AAE 538: Air-Breathing Propulsion

Lecture 4: Fundamentals of Compressible Flow (continued)

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Extensions of 1-D Analysis

Anisentropic Flows

- We began by developing the conservation equations and the state equations from their fully-3D form into a framework that is tractable for analysis of the thermodynamic and physical gas dynamic state of the flow through an engine.
- Using the example of an variable-area duct, we further developed these expressions for the the case of isentropic flow, where

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right) = 0$$

for a perfect gas.

- Assumptions this carries:
  - No heat transfer, no work transfer
    -
  - Ideal processes: i.e. no losses due to the effects of friction, viscosity, …
    -
- We now continue our analysis of compressible flows by diving into these processes which can be very relevant to the analysis of engine flows.
  - Shock Waves
  - Flow with Friction (Fanno)
  - Flow with Heat Transfer (Rayleigh)

Shock Waves

- Shocks are a common irreversibility that occur in supersonic flows; ______________________________.
  - They represent a sharp __________________ in the thermo-physical state of the flow.
  - They can also induce changes in the thermo-chemical state of the flow…
- We typically delineate shock waves into two basic categories, based on orientation to the free-stream.
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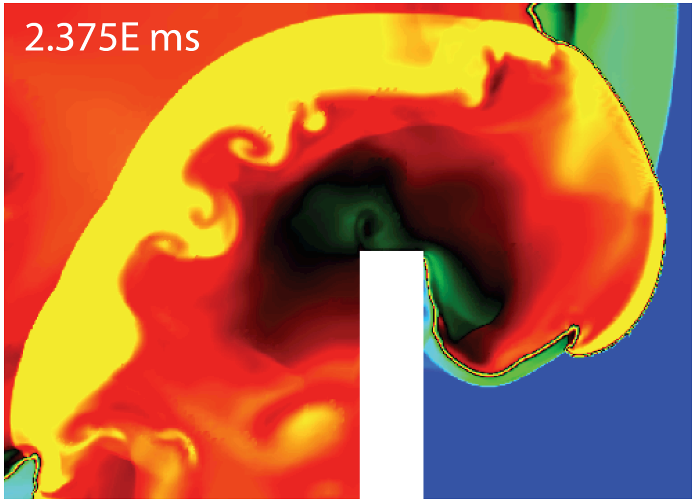
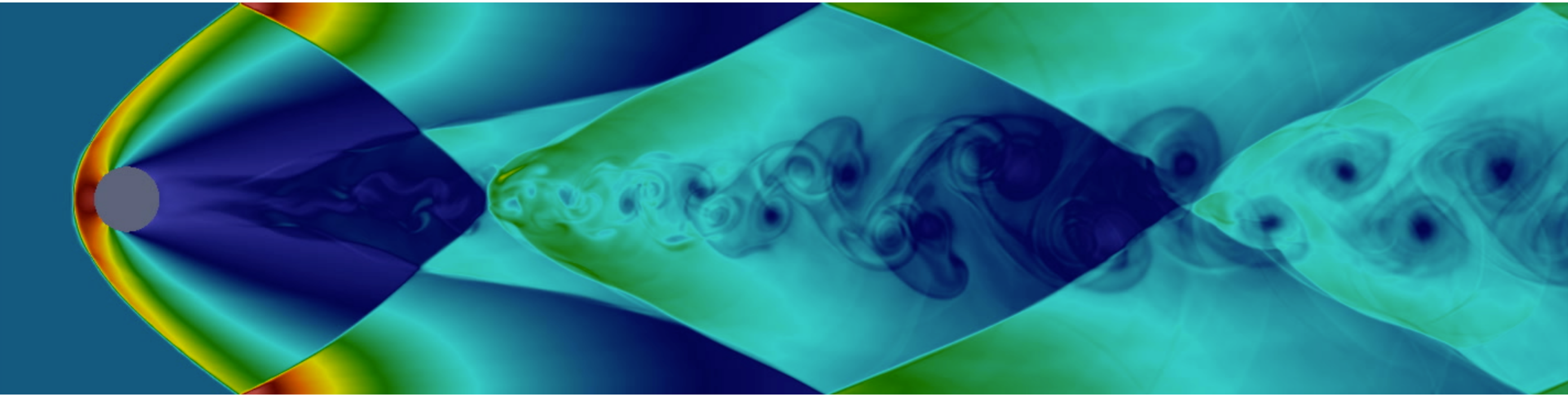


