AAE 538: Air-Breathing Propulsion

Lecture 29: Supersonic Expansion Systems

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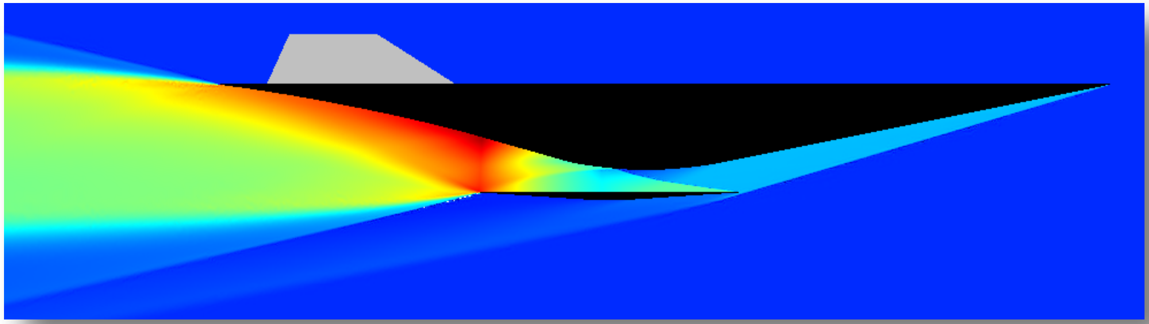
PURDUE AERONAUTICS & ASTRONAUTICS

PURDUE PROPULSION

Introduction

Expansion Systems

- The function of an expansion system is ______________________________ of the flow from the burner exit condition to the engine exit.
  - Reduce ____________________________ to the local (free-stream) static pressure.
  - Accomplish this condition for a wide range of vehicle operating conditions
    - Controllably
    - Reliably
    - Efficiently



- Expansion components are slightly more simple to develop, with respect to the number of requirements and catastrophic consequences for a bad design.
  - Their operation can have enormous influence on the overall operation of the supersonic propulsion system.

Basic Configuration

- As we first discussed with the introduction to supersonic propulsion, highly-supersonic and hypersonic propulsion systems become very-much integrated with the air-frame design.
    - Designers have become very clever in how they accomplish this.
    - Let's build an understanding of these ideas from our classical understanding of (ideal) nozzle design.
- The exhaust nozzle flow is two-dimensional, rather than axisymmetric (or circular)
    -
    -
    -
- In a Scramjet, the flow at the inlet of the exhaust nozzle is
    -

Basic Configuration

Uniform Mach numbers at the inlet and exit

- The exhaust stream is modeled on the assumption of isentropic flow of a calorically-perfect gas.
  - Supporting use of the method of characteristics.
    - Effectively, the characteristics play the same role in the nozzle as the oblique shocks in the compression system.
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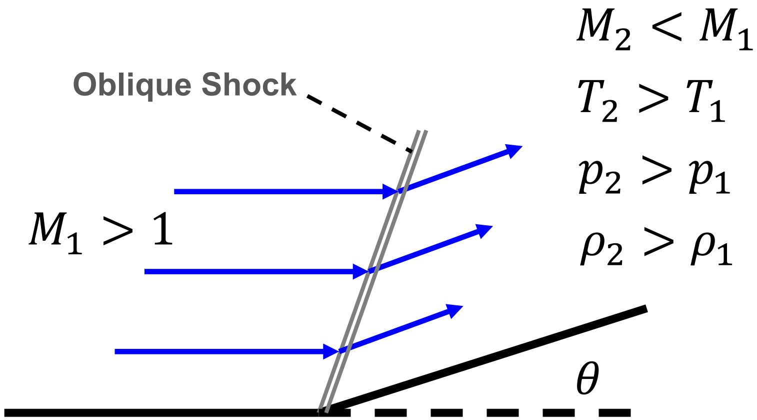
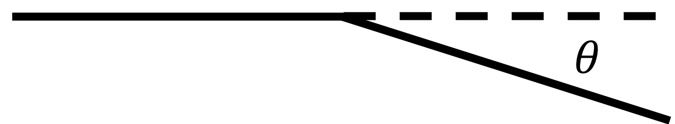
Review of Supersonic Expansion

- When a supersonic flow is turned from itself, an expansion wave is formed.





