Optimal Disc Release Angles

John Key,[1⋆]
[1]Physics Department, University of Dallas, Irving, TX 75062

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ABSTRACT

Discs are common toys and tools for sports that can be thrown around with vastly different results based on how they are thrown. Some aspects of throwing like speed and spin rate are dependent upon the thrower's ability. However, some aspects like release angle can be changed easily. In this paper, I attempt to calculate the optimal release angles to achieve the greatest distance with a throw. A computational simulation was programmed in python that tract the trajectory of discs based on its initial velocities and angles.

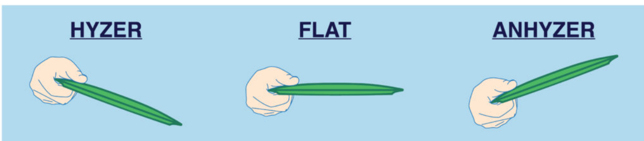

Figure 1. A visualization of roll or ϕ angles. Flat, heizer, and anheizer are associated with a ϕ angle of 0, > 0, and < 0 respectively.

1 INTRODUCTION

Flying discs are gyroscopically stabilized objects that are thrown to fly through the air. Sports with flying discs have become increasingly popular. Ultimate is common across many college campuses, and Disc golf is a great outdoor past time that is increasing in popularity. The nature of spin that stabilizes a disc makes it easy to cut through and remain stable in the wind despite discs' usual small density and high surface area.

With some practice, a person can quickly become familiar with how to throw a disc, and with more practice, can learn how to throw one far. The best way to increase distance when throwing is by increasing velocity. However, this is difficult is more dependent on muscle volume. Knowing how to throw the disc and at what angles is much easier to learn and has similar affect on distance. The primary angles that change are angle of release (AOR), angle of attack (AOA), and roll (ϕ). AOR at what angle off the ground the disc is thrown. It is easy to think of AOR as how high into the air to throw. Angle of attack is the angle that the disc is tilted when thrown of the plane spanned by the disc's velocity. Think of AOA as how nose up the disc is thrown. AOA and AOR can be visualized in Figure 2. Lastly, roll is the angle that the disc is rotated about the x-axis. Think of roll as whether the edge of the disc that is away from the thrower's body is tilted up or down. Roll can be seen and named in Figure 1.

To test how these three angles affect maximum distance in different discs, a python program was written that simulated the physics of the flight of a disc. A disc with certain physical attributes, a throw with certain initial velocities and angles, and a system of differential equations are able to simulate the trajectory of a disc. The equations of motion of gyroscopically stabilized flying objects has long been studied for military and other purposes, but the specific forces on a disc golf disc were not well documented until Hummel (2003) published the most consolidated paper on the physics of disc golf yet, which is now the authority that is commonly referenced. Continuing off of Hummel's research there has been much studied, but most helpfully was Kamaruddin (2011) who used the physiscs simulator that Hummel started, and took real-world wind tunnel data to measure some physical coefficients which Hummel used in her simulator.

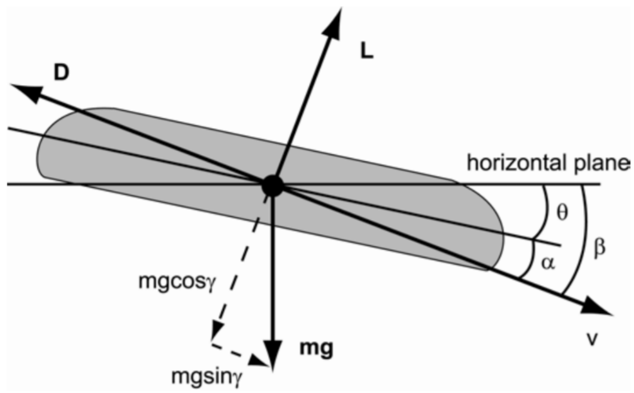

Figure 2. A visualization of angle of attack (AOA), where β is the AOR, and α is the AOA.

2 METHODS

This paper is to meant to discern the best AORs, AOAs, and roll angles for discs to maximize distance of the disc before the disc hits the ground. As mentioned previously and consistent with similar research in the past, many values of the disc's flight are ignored or held constant. Velocity as an initial value is held constant through all of the tests. While the spin of a disc is necessary for stabilization, it does not contribute much to range as noted by Kamaruddin (2011), so spin rate was held constant when applicable. Wind was held constant at $0m/s$, but could be implemented in the software easily. The variables that were changed in testing were the angles aforementioned and the disc used. A python version of the Matlab simulator by Crowther & Potts (2007) was coded with their six degree of freedom model