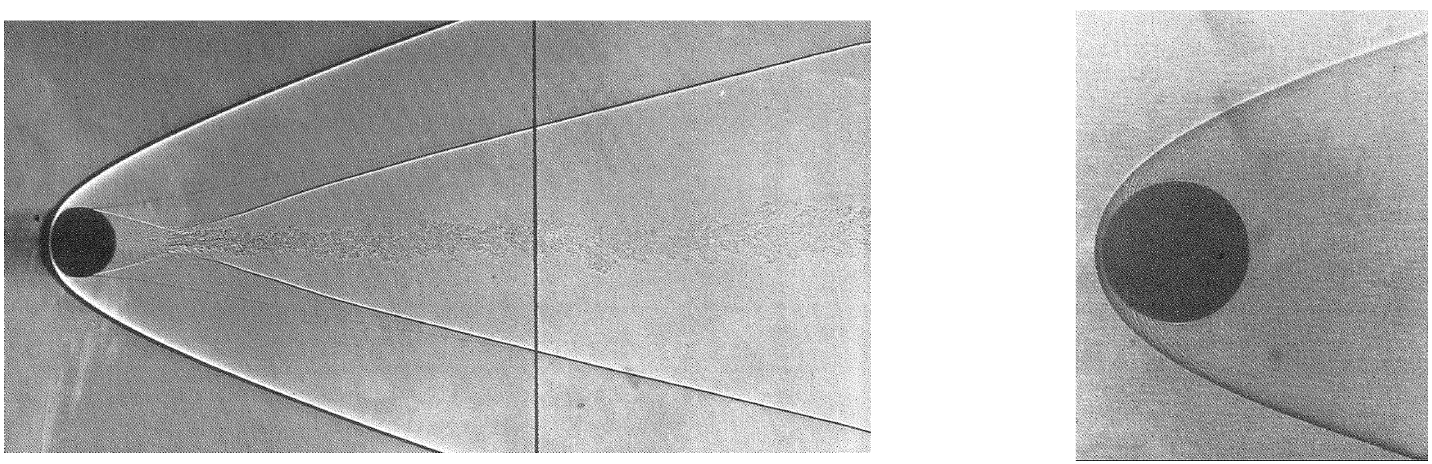
DNS of self-acceleration and DDT channels with obstacles (Oran, 2008, APS)



(beautiful) density field computed using DNS of supersonic flow over a cylinder (Guido Lodato, CORIA, France)

Shock Waves

- Why does the shock form?
  - If an object is introduced into a supersonic flow in the region near the edge sends signals in all directions signaling the presence of the obstruction.
    -
  - If the flow were to remain supersonic, the signals would be swept downstream, never alerting the incoming flow of the obstruction.
  - Instead, the signals (waves) pile up in front of the obstruction and coalesce to form a shock wave.
    -
    - However, it can be shown that for a flow to slow from supersonic to subsonic through an adiabatic process, the entropy would have to decrease. This cannot happen, so a shock forms to affect the attendant jump from ______________________________.