- An expansion fan works to accomplish the exact opposite action as when the fan is turned onto itself. Expansion waves are the antithesis of shock waves.
  - Some key characteristics:
    - An expansion corner is a means to _____________the flow Mach number:
    - The pressure, density, and temperature __________________through an expansion.
    - The fan is a continuous expansion region, composed of an infinite number of Mach waves. It is bounded by forward and rearward Mach lines at angles μ_1 and μ_2, respectively, where

Review of Supersonic Expansion

- Some key characteristics (continued):
  - The streamlines through an expansion fan are smooth,
  - Since expansion takes place through a continuous succession of Mach waves, and $ds = 0$ across each Mach wave,

- When an expansion wave emanates from a sharp corner, it is referred to as a centered expansion fan. We generally refer to these structures as Prandtl-Meyer expansion waves, since these two were the ones that worked out the theory back in the 1907/8 time-frame. What they showed is that, for an infinitesimally-small deflection, $d\theta$, the change in the velocity can be described as
  - This expression is technically an approximation for a finite $d\theta$, becoming a true equality for $d\theta \to 0$.
  -
- Applying our typical assumptions, we can reduce this form to an algebraic expression, which we refer to as the Prandtl-Meyer function:

Review of Supersonic Expansion

- The Prandtl-Meyer Function:

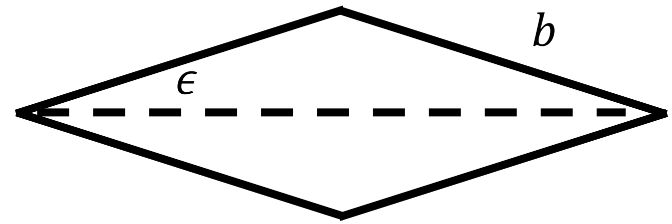
  where

Here, we have now related θ and μ to M_1 and M_2 just like with an oblique shock

- We can either use the ________________________________ or a finite-difference scheme to solve for the details of these spatial structures
  - Characteristics can be understood and contours in this packet of infinitely thin, infinitely weak waves along which the derivative of velocity is indeterminate and across which you have a discontinuity.

(Brief) Note on Shock-Expansion Int.

- Consider a simplified version of an airfoil:
  - Diamond-shaped
  - Zero angle of attack



1-2:

2-3:

3-4:

- If you zoom way out an look at a (flat plate) with a finite angle of attack.

Basic Configuration

Returning to our nozzle…

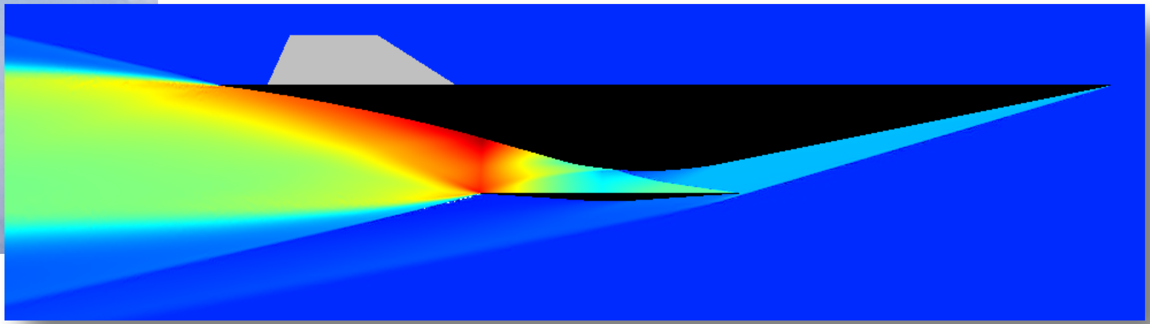
- The nozzles are designed to produce a uniform, parallel flow that maximizes thrust at a target Mach number.
- The design of the minimum length nozzle is achieved by placing a sharp step at the nozzle throat.
    -
    - At the axial locations where each characteristic comes in contact with the wall, the wall angle with respect to the axial flow direction (centerline) is changed in slope
        -
        -
- The characteristics can be used to quantify the important aspects of the flow-field behavior, but first, let's take a look at the principal structures of the nozzle flow.

