

KERNEL TRICK

Quiz: Which equation could come to our rescue?

☐ $x + y$

☐ xy

☐ $x^2 + y^2$

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	$(-3,-3)$	$(3,-3)$	$(-3,3)$	$(3,3)$				

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	$(-3,-3)$	$(3,-3)$	$(-3,3)$	$(3,3)$	$(-1,-1)$	$(1,-1)$	$(-1,1)$	$(1,1)$
$x + y$	-6							

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	$(-3,-3)$	$(3,-3)$	$(-3,3)$	$(3,3)$	$(-1,-1)$	$(1,-1)$	$(-1,1)$	$(1,1)$
$x + y$	-6	0	0	6	-2	0	0	2

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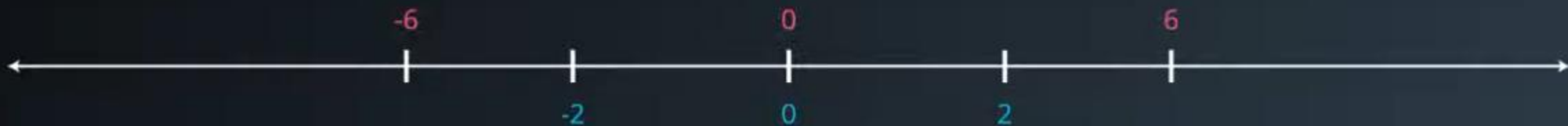


KERNEL TRICK

	$(-3,-3)$	$(3,-3)$	$(-3,3)$	$(3,3)$	$(-1,-1)$	$(1,-1)$	$(-1,1)$	$(1,1)$
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xy								

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	$(-3,-3)$	$(3,-3)$	$(-3,3)$	$(3,3)$	$(-1,-1)$	$(1,-1)$	$(-1,1)$	$(1,1)$
$x + y$	-6	0	0	6	-2	0	0	2
xy	9							

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KERNEL TRICK

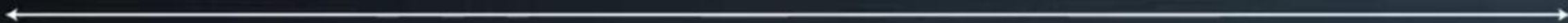
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$x + y$	-6	0	0	6	-2	0	0	2
xy	9	-9						

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KERNEL TRICK

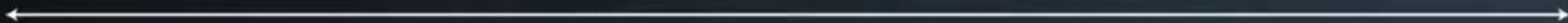
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$x + y$	-6	0	0	6	-2	0	0	2
xy	9	-9	-9	9	1	-1	-1	1

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KERNEL TRICK

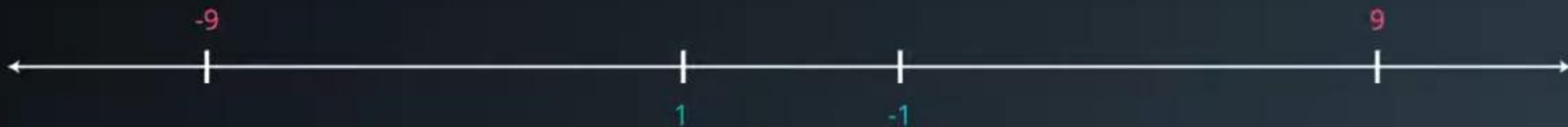
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$x + y$	-6	0	0	6	-2	0	0	2
xy	9	-9	-9	9	1	-1	-1	1
$x^2 + y^2$	18							

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$x + y$	-6	0	0	6	-2	0	0	2
xy	9	-9	-9	9	1	-1	-1	1
$x^2 + y^2$	18	18	18	18	2	2		

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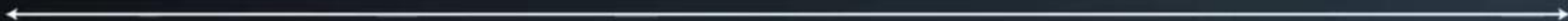
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$x + y$	-6	0	0	6	-2	0	0	2
xy	9	-9	-9	9	1	-1	-1	1
$x^2 + y^2$	18	18	18	18	2	2	2	2

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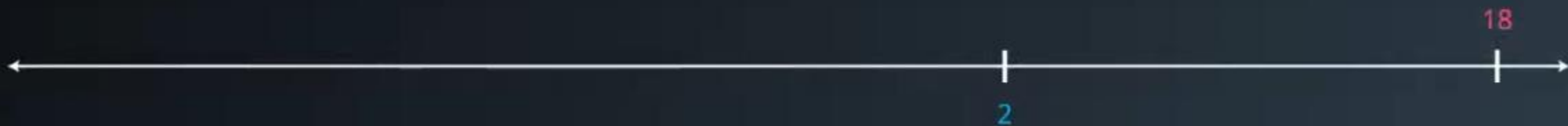
	$(-3,-3)$	$(3,-3)$	$(-3,3)$	$(3,3)$	$(-1,-1)$	$(1,-1)$	$(-1,1)$	$(1,1)$
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xy	9	-9	-9	9	1	-1	-1	1
$x^2 + y^2$	18	18	18	18	2	2	2	2