'An Album of Fluid Motion' (Milton Van Dyke)

A Physical Description of Shock Waves

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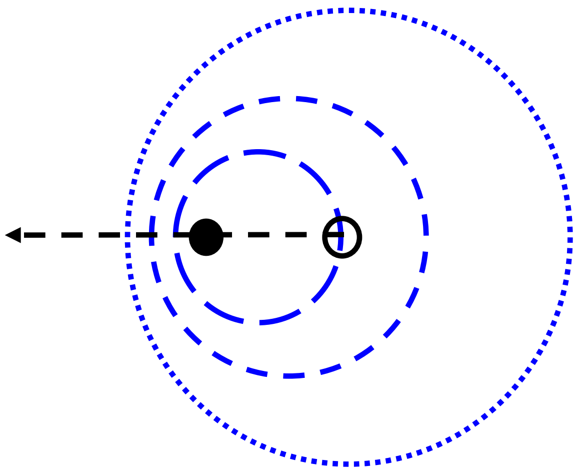
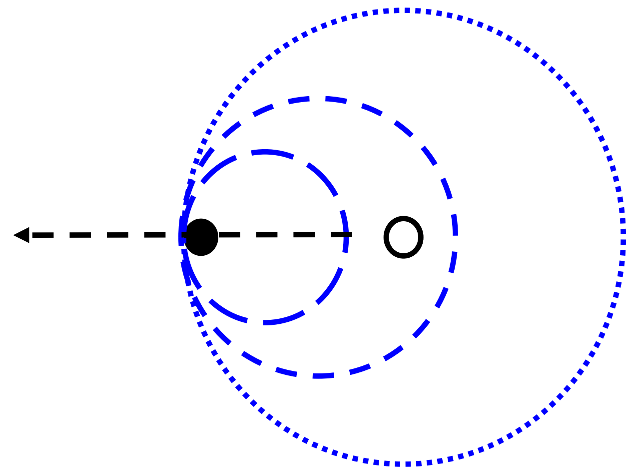
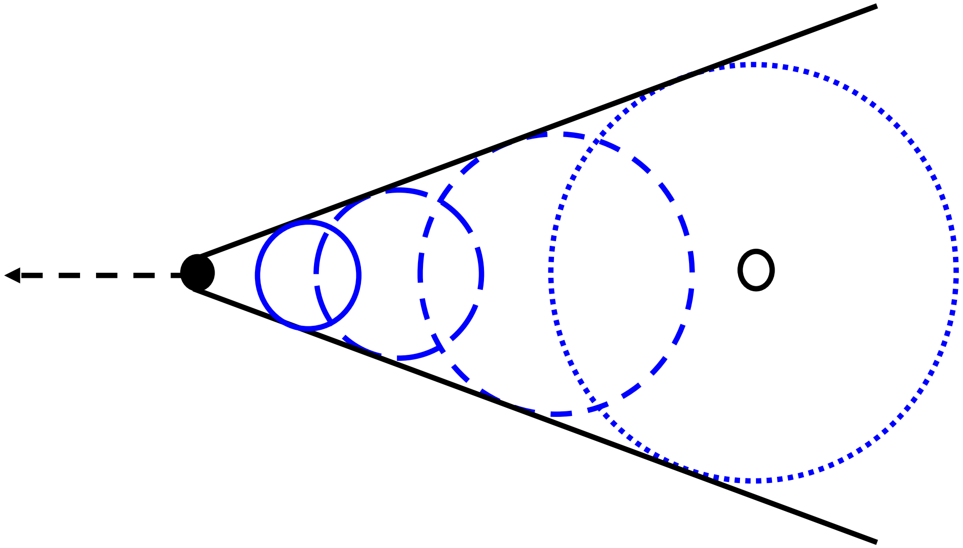
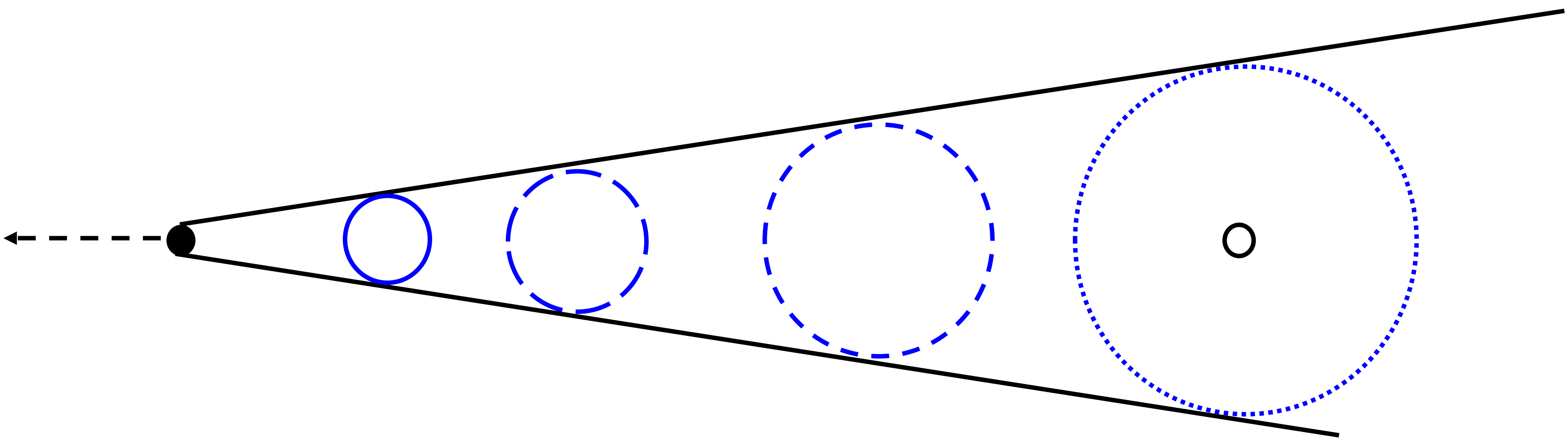
- Treating our vehicle as a particle and thinking about the way the surrounding fluid (air) responds to its motion