Steady Flow Structure

Steady Flow Structure

Typical Nozzle Configuration

- The practical realization of these classical developments is actually very similar; just with half of the geometry.
  - Simply replace the plane of symmetry with a
    - This feature doesn't even need to be very long; extending from the entry to the end of zone IV.
    - The flow has already reached the free-stream static pressure everywhere along the final characteristic.



Expansion Component Analysis

- Expansion components, like compression systems, must be resilient against an enormous range of operating conditions.
  - The performance of these system (influence of non-idealities) over those conditions can vary greatly.
- As before, we will develop the framework for this analysis in the context of ideal performance; laying the foundation for a simple addition/modification to account for losses due to non-ideal influences.
- The key assumptions here are:
  - Ideal behavior (isentropic flow)
  - Calorically-perfect gases
  - Two-dimensional nozzle geometry with 'basic' flow field.

1) Working through the nozzle, we begin by computing the forward Mach wave angle at Zone I; the first structure the flow 'sees'.

Expansion Component Analysis

2)

$$\omega = \sqrt{\frac{\gamma+1}{\gamma-1}}\left\{\arctan\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M_{10}^2-1)}\right) - \arctan\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M_4^2-1)}\right)\right\}$$
$$-\left\{\arctan\left(\sqrt{M_{10}^2-1}\right) - \arctan\left(\sqrt{M_4^2-1}\right)\right\}$$

3) We now compute the Zone III Mach number for the flow turning angle of $w/2$.

$$\frac{\omega}{2} = \sqrt{\frac{\gamma+1}{\gamma-1}}\left\{\arctan\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M_{III}^2-1)}\right) - \arctan\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M_4^2-1)}\right)\right\}$$
$$-\left\{\arctan\left(\sqrt{M_{III}^2-1}\right) - \arctan\left(\sqrt{M_4^2-1}\right)\right\}$$

Expansion Component Analysis

4) With the Zone III Mach number, we can compute the rearward Mach wave angle:

5) Compute the Zone III static pressure

6) Compute the Zone VI Mach wave angle

Expansion Component Analysis

7) From continuity, using the mass flow function ($D(M)$), we compute the ratio of the exit height to the combustor exit area.

8) Compute the ratio of the Zone VI axial length to the entry height

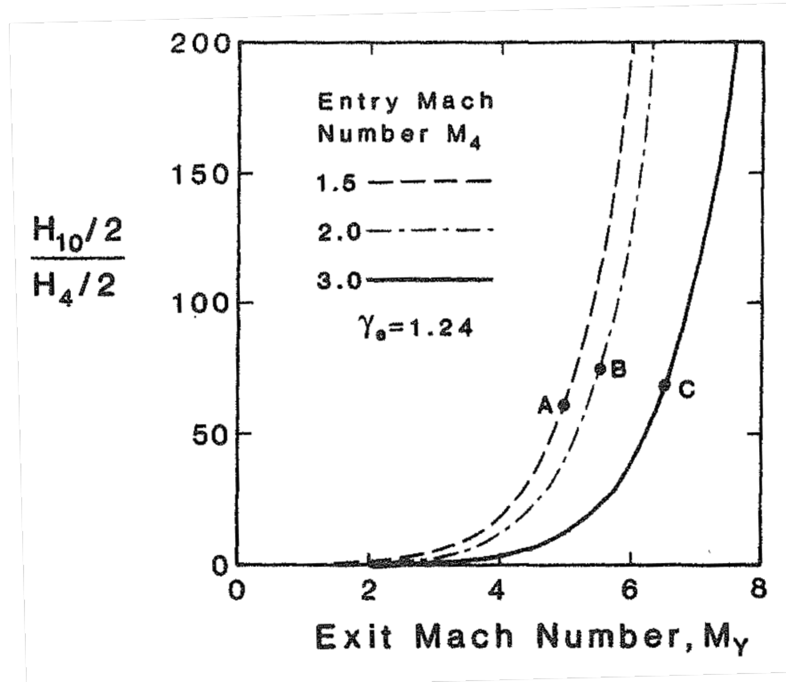
9) Finally, compute the static pressure ratio, assuming constant total pressure (ideal)

