1.20) Demuestre que

$$(\cos\theta + i\sin\theta)^n = \cos n\phi + i\sin n\phi$$

Aplicando inducción

Para $n = 1$

$$(\cos\theta + i\sin\theta)^1 = \cos 1\phi + i\sin 1\phi$$

$$\cos\phi + i\sin\phi = \cos\phi + i\sin\phi$$

✓

Para $n = K$, tomamos de hipótesis

$$(\cos\theta + i\sin\theta)^K = \cos K\phi + i\sin K\phi$$

Para $n = K+1$

$$(\cos\phi + i\sin\phi)^{K+1} = \cos(K+1)\phi + i\sin(K+1)\phi$$

$$\Downarrow$$

$$(\cos\phi + i\sin\phi)^{K}(\cos\phi + i\sin\phi) =$$

$$(\cos K\phi + i\sin K\phi)(\cos\phi + i\sin\phi) =$$

$$[\cos K\phi\cos\phi - \sin K\phi\sin\phi] + i[\sin K\phi\cos\phi + \cos K\phi\sin\phi]$$

$$\cos(K\phi + \phi) + i\sin(K\phi + \phi)$$

$$\cos(K+1)\phi + i\sin(K+1)\phi \quad \checkmark$$

1.22) Demuestre

a) $\cos\phi = \dfrac{e^{i\phi} + e^{-i\phi}}{2}$ ... (1)

i

$$e^{i\phi} = \cos\phi + i\sin\phi \quad \ldots \quad (2)$$

$$e^{-i\phi} = \cos\phi - i\sin\phi \quad \ldots \quad (3)$$

(2) y (3) en (1)

$$\frac{\cos\phi + \cancel{i\sin\phi} + \cos\phi - \cancel{i\sin\phi}}{2}$$

$$\frac{\cancel{2}\cos\phi}{\cancel{2}}$$

$$\cos\phi$$

b) $\sin\phi = \dfrac{e^{i\phi} - e^{-i\phi}}{2} \quad \ldots \quad (1)$

$e^{i\phi}$

$$e^{i\phi} = \cos\phi + i\sin\phi \quad \ldots \quad (2)$$

$$e^{-i\phi} = \cos\phi - i\sin\phi \quad \ldots \quad (3)$$

(2) y (3) en (1)

$$\frac{\cancel{\cos\phi} + i\sin\phi - \cancel{\cos\phi} + i\sin\phi}{2}$$

$$\frac{\cancel{2} i\sin\phi}{\cancel{2}}$$

$$i\sin\phi$$

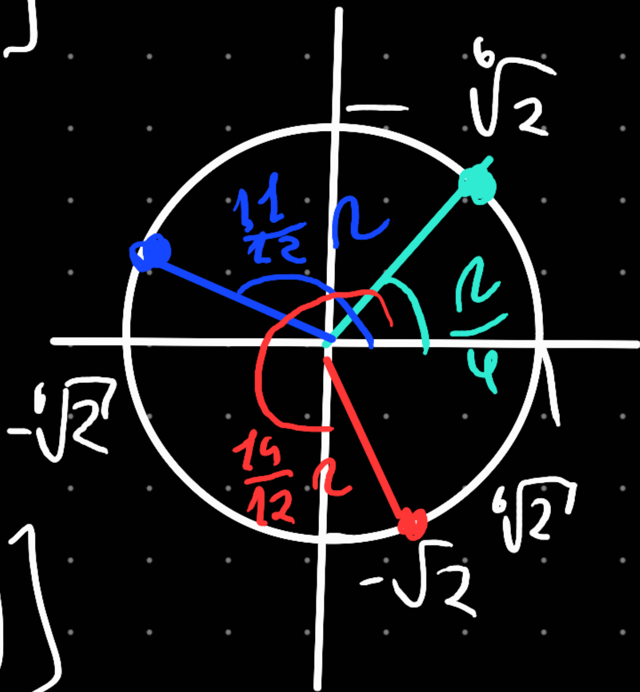
1.24)

a) $(-1+i)^{1/3}$ $\quad r = \sqrt{1^2+1^2} = \sqrt{2} = 2^{1/2}$

$\phi = tg^{-1}\left(\frac{1}{-1}\right) = -\frac{1}{4}\pi$ "o" $\frac{3}{4}\pi$ Para $K = \{0, 1, 2\}$

$$(-1+i)^{1/3} = 2^{1/6}\left[\cos\left(\frac{3\pi/4 + 2K\pi}{3}\right) + i\sin\left(\frac{3\pi/4 + 2K\pi}{3}\right)\right]$$

$$z = \begin{cases} 2^{1/6}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right] \\ 2^{1/6}\left[\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11}{12}\pi\right)\right] \\ 2^{1/6}\left[\cos\left(\frac{19}{12}\pi\right) + i\sin\left(\frac{19}{12}\pi\right)\right] \end{cases}$$

