Problem-Sheet 8

My Solution

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Task1

Given five persons **Herakles**, **Aeson**, **Kerberos**, **Menelaos** and **Apollo** (supposed to be friends) and five islands **Crete**, **Euboea**, **Lesbos**, **Rhodes**, **Chios**.

We want to assign the five biggest Greek islands among five given friends. Each person has individual intentions about island he wants to possess:

Person	**Wants to possess**
Herakles	Crete and Rhodes
Aeson	Lesbos, Rhodes or Chios
Kerberos	Crete and Chios
Menelaos	Euboea or Chios
Apollo	Euboea and Lesbos

Assumption: *Each Island can have exactly ONE owner*

When considering the „wants to possess“ list for each person we observe that some want to possess a set of Islands having a AND relationship, and some a OR-relationship. Thus, in order to solve this task we have to define a fairness term, which tells us how we are going to solve this problem.

So the question is: Is it better to give each person just ONE island or is it better to some persons more than one island (which implies that some will not get any island then)?

I am mentioning this, since some are happy to have either „this island **OR** that island“ whereas some other want to possess „island1, island2 **AND** island3“.

In the following I am going to assume that it is best to statisfy all person to give everyone **at least one** island. Since there are as many islands as there are persons, this implies every person gets exactly one particular island.

Thus we are going to formulate this problem as a **maximum flow problem** where ***each person should get exactly one island fulfilling as much as possible their wishes*** (from their „wants to posses“ wishlist).

Solution:

1. step: Formulate the person-wants-to-possess relationship graph (as a bipartit graph)