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KERNEL TRICK

$$x^2 + y^2 = 2$$

KERNEL TRICK

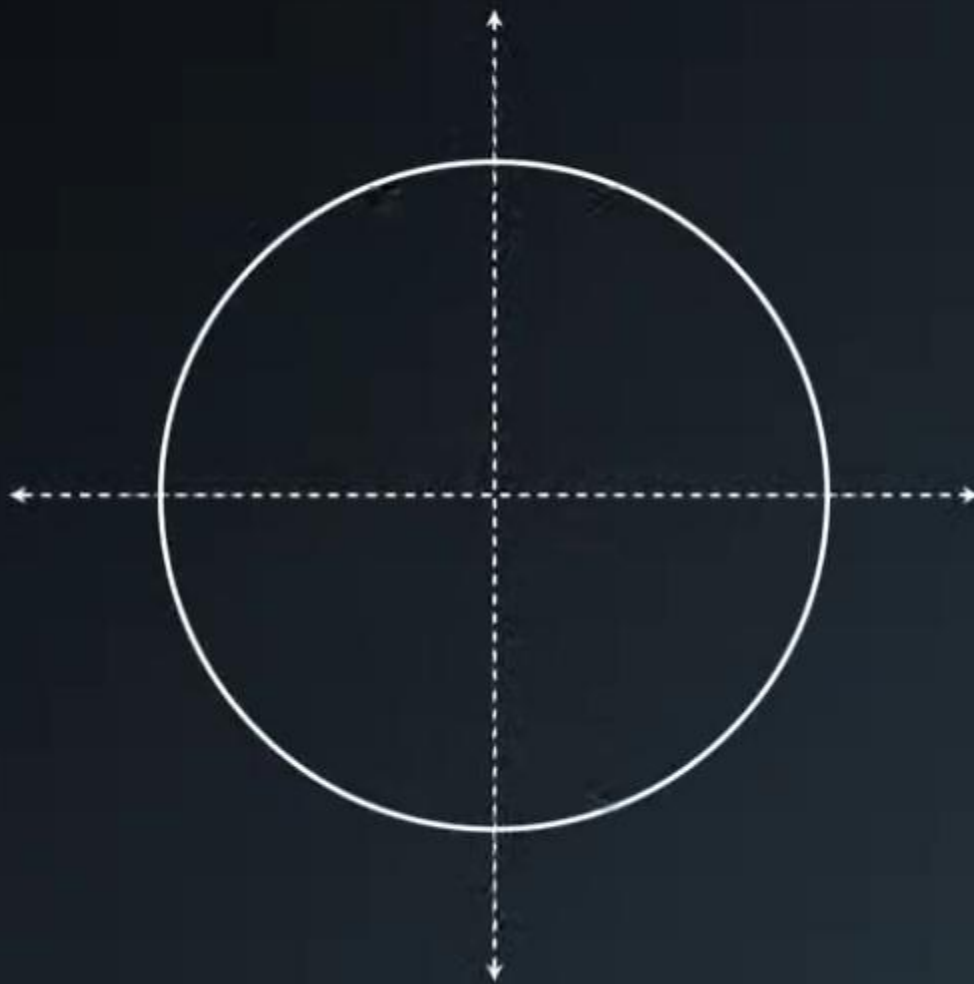
$$x^2 + y^2 = 2$$

$$x^2 + y^2 = 18$$

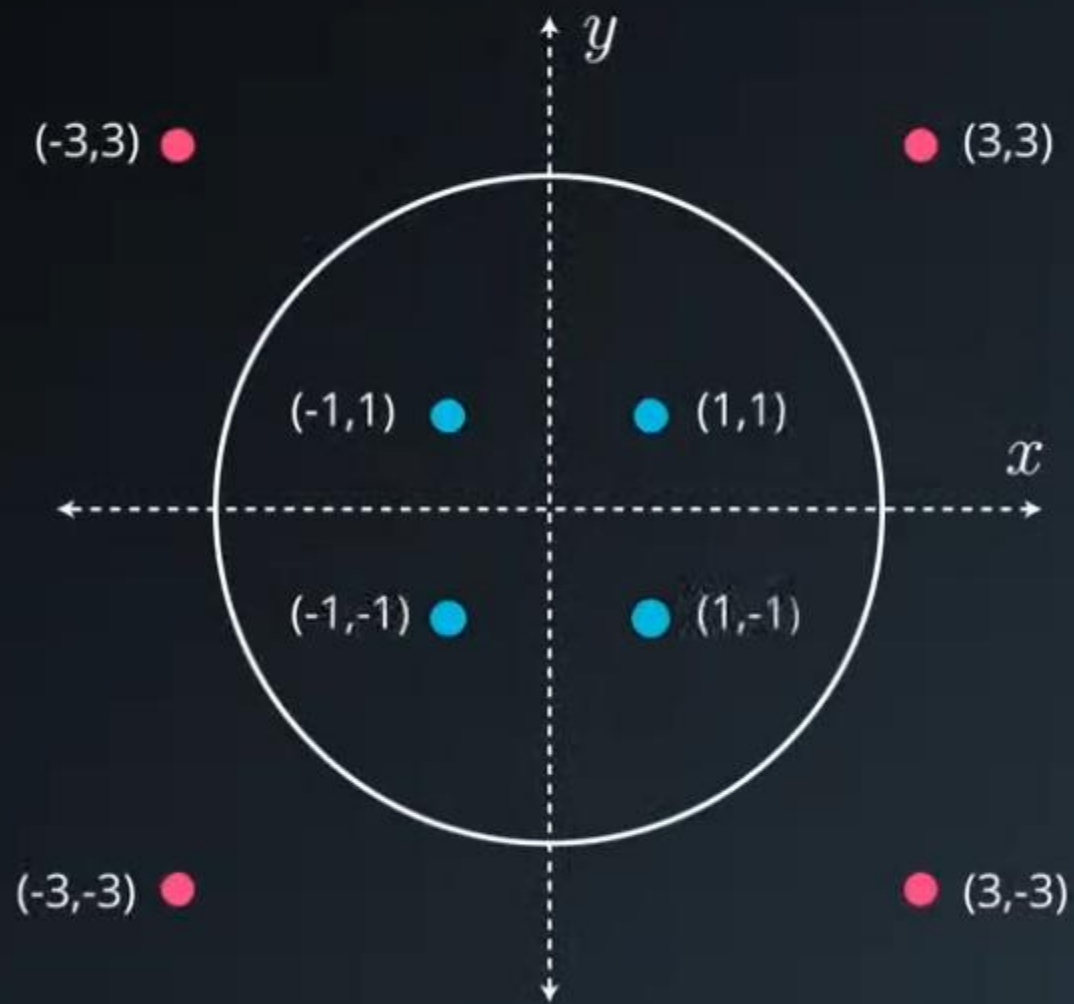
KERNEL TRICK

$$x^2 + y^2 = 2 \quad x^2 + y^2 = 10 \quad x^2 + y^2 = 18$$

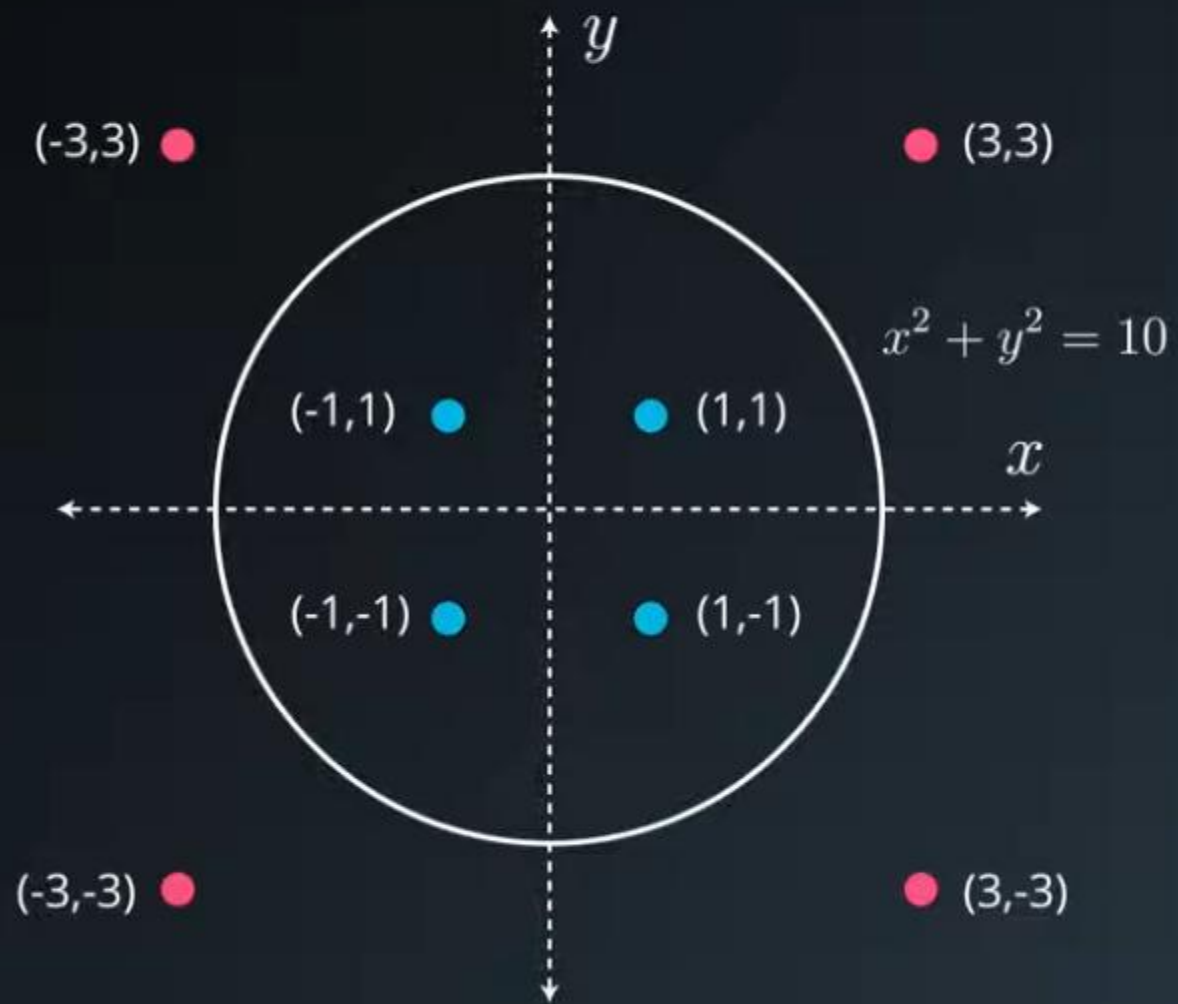
KERNEL TRICK



KERNEL TRICK



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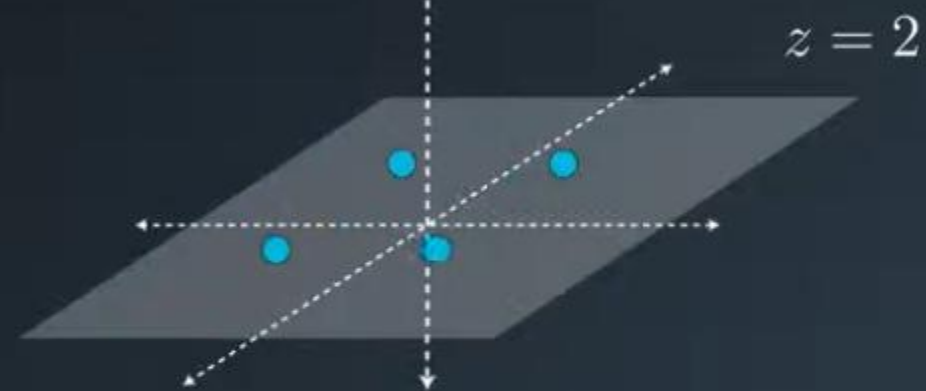


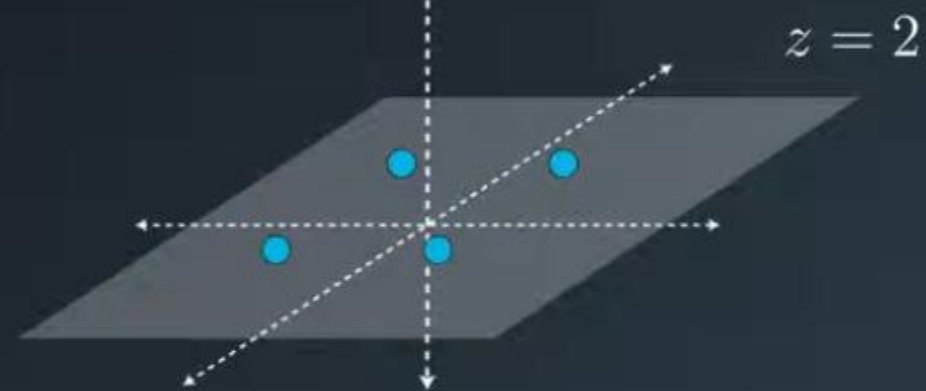
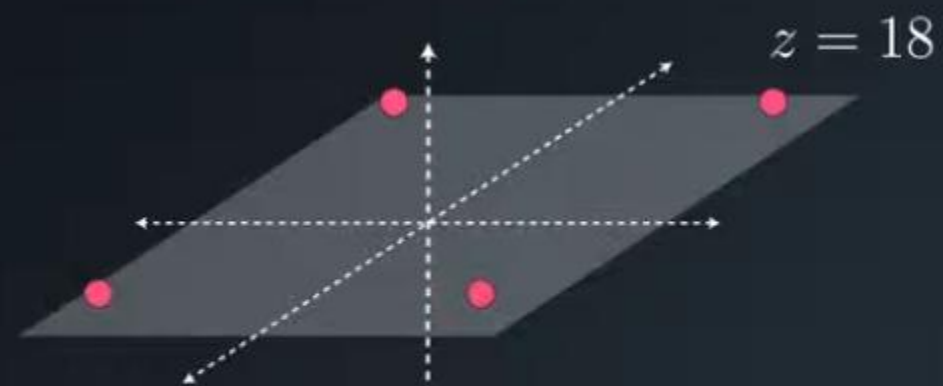


