AAE 538: Air-Breathing Propulsion

Lecture 2: Fundamentals of Compressible Flow

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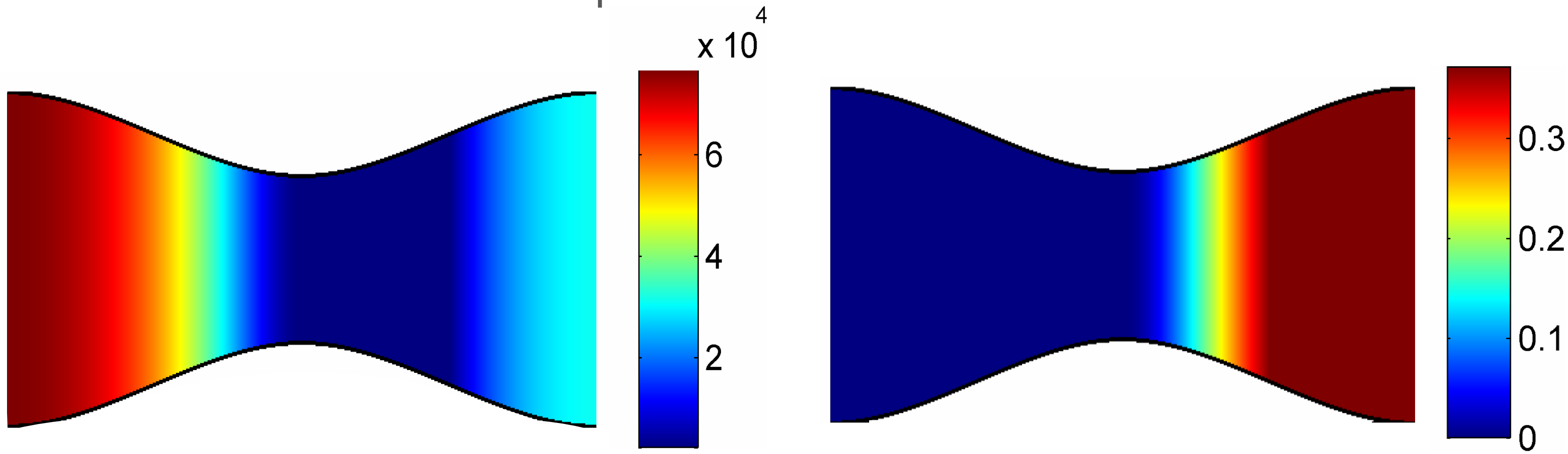
PURDUE PROPULSION

One-Dimensional Flow

- The one-dimensional flow equations are obtained by ______________ the three-dimensional equations across a passage.
  - The process allows us to obtain mean quantities as a function of axial position within a flow. We treat these quantities as ‘constant’ over the cross-section.



An example of a one-dimensional flow of a two-phase fluid through a converging-diverging nozzle for one upstream stagnation and one downstream backpressure: Pressure contours (Pa) on the left and vapor volume fraction contours on the right.

- Many two dimensional effects are present:

- Nevertheless, one-dimensional approximations give a good global overview of the physics in an engine, where full-scale high-fidelity simulations cannot be afforded.

