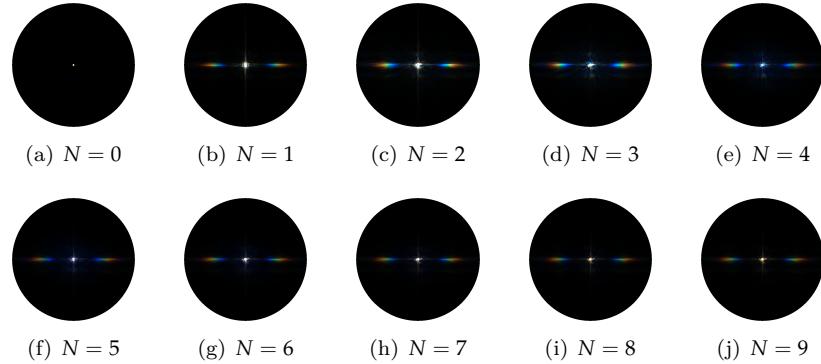


Figure 1.12: Elaphe grating at $65\mu\text{m}$: N Taylor Iterations

Figure 1.13 shows the BRDF maps of the full lambda shading approach applied on the Xenopeltis snake shed, using different θ_i incident angles. When slightly moving the incident angle θ_i , we can observe how the brdf map changes. For higher values of θ_i we start seeing diffraction color contribution on the right side of the BRDF map.

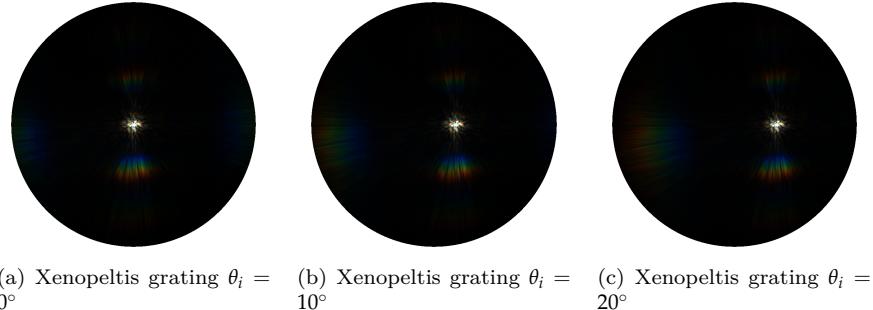


Figure 1.13: BRDF maps for Xenopeltis grating: different θ_i angles

1.2 Rendering Surface Geometries

In this section we are going to present our actual renderings simulating the effect of diffraction caused when a directional light source encounters different nano-scaled surfaces on a given curved snake mesh. We will see that diffraction colors change dramatically with changes in light direction, surface normals and viewing direction, which is typical for diffraction colors observed in nature. For rendering we are going to rely on our full wavelength space sampling approach. Unfortunately, this approach is rather slow and can barely be considered as being interactive performing. Nevertheless, we have introduced some optimizations in order to become interactive in rendering, such as the N_{min}, N_{max} approach, we are going to use this slow approach since this resembles the ground truth and therefore is the most accurate among all our presented approaches. As mentioned we are going to render diffraction on a given snake mesh. Note that we actually just have one particular mesh, for all our renderings we are going to use the same snake mesh which has been produced by 3d scanning an Elaphe snake species, consisting of 11696 vertices and 22950 faces. The reason for that is that it was hard to get a Xenopeltis snake ready for being 3d scanned. In addition, the micro-geometry is highly similar among snake species, it is the geometry of the nano-structures that are highly different among species and that cause the snake to be or not be iridescent. So, even Xenopeltis would not give you very different geometry than Elaphe. Table 1.1 lists the system specifications of the machine I used in order to produce the rendered images.

Processor	Intel i7 CPU 970 @ 3.20 GHz (12 CPUs)
Memory	12288 MB RAM
Graphics Card	GeForce GTX 770

Table 1.1: Hardware specifications of the machine which produced rendered results. Statistics are provided using the tool NVIDIA Geforce Experience.

Figure 1.14 shows renderings produced by the full lambda sampling approach applied on a snake shaped mesh for different given input patches. Due to the fact that a Blazed grating has its maximum intensity for a certain direction and the geometry of the snake mesh is curved which means non-flat, we can expect rather less diffraction color contribution like shown in figure 1.14(b). Differently for our other two gratings, Elaphe and Xenopeltis. For both renderings, we can see color contribution despite the effect of diffraction whereas we see much less colorful patterns for Elaphe 1.14(b) than for Xenopeltis 1.14(c). This also corresponds to the reality, considering the figure ?? as a reference. The nano-scaled surface structure of the species Elaphe as shown in figure

1.2(b) does not look that regular under the electron scanning microscope. This is why it is much less iridescent than the other specie. Xeno has a brownish body with no pattern that makes the iridescence more spectacular than on Ellaphe.