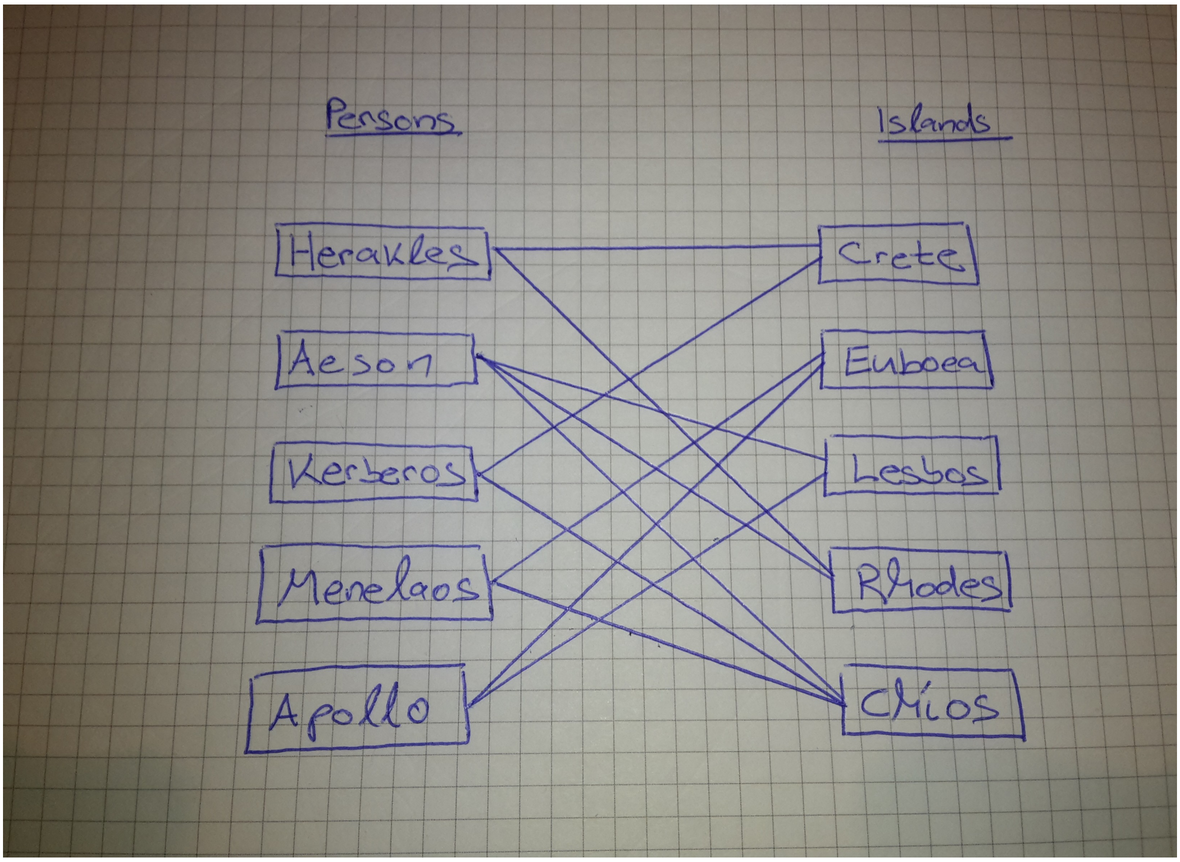

Figure 1.1: Bipartit Graph showing relationship between person and their interests.

2. translate this graph into a flow graph

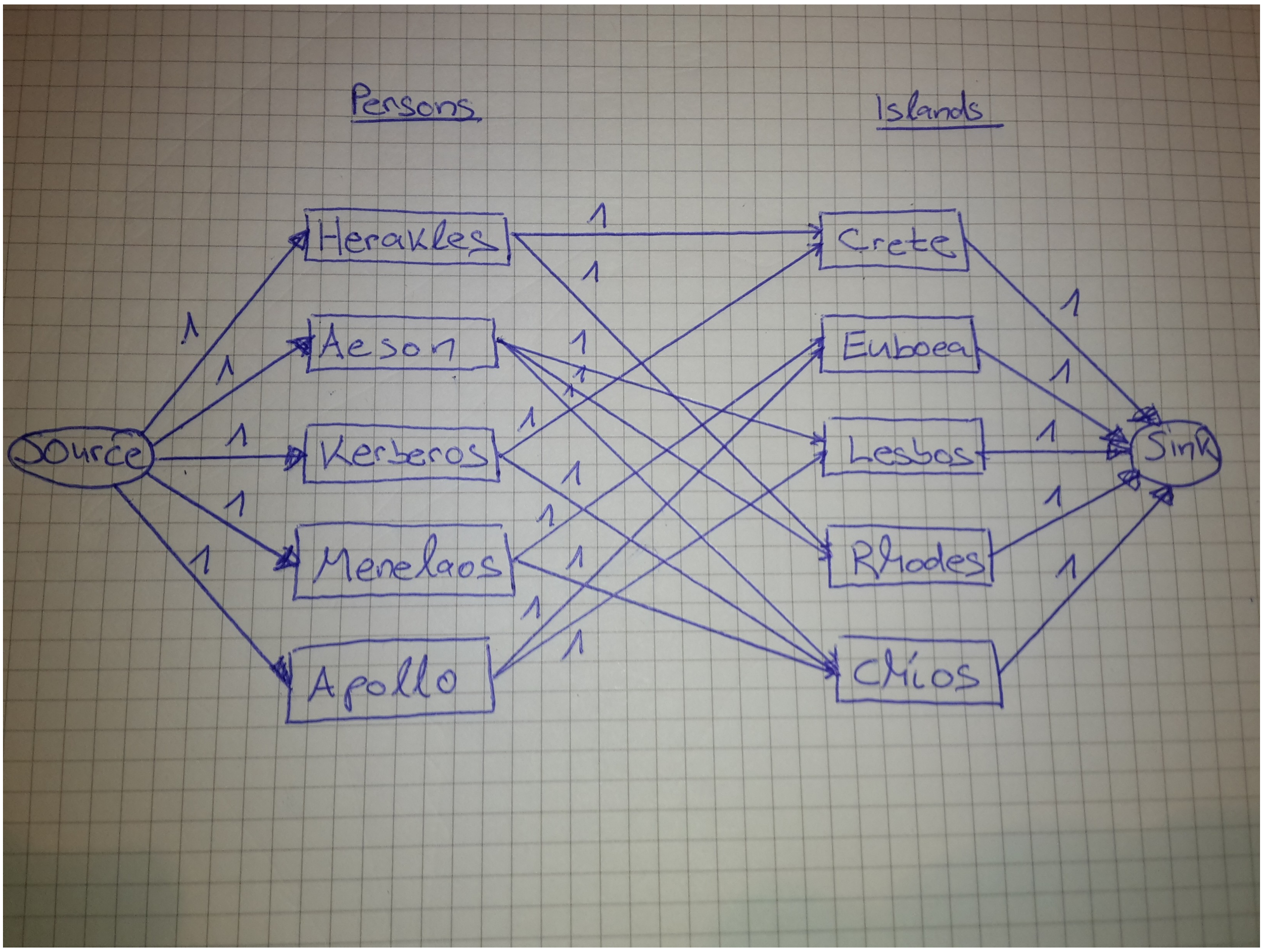

Figure 1.2: Translated version of graph from figure 1.1 into a flow network graph.