$$\frac{p_{10}}{p_4} = \frac{p_0}{p_4} = \left\{ \frac{1 + \frac{\gamma - 1}{2} M_4^2}{1 + \frac{\gamma - 1}{2} M_{10}^2} \right\}^{\frac{\gamma}{(\gamma - 1)}}$$

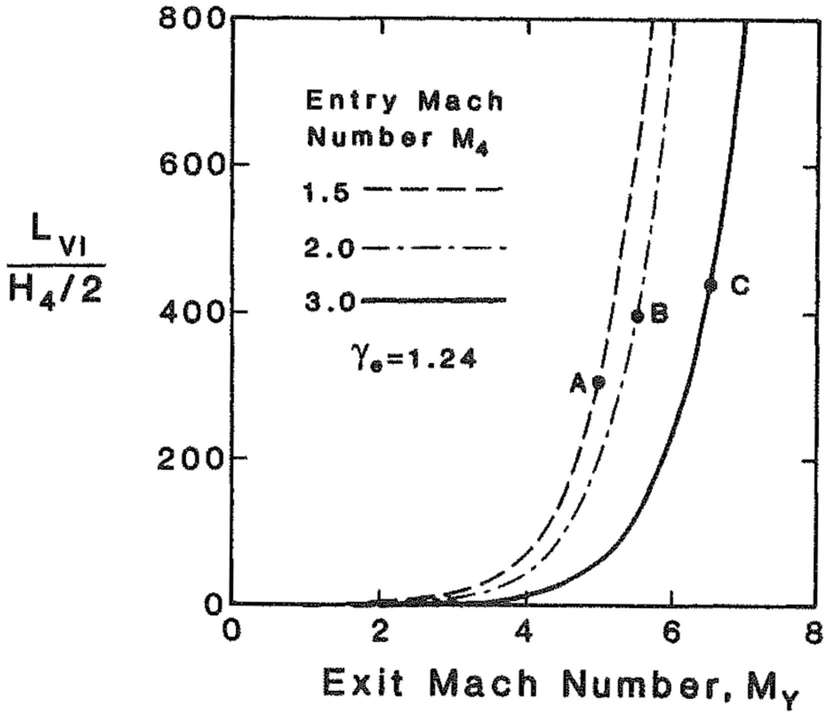
Expansion Component Analysis

- As mentioned above, the expansion system must perform reliably and efficiently over a huge range of operating conditions.
- We can examine the behavior of the key nozzle parameters as a function of the nozzle exit Mach number in the plots below.
  - As a matter of reference, three distinct sets of operating conditions were computed from a simple scramjet cycle analysis and are included on each plot.

