

1.1) Muy fácil

1.2) Demasiado sencillo

1.3)

$$E_{a-b} = \int_a^b \|X(t)\|^2 dt$$

$$P_{a-b} = \frac{1}{b-a} \int_a^b \|X(t)\|^2 dt = \frac{1}{b-a} E_{a-b}$$

a) $E_\infty = \int_0^\infty e^{-4t} dt = \frac{1}{4}$

$$P_\infty = \lim_{T \rightarrow \infty} \frac{1/4}{2T} = 0$$

b) $E_\infty = \int_{-\infty}^\infty \|k^{j\omega t} e^{j\pi/4}\|^2 dt = \left. t \right|_{-\infty}^\infty$

$$P_\infty = \lim_{T \rightarrow \infty} \frac{\cancel{2\pi}}{\cancel{2\pi}} = 1$$

$$c) E_{\infty} = \int_{-\infty}^{\infty} \|\cos(t)\|^2 dt =$$

$$\lim_{T \rightarrow \infty} \frac{2T + \sin 2T}{2} =$$

$$\lim_{T \rightarrow \infty} T + \frac{\sin 2T}{2} \quad \text{since } -\frac{1}{2} \leq \frac{\sin 2T}{2} \leq \frac{1}{2}$$

$$\lim_{T \rightarrow \infty} T - \frac{1}{2} \leq \lim_{T \rightarrow \infty} T + \frac{\sin 2T}{2} \leq \lim_{T \rightarrow \infty} T + \frac{1}{2}$$

$$\therefore E_{\infty} = \infty$$

$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{2T + \sin 2T}{4T}$$

$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2} + \frac{\sin 2T}{4T} \quad \text{since } -1 \leq \sin 2T \leq 1$$

$$\lim_{T \rightarrow \infty} \frac{1}{2} - \frac{1}{T} \leq \lim_{T \rightarrow \infty} \frac{1}{2} + \frac{\sin 2T}{4T} \leq \lim_{T \rightarrow \infty} \frac{1}{2} + \frac{1}{T}$$

$$\therefore P_{\infty} = \frac{1}{2}$$

1.6)

a) No es periódica debido a que la func. $u(t)$ corta la func. en $0 < t$

1.9)

$$\begin{aligned} a) x(t) &= j e^{j \omega_0 t} \\ &= e^{\ln(j)} e^{j \omega_0 t} \\ &= e^{j(10t + \pi/2)} \end{aligned}$$

$$\cos\left(\hat{\omega}t + \frac{\pi}{2}\right) + i \sin\left(\hat{\omega}t + \frac{\pi}{2}\right)$$

$$\hat{\omega} = \frac{2\pi}{\|10\|} = \frac{\pi}{5}$$

b) $x(t) = e^{-t} e^{jt}$

↳ decaimiento

∴ $x(t)$ no es periódica

$$1.10) \quad x(t) = 2\cos(10t+1) - \sin(4t-1)$$

$$\begin{array}{c|cc} 1 & 0 & (2) \\ \hline 5 & 5 & 2 \\ (1) & & (1) \end{array}$$

$$\omega_{\text{común}} = 2 \quad T_{\text{común}} = \frac{2\pi}{\|\omega_{\text{común}}\|}$$

$$= \pi //$$

$$1.13) \quad y(t) = \int_{-\infty}^t \delta(t+z) - \delta(t-z) dt$$

$$y(t) = u(t+z) - u(t-z)$$

$$E_\alpha = \int_{-\infty}^{\infty} \|u(t+z) - u(t-z)\|^2 dt = \int_{-2}^2 dt = 4$$

L_o 14)

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ -2 & 1 < t < 2 \end{cases}$$

Mi enpedo hago XD

$$1.17) \quad y(t) = x(\sin(t))$$

$$a) \quad \text{Para } y(t_0) = x(\sin(t_0))$$

↳

$$-1 \leq \sin(t_0) \leq 1$$

Si $t = -n\pi, n \in \mathbb{N} \rightarrow y(-n\pi) = x(1)$

$x \nearrow$
Future

• El sist. no es causal

$$b) \quad x_1(t) \rightarrow y_1(t) = x_1(\sin(t))$$

$$x_2(t) \rightarrow y_2(t) = x_2(\sin(t))$$

$$x_3(t) = a x_1 + b x_2$$

$$y_3(t) = a y_1(\sin(t)) + b y_2(\sin(t))$$

$$= a y_1(t) + b y_2(t)$$

• El sist. es lineal

L619)

a) $y(t) = t^2 x(t-1)$

$$y(t-t_0) = (t-t_0)^2 x(t-t_0-1)$$

Si $x(t-t_0) \xrightarrow{\delta} t^2 x(t-t_0-1)$

∴ Sistema variable en el tiempo

Por su forma ya vez que es lineal

∴ Sistema lineal

1.20)

$$\chi_a(t) = e^{j2t} \rightarrow y_a(t) = e^{j3t}$$

$$\chi_b(t) = \bar{e}^{j2t} \rightarrow y_b(t) = \bar{e}^{j3t}$$

sistema lineal

a) $\cos(2t) = \frac{1}{2} e^{j2t} + \frac{1}{2} \bar{e}^{-j2t}$

$$\chi_3(t) = \frac{1}{2} \chi_a(t) + \frac{1}{2} \chi_b(t)$$

$$y_3(t) = \frac{1}{2} y_a(t) + \frac{1}{2} y_b(t)$$

$$= \frac{1}{2} \cos(3t)$$

b) $\cos(2t-1) = \frac{1}{2} [e^{j(2t-1)} + \bar{e}^{j(2t-1)}]$

$$= \frac{\bar{e}^j}{2} e^{2jt} + \frac{e^j}{2} e^{-2jt}$$

$$= \frac{\bar{e}^j}{2} \chi_a + \frac{e^j}{2} \chi_b$$

$$y_3 = \frac{e^j}{2} y_a + \frac{e^j}{2} y_b$$

$$= \cos(3t - 1)$$

1.21) Mi oto te voy a dibujar

1.23) Mi oto te voy a dibujar $\times 2$

1.25)

a) $T = \frac{2}{3}\pi \rightarrow$ ángulo

b) $x(t) = e^{j\pi t} e^{j\pi} \text{ inicial}$

$$T = \frac{2\pi}{\omega} = 2$$

c) $x(t) = \cos^2(2t - \pi/3)$

$$= \frac{1}{2} \cos(4t - 2\pi/3) + \frac{1}{2}$$

$$T = \frac{2\pi}{\omega_2} = \frac{\pi}{2}$$

1o27) Muy fácil y largo ya

1o30)

a) $x(t) \xrightarrow{t \rightarrow t-4} x(t-4) \xrightarrow{t \rightarrow t+4} x(t)$

$$y^{-1}(t) = x(t+4)$$

b) $x(t) \xrightarrow{\cos(\cdot)} \cos[x(t)] \xrightarrow{\cos^{-1}(\cdot)} x(t)$

\cos^{-1} no recuperará todo valor de $x(t)$

Solo es invertible para

$$-\frac{\pi}{2} \leq x(t) \leq \frac{\pi}{2}$$

1o32) Página.

$$1.38) \quad f(2t) = \lim_{\Delta \rightarrow 0} f_2(\Delta)$$

$$f_2(t) = \begin{cases} \frac{1}{\Delta} & \Delta \leq t \\ 0 & \text{otherwise} \end{cases} \rightarrow f(2t) = \begin{cases} \frac{1}{\Delta} & \frac{\Delta}{2} \leq t \\ 0 & \text{otherwise} \end{cases}$$