3. find a maximum flow: Paths will tell us who gets which island.

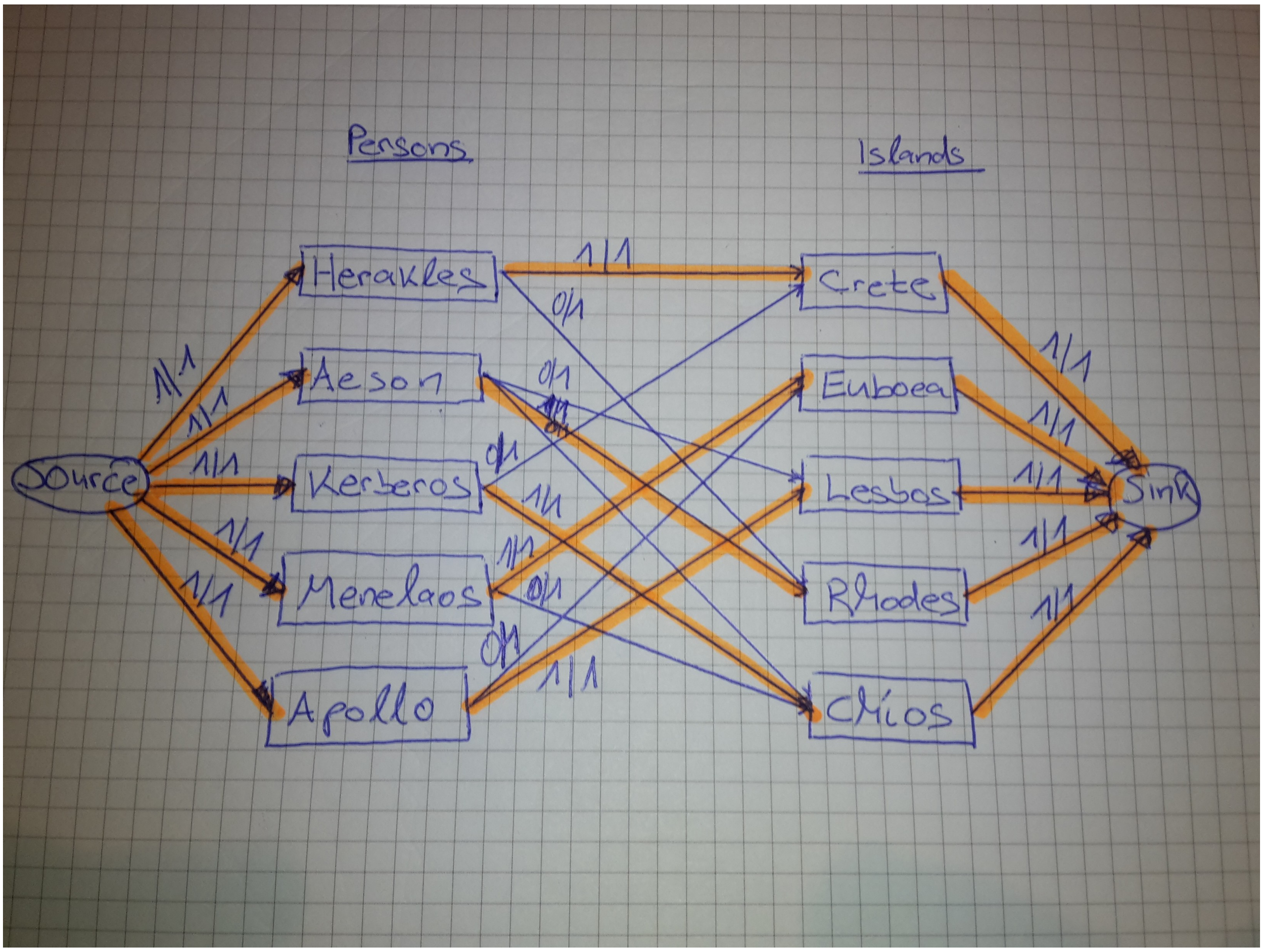

Figure 1.3: A Maximum flow solution of the graph from figure 1.2 found by using ford-fulkerson.

