3.1)

$$T = 8 \rightarrow \omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$x(t) = \frac{4}{2}\left(e^{-j\omega_0 t} + e^{j\omega_0 t}\right) + 4j e^{-3j\omega_0 t} + 4j e^{3j\omega_0 t}$$

$$4\cos(\omega_0 t) - 8\sin(3\omega_0 t)$$

$$4\cos(\omega_0 t) - 8\cos\left(3\omega_0 t - \frac{\pi}{2}\right)$$

$$+ 8\cos\left(3\omega_0 t + \frac{\pi}{2}\right)$$

3.2)

$$T = 5 \rightarrow \omega_0 = \frac{2\pi}{5}$$

$$x(t) = 1 + e^{j\frac{\pi}{4}} e^{2j\omega_0 t} + e^{-j\frac{\pi}{4}} e^{-2j\omega_0 t} + 2e^{j\pi/3} e^{4j\omega_0 t} + 2e^{-j\pi/3} e^{4j\omega_0 t}$$

$$1 + 2\left[\frac{e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}}{2}\right] + 2 \cdot 2\left[\frac{e^{j(4\omega_0 t + \frac{\pi}{3})} + e^{-j(4\omega_0 t + \frac{\pi}{3})}}{2}\right]$$

$$1 + 2\cos\left(2\omega_0 t + \frac{\pi}{4}\right) + 4\cos\left(4\omega_0 t + \frac{\pi}{3}\right)$$

$$1 + 2\sin\left(2\omega_0 t + \frac{3}{4}\pi\right) + 4\sin\left(4\omega_0 t + \frac{5}{6}\pi\right)$$

3.3) $w_0 = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{W_0}$

$T_1 = \frac{2\pi}{\frac{2\pi}{3}} = 3 \quad T_2 = \frac{2\pi}{\frac{5\pi}{3}} = \frac{6}{5}$

$3, 6, 9 \ldots$

$\frac{6}{5}; \frac{12}{5}; \ldots \frac{30}{5} = 6; \ldots$

$T_0 = 6 \rightarrow w_0 = \frac{\pi}{3}$

$$\begin{array}{cc|c} 3 & 6 & 3 \\ (1) & 2 & 2 \\ & (1) & \end{array} \qquad \begin{array}{cc|c} 1 & 5 & (1) \end{array}$$

$$x(t) = 2 + \cos(2w_0 t) + 4\sin(5w_0 t)$$

$$x(t) = 2e^{0jw_0t} + \frac{1}{2}e^{2jw_0t} + \frac{1}{2}e^{-2jw_0t} + \frac{4}{2j}e^{5jw_0t} - \frac{4}{2j}e^{-5jw_0t}$$

$a_0 = 2 \quad a_2 = a_{-2} = \frac{1}{2} \quad a_{-5} = a_5^* = 2j$

3,4)

$$x(t) = \begin{cases} 1,5 & 0 \leq t \leq 1 \\ -1,5 & 1 \leq t < 2 \end{cases}$$

$$T_0 = 2 \longrightarrow w_0 = \pi$$

$$a_n = \frac{1}{2}\left[1,5\int_0^1 e^{-j\pi n t}\,dt - 1,5\int_1^2 e^{-j\pi n t}\,dt\right]$$

Para $n = 0$

$$a_0 = \frac{1}{2}\left[1,5\int_0^1 e^{-j\pi 0 t}\,dt - 1,5\int_1^2 e^{-j\pi 0 t}\,dt\right]$$

$$a_0 = 0$$

Para $\forall n \{n = K \wedge K \in \mathbb{Z} - \{0\}\}$

$$a_k = \frac{1}{2}\left[1,5\int_0^1 e^{-j\pi k t}\,dt - 1,5\int_1^2 e^{-j\pi k t}\,dt\right]$$

$$a_k = \frac{-3\left[e^{-j\pi k t}\Big|_0^1 - e^{-j\pi k t}\Big|_1^2\right]}{4 j\pi K}$$

$$a_k = \frac{-3}{4j\pi k}\left(2e^{-jk\pi} - 1 - e^{-2jk\pi}\right)$$

Si

$$a_k = \frac{3}{4}\left[\int_{-1}^{0} e^{-j\pi kt}\,dt - \int_{0}^{1} e^{-j\pi kt}\,dt\right]$$