2.8)

$$\chi(t) = (t+1)[u(t) - u(t-1)] + (2-t)[u(t-1) - u(t-2)] =$$

$$(t+1)u(t) + \underbrace{(-t-1+2-t)u(t-1)}_{-2t+1} - (2-t)u(t-2)$$

$$(t+2)u(t) + (-2t+1)u(t-1) + (t-2)u(t-2)$$

$$y(t) = \int_{-\infty}^{\infty} \chi(\tau) [f(t-\tau+2) + 2f(t-\tau+1)] d\tau$$

$$= \underbrace{\chi(t+2)}_{(t+3)u(t+2)} + 2 \underbrace{\chi(t+1)}_{[-2(t+2)+1]u(t+1)} + t u(t)$$

$$+ 2 \left[(t+2)u(t+1) + (-2(t+2)+1)u(t) + (t-1)u(t-1) \right]$$

$$= (t+3)u(t+2) + 4u(t+1) - (3t+3)u(t) + (2t-3)u(t-1)$$

2.11)

$$y(t) = \int_{-\infty}^{\infty} u(t-\tau-3) e^{-3\tau} u(\tau) d\tau$$

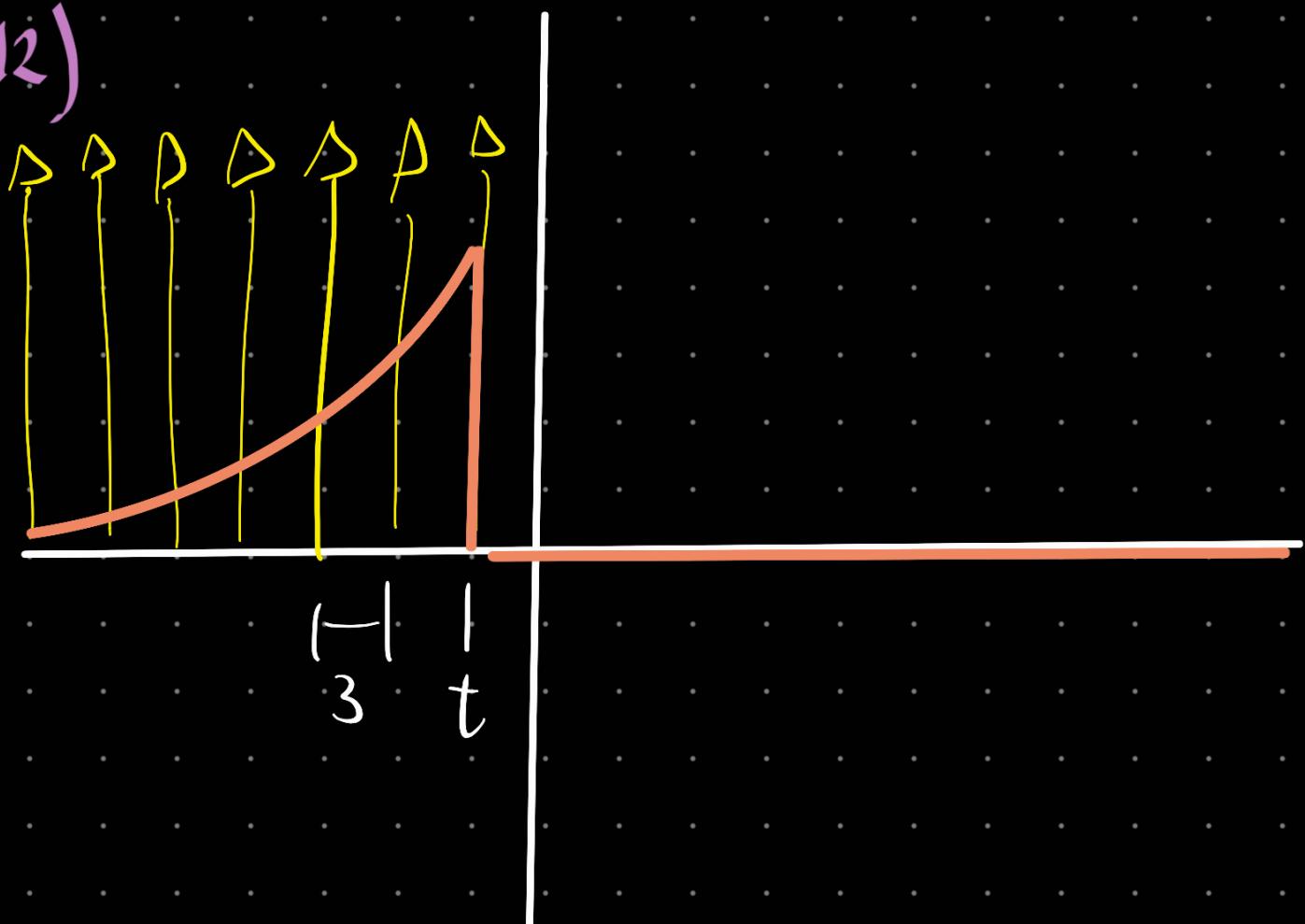
$$\int_{-\infty}^{\infty} u(t-\tau-5) e^{-3\tau} u(\tau) d\tau$$

$$= u(t) \left[u(t-3) \int_0^{t-3} e^{-3x} dx - u(t-5) \int_0^{t-5} e^{-3x} dx \right]$$

$$\frac{1}{3} \left\{ u(t-3) \left[1 - e^{-3(t-3)} \right] + u(t-5) \left[e^{-3(t-5)} - 1 \right] \right\}$$

$$g(x) = \frac{\partial x}{\partial t} * h = \frac{\partial y}{\partial t}$$

2.12)



$$y(t) = e^{-(t+6)} u(t+6) + e^{-(t+3)} u(t+3) + \dots$$

$$e^{-t} \left(e^0 + e^{-3} + e^{-6} + \dots \right)$$

$$\sum_{k=0}^{\infty} e^{-3k} = \frac{e^0}{1 - e^{-3}}$$

2.14)

a) $\int_{-\infty}^{\infty} |e^t e^{j\omega t} u(t)| dt$

$\hookrightarrow t \geq 0$

$$\int_0^{\infty} |e^t| dt = 1 \rightarrow \text{estable}$$

b) $\int_{-\infty}^{\infty} |e^{-t} \cos(2t) u(t)| dt$

$$0 \leq \int_0^{\infty} |e^{-t}| |\cos(2t)| dt \leq \int_0^{\infty} e^{-t} dt = 1$$

\hookrightarrow

$0 \leq I \leq 1 \rightarrow \text{estable}$

2.17)

a) $y' + 4y = x \quad y(0) = 0$

$$e^{\int 4 dt} = e^{4t}$$

$$y(t) = \frac{\int x(t) e^{4t} dt - \int x(t) e^{4t} dt}{e^{4t}} \Big|_{t=0}$$

$$\int x(t) e^{4t} dt = \frac{e^{3t(1+j)}}{3+3j}$$

$$y(t) = \frac{e^{3(1+j)t}}{3(1+j)e^{4t}} - \frac{1}{3(1+j)} e^{4t}$$

$$y(t) = \frac{1-j}{6} \left[e^{(-1+3j)t} - e^{-4t} \right]$$

b) La misma cagada que el a

$$2.20) \quad \int_{-\infty}^{\infty} f(t) \delta(t) dt$$

$$a) \quad \int_{-\infty}^{\infty} u_0(t) \cos(t) dt = \omega s(0) = 2$$

$$b) \quad \int_{-\infty}^{\infty} \sin(2\pi t) \delta(t+3) dt = 0$$

$$c) \quad \int_{-5}^{\infty} u_1(1-\tau) \cos(2\pi\tau) d\tau =$$

$$\left| \int_{-\infty}^{\infty} u_1(t-\tau) \cos(2\pi\tau) [u(\tau+5) - u(\tau-5)] d\tau \right| = t \rightarrow 1$$

$$u_1(t) * \left[\cos(2\pi t) (u(t+5) - u(t-5)) \right] \Big|_{t \rightarrow 1} =$$

$$\frac{d}{dt} \left[\cos(2\pi t) (u(t+5) - u(t-5)) \right] \Big|_{t \rightarrow 1} =$$

$$-2\pi \sin(2\pi t) (u(t+5) - u(t-5)) + \left[\cos(2\pi t) (\delta(t+5) - \delta(t-5)) \right] \Big|_{t \rightarrow 1}$$

0

2.42)

$$\begin{aligned}y(t) &= \int_{-\omega_0\zeta}^{\omega_0\zeta} e^{j\omega_0(t-\tau)} d\tau = \\&= \frac{1}{\omega_0 j} \left(e^{j\omega_0(t+\omega_0\zeta)} - e^{j\omega_0(t-\omega_0\zeta)} \right) \\&= \frac{e^{j\omega_0 t}}{\omega_0 j} \left(e^{\omega_0\zeta j\omega_0} - e^{-\omega_0\zeta j\omega_0} \right)\end{aligned}$$

$$y(t) = \frac{2e^{j\omega_0 t}}{\omega_0} \sin\left(\frac{\omega_0}{2}\zeta\right)$$