The notation 1/1 indicates there is a flow of 1 given the capacity of 1. 0/1 tells us there is a flow of zero, given the capacity of 1 along a given arc.

For better visibility I indicated existing flows in orange color (see figure 1.3).

By the end of the day, this flow is modelling a binary decision probem: Does person p_r receive island I_k? If yes, there is flow, otherwise there is no flow. Note that an arc is modelling the interest of a particular person (all islands a person is connected to belong this his whish-list (wants to posses)).

Hence, one Solution using the result from step 3 for this Problem is the following:

Person	Receives Island
Herakles	Crete
Aeson	Rhodes
Kerberos	Chios
Menalaos	Euboea
Apollo	Lesbos

Task2

Given a flow graph. The capacity on each edge (edge = highway) denotes the number of lanes in a highway from node u to node v (u and v are supposed to denote cities – source and destionation).

Criminals are transporting contraband from A to F. The police has to assign as many police officers to a highway as there are lanes on a highway in order to check for contraband along this considered highway. What is the minimum number of police officers needed for the following graph in figure 2.1?

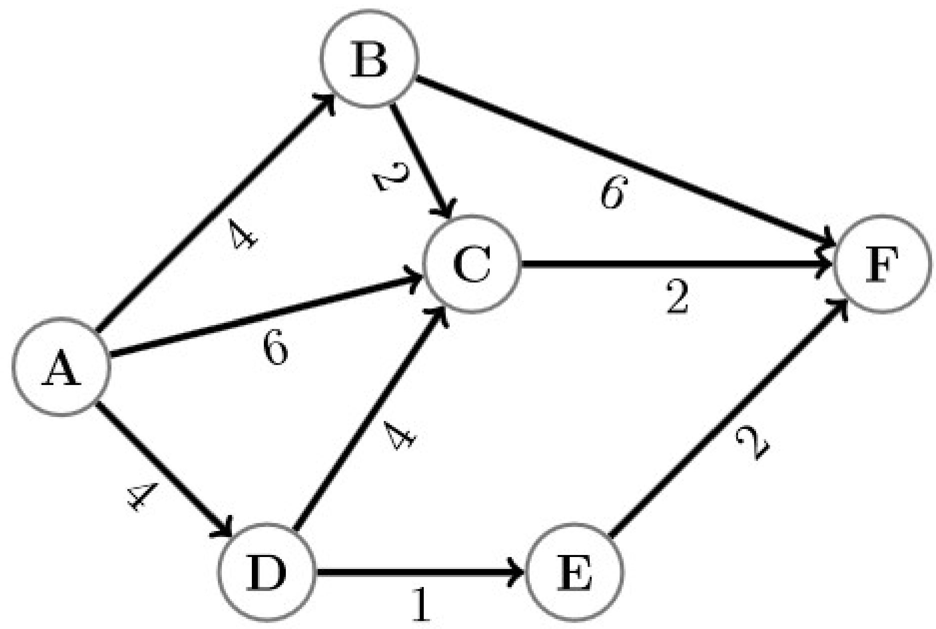

Figure 2.1: Given Road network

Solution: Find the max flow in this graph. The number of the minimum cut corresponds to the max flow (according to the max flow min cut theorem). The minumum cut corresponds to the minumum number of police officers needed.