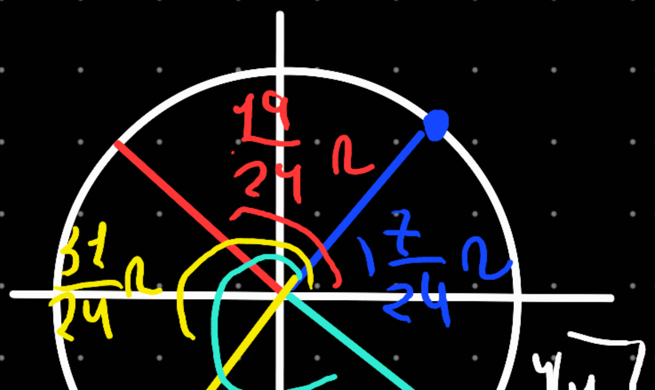


b) $(-2\sqrt{3} - 2i)^{1/4}$ $\quad r = \sqrt{4\cdot 3 + 4} = 4$

$\theta = tg^{-1}\left(\frac{2}{2\sqrt{3}}\right) = \frac{7}{6}\pi$ "o" $\frac{1}{6}\pi$

$$z^{1/4} = 4^{1/4}\left[\cos\left(\frac{7/6 + 2K}{4}\pi\right) + i\sin\left(\frac{7/6 + 2K}{4}\right)\right]$$

$$z = \begin{cases} K=0, \ 4^{1/4}\left[\cos\left(\frac{7\pi}{24}\right) + i\sin\left(\frac{7\pi}{24}\right)\right] \\ K=1, \ 4^{1/4}\left[\cos\left(\frac{19}{24}\pi\right) + i\sin\left(\frac{19}{24}\pi\right)\right] \end{cases}$$



$$k=2,\quad 4^{1/4}\left[\cos\left(\frac{31}{24}\pi\right)+i\sin\left(\frac{31}{24}\pi\right)\right]$$

$$k=3,\quad 4^{1/4}\left[\cos\left(\frac{43}{24}\pi\right)+i\sin\left(\frac{43}{24}\pi\right)\right]$$

1.31)

$$\begin{cases} az^2+bz+c=0 \\ a\neq 0 \end{cases}$$

$$4aaz^2+4abz+4ac=0$$

$$(2az)^2+2(2ba)z=-4ac$$

$$(2az)^2+2(2ba)z+b^2=b^2-4ac$$

$$(2az+b)^2=b^2-4ac$$

$$2az+b=\pm\sqrt{b^2-4ac}$$

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1.34)

$$6z^4 - 25z^3 + 32z^2 + 3z - 10 = 0$$

Mediante div. sintetica

	6	-25	32	3	-10
$-\frac{1}{2}$		-3	14	-23	10
	6	-28	46	-20	0
$\frac{2}{3}$		4	-16	20	
	6	-24	30	0	

$$\left(z + \tfrac{1}{2}\right)\left(z - \tfrac{2}{3}\right)\left(6z^2 - 24z + 30\right)$$

Mediante la fórmula cuadrática

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{24 \pm \sqrt{24^2-4\cdot 6\cdot 30}}{2\cdot 6}$$

$$2 \pm \frac{\sqrt{-144}}{12}$$

$$2 \pm \frac{\cancel{12}\, i}{\cancel{12}} = 2 \pm i$$

$$z = \begin{cases} -1/2 \\ 2/3 \end{cases}$$

$2 \pm i$

1,50) $z^2(1-z^2)=16$

$-z^4+z^2-16=0 \quad \cdot(-1)$

$z^4-z^2+16=0$

$$z^2=\frac{1\pm\sqrt{1^2-4.16}}{2}$$

$$z^2=\frac{1}{2}\pm\frac{3}{2}\sqrt{7}\,i$$

Para $z^2=\frac{1}{2}+\frac{3}{2}\sqrt{7}\,i$

Mediante la ecuación

$$\sqrt{a+bi} = \pm\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} \pm i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}$$

$$\pm\frac{3}{2} \pm \frac{\sqrt{7}}{2}i$$

De manera análoga, para

$$\sqrt{a-bi} = \pm\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} \mp i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}$$

$$\pm\frac{3}{2} \mp \frac{\sqrt{7}}{2}i$$

1.53)