One-Dimensional Flow

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- In general, the one-dimensional approximation is less accurate for ______________________ than for ______________________.
  - For more well-resolved (higher fidelity) solutions that include local multi-dimensional effects, computational fluid dynamics methods must be used.
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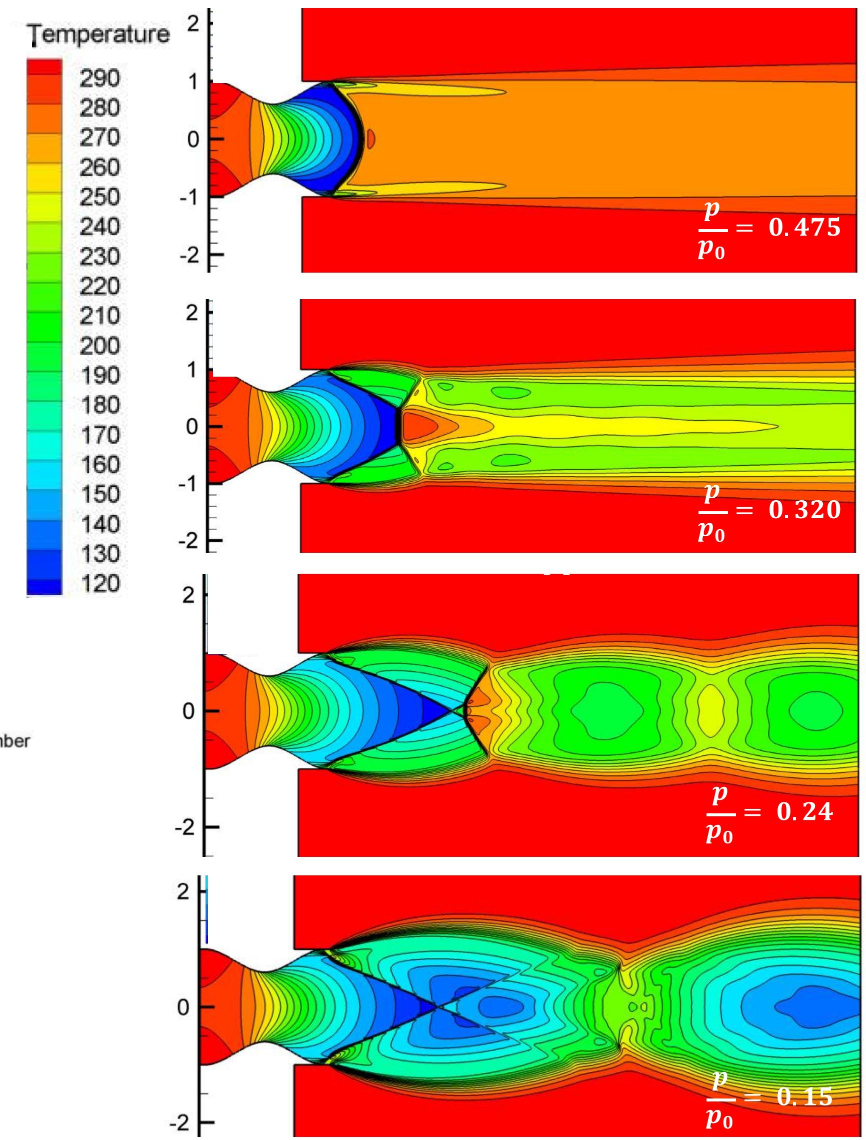
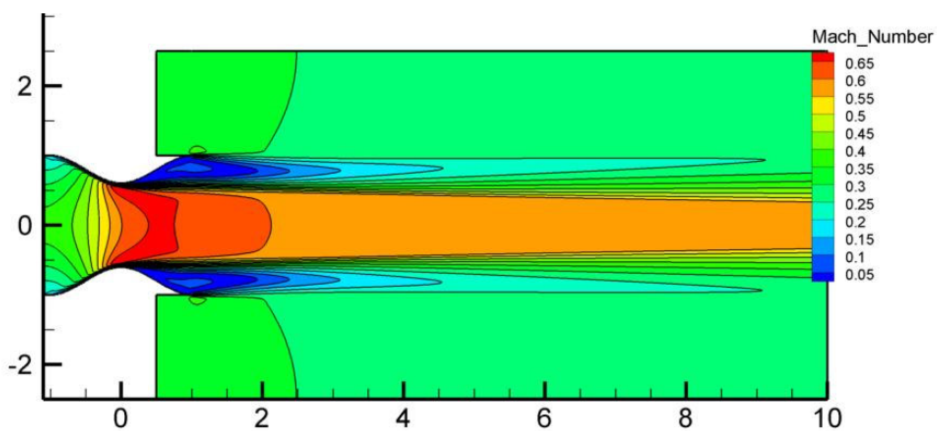


