2.10) $x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{o.c} \end{cases}$

$h(t) = x(t/\alpha) \quad 0 \le \alpha \le 1$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) \cdot x\left(\frac{\tau}{\alpha}\right) d\tau$$

$\to 0 \le t-\tau \le \alpha$

$t \ge \tau \ge t-\alpha$

$0 \le \frac{\tau}{\alpha} \le 1$

$\to 0 \le \tau \le \alpha$

$$y(t) = \int_0^{\alpha} x\left(\frac{\tau}{\alpha}\right) d\tau + \int_{t-\alpha}^{t} x(\tau) d\tau$$

$$= \tau \Big|_0^{\alpha} + \tau \Big|_{t-\alpha}^{t}$$

$0 \le \frac{t-\tau}{\alpha} \le 1$

$t \ge \tau \ge t-\alpha$

$$x(\tau)\begin{cases} 1 & 0\le \tau \le 1 \\ 0 & \text{O.C.} \end{cases}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\cdot x\left(\frac{t-\tau}{\alpha}\right) d\tau$$

$1 \quad 0 \le \tau \le 1$

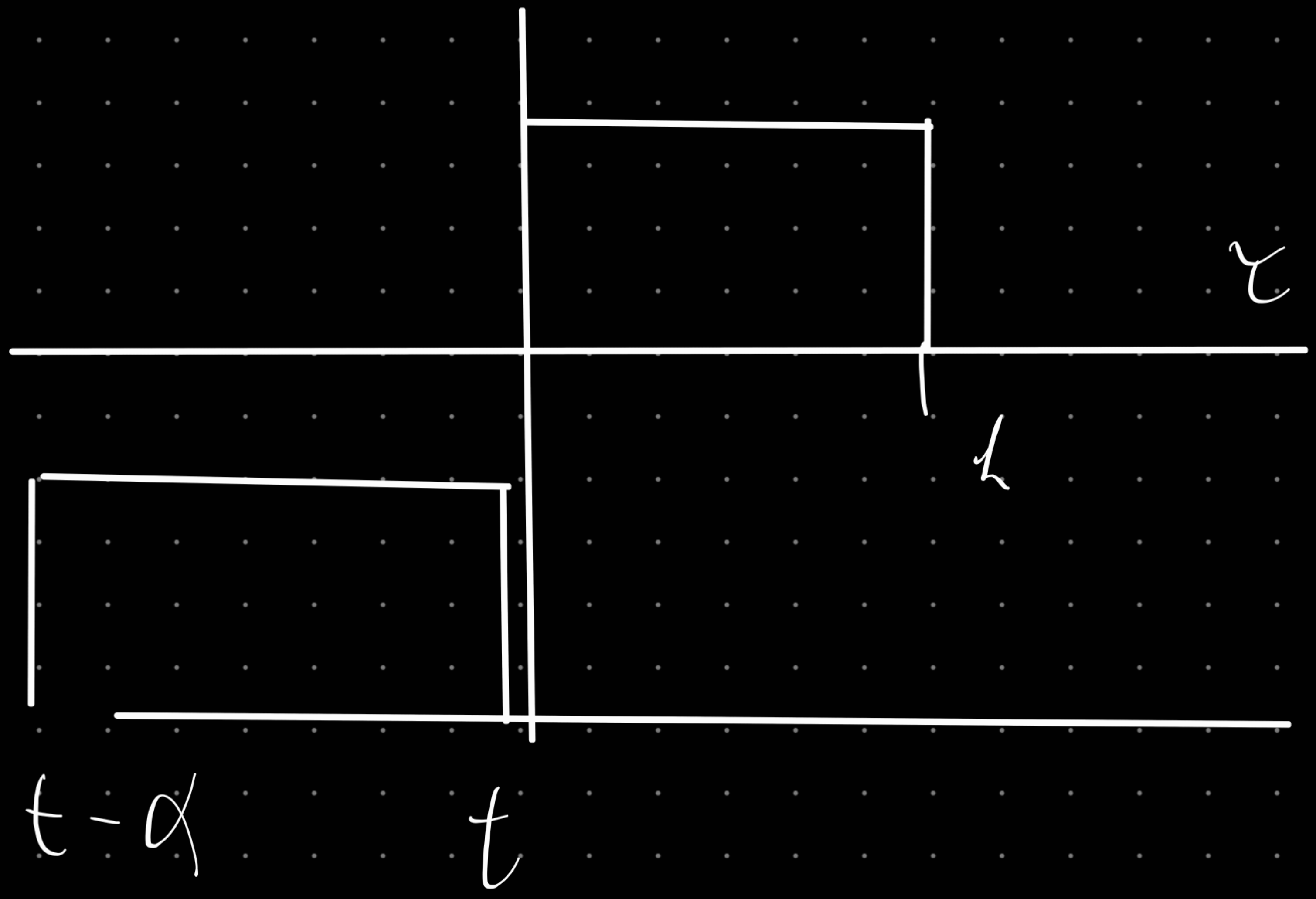
$0 \quad \text{O.C.}$

$1 \quad 0 \le \frac{t-\tau}{\alpha} \le 1$

0

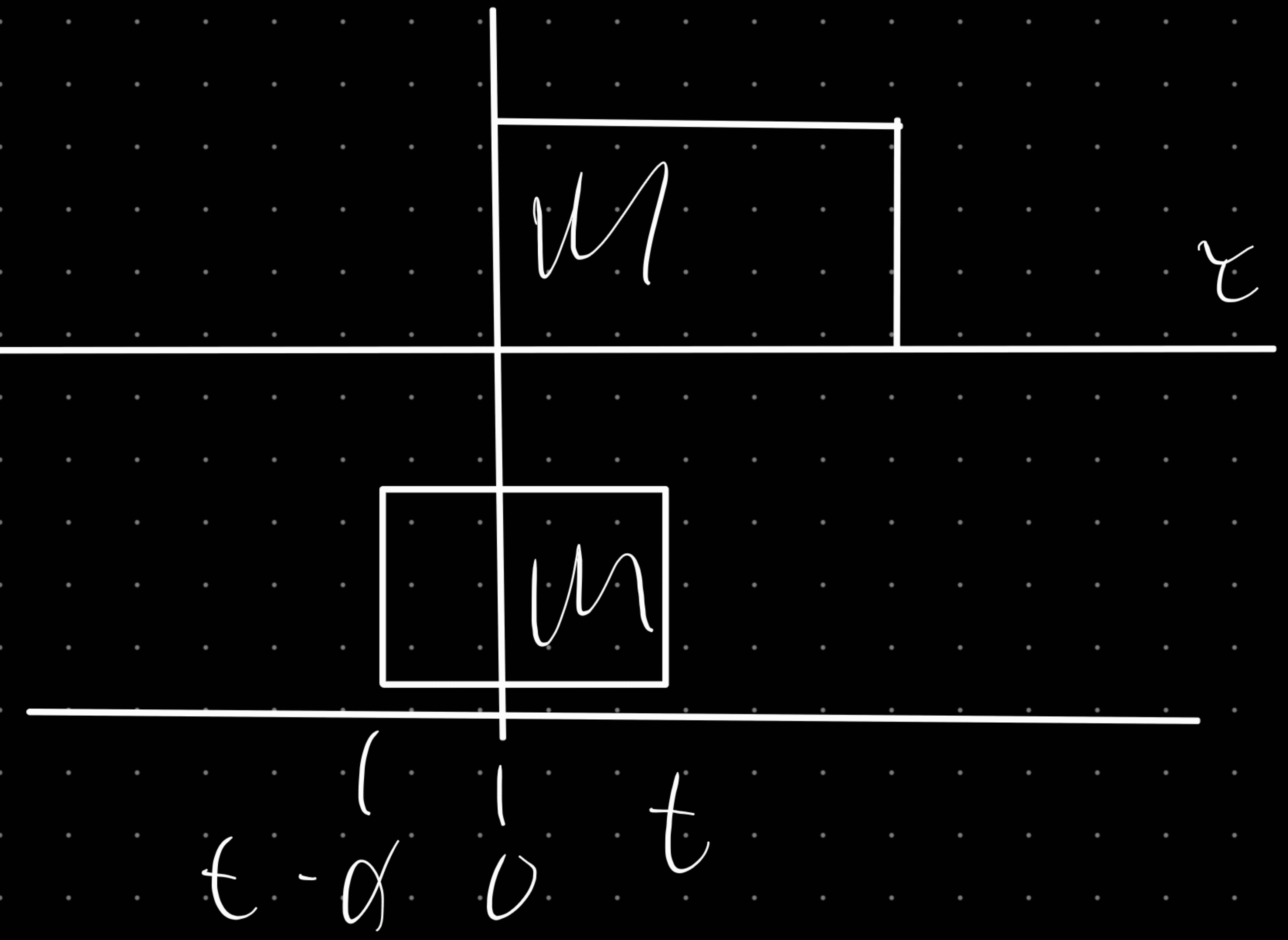
$t \ge \tau \ge t-\alpha$

Para $t < 0$



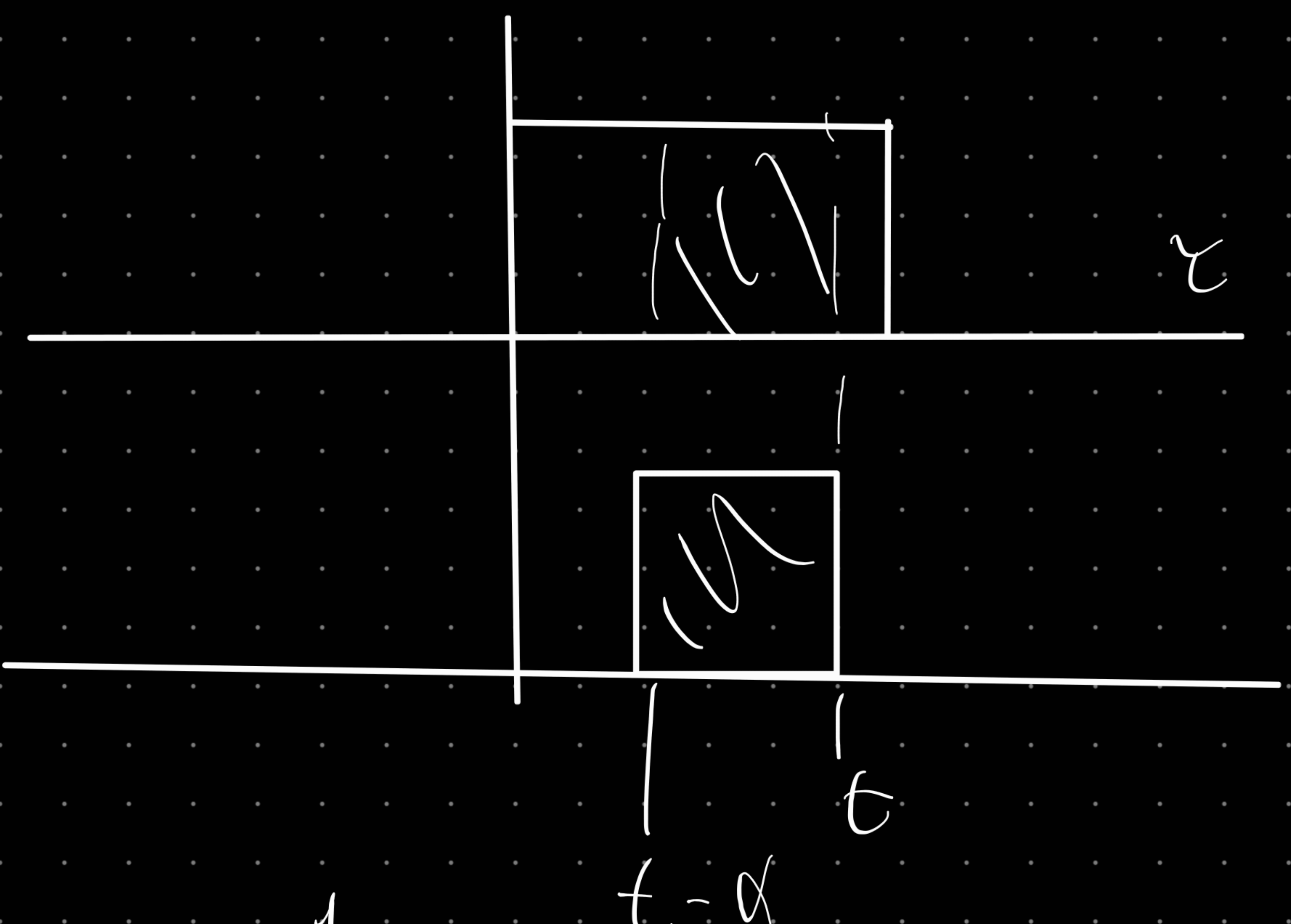
$y(t) = 0$