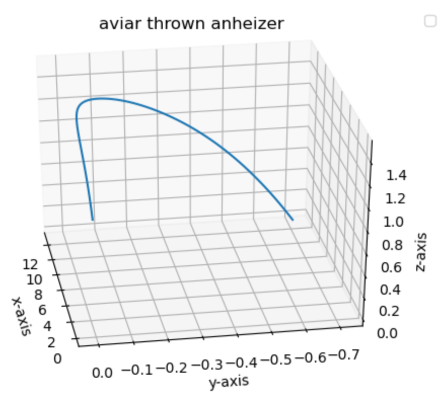

Figure 3. The 3D flight of the Aviar disc thrown with an anheizer angle. The axis are automatically scalled to see change in direction. The disc was thrown $1m$ off the ground.

and the differential equations of Hummel (2003). Additionally, disc attributes from Kamaruddin (2011) were used for each disc that was tested. All of these were combined into one program that simulated disc flights and compared the effects of the angles on flight range. The program is made up of four python files: simulation.py, disc.py, throw.py, and flight.py. The simulation.py file calculates and plots the trajectory and stores the data in flight.py. Simulation objects store a disc object and throw object. An example flight can be seen in 3. Once the program was assembled varying tests were run changing certain variables. The three angles were tested independently and multiple discs were used to check consistency.

2.1 Equations

The primary equations of motion used were from Hummel (2003). The first three are used to calculate linear forces on the disc. They are the lift (L) and drag (D) forces due to the disc cutting through the air.

$$L = C_L \frac{1}{2}\rho v^2 A \quad (1)$$

$$D = C_D \frac{1}{2}\rho v^2 A \quad (2)$$

Where ρ, v, and A are air density, speed, and planform area (area seen from above) respectively. The coefficients are results of the physical shape and material of the disc. They are also functions of the angle of attack (α) and angle of attack at zero lift (α_0). They are as follow.

$$C_L = C_{L_0} + C_{L_\alpha}\alpha \quad (3)$$

$$C_D = C_{D_0} + C_{D_\alpha}(\alpha - \alpha_0) \quad (4)$$

The equations for angular moments with their coefficients are as follows.

$$R = C_R \frac{1}{2}\rho v^2 A \quad (5)$$

$$M = C_M \frac{1}{2}\rho v^2 A \quad (6)$$

$$N = C_N \frac{1}{2}\rho v^2 A \quad (7)$$

$$C_R = C_{R_r} r + C_{R_p} p \quad (8)$$

$$C_M = C_{M_0} + C_{M_\alpha}\alpha + C_{M_q} q \quad (9)$$

$$C_N = C_{N_r} r \quad (10)$$

Where R, M, N, p, q, and r are roll, pitch, and yaw angular moments and velocities. Using the above moments. The angular accelerations can be calculated using the following equations according to Hummel (2003).

$$\ddot{\phi} = \frac{(R + I_{xy}\dot{\theta}\dot{\phi}sin\theta - I_z\dot{\theta}(\dot{\phi}sin\theta + \dot{\gamma}) + I_{xy}\dot{\theta}\dot{\phi}sin\theta)cos\theta}{I_{xy}} \quad (11)$$

$$\ddot{\theta} = \frac{(M + I_z\dot{\phi}cos\theta(\dot{\phi}sin\theta + \dot{\gamma}) - I_y\dot{\phi}\dot{\phi}cos\theta sin\theta)}{I_{xy}} \quad (12)$$

$$\ddot{\gamma} = \frac{(N - I_z\ddot{\phi}sin\theta + \dot{\theta}\dot{\phi}cos\theta)}{I_z} \quad (13)$$

3 RESULTS

After completing the tests, the results produced by the simulator is as follows. Time steps for the OEDs were held constant at $0.01s$ for all tests. The results did not change when reducing the time step past that size.

For varying the ϕ or heizer angle, a specific disk was used whose coefficients were calculated by Hummel (2003). The relationship between ϕ and range is shown in 4 below.

Next, the angle of attack (AOA) was tested. For this test, multiple discs were tested in case putters and drivers reacted differently to the change in AOA. The results are shown below in 5.

Lastly, the angle of release (AOR) was tested. The calculations were done similarly to the test above of the AOA. the results are in 6.

4 ANALYSIS

The three tests were done with varying success. Firstly the roll, ϕ, or heizer angle found that an extremely anheizer angle was favourable. This is as expected, but the result of $-48.7deg$ was greater than expected. As discs fly, they tend to rotate towards a $+\phi$ angle. To counteract that roll, discs are thrown with a $-\phi$ angle to achieve maximum flight. $-48.7deg$ is not out of the realm of possible throws. Kamaruddin (2011) found that roll angle greatly decreases maximum distance after $-30deg$. That does occur in 4, but not until after $-50deg$. The error in this test seemed to skew data towards a more $-\phi$ angle by approximately $-20deg$.