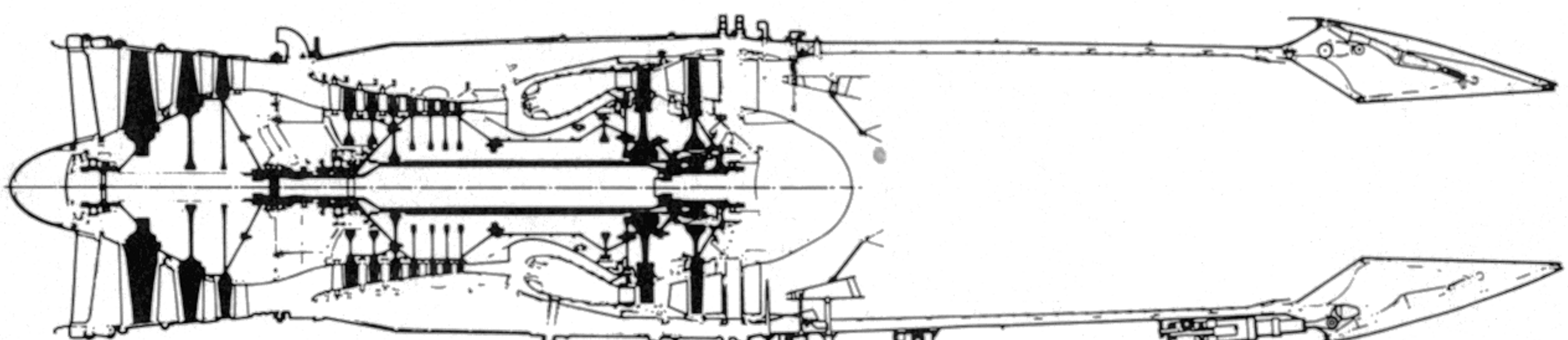


Control Volume Analysis

- Consider a control volume through all, or some part of an engine

The complete, exact picture of the physics occurring within that select volume cannot be known or fully-analyzed at this time. Despite that fact, engines still run and we still have to keep making them run better

  - To make this analysis a tractable endeavor we make assumptions about what is happening inside of this volume.
    -
    -



Cross-section of an afterburning military turbofan engine

Control Volume Analysis

- Reynolds Transport Theorem provides a general form to translate the conservation laws of mechanics and thermodynamics from a ________________________ to a ______________________ framework.
- For any arbitrary control volume (which can be moving)
  - _____________________________ can cross the control surface boundaries
  - ________________ can act on the surface of the control volume as well as on the mass in the control volume.

Control Volume Analysis

- First set of assumptions to reduce the nonlinear partial differential equations to an integral form relative to an inertial frame of reference.

- Defining some nomenclature for our one-dimensional analysis:
  - Subscripts
    - i = inlet plane
    - e = exit plane

Velocity (Axial)	u_i	u_e
Density	ρ_i	ρ_e
Pressure	p_i	p_e
Temperature	T_i	T_e
Enthalpy	h_i	h_e
Internal Energy	e_i	e_e
Entropy	s_i	s_e
Speed of Sound	a_i	a_e
Mach Number	M_i	M_e

Control Volume Analysis

Conservation Equations for a General Fluid

- There are **nine** variables. We need **nine** equations.
  1. Conservation of Mass (continuity)

$$0 = \frac{\partial}{\partial t} \oiiint_V \rho dV + \oiint_S \rho \vec{u} \cdot \vec{n} dS$$

  2. Conservation of Momentum (Newton's Second Law)

$$\vec{F}_E - \oiint_S p\hat{n}dS + \oiint_S \vec{\sigma} dS + \oiiint_V \rho \vec{f} dV$$

$$= \frac{\partial}{\partial t} \oiiint_V \rho \vec{u} dV + \oiint_S \rho \vec{u}(\vec{u} \cdot \vec{n}) dS$$