	M_0	M_4	M_{10}	$\frac{u_{10}}{u_0}$	$\frac{u_4}{u_{10}}$
A	8	1.5	5	1.3	0.55
B	10	2.0	5.5	1.25	0.7
C	18	3.0	6.5	1.20	0.8

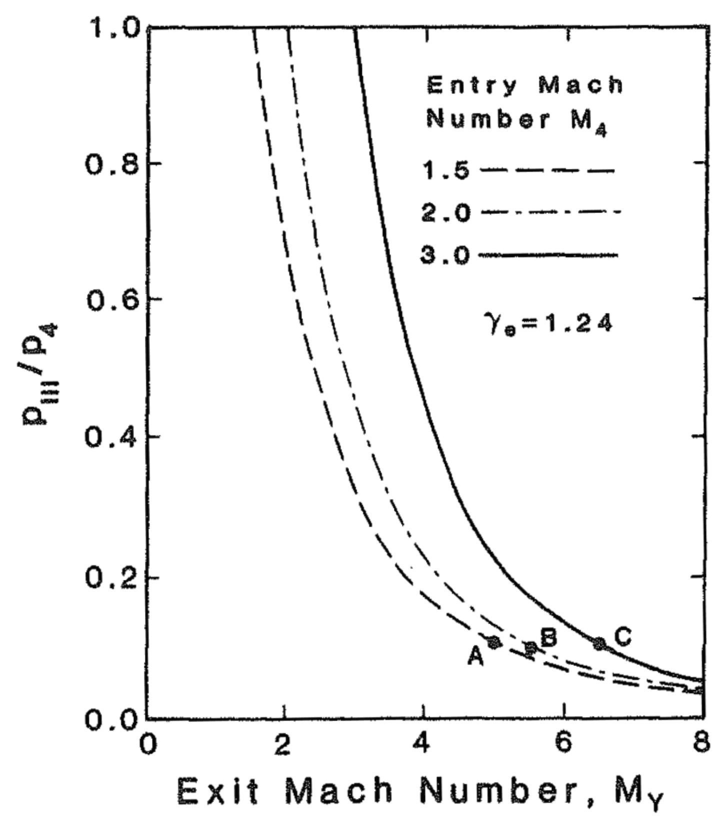


Expansion Component Analysis



- The length of the expansion system to achieve complete expansion is an enormous multiple

- The ratio of the static pressure in Zone III to the entry static pressure shows



Expansion System Performance

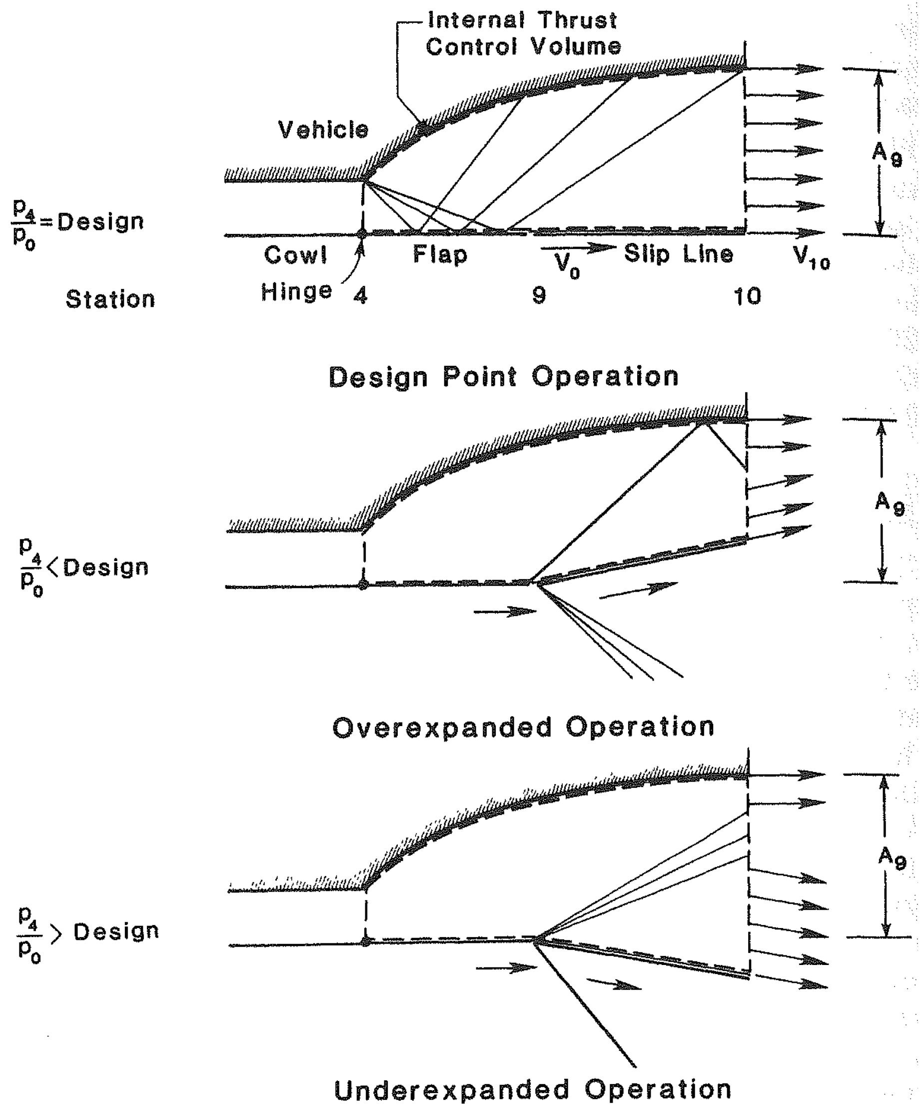
- The function of an expansion system is to convert the thrust potential of the flow exiting the combustor into thrust. It stand to reason that all of our performance measures can be traced back to this, as a primary metric.
    -
    -
    -