$$a_k = \frac{3}{-4j\pi k}\left(1 - \frac{e^{j\pi k} - e^{-j\pi k}}{2} \cdot 2 + 1\right)$$

$$\frac{3j}{2\pi k}\left[1 - \cos(\pi k)\right]$$

$$T_0 = \frac{\text{MCM do num}}{\text{MCD do den}}$$

$$w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\frac{LCM(A)}{MCD(B)}} =$$

$$w_0 = \frac{gcd(Lista\_num)}{lcm(Lista\_den)}$$

3.6)

a) Si $x(t)$ es real $\to a_k = a_{-k}^*$

1) No, ya que $k \in [0, 1\omega]$

2) $\cos(k n) \stackrel{?}{=} \cos(-kn)^*$

$\cos(kn) \stackrel{?}{=} \cos(kn)^*$

$\hookrightarrow$ True $\to$ Es real

3) $j\sin\left(\frac{k}{2}n\right) \stackrel{?}{=} j\sin\left(-\frac{k}{2}n\right)^*$

$j\sin\left(\frac{k}{2}n\right) \stackrel{?}{=} \left[-j\sin\left(\frac{k}{2}n\right)\right]^*$

$j\sin\left(\frac{k}{2}n\right) \stackrel{?}{=} j\sin\left(\frac{k}{2}n\right)$

$\hookrightarrow$ True $\to$ Es real

b) Si $x(t)$ es par $\to a_k$ es par

$\downarrow$

$a_k \stackrel{?}{=} a_{-k}$

1) No

2) Si

3) No

3.15) $\omega_0 = \dfrac{2\pi}{\frac{\pi}{6}} = 12$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(e^{jk\omega_0}) e^{jk\omega_0 t}$$

$$|j\omega k| > 100$$

$$|k|\,\omega_0 > 100$$

$$|k| > 8,3$$

$$k \in \mathbb{Z} - [-8 ; 8]$$

3.21) $T_0 = 8 \rightarrow \omega_0 = \pi/4$

$$a_1 = j = a_{-1}^* \rightarrow a_{-1} = -j$$

$$a_5 = a_{-5} = 2$$

$$x(t) = j e^{j\omega_0 t} - j e^{-j\omega_0 t} + 2e^{5j\omega_0 t} + 2e^{-5j\omega_0 t}$$

$$= 2j\, j \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right) + 2\cdot 2\left(\frac{e^{5j\omega_0 t} + e^{-5j\omega_0 t}}{2}\right)$$

$$-2\sin(\omega_0 t) + 4\cos(5\omega_0 t)$$

$$2\sin(\omega_0 t + \pi) + \quad // \quad //$$

$$2\cos\left(\omega_0 t + \frac{\pi}{2}\right) + 4\cos(5\omega_0 t)$$

3.22)

a)

a) $x(t) = t \quad -1 \leq t \leq 1 \quad w_0 = \frac{2\pi}{2}$

$$a_n = \frac{1}{2}\int_{-1}^{1} t\, e^{-j\overbrace{n w_0}^{a} t}\, dt$$

Para $n \neq 0 \quad \wedge \quad a = n w_0$

$$a_n = \frac{1}{2a^2}\left[ja\,2\,\underline{\left(\frac{e^{ja}+e^{-ja}}{2}\right)} - 1\,2j\,\underline{\left(\frac{e^{ja}-e^{-ja}}{2j}\right)}\right]$$

$$a_n = \frac{1}{2a^2}\left[\cancel{2}aj\cos(a) - \cancel{2}j\sin(a)\right]$$

$\cos(a) \to (-1)^k$ $\quad$ $\sin(a) \to \sin(n\pi) \to 0$

Siempre

$$\frac{j}{n\pi}(-1)^k$$

$$x(t) = \sum_{k=-\infty}^{\infty}\left[\begin{cases} 0, & k=0 \\ \frac{j}{n\pi}(-1)^k, & \text{o.c} \end{cases}\right] e^{jk\pi t}$$

b-c) Comeme los huevos

d) $T_0 = 2 \to \omega_0 = \pi$

$$x(t) = \begin{cases} \delta(t) & , t = 0 \\ -2\delta(t-1) & , t = 1 \end{cases}$$

$$a_k = \frac{1}{2} \int_{-0,5}^{1,5} \left[\delta(t) - 2\delta(t-1)\right] e^{-jk\omega_0 t} \, dt$$