Oblique Shock Waves





Shock Waves

How do we analyze them?

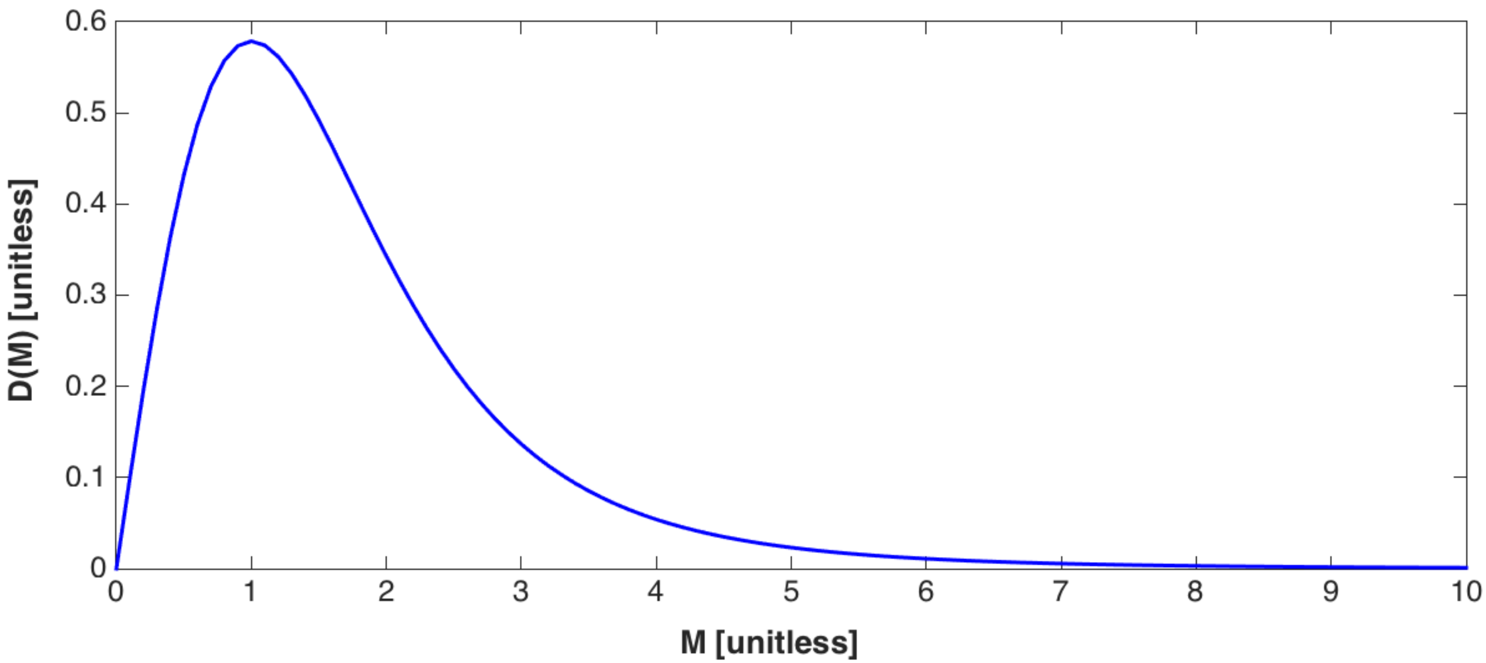
- A shock-wave is a nearly discontinuous jump in the thermo-physical state of the fluid.
  - This jump occurs over a distance of a few mean free paths.
  - Entropy
  - The velocity