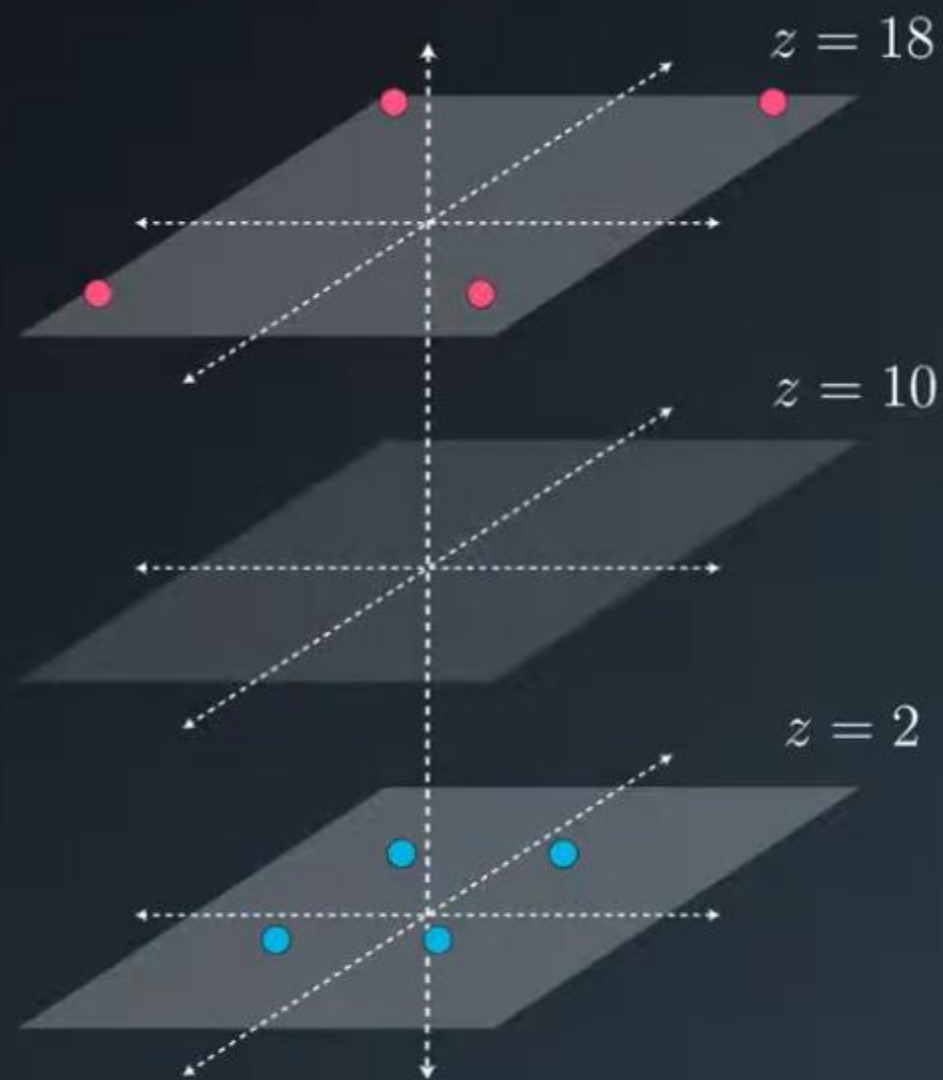


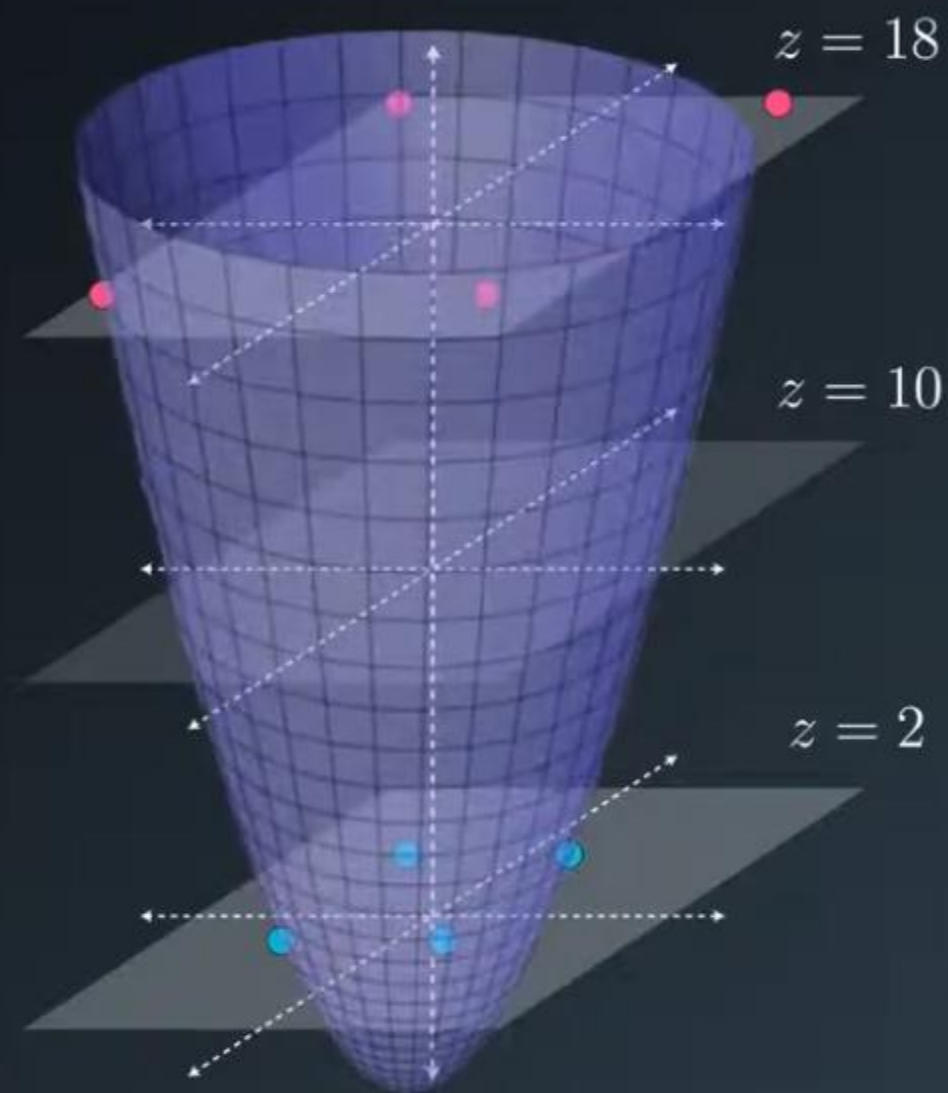




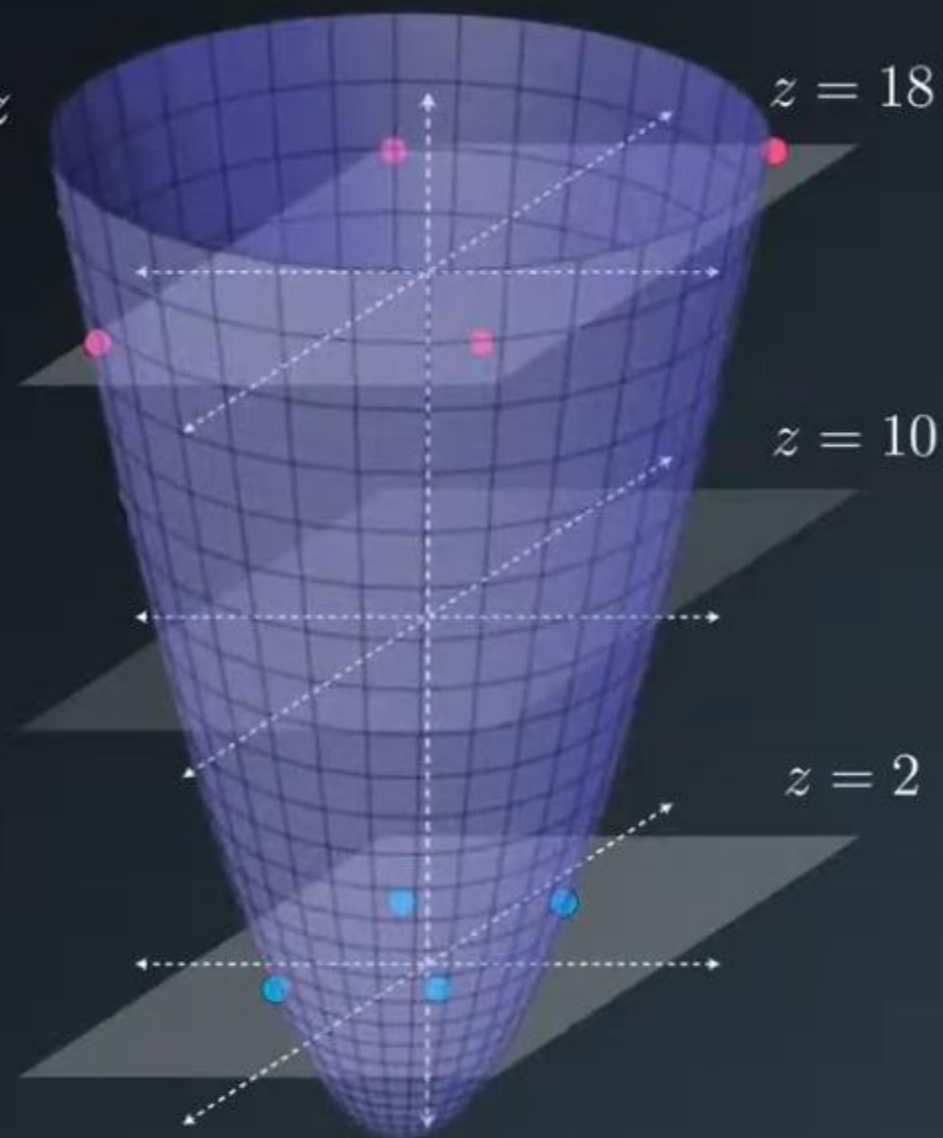




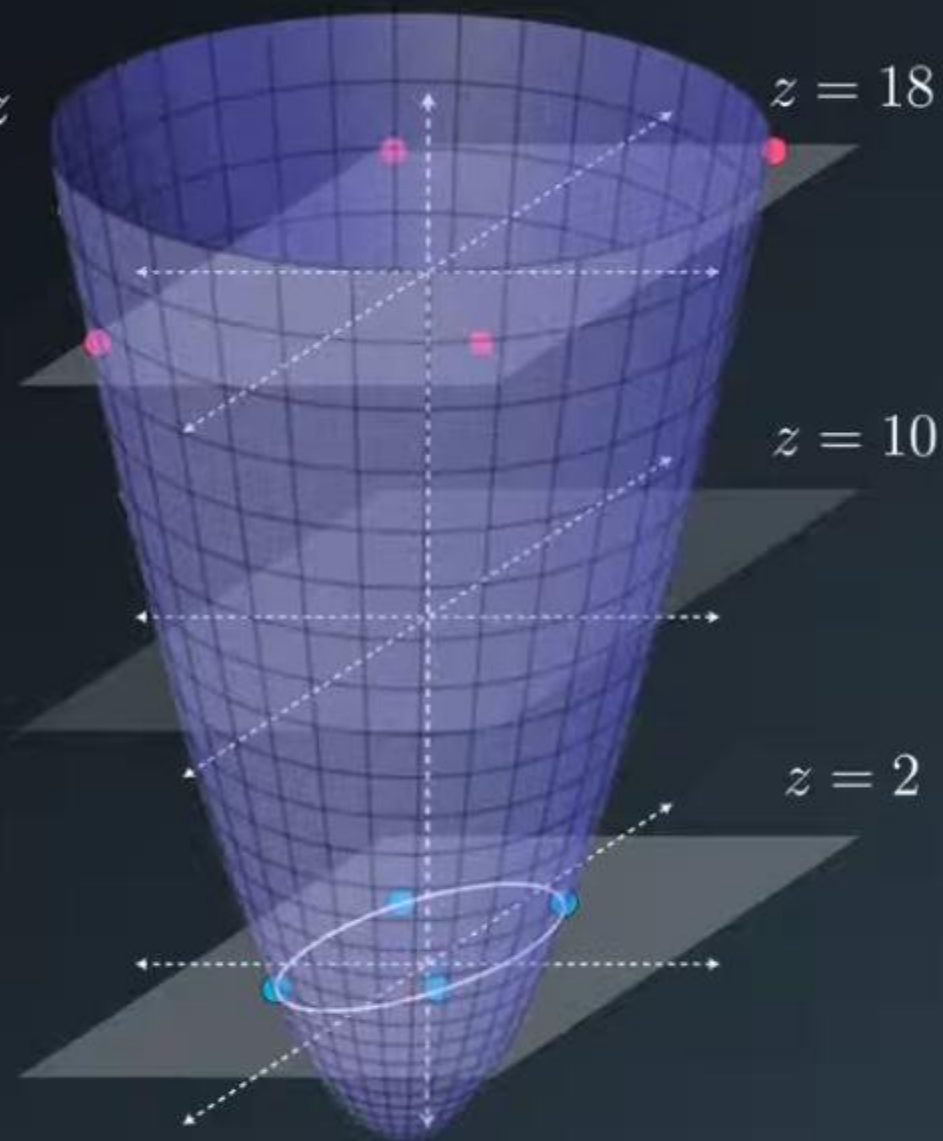




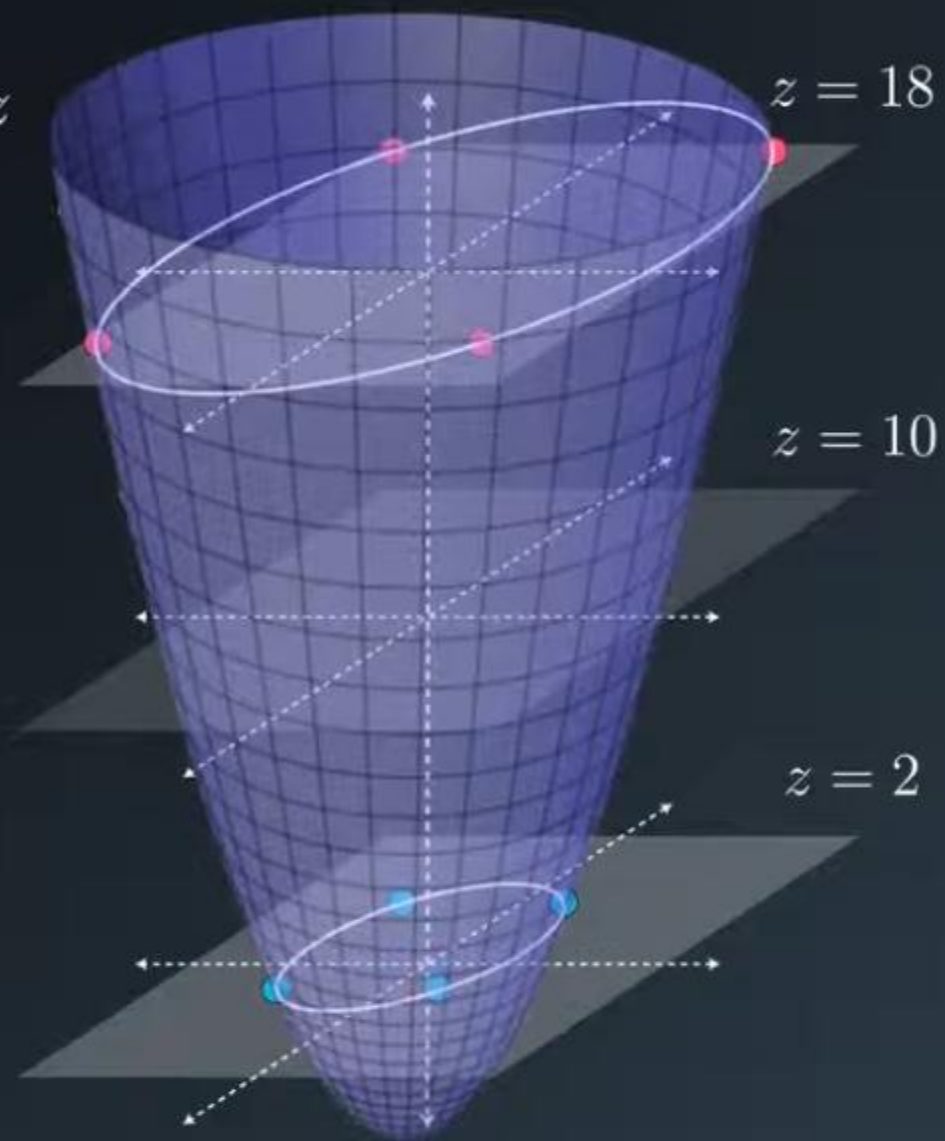
$$x^2 + y^2 = z$$



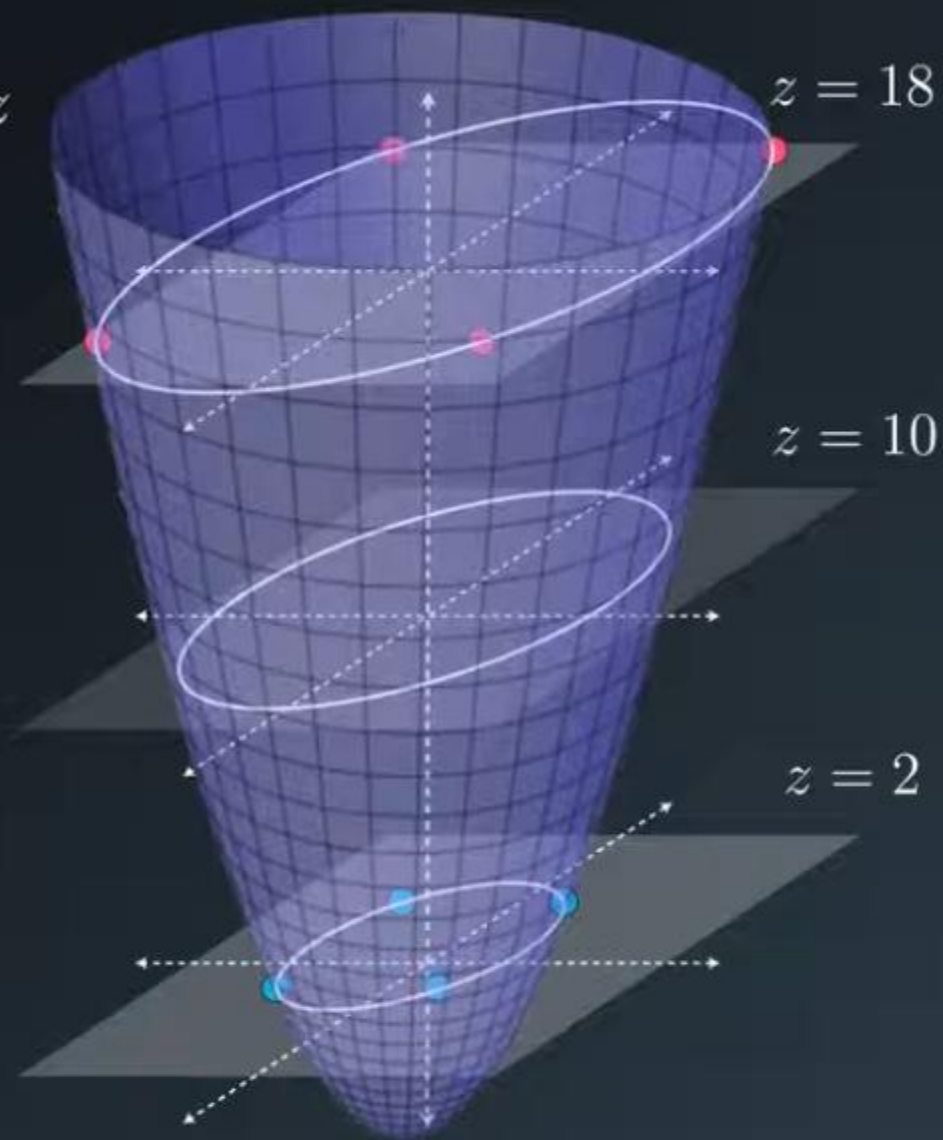
$$x^2 + y^2 = z$$



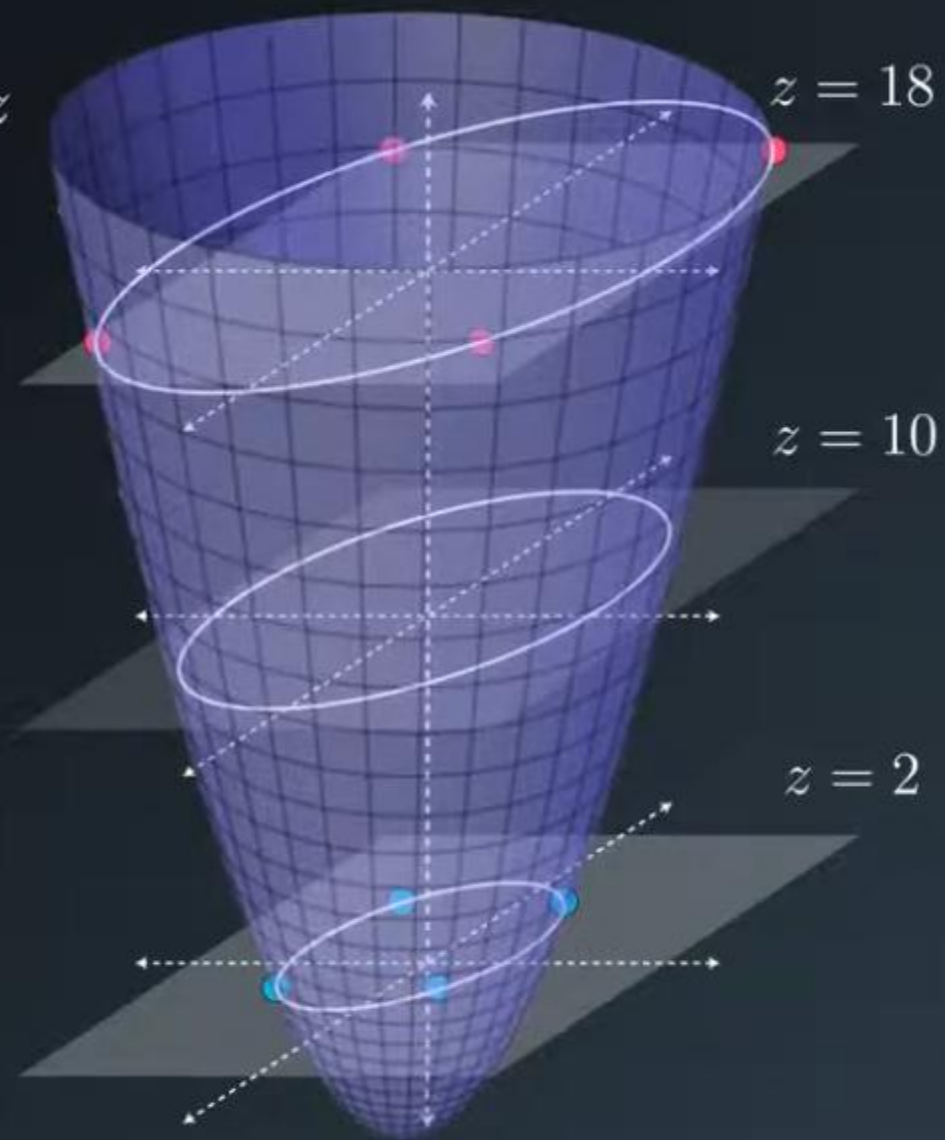
$$x^2 + y^2 = z$$



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$$x^2 + y^2 = z$$



KERNEL TRICK

KERNEL TRICK

2 Dimensions

5 Dimensions

$$(x, y) \longrightarrow (x, y, x^2, xy, y^2)$$

KERNEL TRICK

2 Dimensions

$$\begin{pmatrix} x \\ y \end{pmatrix}$$
$$(2, 3)$$



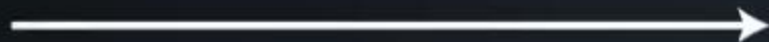
5 Dimensions

$$\begin{pmatrix} x \\ y \\ x^2 \\ xy \\ y^2 \end{pmatrix}$$