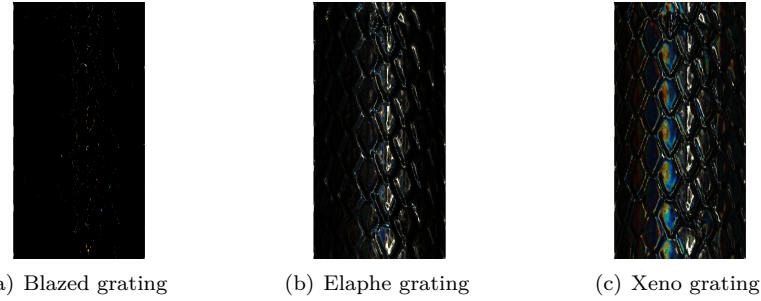


Figure 1.14: Diffraction of different snake skin gratings rendered on a snake geometry

Figure 1.15 contains a summary-collection of subfigures for rendering the effect of diffraction produced by the full lambda shading approach with all its required components, applied on our snake mesh, using the Elaphe nano-scaled surface structure. Subfigure 1.15(b) shows the final diffraction color-contribution result with texture-blending. We only see little diffraction color contribution in this subfigure which resembles quite well to the reality as shown in figure ???. In subfigure 1.15(d) we see the light cone in order to show the direction of the light source besides the rendered results. Subfigure 1.15(e) is a sample Fourier image of Elpahe's nanosclae surface structure 1.15(d).

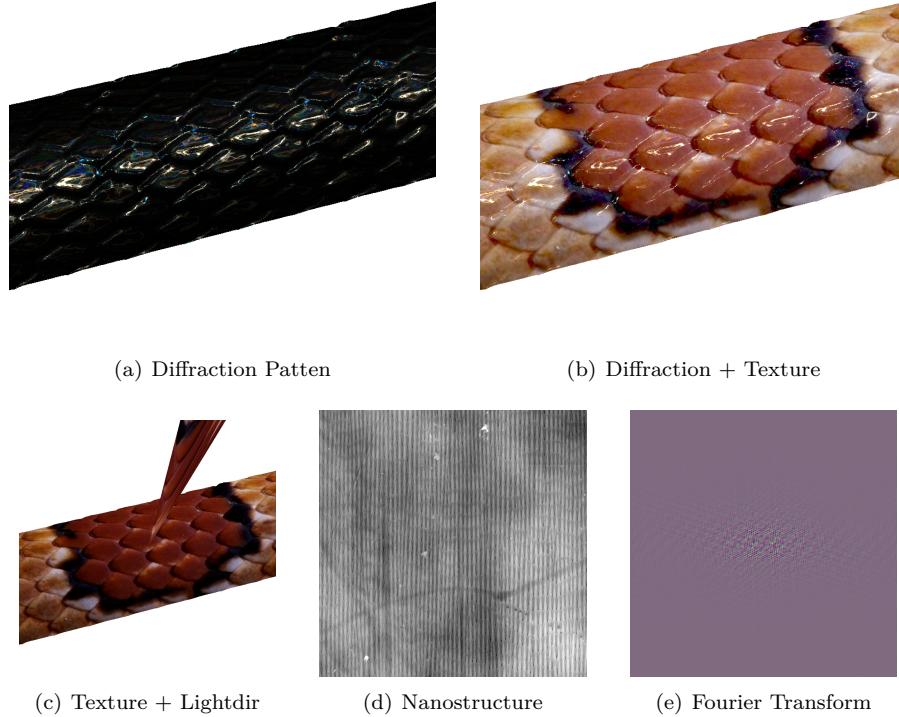


Figure 1.15: Diffraction for Elaphe snake skin

Like in the previous figure this figure 1.16 also shows a summary-collection of subfigures for the effect of diffraction with all its involved components but this time for the Xenopeltis snake surface. For texture blending we use the same texture like we used for Elaphe. For Xenopeltis see quite a lot color contribution due the phenomenon of diffraction like shown in figure 1.16(b). Comparing this to a real image ?? we notice much resemblance regarding the reflectance strength and colorful pattern.