$$a_k = \frac{1}{2}\left(e^{-jk\omega_0 \cdot 0} - 2e^{-jk\omega_0 \cdot 1} \right)$$

$$a_k = \frac{1}{2} - e^{-jk\omega_0}$$

$$a_k = \frac{1}{2} - \left[\cos(k\pi) - \cancel{j\sin(k\pi)}^{\,0}\right]$$

$$a_k = \frac{1}{2} - (-1)^k$$

$$x(t) = \sum_{k=-\infty}^{\infty} \left[\frac{1}{2} - (-1)^k\right] e^{jk\pi t}$$

e-f) Ir al [b-c]

b) $x(t) = e^{-t} \quad -1 < t < 1$

$T = 2 \rightarrow w_0 = \pi$

$$a_k = \frac{1}{2} \int_{-1}^{1} e^{-t} e^{-jkw_0 t} \, dt$$

$k = 0$

$$a_0 = \frac{1}{2} \int_{-1}^{1} e^{-t} \cancel{e^{-j0w_0 t}} \, dt \rightarrow 1$$

$$a_0 = \frac{1}{2} \int_{-1}^{1} e^{-t} \, dt = \frac{e^2 - 1}{2e}$$

Para $k \neq 0$

$$a_k = \frac{1}{2} \int_{-1}^{1} e^{-t} e^{-jkw_0 t} \, dt$$

$$a_k = \frac{1}{2} \int_{-1}^{1} e^{-\overbrace{(1+jkw_0)}^{\alpha} t} \, dt$$

$a_k =$

$$a_k = \frac{1}{2\alpha}\left(e^{\alpha} - e^{-\alpha}\right)$$

$$a_k = \frac{1}{2\alpha}\left[e^{(1+jk\omega_0)} - e^{-(1+jk\omega_0)}\right]$$

$$\frac{1}{2\alpha}\left[e \cos k\pi - e^{-1}\cos k\pi\right]$$

$$\frac{(-1)^k}{2(1+jk\omega_0)}\left(e^{1} - e^{-1}\right)$$

c)

$$x(t) = \begin{cases} \sin\pi t & 0 \le t \le 2 \\ 0 & 2 < t \le 4 \end{cases}$$

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_k = \frac{1}{4}\int_0^2 \sin\pi t\, e^{-jk\omega_0 t}\,dt$$

$$a_k = \frac{1}{4}\,\frac{1}{2j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^{-jk\omega_0 t}\,dt$$

$$a_k = \frac{1}{8j}\left(\int_0^2 e^{j\overbrace{(\pi-k\omega_0)}^{\alpha}t} - e^{-j\overbrace{(\pi+k\omega_0)}^{B}t}\,dt\right.$$

$$a_k = \frac{1}{8j}\left(\frac{j - je^{2j\alpha}}{\alpha} + \frac{-je^{-2j\beta} + j}{\beta} \right)$$

$$a_k = \frac{1}{8j}\left\{ \frac{j - j\left[\cos\left(2\pi - 2k\frac{\pi}{2}\right) + j\sin(2\pi - k\pi)\right]}{\alpha} + \frac{j - j\left[\cos(2\pi + k\pi) + j\sin(2\pi + k\pi)\right]}{\beta} \right\}$$

$$a_k = 2\cdot\frac{1}{8j}\left[\frac{j - j(-1)^k}{\left(\pi - k\frac{\pi}{2}\right)\cdot 2} + \frac{j - j(-1)^k}{\left(\pi + k\frac{\pi}{2}\right)\cdot 2} \right]$$

$$a_k = \frac{1}{4\cancel{j}}\left\{ \frac{\cancel{j}\left[1 - (-1)^k\right]}{\pi(2-k)} + \frac{\cancel{j}\left[1 - (-1)^k\right]}{\pi(2+k)} \right\}$$

$$a_k = \frac{1}{4\pi}\left[\frac{1 - \overbrace{(-1)^k}^{\to a}}{2-k} + \frac{1 - \overbrace{(-1)^k}^{\to a}}{2+k} \right]$$

$$a_k = \frac{1 - (-1)^k}{\pi(4 - k^2)}$$

3.26) $a_k = \begin{cases} 2 & k=0 \\ j\left(\frac{1}{2}\right)|k| & \text{O.C.} \end{cases}$

a) ¿ $a_k = a^*_{-k}$?