$$(2, 3, 4, 6, 9)$$

KERNEL TRICK

2 Dimensions

$$\begin{pmatrix} x \\ y \end{pmatrix}$$
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5 Dimensions

$$\begin{pmatrix} x \\ y \\ x^2 \\ xy \\ y^2 \end{pmatrix}$$
$$(2, 3, 4, 6, 9)$$

4-dimensional
boundary
hyperplane

KERNEL TRICK

2 Dimensions

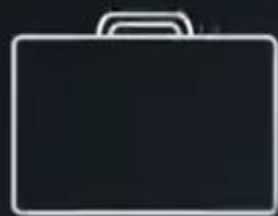
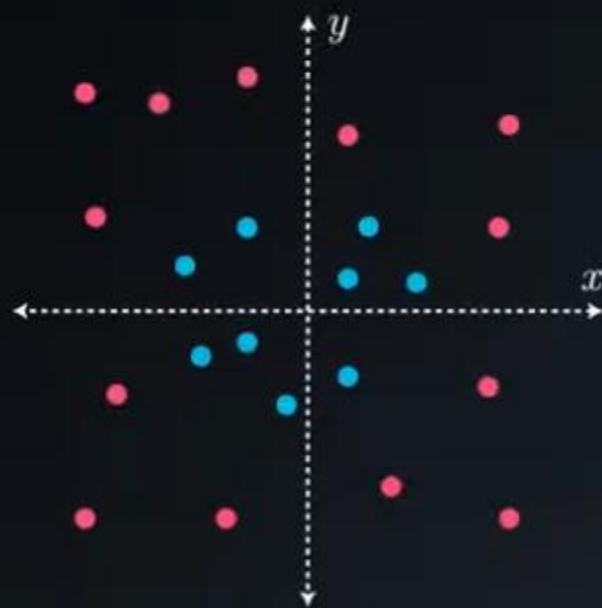
5 Dimensions

$$\begin{array}{ccc} (x, y) & \longrightarrow & (x, y, x^2, xy, y^2) \\ (2, 3) & & (2, 3, 4, 6, 9) \end{array}$$

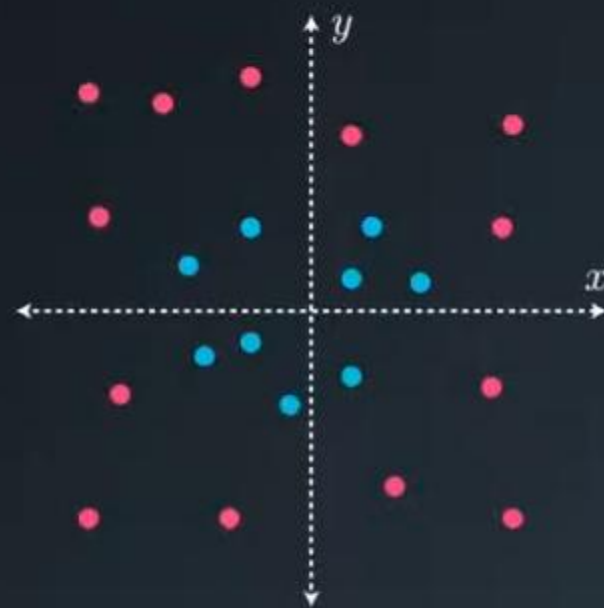
Degree 2
Polynomial
boundary

4-dimensional
boundary
hyperplane

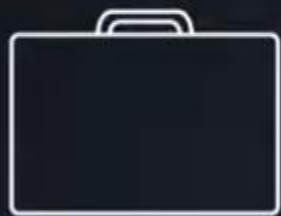
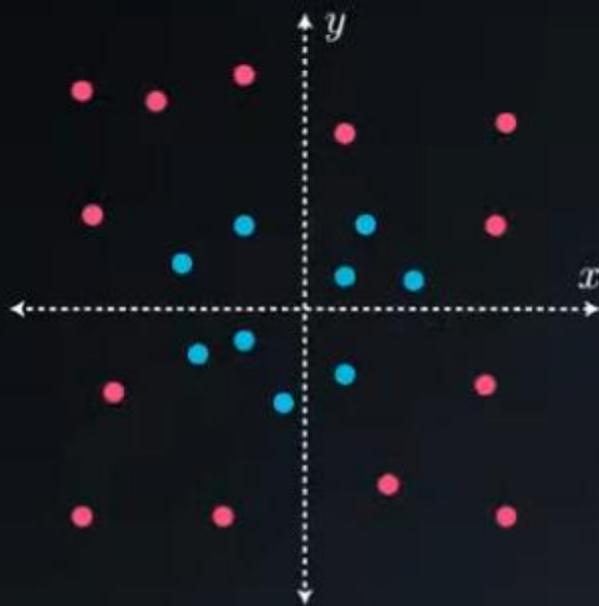
KERNEL