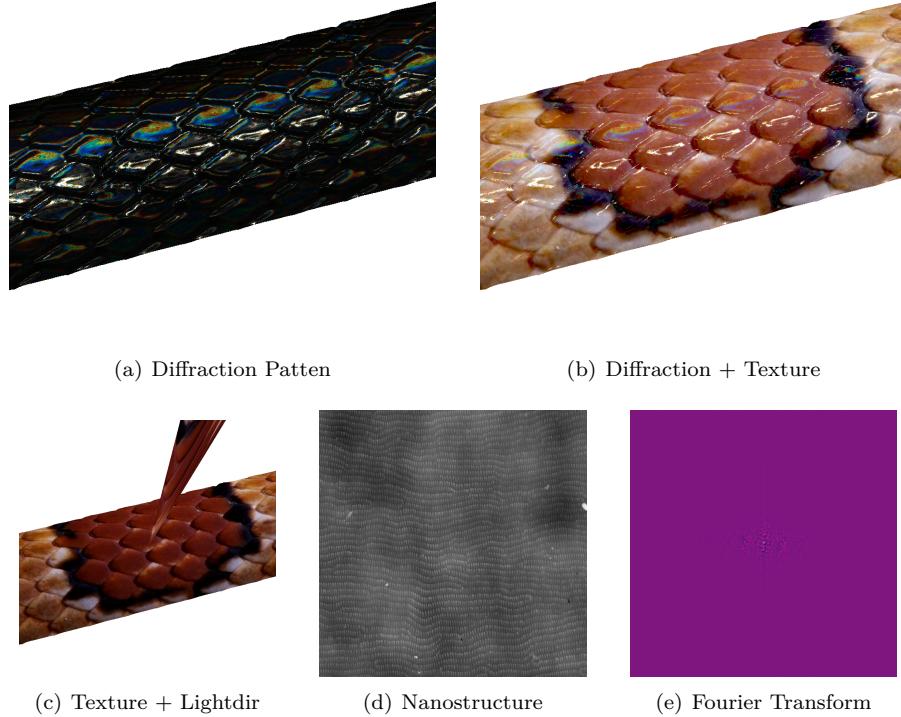


Figure 1.16: Diffraction for Xeno snake skin

Figure 1.17 shows the diffraction pattern of a Elaphe snake shed for different zoom levels for fixed incident light and viewing direction using the full lambda sampling approach. From those different close up perspectives it would appear the complexity of the colorful diffraction pattern.