Control Volume Analysis

Conservation Equations for a General Fluid

3. Conservation of Energy (First Law of Thermodynamics)

$$\dot{Q}_{in} - \dot{W}_{out} =$$

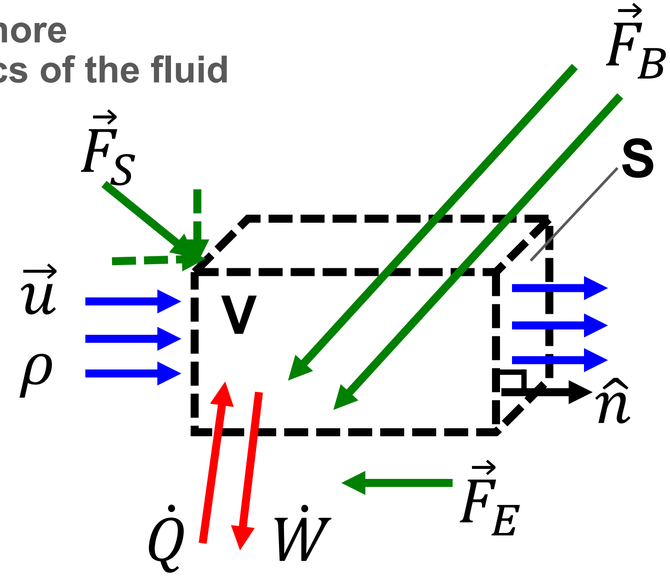
$$\frac{\partial}{\partial t} \oiiint\limits_{V} \rho \left(e + \frac{u^2}{2} + gz \right) dV + \oiint\limits_{S} \rho \left(h + \frac{u^2}{2} + gz \right) (\vec{u} \cdot \hat{n}) dS$$

Control Volume Analysis

State Equations for a General Fluid

- There are nine variables. We need nine equations.

We now have the conservation equations reduced, but we need more information about the thermodynamics and physical gas dynamics of the fluid



4. Equation of State:

5. Caloric Equation of State (Enthalpy)

6. Internal Energy

7. Entropy (Gibbs' Equation)

8. Speed of Sound:

9. Mach Number

Control Volume Analysis

State Equations for a General Fluid

- Now that we have obtained algebraic forms of the conservation equations, we seek to further reduce the complexity of the state relationships as well:
  - Assume Ideal Gas Behavior
    -
    -