- The areas associated with the two locations (before and after the shock) are essentially identical even in a variable area duct.

- Looking at the 1-D form of the continuity equation for an ideal gas:

Shock Waves

- Remember, the mass flow function is double-valued.
  - It has a maximum at $M = 1$ and a minimum at $M = 0, \infty$.
  - For any value of $D(M)$, there will be two solutions for the Mach number.



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Shock Waves

- We can ascertain from the energy equation that the stagnation temperature ________________________ across a shock.
  - The volume of the shock is negligible ('zero' thickness) so that heat cannot be added and work cannot be done across the shock.

- The absence of heat and work transfer do not imply that the ____________________ is constant because the entropy increases across a shock.
- Approximating the fluid as a perfect gas with constant properties, the continuity equation becomes

where provisions are left for variability in the stagnation pressure due to entropy change across the shock. i.e. the shock is not an ideal process, so the stagnation pressure must be retained in the mass flow relation for this control volume.

Shock Waves

- The very short distance between the two sides of the shock also indicates that ________________________________.
  - Dropping them from the momentum equation gives the following:

- Dividing the continuity equation by the momentum equation leads to the conditions across the normal shock:

  which simplifies to:

Shock Waves

Given this relation, we can express the conditions across the shock in terms of Mach number, only:

$$\frac{M_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{1}{2}}}{(1+\gamma M_1^2)} = \frac{M_2\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{1}{2}}}{(1+\gamma M_2^2)}$$

- Now, given the Mach number on one side of the shock, the Mach number on the other side of the shock can be computed using iterative methods.
  -
- Some observations:
  - One possible solution is that
  - There is no
    - Conservation of mass and energy can be satisfied with both compression and expansion shocks.
    - To ensure that our analysis is physical, we need to consider the ________________________ across the shock as well.

Shock Waves

- Returning to the continuity expression, once the Mach number is known on both sides of the shock, we can solve for the stagnation pressure ratio where,

$$\dot{m}\sqrt{T_o}\sqrt{\frac{R}{\gamma}} = p_{o,1}A_1D_1 = p_{o,2}A_2D_2 \quad \rightarrow$$

where the stagnation pressure ratio will have multiple solutions that are readily determined from the known Mach numbers on both sides of the shock. The correct solution is determined from the entropy relation, where

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

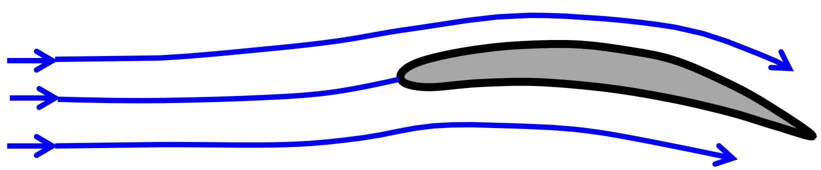
or, written in terms of ______________________________ :

Shock Waves

It's important to realize that the entropy of the stagnation state is identical to the entropy of the moving fluid.