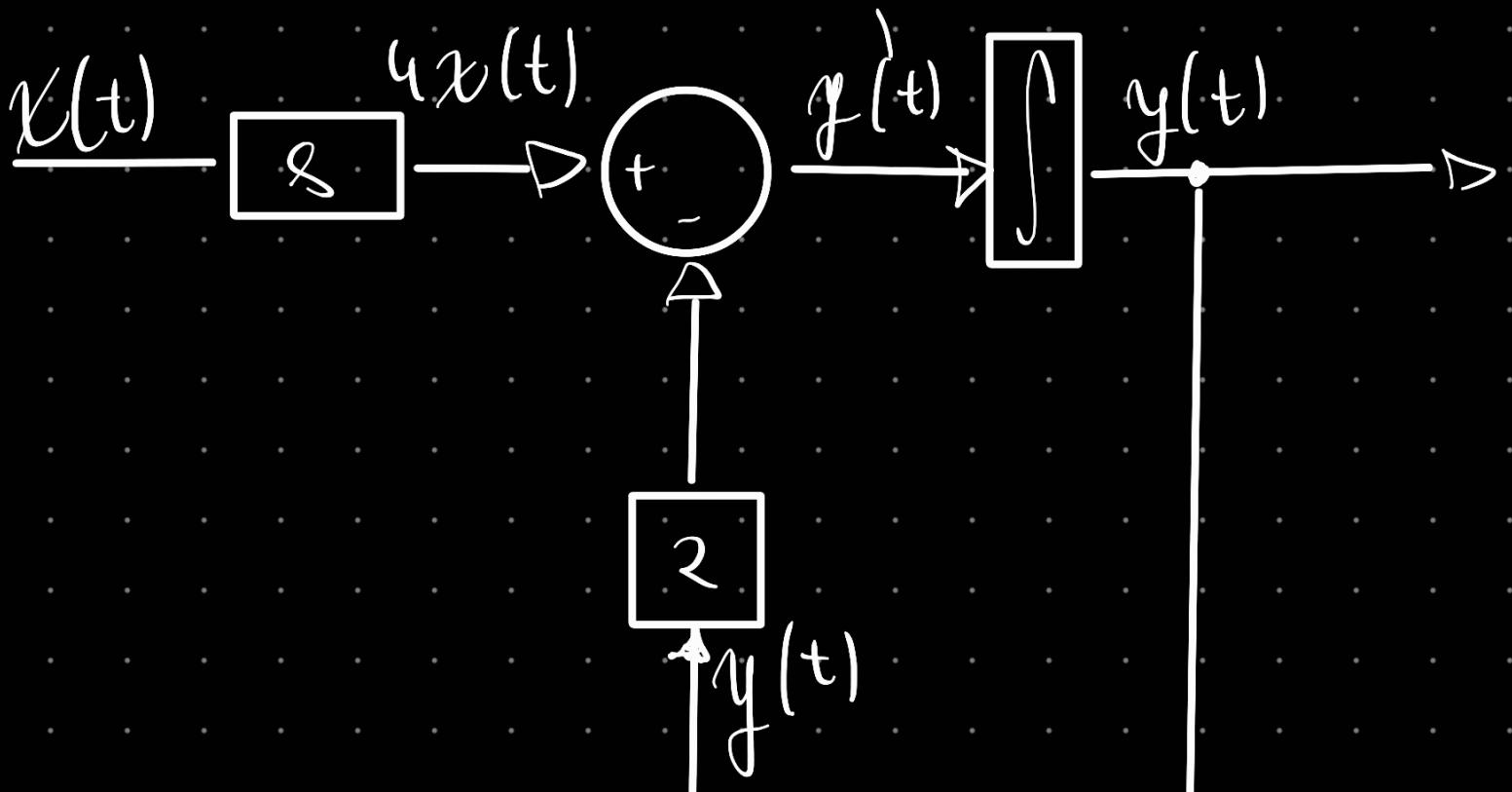
$$y'(0) = \frac{2}{\omega_0} \sin\left(\frac{\omega_0}{2}\zeta\right) = 0$$

$$\frac{\omega_0}{2} = \pi n$$

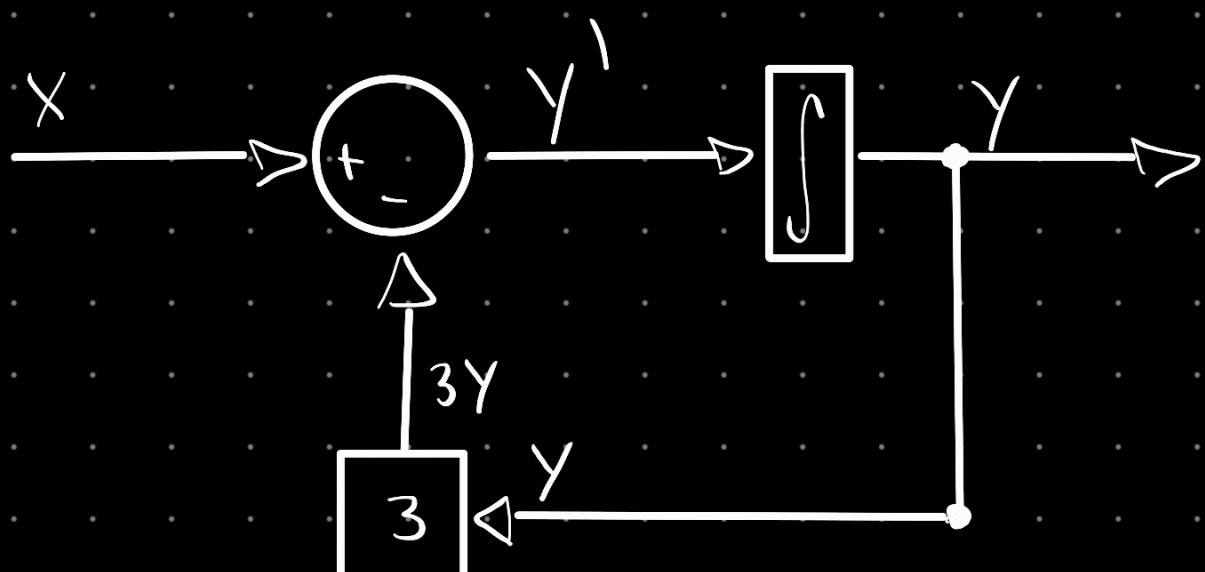
$$\omega_0 = 2\pi n$$

2.39)