$$\rho = \rho(p, T)$$

$$h = h(p, T)$$

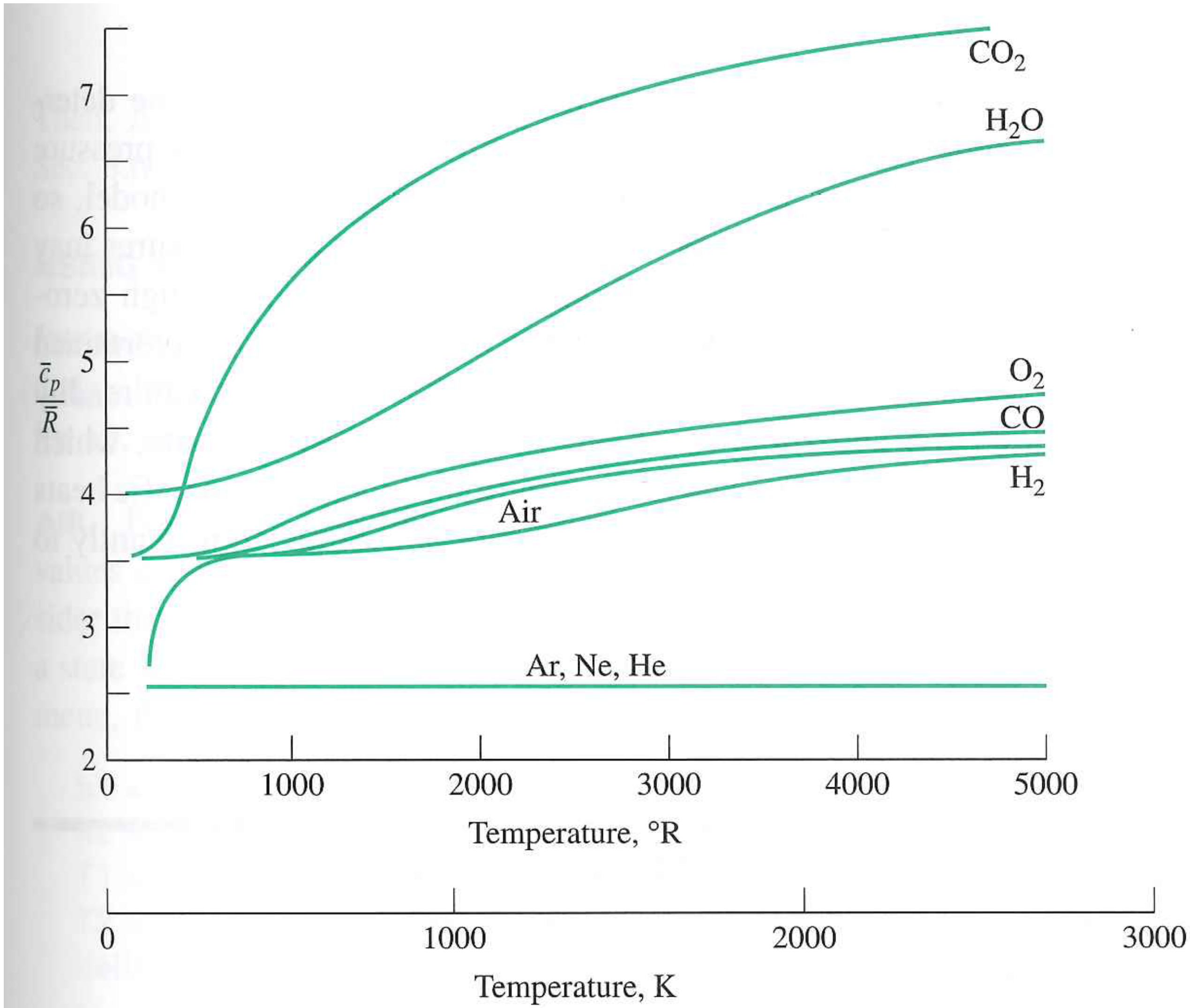
- From the last expression, we obtain

Control Volume Analysis

Variation of the Specific Heat with Temperature

Specific Heats

As a Function of Temperature



Moran and Shapiro, 2004

Control Volume Analysis

State Equations for a General Fluid

- Assume ______________________________
  - For simplicity in the classroom and homework problems, we will often use an appropriately averaged constant value of the specific heat corresponding to the temperature range of interest in the current application.
  - Using a locally constant value allows us to omit the reference value and express the enthalpy as:

- From the definition of the internal energy, $e = h - p/\rho$, we can then relate the differential of the enthalpy to the differentials of the internal energy, pressure, and density as:

Control Volume Analysis

State Equations for a General Fluid

- Combining this with the differential of the

  The enthalpy – internal energy relation becomes

  Where we have used the fact that:

  - Again, for an approximately constant local value of the specific heat, the internal energy can be written:

Control Volume Analysis

State Equations for a General Fluid

- We also, now introduce the ratio of specific heats: $$\gamma = \frac{c_p}{c_v}$$

  And these subsequent, useful relations:

  And, similarly:

- Returning to entropy, the Gibbs' Relation for the entropy of a perfect gas becomes:

$$Tds = c_p dT - \frac{dp}{\rho}$$

These relationships are among the most useful for working with perfect gases

Control Volume Analysis

State Equations for a General Fluid

- With these developments, we can now find our familiar algebraic relationship for the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s =$$