- We can gain some qualitative insight to the nature of nozzle performance just by looking at the plume.

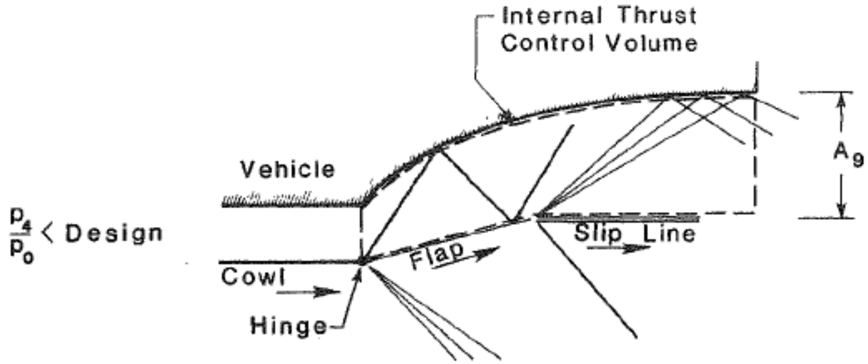
- Physically, we understand that performance is maximized when we are able to maximize the

  ____

  of the gases exiting the propulsion system.

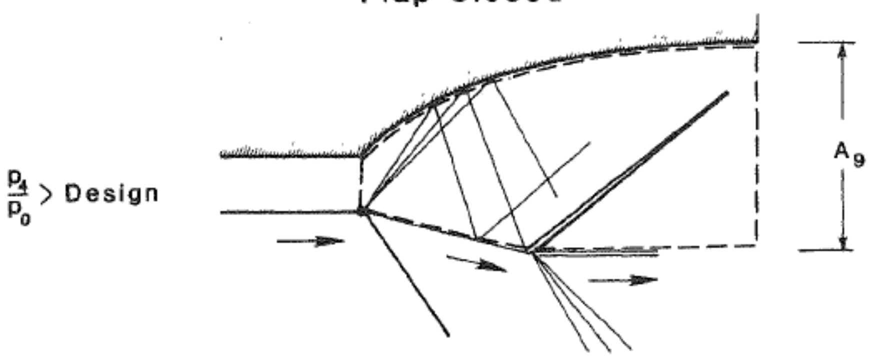


Expansion System Performance



Overexpanded Operation

Flap Closed



Underexpanded Operation

Flap Open

https://nescacademy.nasa.gov/category/3/sub/1 'Fundamentals of aircraft engine control'

Course Conclusion