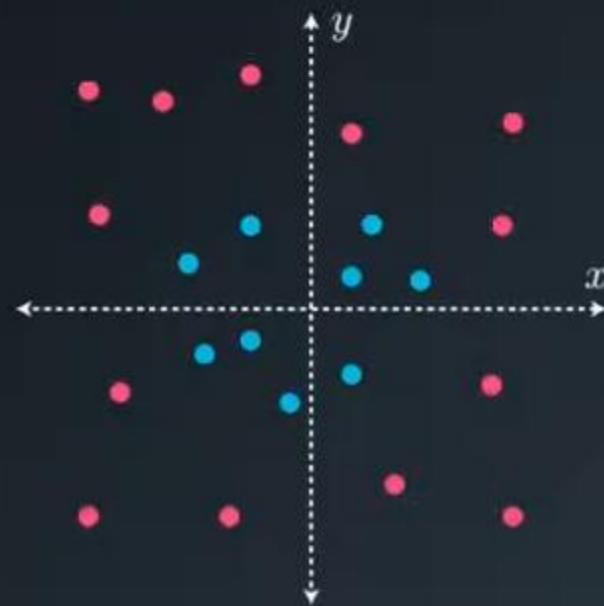
Linear Kernel



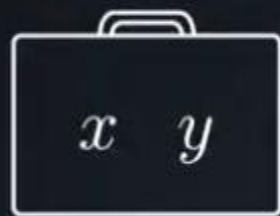
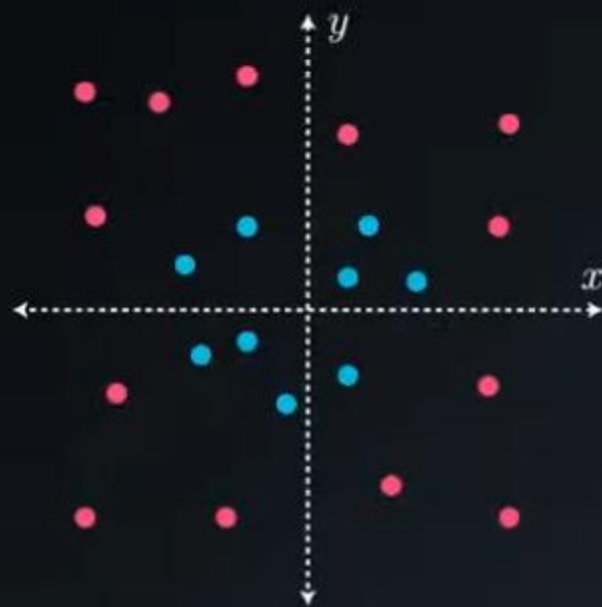
KERNEL



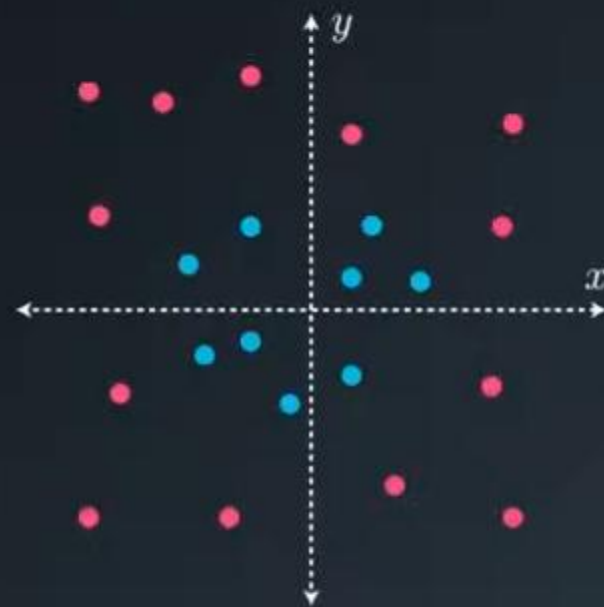
Linear Kernel



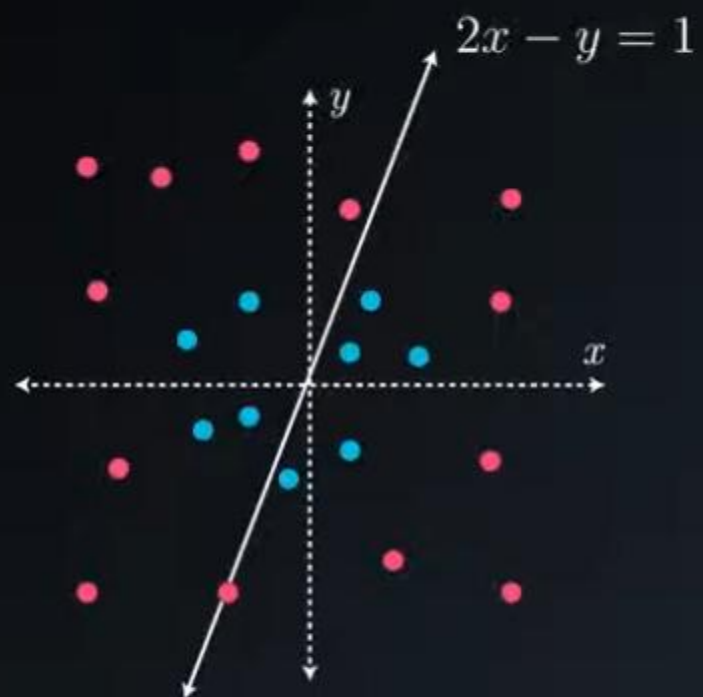
KERNEL



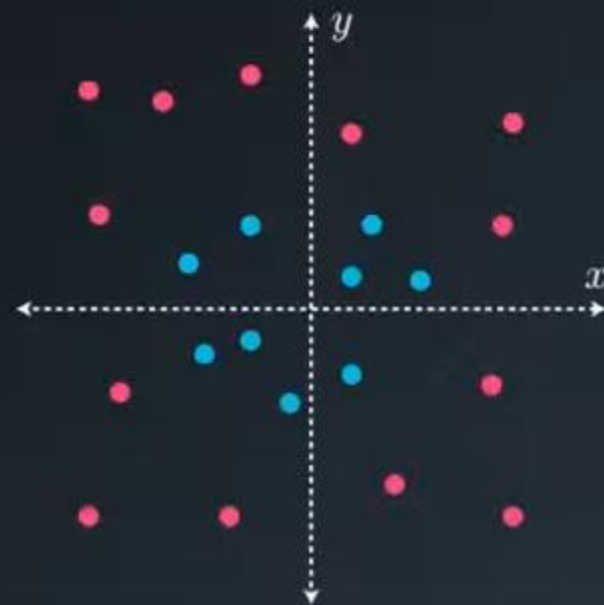
Linear Kernel



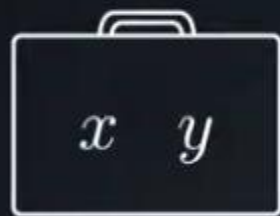
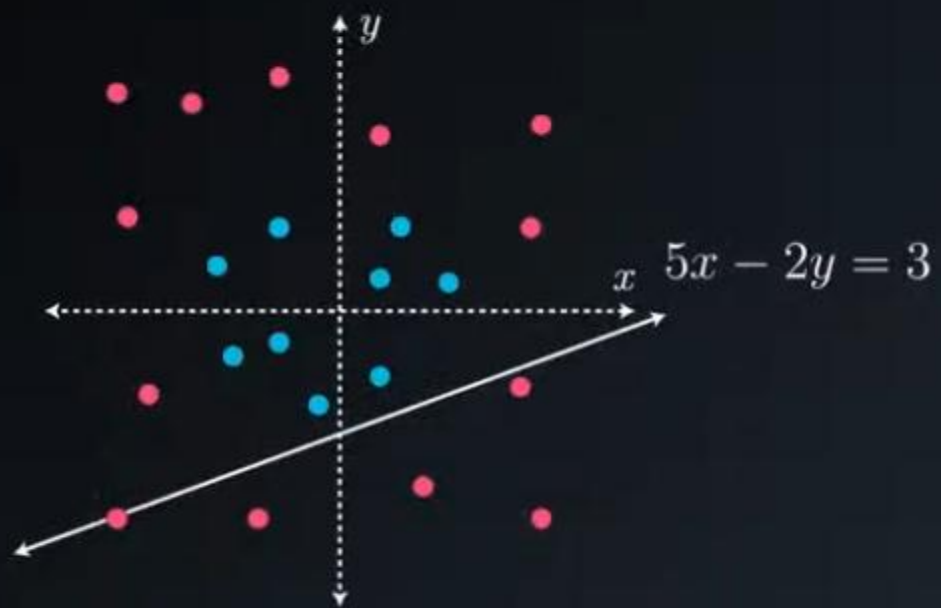
KERNEL