Next, the AOA test in 5 produced the most unexpected results. The test was only done in the range $0deg$ and $20deg$, because in

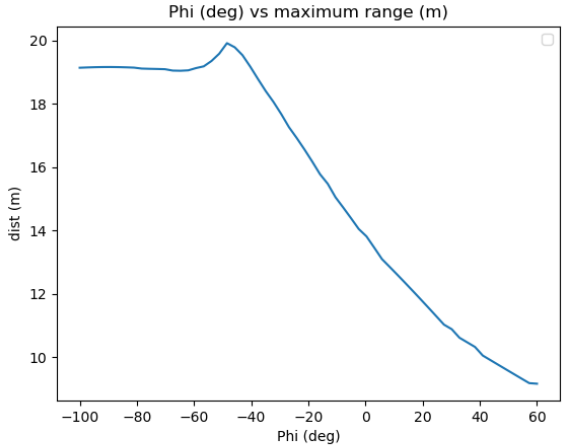

Figure 4. The relationship between ϕ angle at release and maximum distance. Initial velocity was kept at $20m/s$, AOR at $10deg$, and AOA $10.5deg$. The maximum range was reached when ϕ was $-48.7deg$.

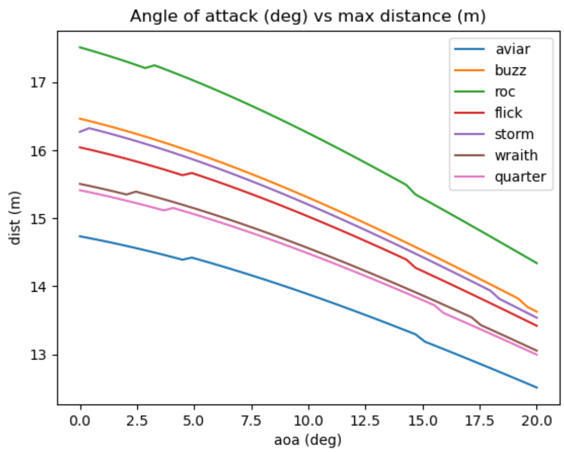

Figure 5. The relationship between AOA at release and maximum distance. Initial velocity was kept at $20m/s$, AOR at $15deg$, and ϕ at $0deg$.

field tests, a − AOA is unrealistic. It is a throw with the nose of the disc tilted down, the disc cannot fly this way. The test indicated that a smaller AOA towards flat produced the greatest range, but the test also indicated that a negative AOA would have been better. The discontinuous plotted lines are also indicative of some simulation error. This test would have to be checked by other tests, because the data is far from expected.

Finally, the AOR test in 6 the best results. It was expected that some angle up on release would be beneficial above 0, but at a certain angle the disc would stall out. This test used multiple discs and they all operated as expected. The driver discs all flew further than the putters. The test found that the best launch angles were between $27deg$ and $30deg$ depending on the disc. Angles greater than those ended up not going as far due to stalling out as it is called. This is as expected. However, Kamaruddin (2011) found that $10deg$ to $15deg$ was optimal for AOR. Again, there seems to be a skew of results while the shape of results is similar.

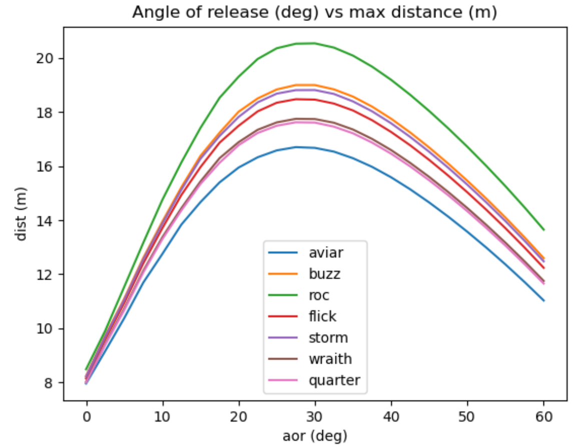

Figure 6. The relationship between AOR at release and maximum distance. Initial velocity was kept at $20m/s$, AOA at $1deg$, and ϕ at $0deg$. The maximum ranges were reached when AOR was between $27deg$ and $30deg$.

Throughout the tests, it was found that, while the discs all flew differently, they all had similar optimal angles.

5 CONCLUSIONS

The exact values of the results may not be completely applicable to real world performance. Discs are always slightly beat up and change flight characteristics. Other variables are never completely constant, especially wind which will greatly change the flight of a disc. However, knowing that there is a certain optimal AOR for distance is important. Whenever throwing that optimal angle can be quickly figured out with some test throws depending on the wind. Again the amount of heizer angle to throw the disc with can be quickly be recalculated if you know what to look for. The most helpful result of this paper is understanding the equations of motion behind the disc. Realizing that all of the forces and moments depend upon AOA chnages ones understanding of disc flight and explains why discs fall to the ground slowly the way they do.

REFERENCES

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Hummel S., 2003, UC Davis, 1, 1
Kamaruddin N. M., 2011, U of Manchester, 1, 1

APPENDIX A: SIMULATOR PROGRAM

Code can be found on https://github.com/simpleJavaJohn/Disc-Flight-Simulator with python files, text files, and some tests ran. The primary file with physics simulation is simulation.py. The code didn't show up well in this PDF.

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