Para $0 < t < \alpha$



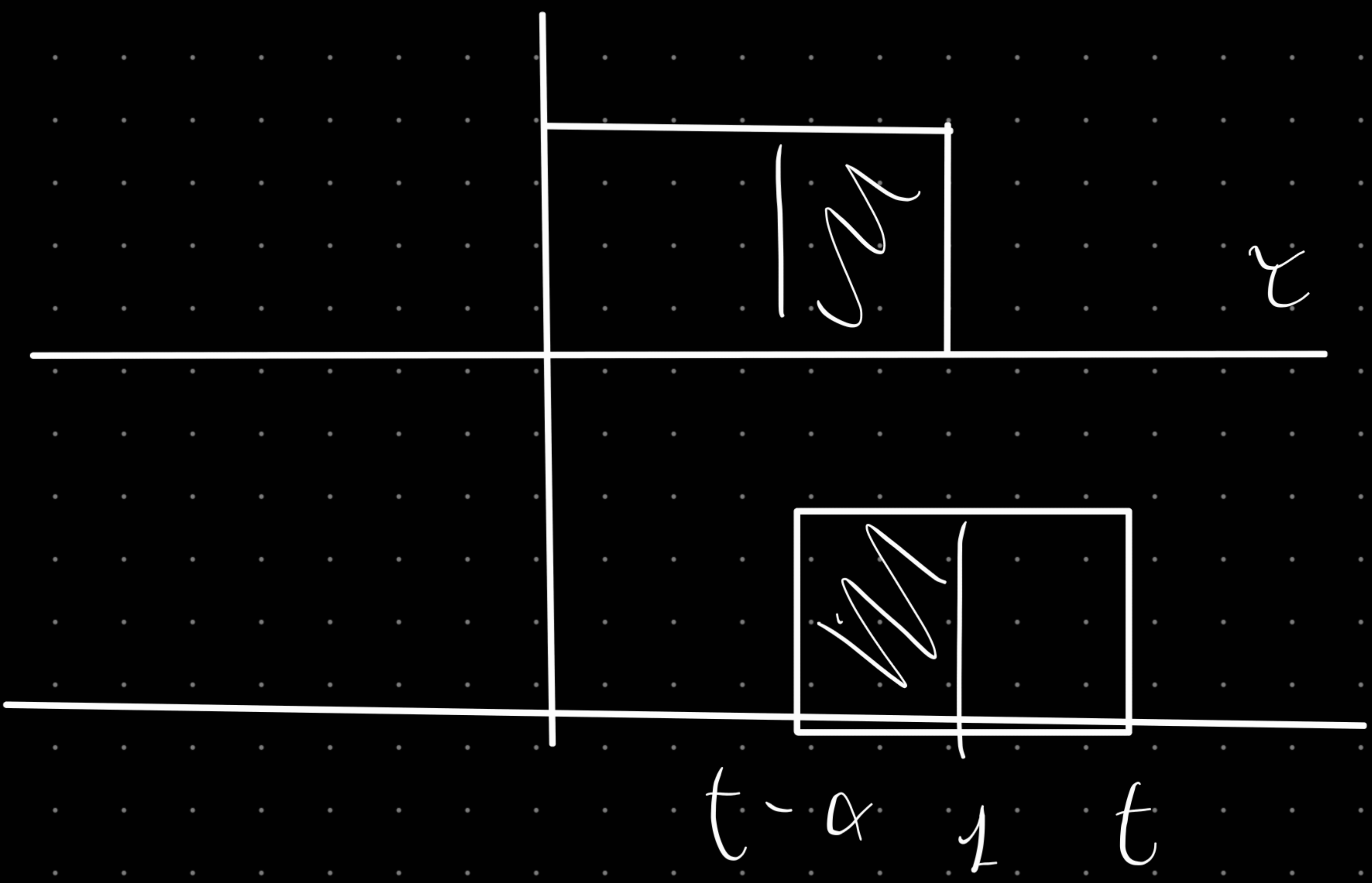
$$\int_0^t d\tau = \tau \Big|_0^t = t$$

Para $\alpha < t < 1$



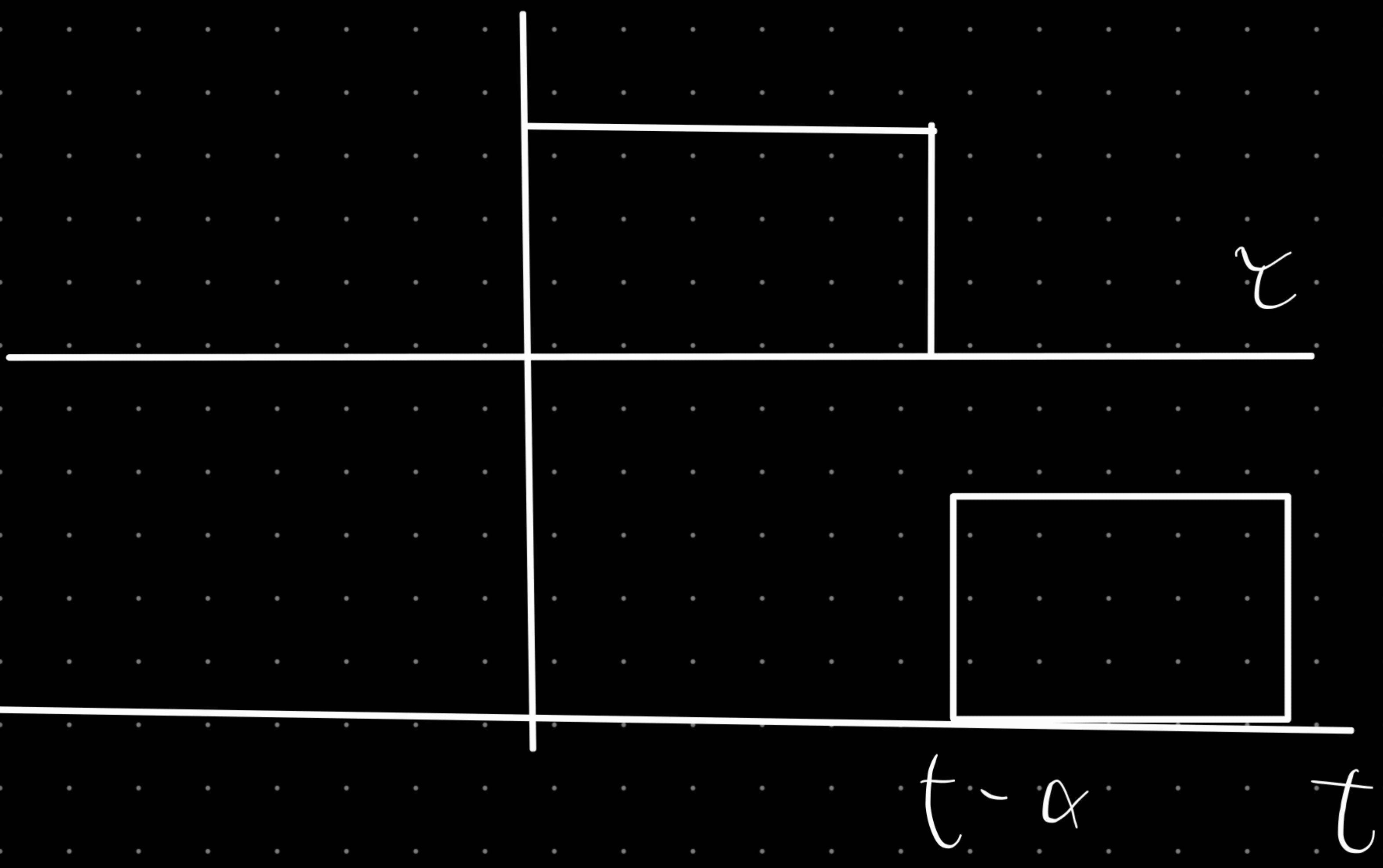
$$\int_{t-\alpha}^{t} d\tau = \tau \Big|_{t-\alpha}^{t} = \cancel{t} + \alpha - \cancel{t}$$