Linear Kernel

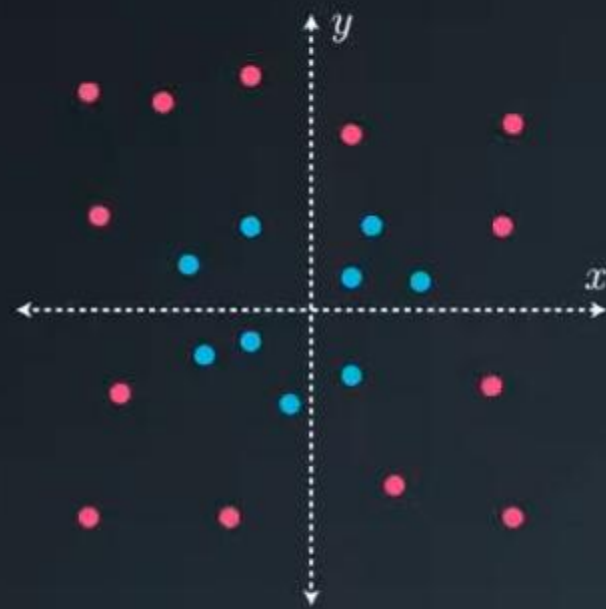


KERNEL

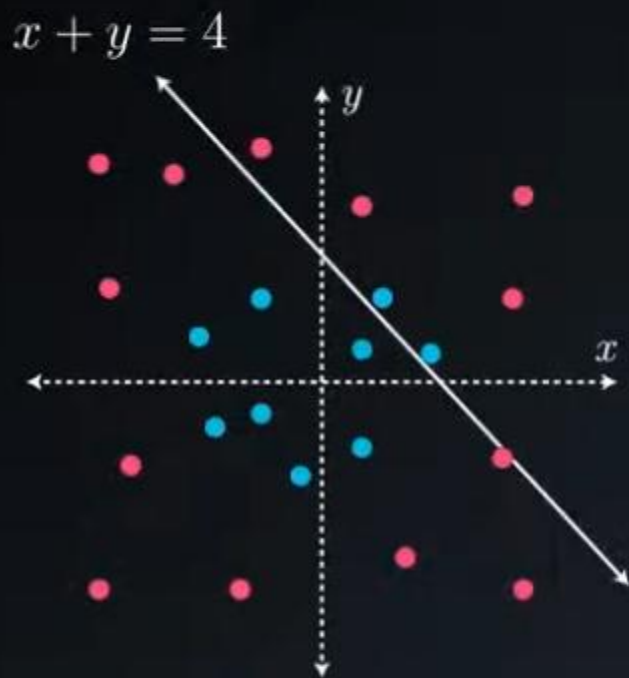


$$2x - y = 1$$

Linear Kernel

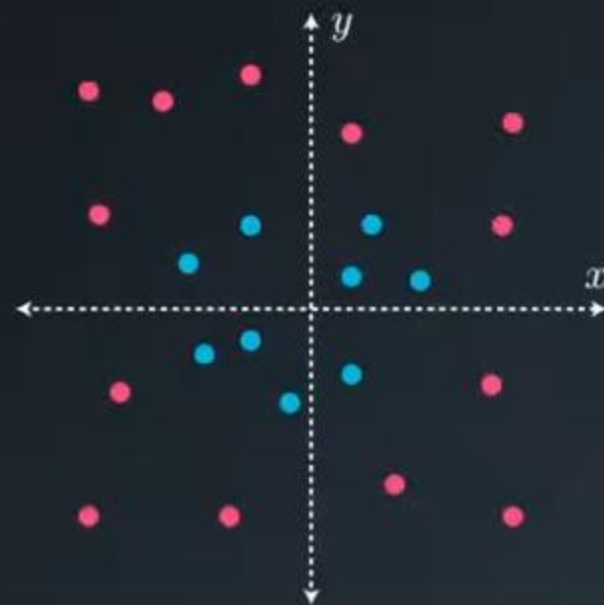


KERNEL

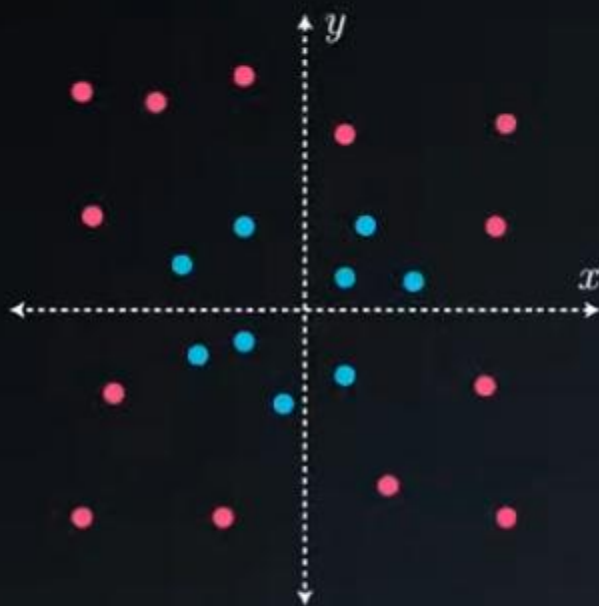


$$\begin{aligned} 2x - y &= 1 \\ 5x - 2y &= 3 \end{aligned}$$

Linear Kernel

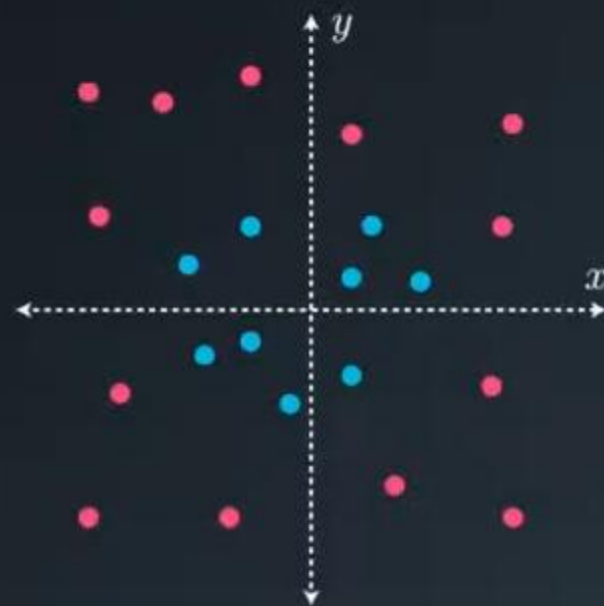


KERNEL

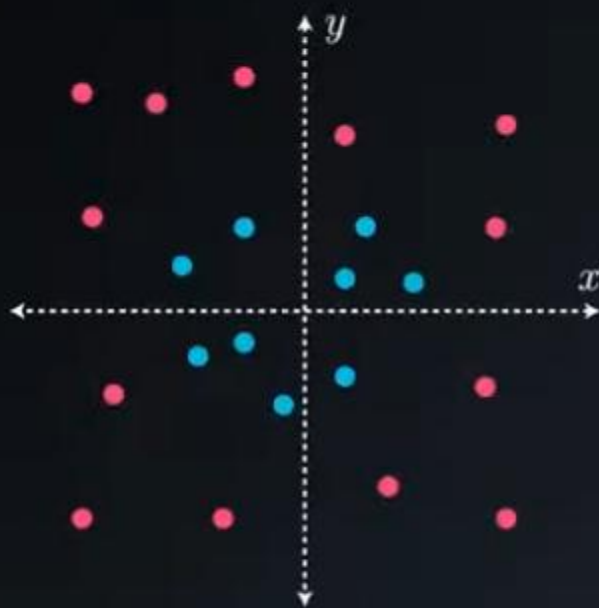


$$\begin{aligned}2x - y &= 1 \\5x - 2y &= 3 \\x + y &= 4\end{aligned}$$

Linear Kernel

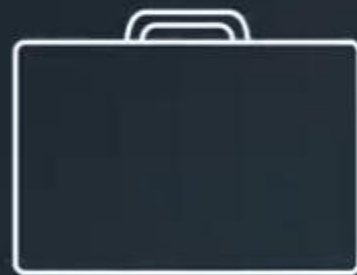
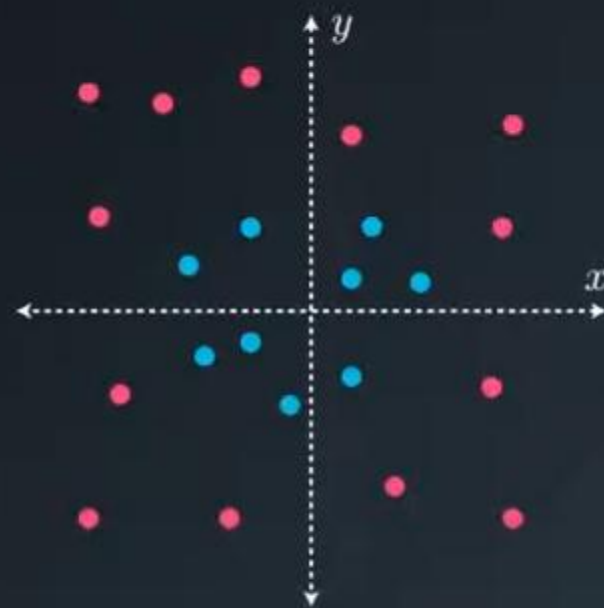


KERNEL



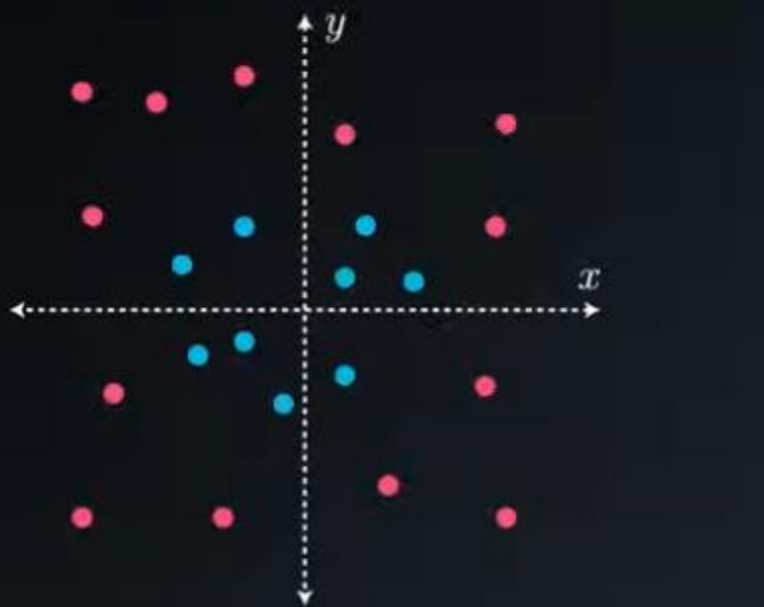
$$\begin{aligned}2x - y &= 1 \\5x - 2y &= 3 \\x + y &= 4\end{aligned}$$

Linear Kernel



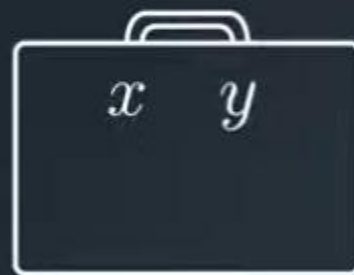
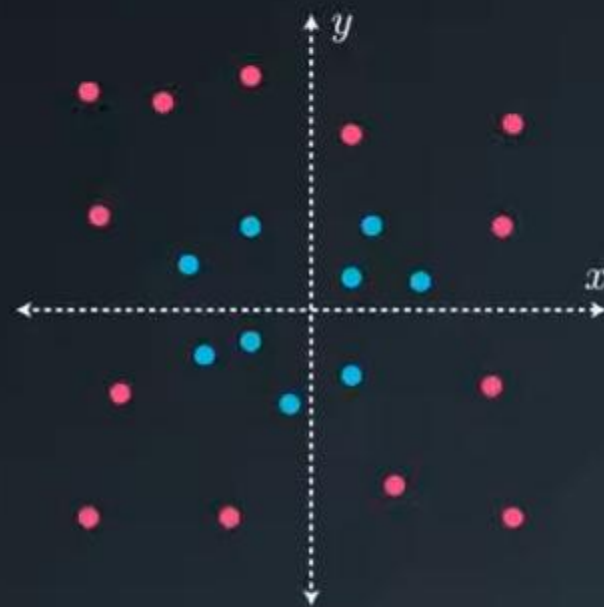
Polynomial Kernel

KERNEL



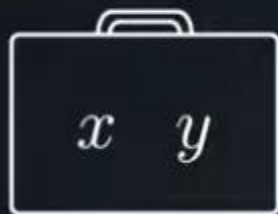
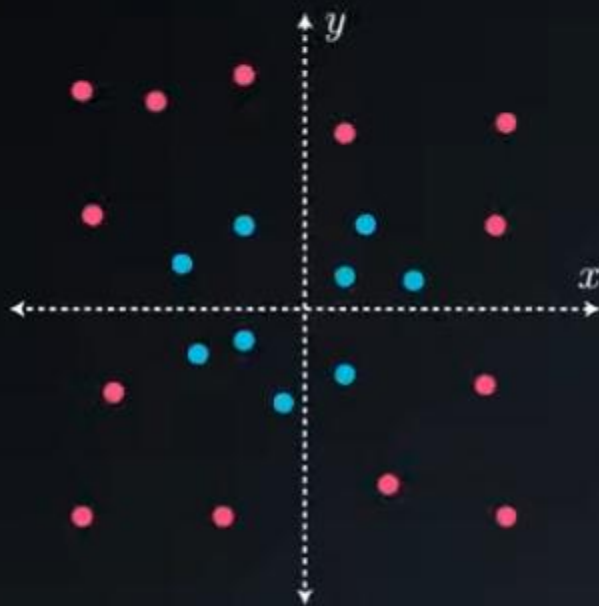
$$\begin{aligned} 2x - y &= 1 \\ 5x - 2y &= 3 \\ x + y &= 4 \end{aligned}$$

Linear Kernel



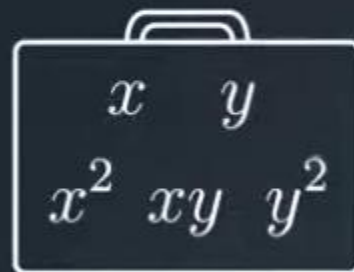
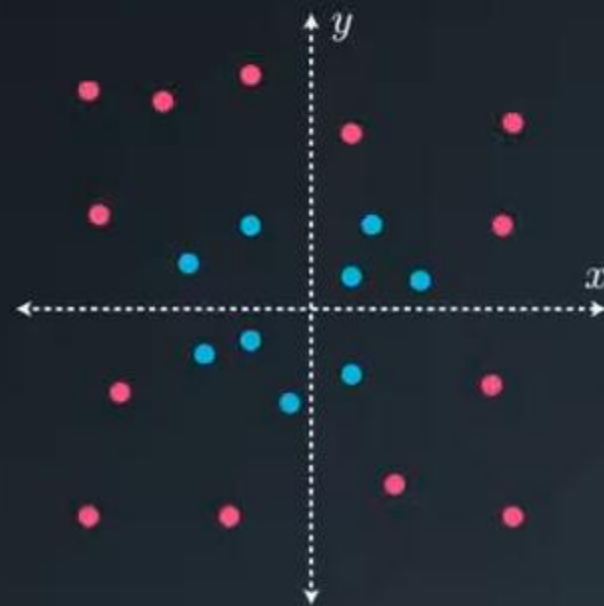
Polynomial Kernel