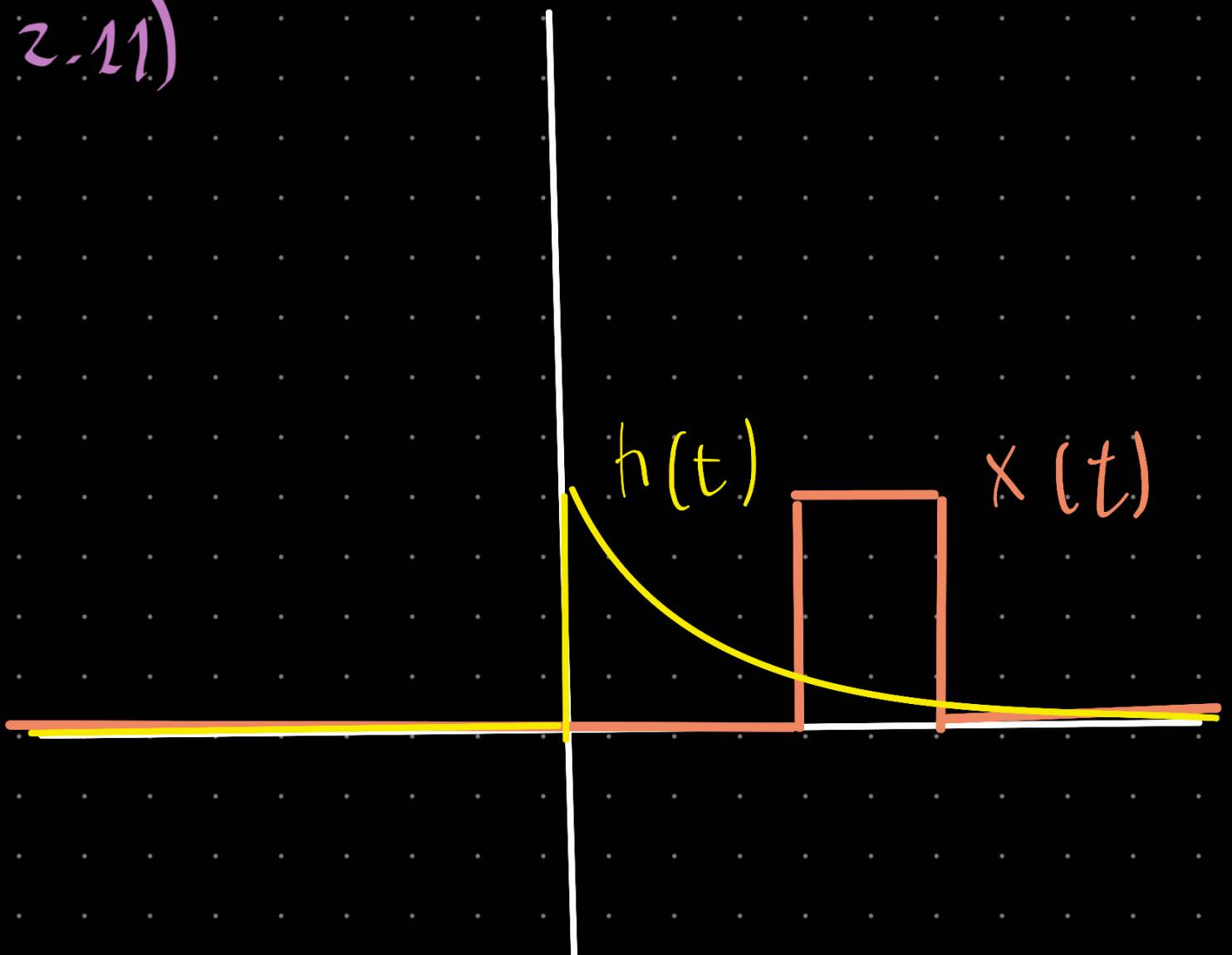
$$y' = -2y + 8x$$



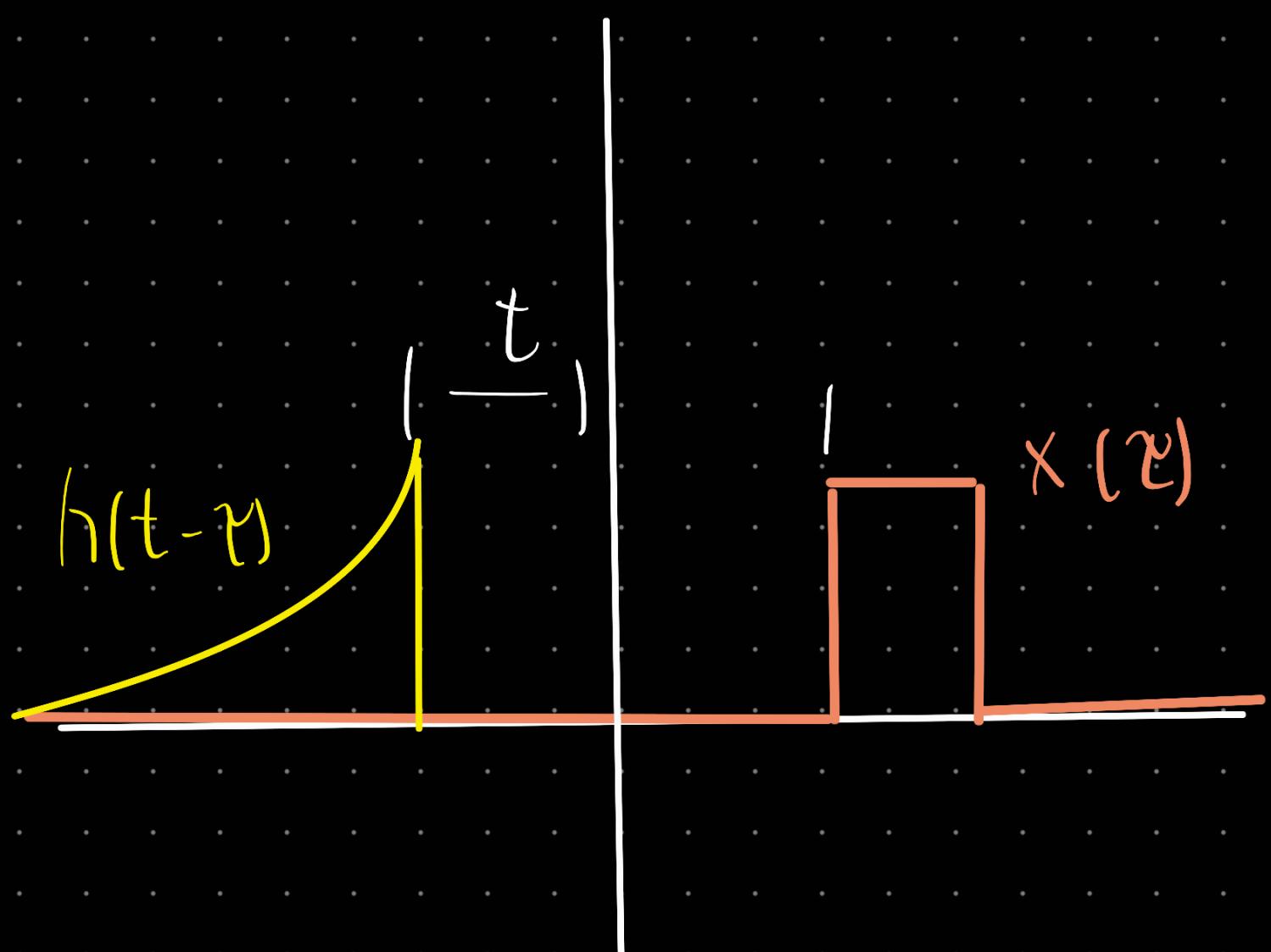
$$y' = x - 3y$$



2.11)