$$a^*_{-k} = -j\left(\frac{1}{2}\right)|-k|$$

$$a^*_{-k} = -j\left(\frac{1}{2}\right)^{|k|} \neq a_k$$

$\therefore$ No es Real

b) ¿ a_k es Par? Sí $x(t)$ es par

c) ¿ $a'_k = jkw_0 a_k$ es par?

$$a'_k = jkw_0 \cdot j\left(\frac{1}{2}\right)^{|k|}$$

$$a'_k = -\left(\frac{1}{2}\right)^{k} k w_0 \rightarrow x'(t) \text{ es Impar}$$

3.40) Si $x(t) \rightarrow a_k$:

a) $x(t-t_0) + x(t+t_0) \rightarrow x(t-(-t_0))$

$$a_k e^{-jk\omega_0 t_0} + a e^{jk\omega_0 t_0}$$

b) $Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-x)]$

$$\frac{1}{2}[a_k + a_{-k}]$$

c) $Re\{x(t)\} = \frac{1}{2}[x(t) + x^*(t)]$

$$\frac{1}{2}[a_k + a^*_{-k}]$$

d) $x''(t)$

$$a'_k = jk\omega_0 a_k$$

$$a''_k = jk\omega_0[jk\omega_0 a_k]$$

$$a_k = -(k\omega_0)^2 a_k$$

e) $x(3t-1)$ $\qquad w_0 = \frac{2\pi}{T_0/3} = \frac{6\pi}{T_0}$

$a_k e^{-jk3w_0}$

3.44) $x(t)$ es Real $\rightarrow$ $a_k = a_k^*$

$x(t)$ es periódica $\wedge$ $T = 6 \rightarrow w_0 = \frac{\pi}{3}$

$a_k = 0 \qquad k = 0 \wedge k > 2$

$a_k = a_k^* \rightarrow \forall a_k \{ k \notin [-2,2] - \{0\} \rightarrow a_k = 0$

$$x(t) = \sum_{k=-2}^{k=2} a_k e^{jkw_0t} = a_1 \cos(w_0 t) + a_2 \cos(2w_0 t)$$

$$x(t) = -x(t-3)$$

$$a_1\cos(w_0t) + a_2\cos(2w_0t) = -a_1\cos(w_0t - \cancel{3}\tfrac{\pi}{\cancel{3}}) - a_2\cos(2w_0t - 2\pi)$$

$$a_1\cos(w_0t) + a_2\cos(2w_0t) = a_1\cos(w_0t - \cancel{3}\tfrac{\pi}{\cancel{3}}) - a_2\cos(2w_0t - 2\pi)$$

$\hookrightarrow a_2 = -a_2 \rightarrow a_2 = 0$

$$\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{1}{2} = \sum_{k=-2}^{2} |a_k|^2$$

$$\cancel{|a_{-2}|^2} + |a_{-1}|^2 + |a_1|^2 + \cancel{|a_2|^2} = \frac{1}{2}$$

$$2|a_1|^2 = \frac{1}{2}$$

$$|a_1|^2 = \pm\frac{1}{4} = \pm\frac{1}{2}$$

$A = 1/2 \qquad C = 0$

$B = \pi/3$

$a_1 > 0 \rightarrow a_1 = \frac{1}{2}$

3.45)

$$Ev\{x(t)\} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \frac{1}{2}\left(x(t)+x(-t)\right)$$

$$a_k = \frac{1}{2}\left(a_k + a_{-k}\right)$$

$$\cancel{\tfrac{1}{2}}\, a_k = \cancel{\tfrac{1}{2}}\, a_{-k} \rightarrow a_k = a_{-k}$$

↳ a_k es par

↳ $x(t)$ es par

$$x(t) = x(-t)$$

$$a_0 + 2\sum_{k=1}^{\infty} B_k \cos k\omega_0 t - C_k \sin k\omega_0 t = a_0 + 2\sum_{k=1}^{\infty} B_k \cos -k\omega_0 t - C_k \sin -k\omega_0 t$$

$$C_k \quad = -C_k = 0$$

$$B_k = -B_{-k}$$

→ es real y par

$$Ev\{x(t)\} \rightarrow a_k = B_k \qquad a_0 = a_{x_0}$$

$$Od\{x(t)\} \rightarrow B_k = j\begin{cases} -C_k & k>0 \\ C_k & k<0 \\ 0 & k=0 \end{cases}$$

↳ es imag. e Impar