KERNEL



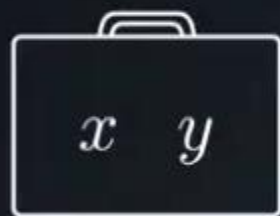
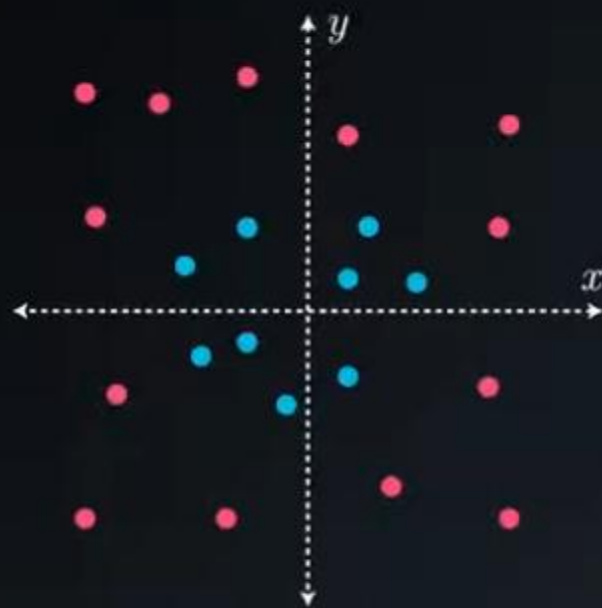
$$\begin{aligned} 2x - y &= 1 \\ 5x - 2y &= 3 \\ x + y &= 4 \end{aligned}$$

Linear Kernel



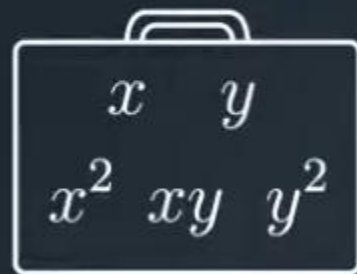
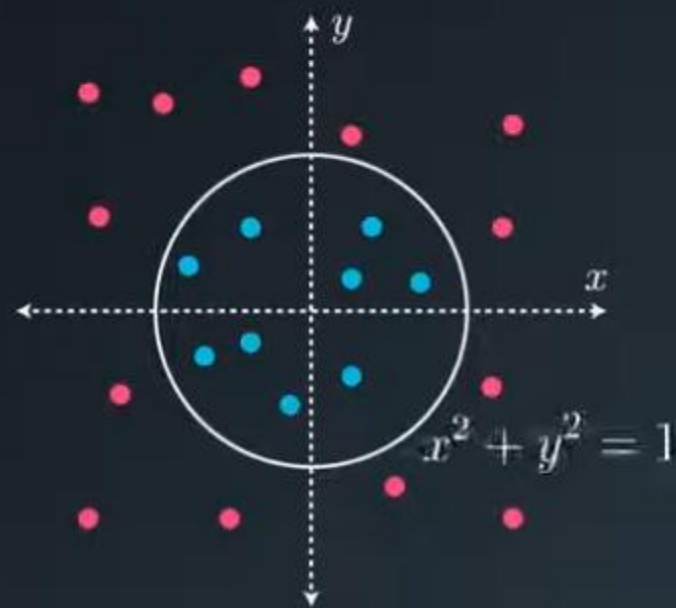
Polynomial Kernel

KERNEL



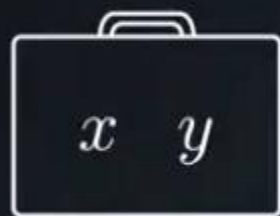
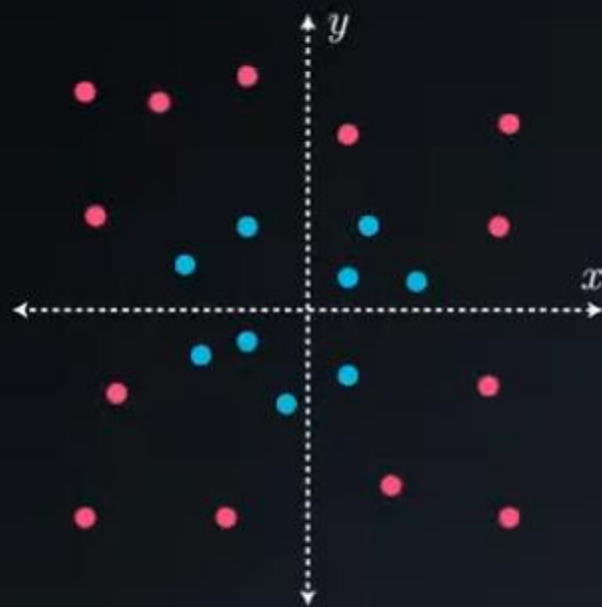
$$\begin{aligned} 2x - y &= 1 \\ 5x - 2y &= 3 \\ x + y &= 4 \end{aligned}$$

Linear Kernel



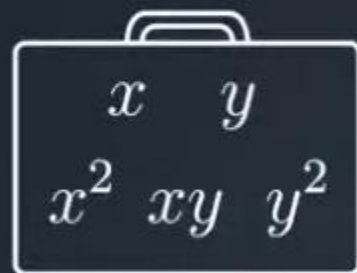
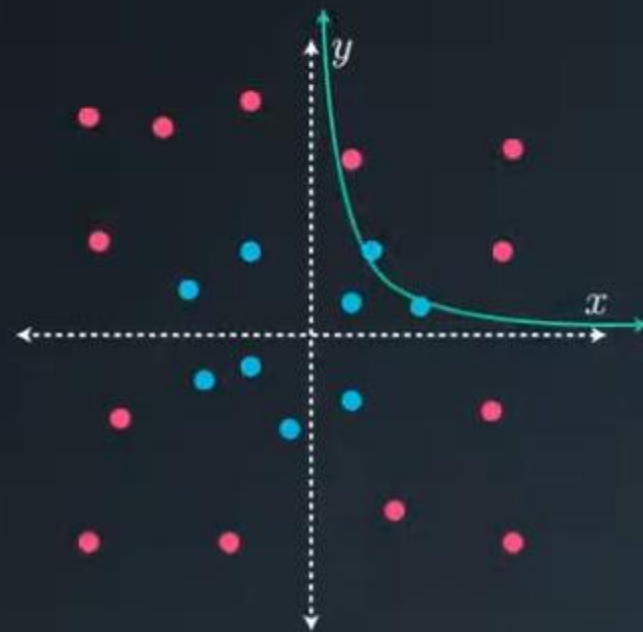
Polynomial Kernel

KERNEL



$$\begin{aligned} 2x - y &= 1 \\ 5x - 2y &= 3 \\ x + y &= 4 \end{aligned}$$

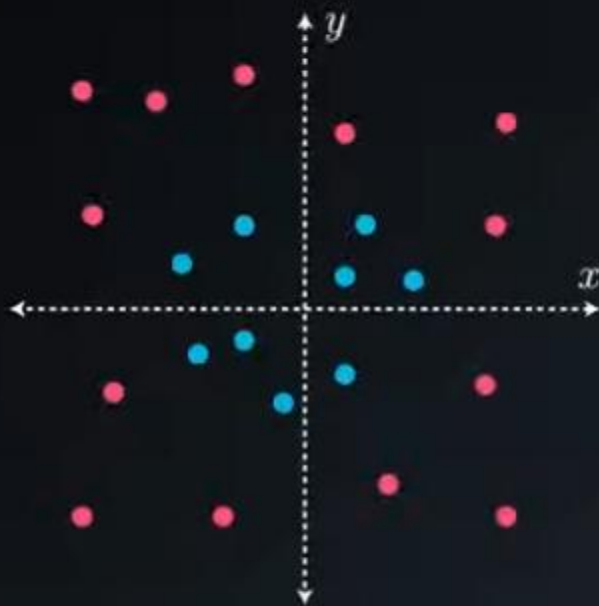
Linear Kernel



$$x^2 + y^2 = 1$$

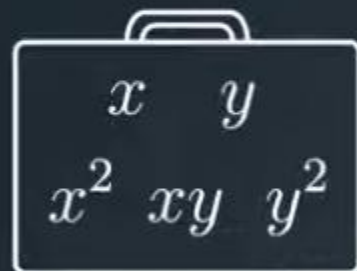
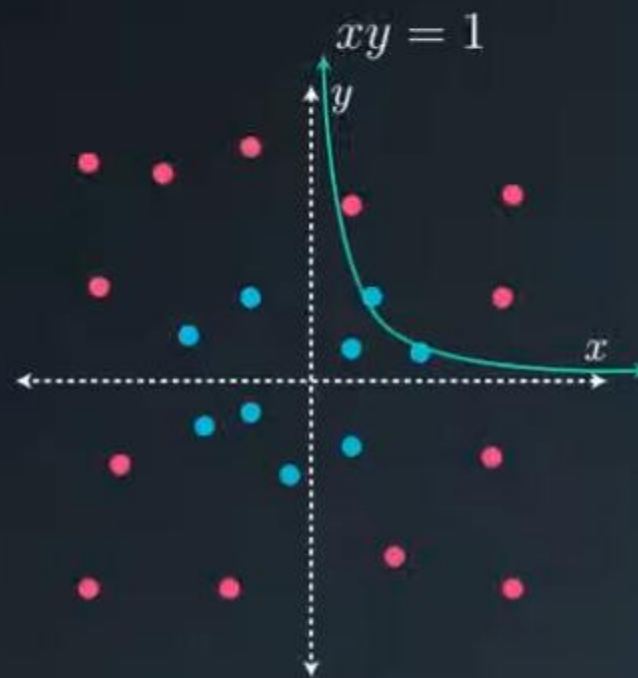
Polynomial Kernel

KERNEL



$$\begin{aligned} 2x - y &= 1 \\ 5x - 2y &= 3 \\ x + y &= 4 \end{aligned}$$

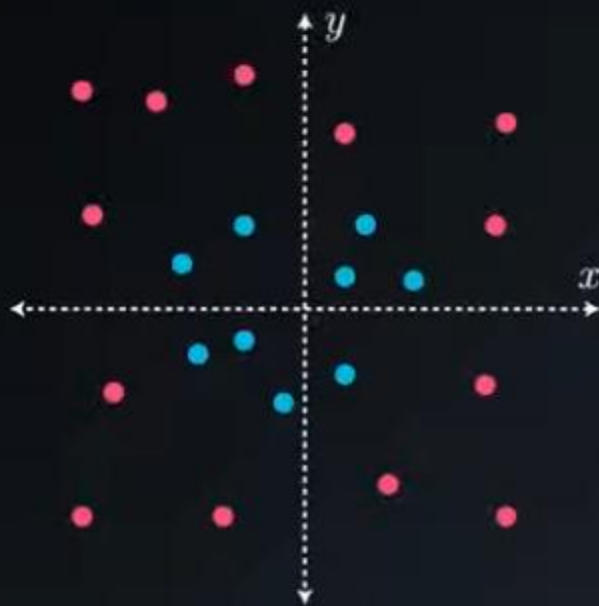
Linear Kernel



$$x^2 + y^2 = 1$$

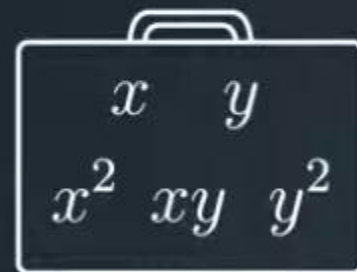
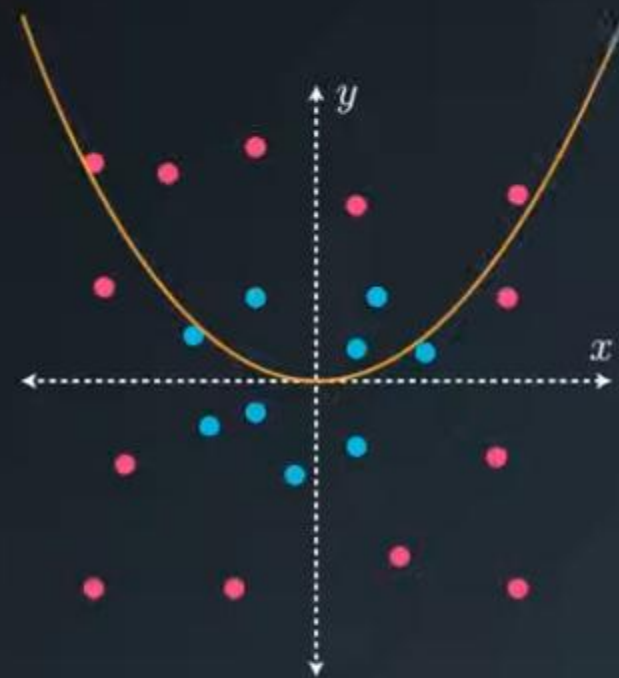
Polynomial Kernel