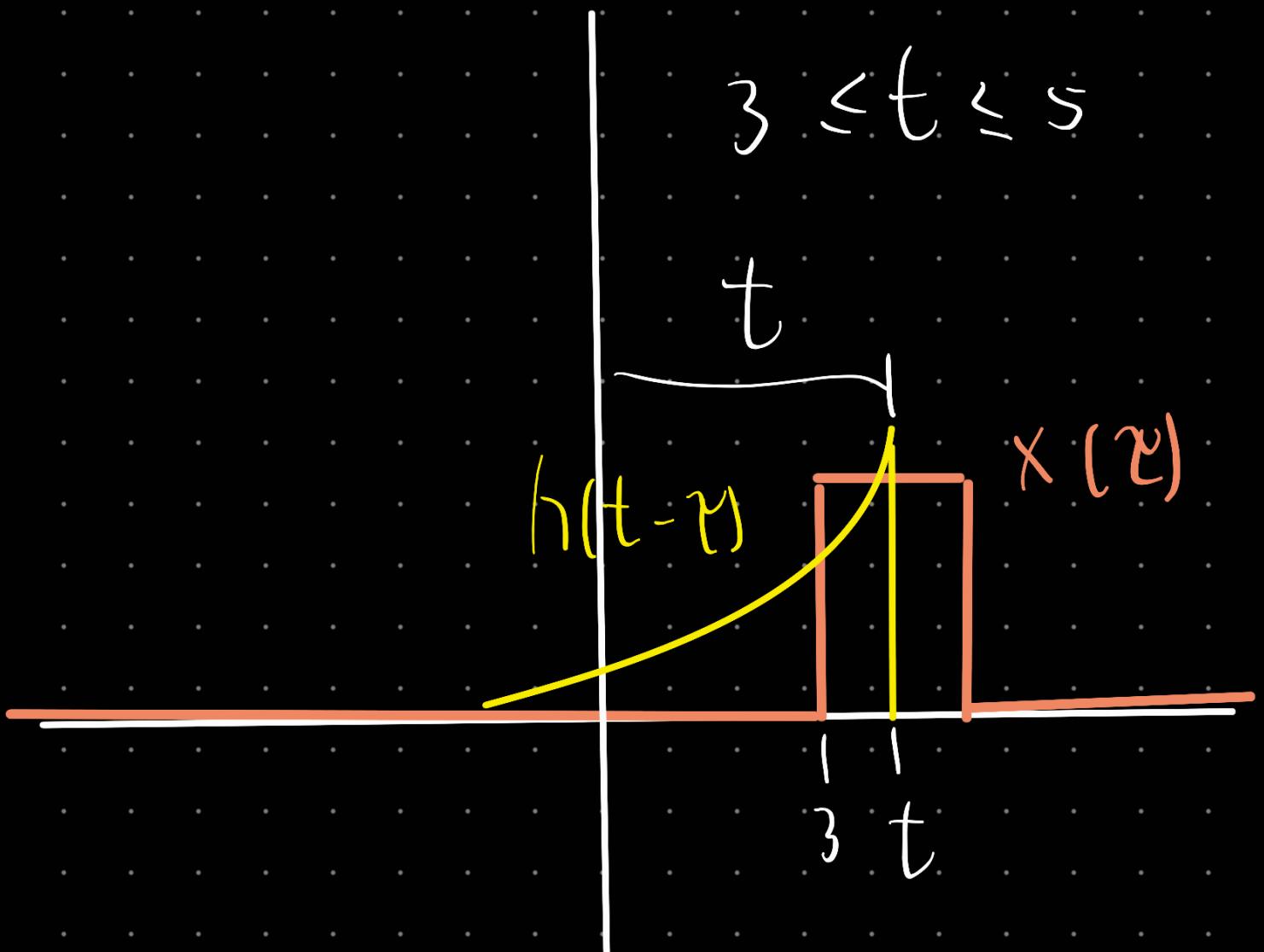


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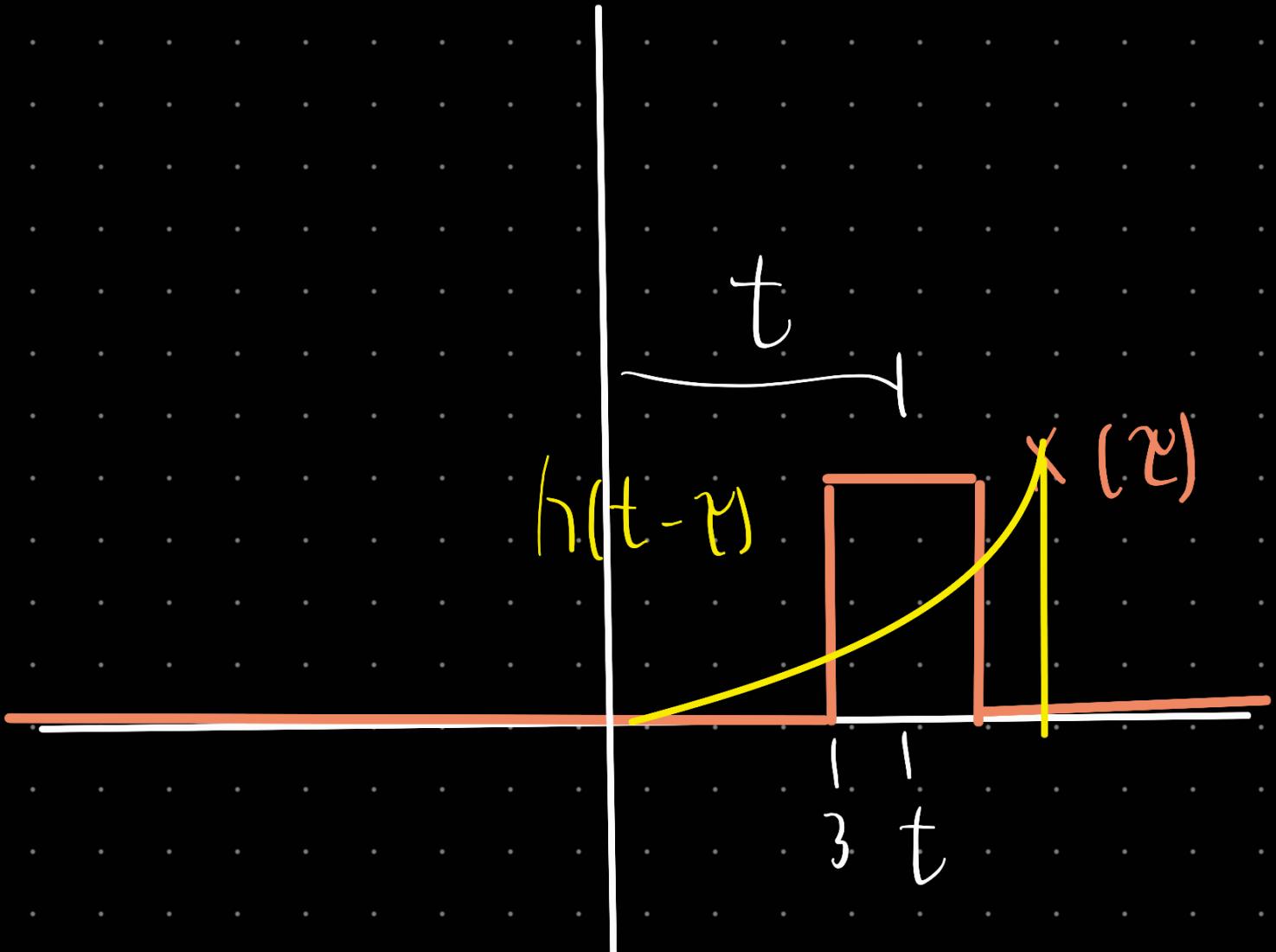
$$t \leq 3$$

$$y(t) = 0$$



$$\int_3^t 1 \cdot e^{-3(t-\tau)} d\tau =$$

$$t \geq 5$$



$$\int_3^5 e^{-3(t-\tau)} d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} u(\tau-3) e^{-3(t-\tau)} u(t-\tau) d\tau - \int_{-\infty}^{\infty} u(\tau-5) e^{-3(t-\tau)} d\tau$$

$$= u(t) \left[u(t-3) \int_3^t e^{-3(t-\tau)} d\tau - u(t-5) \int_5^t e^{-3(t-\tau)} d\tau \right]$$

$$x(t) \xrightarrow{S} y(t)$$

$$x'(t) \xrightarrow{S} y'(t)$$

$$\int_{-\infty}^{\infty} f(\tau) \delta'(t-\tau) d\tau = f(t-0)$$

$$\int_{-\infty}^{\infty} w_s(t) \int_{-\infty}^t (t-s) dt = 0$$

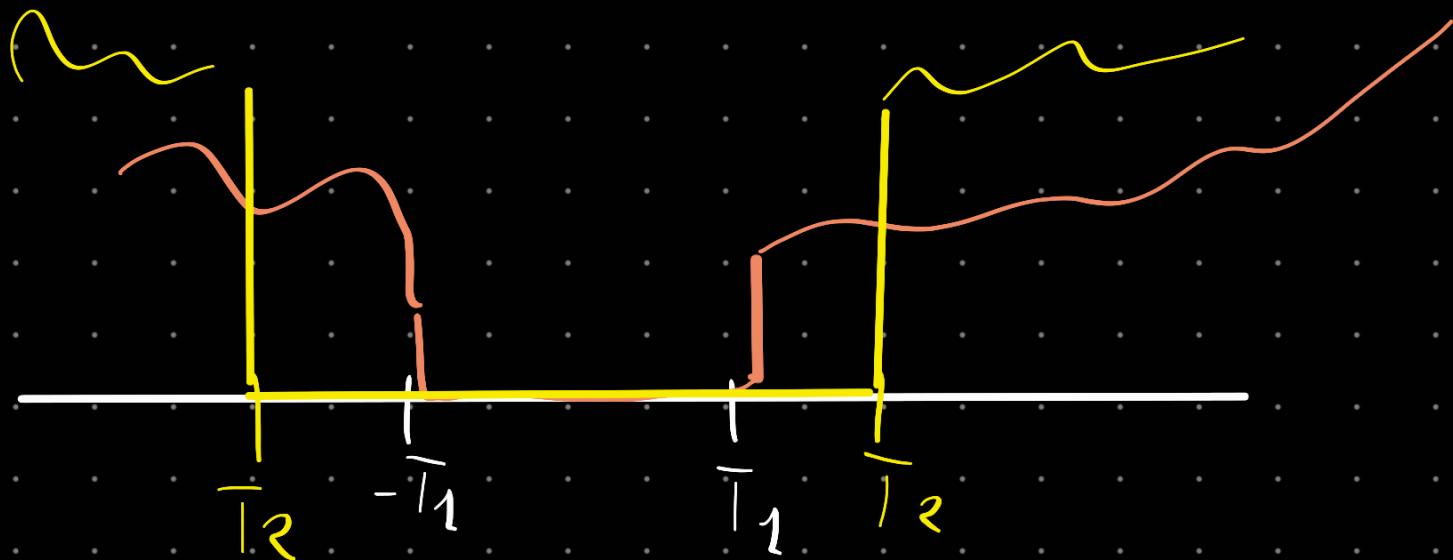
$$\int_{-\infty}^{\infty} w_s(t) \int_{-\infty}^{t-8} (t-8) dt = 0$$