When solving for the max-flow (using the ford-fulkerson algorithm) we get:

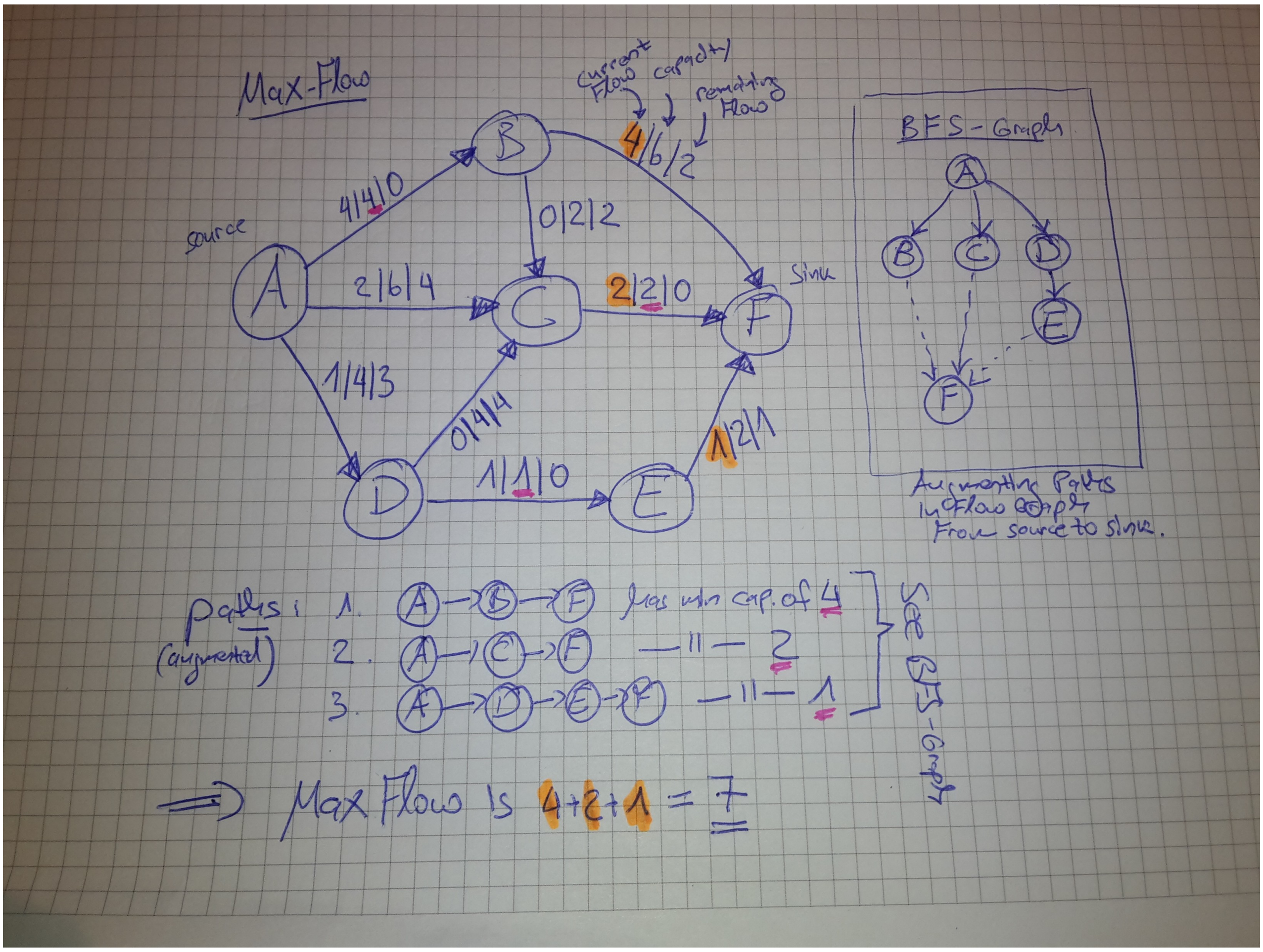

Figure 2.2: Maximum Flow Solution of the Roead Network from figure 2.1

In figure 2.2, we see taht the maximum flow is equal to 7. Therefore, the minumum cut is also equal to seven. Thus, there are **at least 7 police officers needed**.

Note that for finding augmented paths I used the BFS algorithm to find paths from source to sink. See figure 2.2. The pink marking is indicating the minimum capacity within a particular augmenting path. Orange is marking the flow through a augmented path according to the minumum cap. The sum of all flows to the sink is the max flow.

Remark: one particular **MIN CUT** in the graph from figure 2.1 is the following: **AB, CF, DE (i.e. Police has to control those highway routes) (their capacity sum is also 7, this is hereby a min cut).**