a) $(4-3i)+(2i-8)$

$-4-i$

b) $3(-1+4i)-2(7-i)$

$-3+12i-14-2i$

$-17+14i$

c) $(3+2i)(2-i)$

$6-3i+4i+2$

$8+i$

d) $(i-2)\left[2(1+i)-3(i-1)\right]$

$(2+2i-3i+3)$

$(i-2)(5-i)$

$5i+1-10+2i$

$$-4+7i$$

e) $\dfrac{2-3i}{4-i} \cdot \dfrac{4+i}{4+i}$

$$\frac{8+2i-12i+3}{17}$$

$$\frac{11}{17}-\frac{10}{17}i$$

f) $(4+i)(3+2i)(1-i)$

$(12+8i+3i-2)$

$(10+11i)(1-i)$

$10-10i+11i+11$

$$21 + i$$

9) $$\frac{(2+i)(3-2i)(1+2i)}{(1-i)^2}$$

$$\frac{(6-4i+3i+2)(1+2i)}{(\cancel{1}-2i\cancel{-1})}$$

$$\frac{(8-i)(1+2i)}{-2i} \cdot \frac{i}{i}$$

$$\frac{(8+16i-i+2)\,i}{2}$$

$$(10+15i)\,\frac{i}{2}$$

$$5i-\frac{15}{2}$$

h) $(2i-1)^2\left[\frac{4}{1-i}+\frac{2-i}{1+i}\right]$

$$(-4-4i+1)\left[\frac{4+4i+(2-i)(1-i)}{(1-i)(1+i)}\right]$$

$$(-3-4i)\left(\frac{4+4i+2\underbrace{-2i-i}_{-3i}-1}{1^2+1^2}\right)$$

$$\left(-3-4i\right)\left(\frac{5+i}{2}\right)$$

$$\frac{-15-3i-20i+4}{2}$$

$$-\frac{11}{2}-\frac{23}{2}i$$

i) $$\frac{i^{4}+i^{9}+i^{16}}{2-i^{5}+i^{10}-i^{15}}$$

$$\frac{(-1)^{2}+(-1)^{4}i+(-1)^{8}}{2-(-1)^{2}i+(-1)^{5}-(-1)^{7}\cdot i}$$

$$\frac{1+i+1}{2-\cancel{i}-1+\cancel{i}}$$

$$\frac{2+i}{1} = 2+i$$

1.80)

a) $(a,b)^{1/n}$ se define como

$$(a,b)^{1/n} = \sqrt{a^2+b^2}\left(\cos\left[\frac{tg^{-1}\left(\frac{b}{a}\right)+2k\pi}{n}\right]; \sin\left[\frac{tg^{-1}\left(\frac{b}{a}\right)+2k\pi}{n}\right]\right)$$

donde $K = \{1, 2, \ldots, n-1\}$

b) $(a+bi)^{1/2} = (x+yi)$

$$a + bi = x^2 + 2xyi - y^2$$

$$\begin{cases} x^2 - y^2 = a & (1) \\ 2xy = b & (2) \end{cases}$$

$(1)^2 + (2)^2$

$$a^2 + b^2 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2$$

$$a^2 + b^2 = x^4 + 2x^2y^2 + y^4$$

$$a^2 + b^2 = (x^2 + y^2)^2$$

$$x^2 + y^2 = \sqrt{a^2 + b^2} \quad (3)$$

(1) y (3)

$$x^2 + x^2 - a = \sqrt{a^2 + b^2}$$

$$x = \pm\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}}$$

(1) y (3)

$$y^2+y^2+a = \sqrt{a^2+b^2}$$

$$y = \pm\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}$$

1.83) Para $a+bi = r\angle\phi$

$r = \sqrt{a^2+b^2}$ y $\phi = tg^{-1}\left(\frac{b}{a}\right)$

a) $-3-4i = 5\,\underline{\lfloor -53{,}13}$

b) $1 - 2i = \sqrt{5} \angle -63{,}435$

1,87)

$$z = Re^{i\phi}$$

$$z = R(\cos\phi + i\sin\phi)$$

$$|e^{it}| = \left|e^{iR(\cos\phi + i\sin\phi)}\right|$$

$$= \left|e^{-R\sin\phi}\, e^{iR\cos\phi}\right|$$

$$= \left|e^{-R\sin\phi}\right| \cdot \left|e^{iR\cos\phi}\right|$$

Sabemos que $|e^{i\phi}|$ para cualquier ϕ es siempre igual a 1.

$$|e^{iz}| = |e^{-R\sin\phi}| \cdot \overbrace{|e^{iR\cos\phi}|}^{1}$$

$$|e^{iz}| = |e^{-R\sin\phi}|$$

$\hookrightarrow$ siempre (+)

$$|e^{iz}| = e^{-R\sin\phi}$$