- The stagnation pressure is defined as the pressure that is reached when the flow is isentropically-stagnated.
- The stagnation temperature is the temperature that is reached when the flow is stagnated by either a real or an ideal process.

$$s_2 - s_1 = c_p \ln\left(\frac{T_{o,2}}{T_{o,1}}\right) - R \ln\left(\frac{p_{o,2}}{p_{o,1}}\right)$$



- Therefore, **by definition,** an ideal process that takes the fluid from the flowing condition at location 1 to the companion stagnation conditions for which the velocity is zero, will require have___________________ .
-

Shock Waves

- Combining the energy equation, the entropy relation reduces to:

- For a shock to be physical, the entropy must increase.
  -
  - This result indicates that the stagnation pressure must, therefore, decrease across a shock: i.e.

- From the momentum equation, the $G(M)$ function must _______________ across the shock since $p_{o,1}A_1G_1 = p_{o,2}A_2G_2$ and $A_1 = A_2$.
  - Recalling that $G(M)$:
    - starts from unity when the $M = 0$,
    - reaches a peak at $M = 1$,
    - decreases to zero as the Mach number goes to infinity
  - This is satisfied when the Mach number

Shock Waves

- To compute the static pressure change across a shock, we express the static pressure ratio in terms of the stagnation pressure ratio using the Mach number relation.

$$\frac{p_2}{p_1} = \frac{p_{o,2}}{p_{o,1}}\left(\frac{1+\frac{(\gamma-1)}{2}M_1^2}{1+\frac{(\gamma-1)}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}}$$

Replacing the stagnation pressure ratio by the ratio of the G function.

where

$$G(M) = \frac{(1+\gamma M^2)}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

- Thus, if the Mach number in front of the shock is supersonic and the downstream of the shock is subsonic,

Shock Waves

- In a similar manner to the pressure ratio, we can develop an expression for the temperature ratio across a shock using the stagnation to static temperature ratio function of Mach number,

$$\frac{T_2}{T_1} = \frac{T_{o,2}}{T_{o,1}} \left(\frac{1 + \frac{(\gamma - 1)}{2} M_1^2}{1 + \frac{(\gamma - 1)}{2} M_2^2} \right)^{\frac{\gamma}{\gamma - 1}}$$

  where we have applied the fact that the stagnation temperature is constant across the shock.

- We see that the static temperature ratio across a shock must also be great than unity.
  - This also makes sense from consideration of entropy rise in terms of kinetic energy of gas molecules
    -
    -
  - Effectively, the shock converts
  - The increase in randomness and decrease in order is associated with the increase in entropy.