Para $1 < t < 1+\alpha$

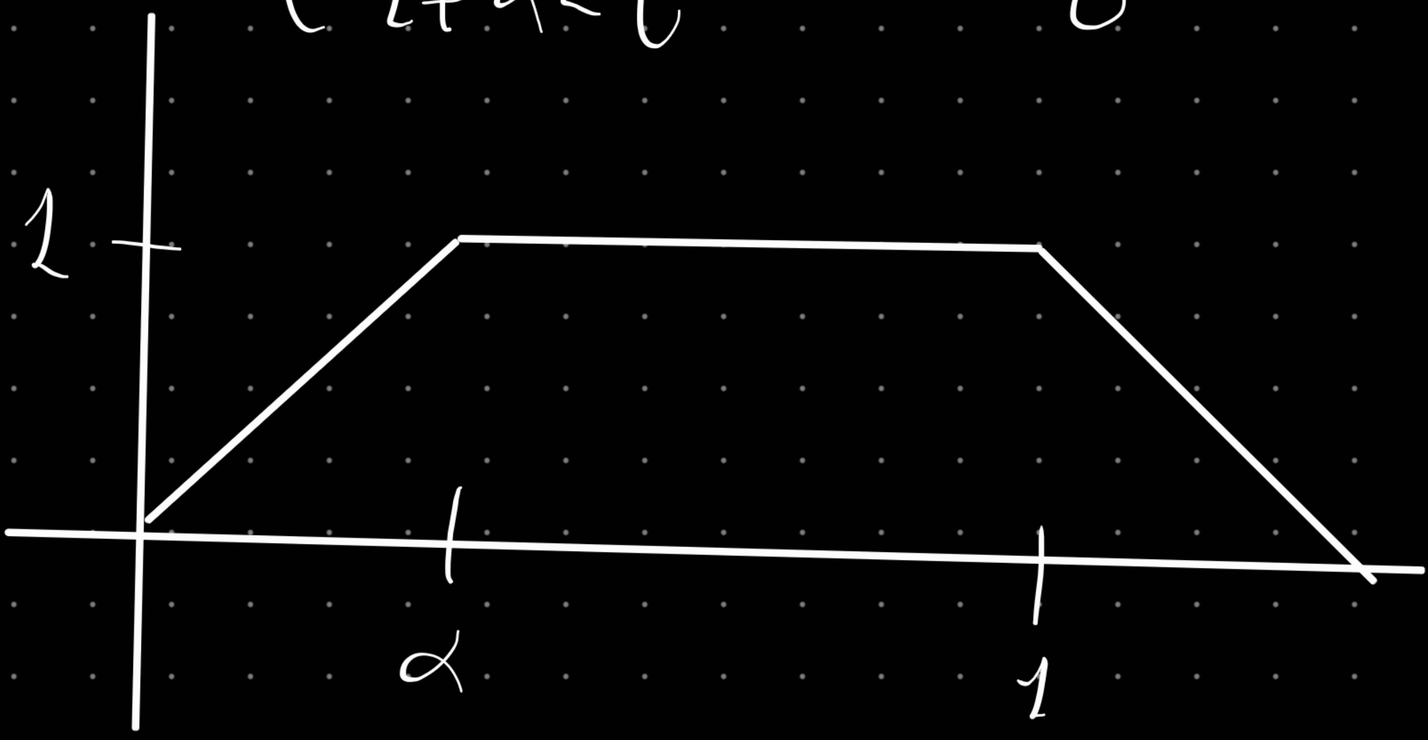


$$\int_{t-\alpha}^{1} d\tau = 1 - t + \alpha$$

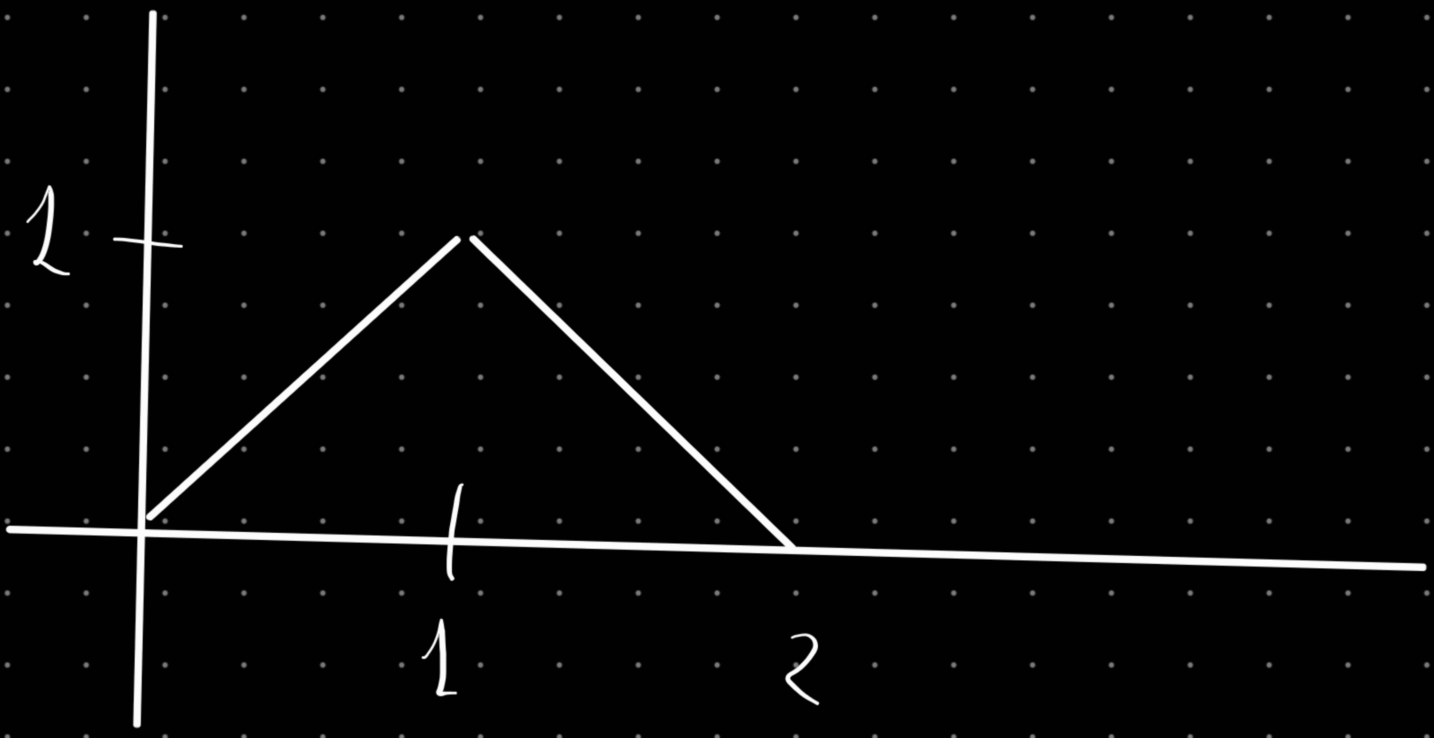
Para $1+\alpha < t$



$$y(t) = \begin{cases} t<0 & 0 \\ 0<t<\alpha & t \\ \alpha<t<1 & \alpha \\ 1<t<1+\alpha & 1-t+\alpha \\ 1+\alpha<t & 0 \end{cases}$$



b) $\alpha = 1$ entonces



$$y(t) = \begin{cases} t<0 & 0 \\ 0<t<1 & t \\ 1<t<2 & 1-t \\ 2<t & 0 \end{cases}$$

$$y'(t) = \begin{cases} t<0 & 0 \\ 0<t<1 & 1 \\ 1<t<2 & -1 \\ 2<t & 0 \end{cases}$$

$\Downarrow$

3 discontinuidades

2.22)

a)

I) $\alpha \neq \beta$

$$y(t) = \int_{-\infty}^{\infty} e^{-\alpha\tau} \cdot e^{-\beta(t-\tau)} \, u(\tau)\, u(t-\tau)\, d\tau$$

$u(\tau) \to 0$, $u(t-\tau) \to t$

solo está activo de 0 a t

$$= u(t) \int_{0}^{t} e^{-\alpha\tau} \cdot e^{-\beta(t-\tau)} \, d\tau$$

$$= u(t) \int_{0}^{t} e^{-\alpha\tau - \beta(t-\tau)} \, d\tau$$

$$= u(t)\, e^{-\beta t} \int_{0}^{t} e^{(\beta-\alpha)\tau} \, d\tau$$

$$= \frac{e^{-\beta t}}{\beta-\alpha} \left(e^{(\beta-\alpha)t} - 1 \right) u(t)$$