- And the Mach number is already expressed in terms of an algebraic relationship that is unaffected by all of these developments

$$\Rightarrow \quad M = \frac{u}{a}$$

Iso-Energetic Flows

…and the Stagnation Enthalpy

- To define the stagnation properties, we can first return to the general equation of state:

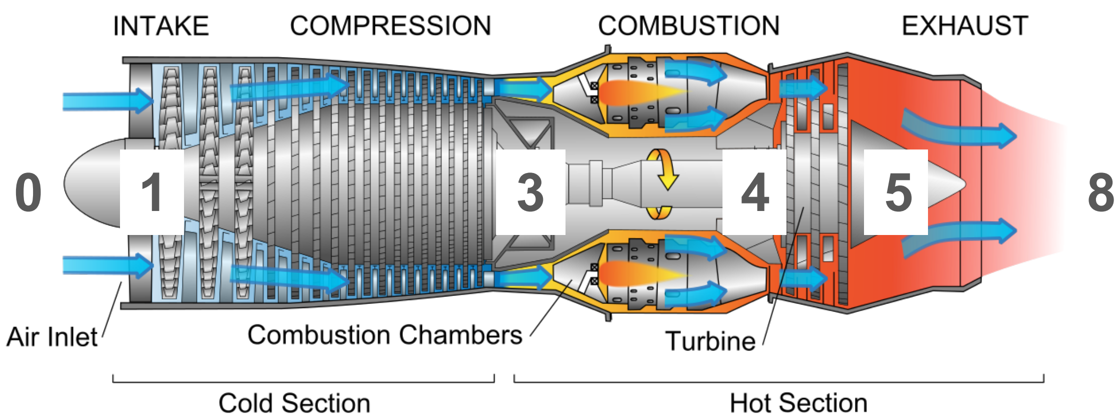
  And apply the energy equation to 1-D flow with no work or heat addition: $\dot{W} = \dot{Q} = 0$
  The energy equation then becomes:

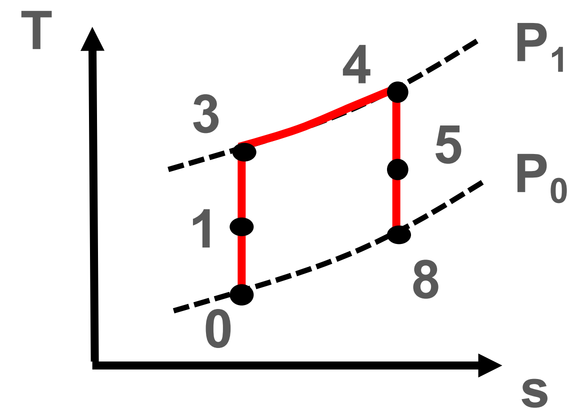
  Applying continuity,

Iso-Energetic Flows

…and the Stagnation Enthalpy

- Flow with no heat addition of work done is referred to as ____________________.
  - This approximation is typically appropriate for ____________________ and __________________________ in air-breathing engines.





- Assuming iso-energetic flow does not imply that the component is ________________; it does allow for losses and non-idealities with accompanying increases in entropy.
- The constant stagnation enthalpy result holds for any type of fluid (gas or liquid – regardless of its equation of state) so long as no work is done and no heat is added.