KERNEL



$$\begin{aligned}2x - y &= 1 \\5x - 2y &= 3 \\x + y &= 4\end{aligned}$$

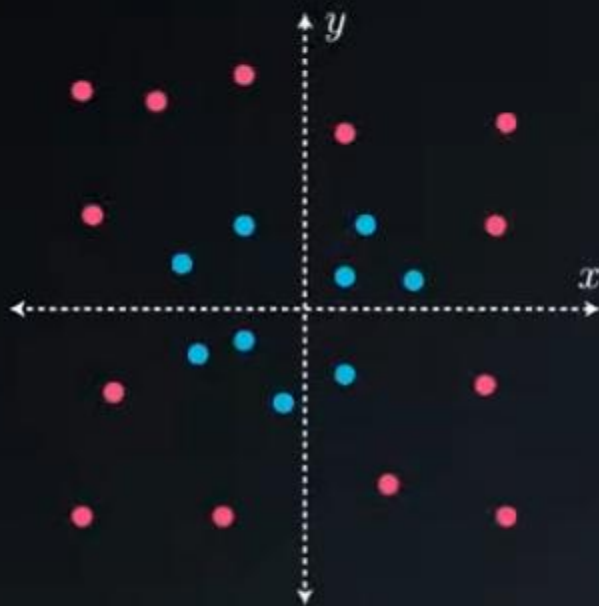
Linear Kernel



$$\begin{aligned}x^2 + y^2 &= 1 \\xy &= 1\end{aligned}$$

Polynomial Kernel

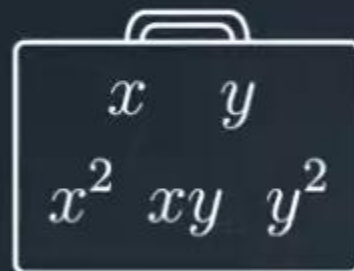
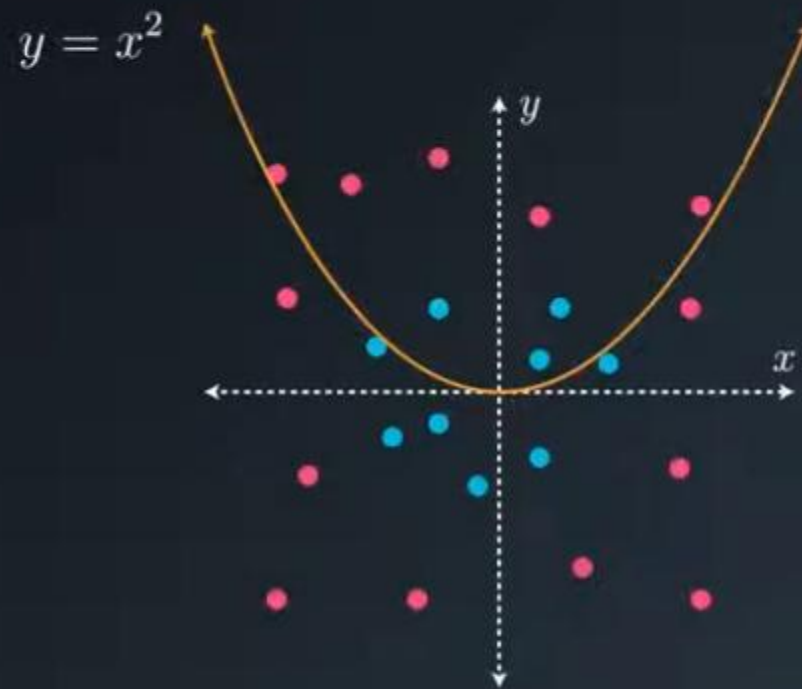
KERNEL



x y

$$\begin{aligned} 2x - y &= 1 \\ 5x - 2y &= 3 \\ x + y &= 4 \end{aligned}$$

Linear Kernel

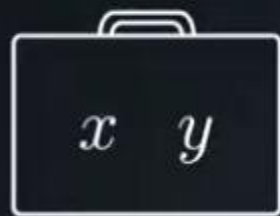
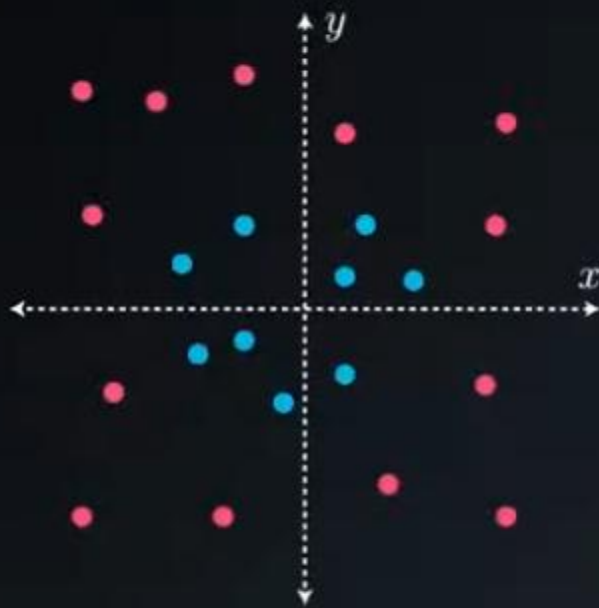


x y
 x^2 xy y^2

$$\begin{aligned} x^2 + y^2 &= 1 \\ xy &= 1 \end{aligned}$$

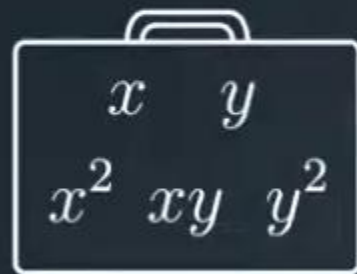
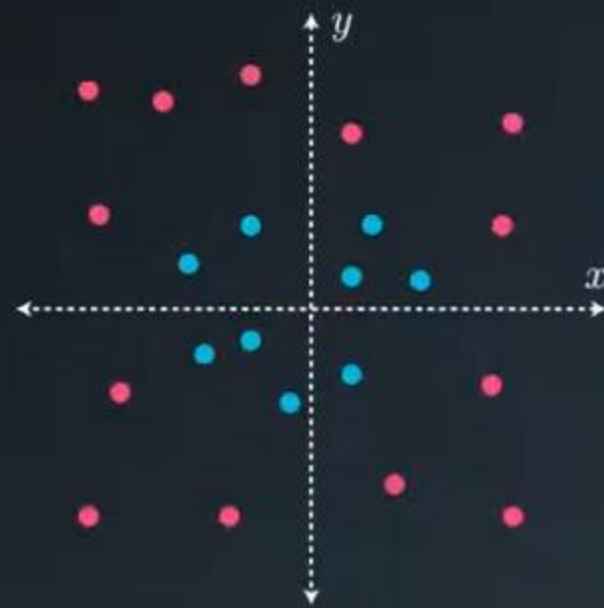
Polynomial Kernel

KERNEL



$$\begin{aligned}2x - y &= 1 \\5x - 2y &= 3 \\x + y &= 4\end{aligned}$$

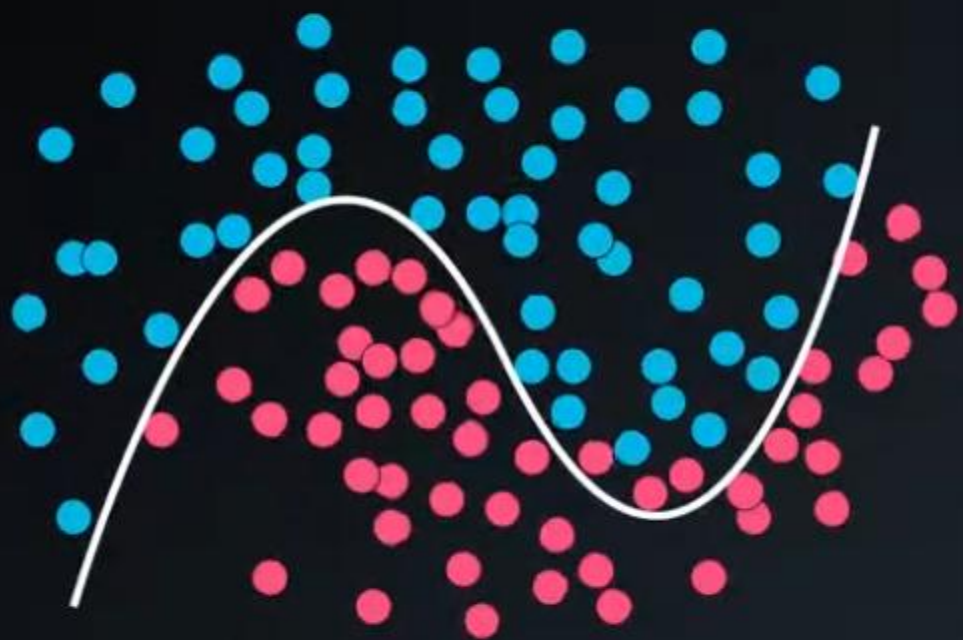
Linear Kernel



$$\begin{aligned}x^2 + y^2 &= 1 \\xy &= 1 \\y &= x^2\end{aligned}$$

Polynomial Kernel

KERNEL TRICK

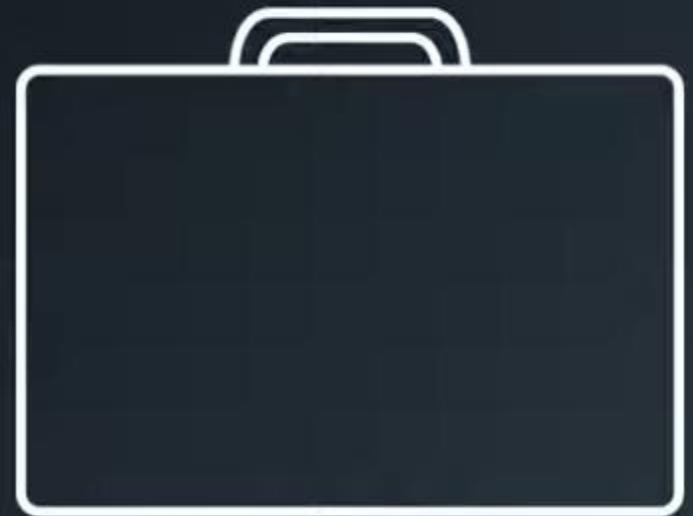


$$y = x^3 + 2x^2 - x - 2$$

KERNEL TRICK



$$y = x^3 + 2x^2 - x - 2$$



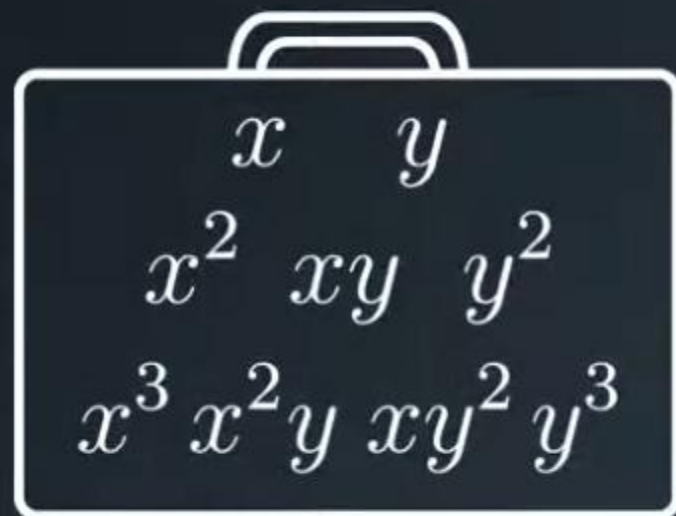
Polynomial Kernel (degree 3)



KERNEL TRICK



$$y = x^3 + 2x^2 - x - 2$$



Polynomial Kernel (degree 3)