Example

Jump Conditions Across a Normal Shock

Example

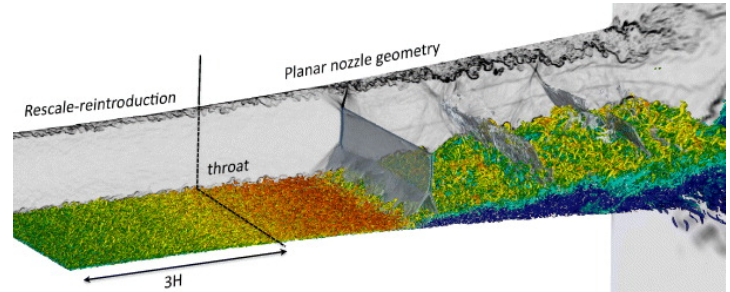
Jump Conditions Across a Normal Shock

Example

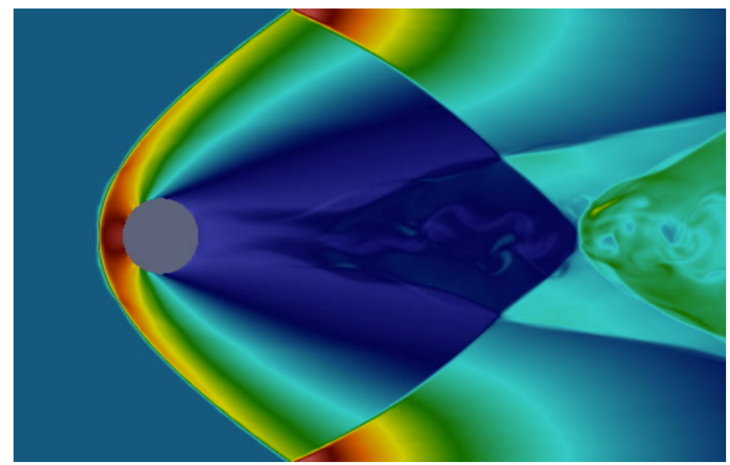
Jump Conditions Across a Normal Shock

Oblique Shock Waves

- Unlike a normal shock, for which the downstream flow remains in the same direction (orthogonal to the plane of the shock), an oblique shock deflects the flow through an angle θ.
- Oblique shocks occur when a supersonic flow is forced to change direction due to some change in boundary condition, such as:
  - A ramp in the wall of a supersonic wind tunnel
  - The leading edge of an engine inlet.
- For an oblique shock, the upstream flow is always supersonic, but the downstream Mach number may by subsonic, sonic, or supersonic depending on the conditions.



(Olsen and Lele, 2008, APS)



(Lodato)

Oblique Shock Waves

- In this course, we will treat oblique shock waves in a two-dimensional sense
  - Straight, but inclined at an angle to the upstream flow

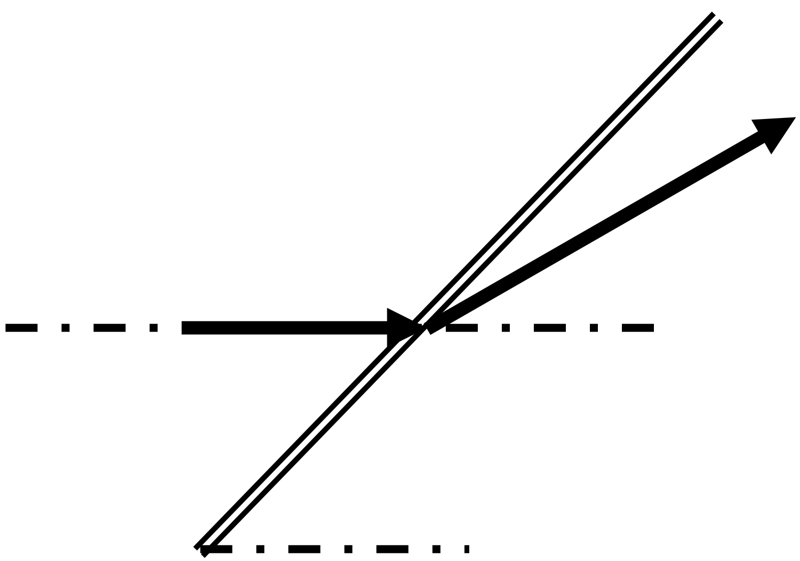
- Summarizing property changes across a normal shock properties

Property	Effect
Entropy, s	
Stagnation temperature, T_o	
Stagnation pressure, p_o	
Mach number, Ma	
Static Temperature, T	
Static Pressure, p	
Velocity, u	
Density, ρ	

Oblique Shock Waves

- The oblique shock relations are derived from the normal shock relations by noting that the oblique shock can not impose a momentum change in the direction parallel to the plane in which it lies.

  - The scalar jump conditions can be determined using the normal shock calculations for the ______________________________.
  - The flow turning angle and shock angle are related to the upstream (free-stream) Mach number with the $\theta - \beta - M$ relation



Oblique Shock Waves

- Shock losses across oblique shock waves are a function of the Mach number normal to the wave, so we can find the change in stagnation pressure, for example, by using the normal shock relations with the normal Mach number

  Where β has been calculated, for the given geometry (defining θ) and M_1.

- The normal shock relations also give us the normal Mach number behind the shock, M_{2n}. This quantity can be related to the actual M_2 value through geometry

- Treatment of property changes across the oblique shocks follow directly from the normal shock relations in this way.