Iso-Energetic Flows of Perfect Gases

Stagnation Temperature and Pressure

- If we return to the assumption of perfect gas behavior and take a proper local average of the specific heat, we can define the stagnation temperature as:

  Dividing through by the (constant) average value of the specific heat, we define

  A simple modification provides a very useful expression between the stagnation temperature, the static temperature, and the Mach number:

Iso-Energetic Flows of Perfect Gases

Stagnation Temperature and Pressure

Thus, the ratio of the stagnation temperature to the temperature (or the 'static' temperature) is:

- From a physical viewpoint, the stagnation enthalpy is the enthalpy reached when a flow with Mach number, M, is stopped without the addition/removal of heat or work.
  - When a constant specific heat is assumed, the same statement can be made for the ______________________________.
  - Therefore, assuming a constant specific heat, the stagnation temperature is __________________ for an iso-energetic flow.
- The **stagnation pressure** is the corresponding pressure reached when a fluid flow is stopped without ________________________________.
  - Specifically, the stagnation pressure is the pressure reached when the flow is stopped, __________________.

Iso-Energetic Flows of Perfect Gases

Stagnation Temperature and Pressure

- Consider a vehicle traveling in the atmosphere at a Mach number, M.
  - The external temperature 'felt' by the vehicle is:
    -
- Changing coordinates from the earth to the vehicle gives us a steady-flow problem in which uniform flow is approaching the aircraft at the flight speed.
  - Upon reaching the nose of the aircraft, the flow along the stagnation streamline must be decelerated to zero iso-energetically. Accordingly, temperature reached by the fluid is the stagnation temperature.
    -
  - For an isentropic process, the fluid at point 2 has the following properties:

Iso-Energetic Flows of Perfect Gases

Stagnation Temperature and Pressure

- Note the key difference between the two definitions:
  - The stagnation temperature is defined as the temperature reached when the fluid is brought to rest by a __________________________.
  - The stagnation pressure is is the pressure reached when the fluid is brought to rest in an ________________________________.

- The fact that the static temperature at point 2 is equal to the stagnation temperature comes from _________________________.
- The fact that the static pressure at point 2 is equal to the stagnation pressure arises from the ________________________ where, for a perfect gas:

Iso-Energetic Flows of Perfect Gases

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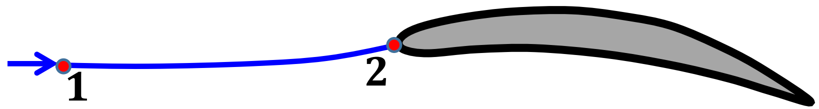
Stagnation Temperature and Pressure

M_1
$T_{o,1}$
$p_{o,1}$



Plugging in for our isentropic process between points 1 and 2 and rearranging

Taking exponentials of both sides and rearranging gives:

- Here, the second law of thermodynamics ensures that γ is greater than unity
  - Thus, the exponent on temperature is always greater than unity
  - This implies that

Iso-Energetic Flows of Perfect Gases

Stagnation Temperature and Pressure

- With this complete, we can now derive an expression for the stagnation to static pressure ratio in terms of Mach number:

- When a pitot probe is inserted into the flow, the velocity at the inlet is brought to rest.
  - Although this process is obviously a real process the includes frictional effects and entropy increase, the change in entropy is small for ___________________.
    - The probe gives a fairly accurate measure of the stagnation pressure
  - In supersonic flows, the stagnation pressure will be larger than the indicated measurement because of the presence of a _____________ in front of the probe.
    - The freestream stagnation pressure, in this case, must be computed through the known jump-conditions across the shock.

Stagnation Pressure for General Fluids

- For a general fluid, including a perfect gas with variable specific heats, the definition of the stagnation pressure becomes more complicated.
- Starting from the general form of the entropy relation:

$$Tds = dh - dp/\rho$$

and defining an isentropic process between states 1 and 2, we have

$$s_2 - s_1 = \int \frac{dh}{T} - \int \frac{dp}{\rho T}$$

- To complete this integration, we need to use the equation of state information,

$$\rho = \rho(p, T) \qquad and \qquad h = h(p, T)$$

which requires ____________________________ to obtain integrals that have only a single thermodynamic variable.

- While the procedure is straight-forward, it doesn't really introduce any important new concepts for this class.
    - We'll limit our attention to ideal gases in 538, but you can always come back to this if you run into a problem such this treatments are necessary.