$$\partial(t-3) - \partial(t-5)$$

$$\begin{aligned} & \int_{-\infty}^{\infty} \partial(\tau-3) e^{3(t-\tau)} u(t-\tau) d\tau - \int_{-\infty}^{\infty} \partial(\tau-5) e^{5(t-\tau)} u(t-\tau) d\tau \\ &= e^{-3(t-3)} u(t-3) - e^{-5(t-5)} u(t-5) \end{aligned}$$

$$T + 0,5 \geq 0$$

$$T \geq -0,5$$



$$-T_1 \geq t \quad T \leq t$$

$$x(t) = f(t) [u(-T_1 - t) + u(t - T_1)]$$

$$h(t) = g(t) [u(-T_2 - t) + u(t - T_2)]$$

$$-T_1 - \tau \geq 0 \rightarrow -T_1 \geq \tau$$

$$\int f(\tau) [u(T_1 + \tau) + u(\tau - T_1)] g(t - \tau) [u(-T_2 - t + \tau) + u(t - \tau - T_2)]$$

$$-T_1 - \tau \geq 0 \wedge \tau - T_1 \geq 0$$

+

$$-T_2 - t + \tau \geq 0 \wedge t - \tau - T_2 \geq 0$$

$$-T_1 \geq \tau \geq T_2 + t \wedge t - T_2 \geq \tau \geq T_1$$

$$-T_1 \geq T_2 + t \wedge t - T_2 \geq T_1$$

$$-T_1 - T_2 \geq t \geq T_2 + T_1$$

$$t \geq |T_1 + T_2|$$

$$f(t) = \cos(4t+2) + \sin(2t-\pi)$$

$$MCM(1,2) = \omega_f = 2$$

$$\int_{-\infty}^{t-s} \delta(\tau) d\tau = u(t-s)$$

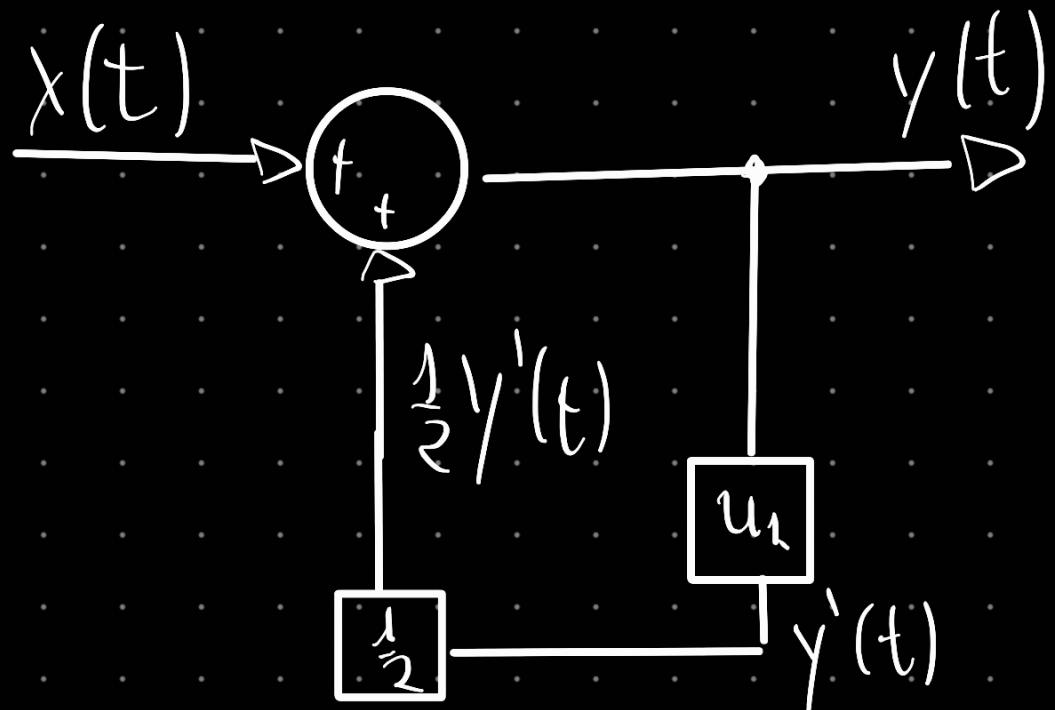
$$\int_{-\infty}^t \delta(t-\tau) d\tau =$$

$$-\int_{\infty}^0 \delta(u) du \quad \begin{aligned} u &= t - \tau \\ du &= -d\tau \end{aligned}$$