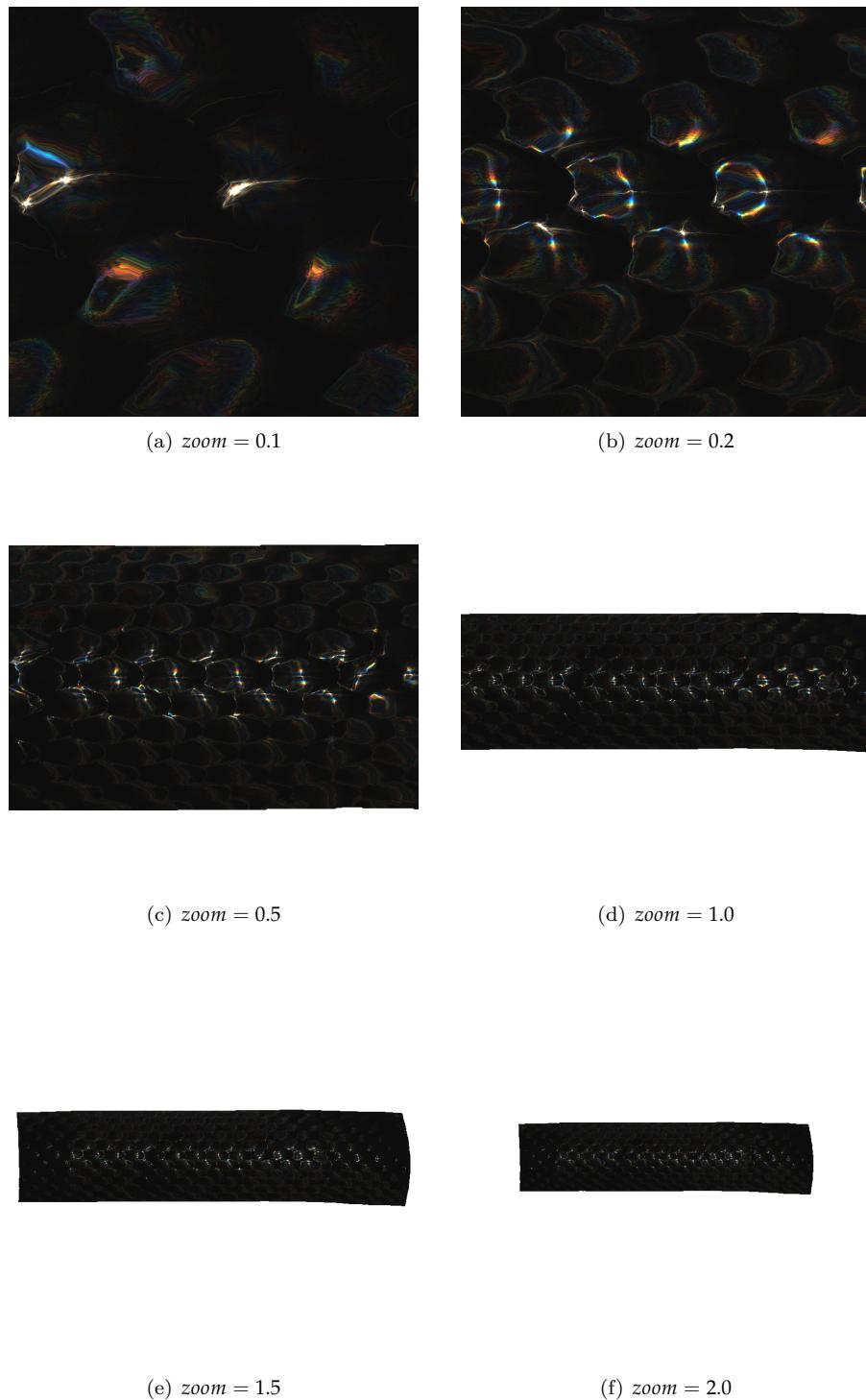


Figure 1.17: Diffraction on Elaphe snake skin grating: Different camera zoom levels

Figure 1.18 shows how the diffraction pattern changes when slightly moving the incident light direction. Which gives us an impression what kind of complex, perspective-dependent pattern the phenomenon of diffraction may cause.

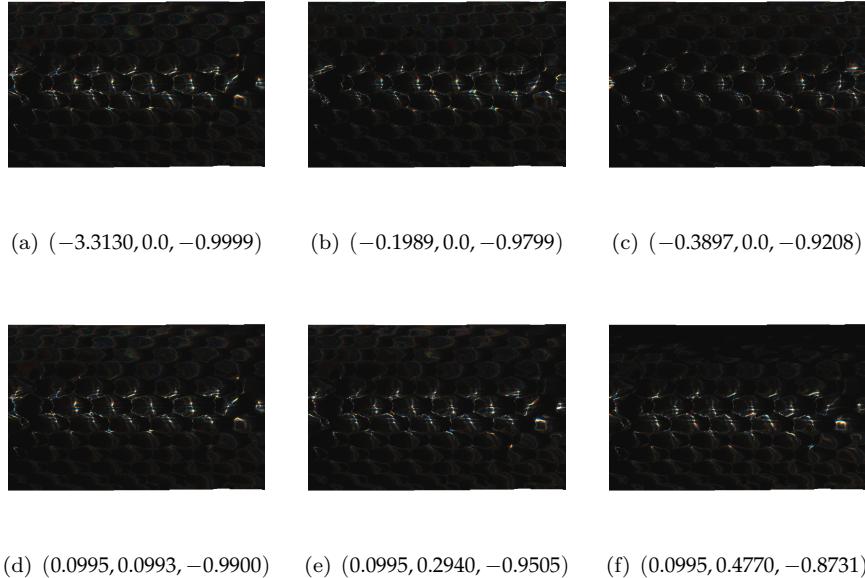


Figure 1.18: Diffraction on Elaphe snake skin grating: Different light directions

Figure 1.19 shows a photo of an experimental setup for demonstrating the effect of diffraction using a Elaphe snake grating. The exact parameters for the experimental setup are unknown. Nevertheless this image gives us an impression of how close our model is to the reality comparing it with our simulated results since we notice similar diffraction patterns for our simulated results using an Elaphe snake shed.

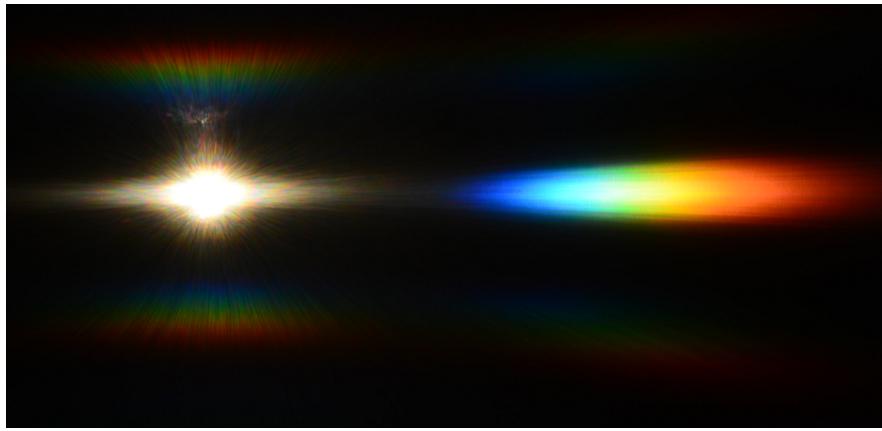


Figure 1.19: Diffraction Elaphe: experimental setup

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List of Algorithms

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Erklärung

gemäss Art. 28 Abs. 2 RSL 05

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Master

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