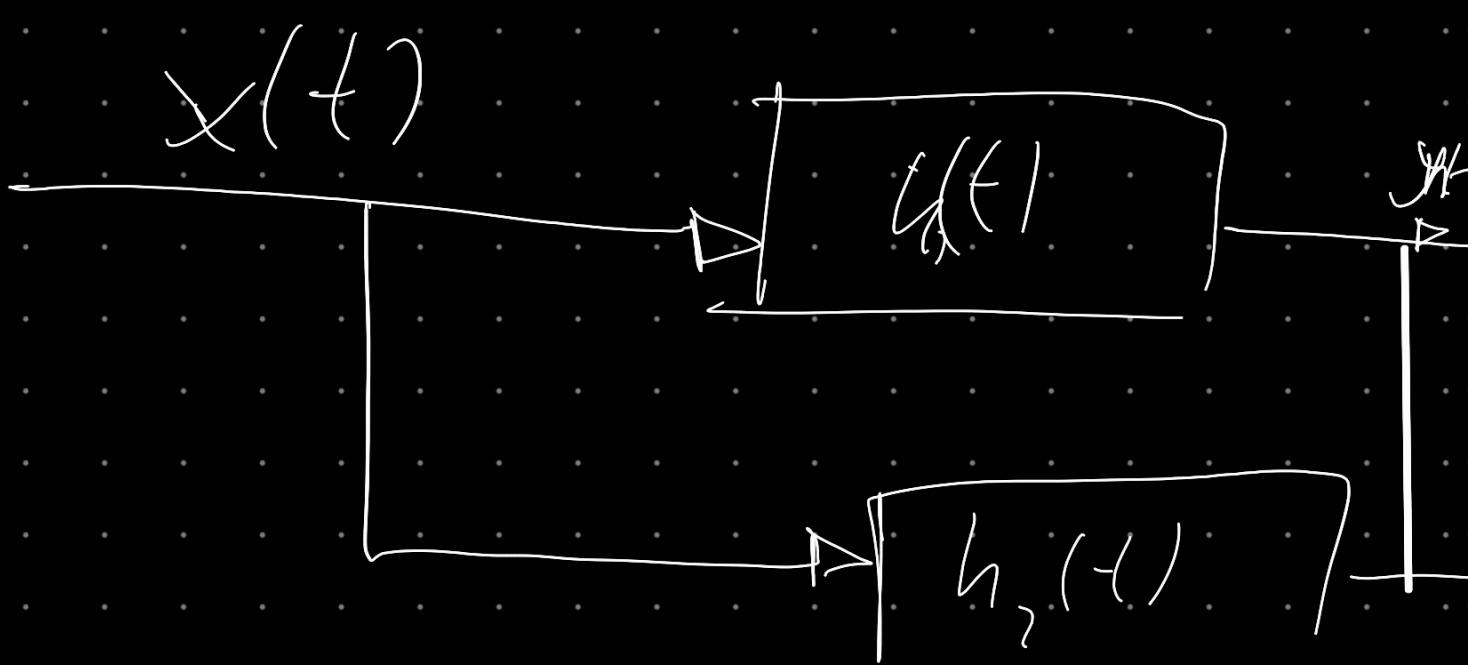
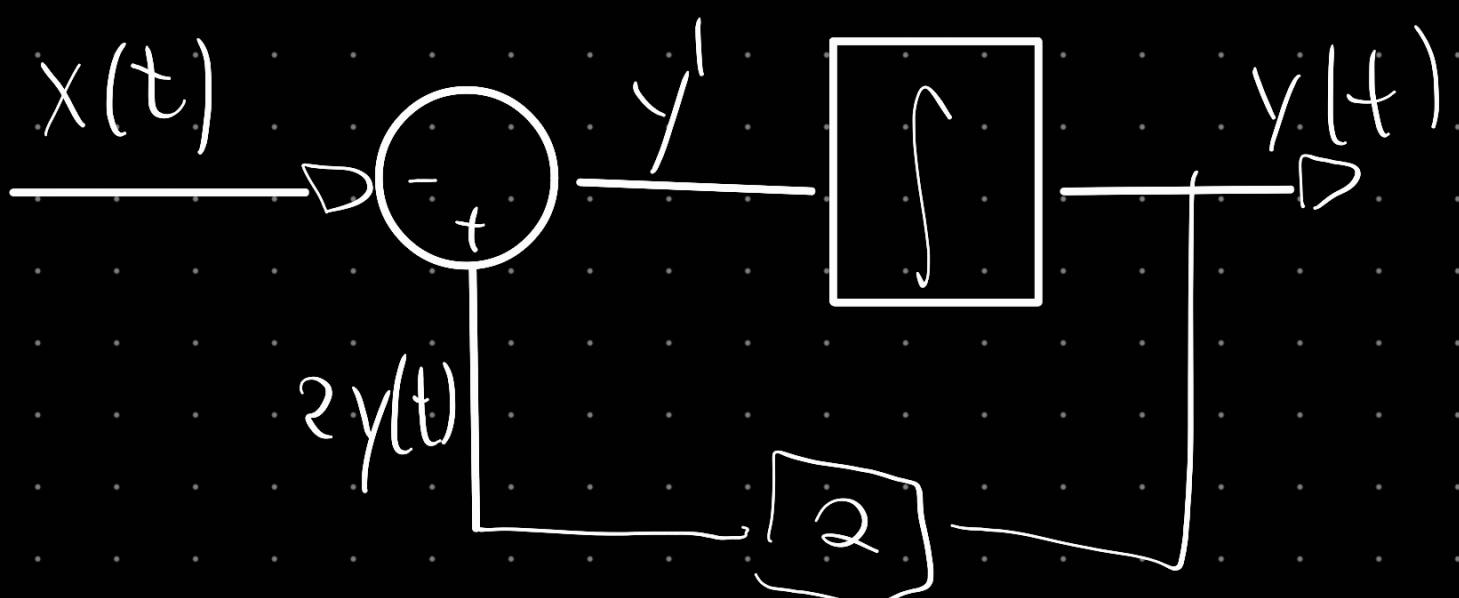
$$\int_0^\infty \delta(u) du = 1$$



$$y(t) = \frac{1}{2} y'(t) + x(t)$$



$$y'(t) = 2y(t) - 2x(t)$$



$$y(t) = x(t-\tau) + x(\tau-t)$$

$$x(t) \xrightarrow{S} x(t-\tau) + x(\tau-t)$$

$$x(0) \rightarrow x(-\tau) + x(\tau)$$

Time reversal
inversion

$$x(t) \xrightarrow{S} x(\sin t)$$

$$x(n) \rightarrow x(\sin n)$$

$$x(0)$$

$$x(t-t_0) \rightarrow y(t-t_0)$$

$$x(t) \rightarrow \omega s(3t) x(t)$$

$$x(t-t_0) \rightarrow \omega s(3t) x(t-t_0)$$

$$y(t-t_0) = \omega s(3(t-t_0)) x(t-t_0)$$

$$-\beta \leq x(t) \leq \beta$$

$y(t) \rightarrow \text{Estab} | \emptyset$

$$-2\beta \leq x(t-2) + x(2-t) \leq 2\beta$$

$$-\frac{1}{\beta} \leq \frac{1}{x(t)} \leq \frac{1}{\beta}$$

$$\beta \rightarrow 0$$

$$-\infty \leq \frac{1}{x(t)} \leq \infty$$

$$\frac{t^2}{2} + \infty$$

$$y(t) = X(2t) - X(-\infty)$$

$$\text{Si } x(t) = t$$

$$x(t) \rightarrow \omega(x(t-\tau)) \xrightarrow{\omega_0^{-1}(x(t+\tau))} x(t)$$

$$\text{cosh} \frac{2\pi}{\omega_0}$$

$$T_{sec/t} = \frac{\pi}{\omega_0}$$

$$\frac{2\pi}{4} =$$

$$b) e^{j(n\tau - 1)} = e^{jn\tau} e^{-j} \xrightarrow{\text{cosh}} u_{\text{cosh}}$$

$$(1) \quad (1) \quad | \quad (1)$$

$$[\omega(n\tau) + i \sin(n\tau)] e^{-j}$$

$$\cos(\beta\tau + \delta) + \sin(\alpha\tau - \alpha)$$

$$T = \text{MCM}(T_1, T_2) = \frac{2\pi}{\omega}$$

$$\omega = \text{MCM}(\omega_1, \omega_2)$$

$$\begin{array}{r|rr} 8 & 36 & 2 \\ 4 & 18 & 2 \\ 2 & 9 & \end{array} \left. \right\} 4$$

$$x_1 \rightarrow y_1$$

$$x_2 \rightarrow y_2$$

$$x_3 = ax_1 + bx_2$$

$$x_3 \rightarrow y_3 = ay_1 + by_2$$

$$y(t) = \omega s[x(t)]$$

$$x_1(t) \rightarrow y_1(t) = \cos[x_1(t)]$$

$$x_2(t) \rightarrow y_2(t) = \cos[x_2(t)]$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

$$y_3(t) = ay_1(t) + by_2(t)$$

$$y_3(t) = a \cos(x_1(t)) + b \cos(x_2(t))$$

→ obj

$$x_3(t) \rightarrow y_3(t) = \omega s[x_3(t)]$$

No Linear

$$= \omega s(ax_1(t) + bx_2(t))$$

$$= \omega s(ax_1) \omega s(bx_2) \sin$$

invariante en el tiempo

$$x(t) \rightarrow y(t) = \cos[x(t)]$$

$$x(t-t_0) \rightarrow y(t-t_0) = \cos[x(t-t_0)]$$

$$x(t) \rightarrow y(t) = -\tan[x(t)]$$

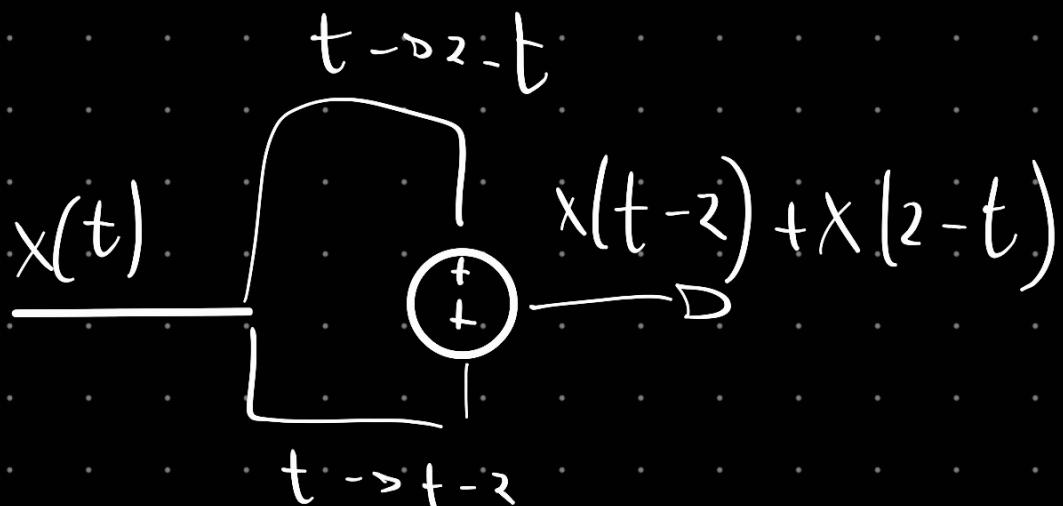
$$y(t-t_0) = -(\underline{t} - t_0) \cos[x(t-t_0)]$$

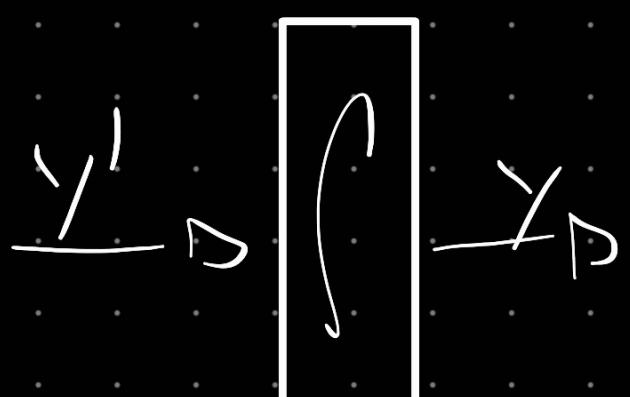
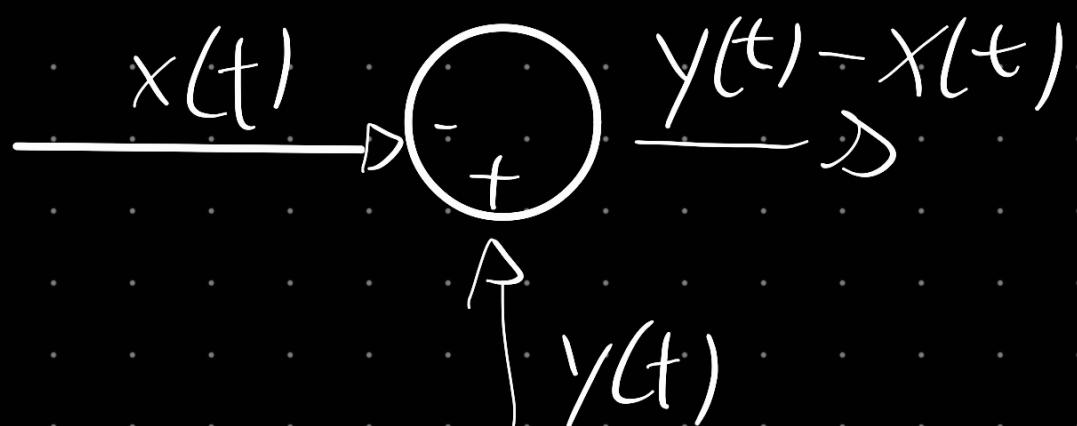
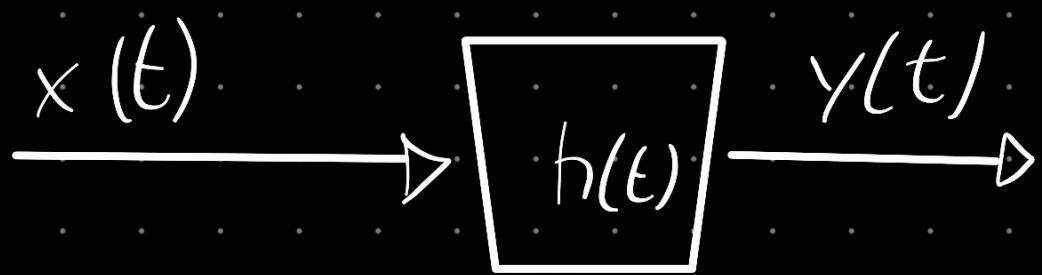
$$x(t-t_0) \rightarrow y(x(t-t_0)) = -t \cos[x(t-t_0)]$$

$$x(t) \rightarrow y(t) = x(t-3)$$

$$x(t) \xrightarrow{t \rightarrow t-3} x(t-3) \xrightarrow{t \rightarrow t+3} x(t)$$

$$x(t) \rightarrow x(t-2) + x(2-t) \rightarrow$$



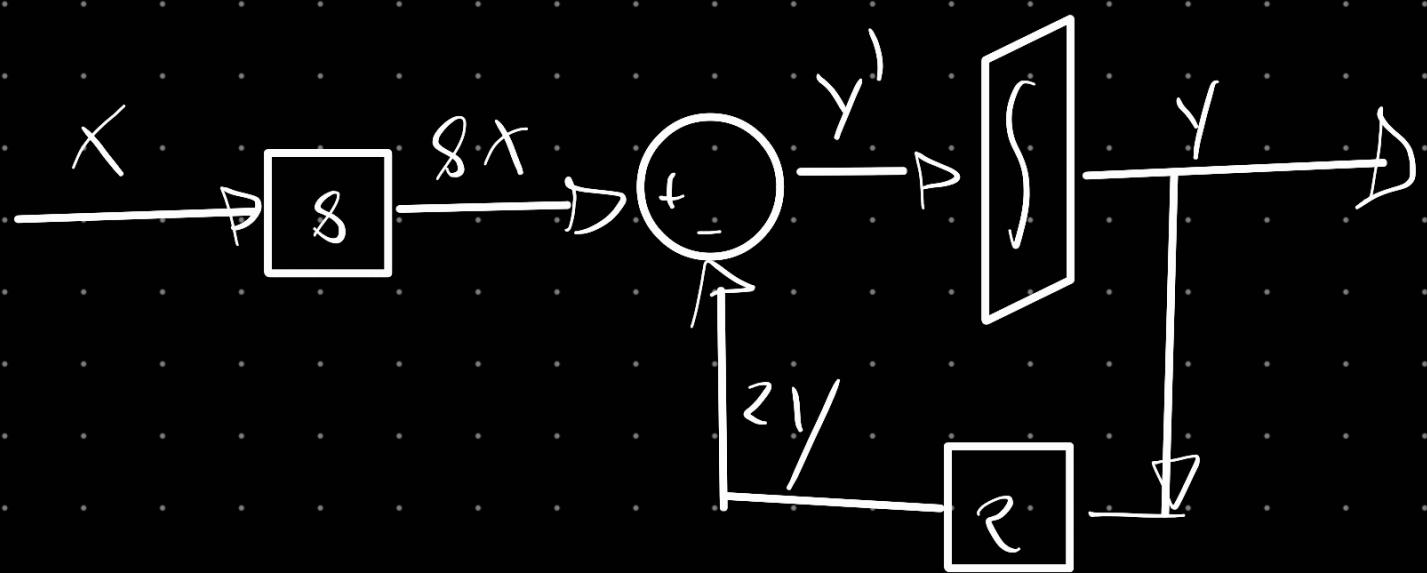


$$y' + p(t)y = x(t)$$

$$u(t) = e^{\int p(t) dt}$$

$$y = \frac{1}{u(t)} \left(\int u(t)x(t) \right)$$

$$a) \quad y' = 8x - 2y$$



$$y' = 8x - 2y$$

$$y' + 2y = 8x$$

$$y' + p(t)y = x(t)$$

$$z(t) = e^{\int p(t) dt}$$

$$y = \frac{1}{z(t)} \left(\int z(t) x(t) dt \right)$$

$$y' + 2y = x(t)$$

$$z(t) = e^{\int 2 dt} = e^{2t}$$

$$y = \frac{1}{e^{2t}} \left(\int e^{2t} x(t) dt \right)$$

$$\int e^{2t} e^{3t} u(t) dt$$

$$y(t) = \frac{1}{e^{2t}} \left(\frac{1}{5} e^{5t} u(t) + C \right)$$

$$y(0) = 0 = \frac{1}{0.25} \left(\underbrace{\frac{1}{5} e^{5.0}}_{U(0)} + C \right)$$

$$C = -\frac{1}{2}$$

$$\int f(t) \delta(t-\alpha) = f(\alpha)$$

$$\int f(t) u(t-\alpha) dt = u(t-\alpha) \int \underbrace{f(\tau)}_{x(\tau)} \delta(t-\tau-\alpha) d\tau$$

$$\int f(\tau) u(\tau-a) \Big|_{\tau=s}^{t-b}$$

$$f(t-b) u(t-b-a)$$