II) $\beta = \alpha$

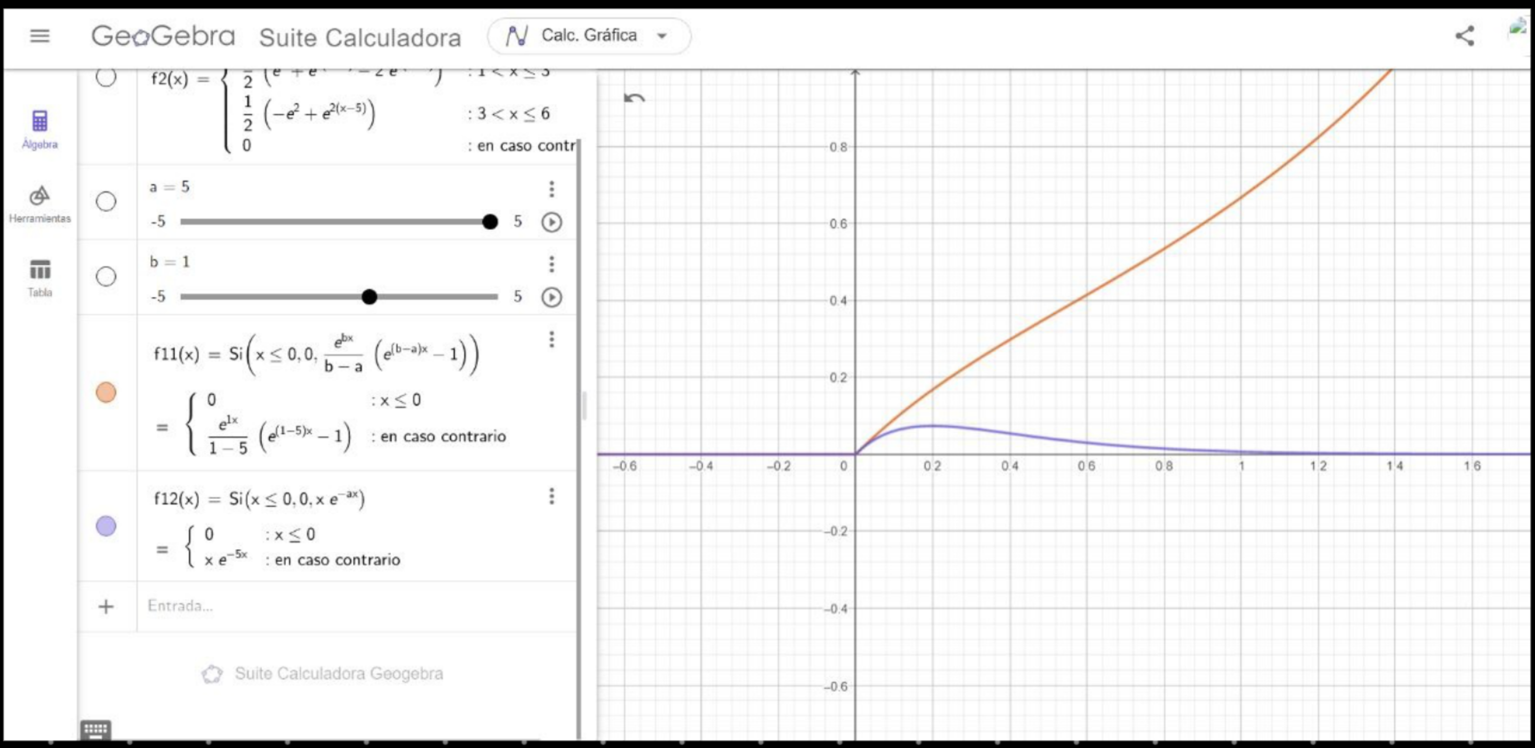
$$y(t) = \int_{-\infty}^{\infty} e^{-\alpha\tau} \cdot e^{-\alpha(t-\tau)}\, \overset{\to 0}{u(\tau)}\, \overset{\to t}{u(t-\tau)}\, d\tau$$

→ solo esta activo de 0 a t

$$u(t) \int_0^t \cancel{e^{-\alpha\tau}} \cdot e^{-\alpha(t-\cancel{\tau})}\, d\tau$$

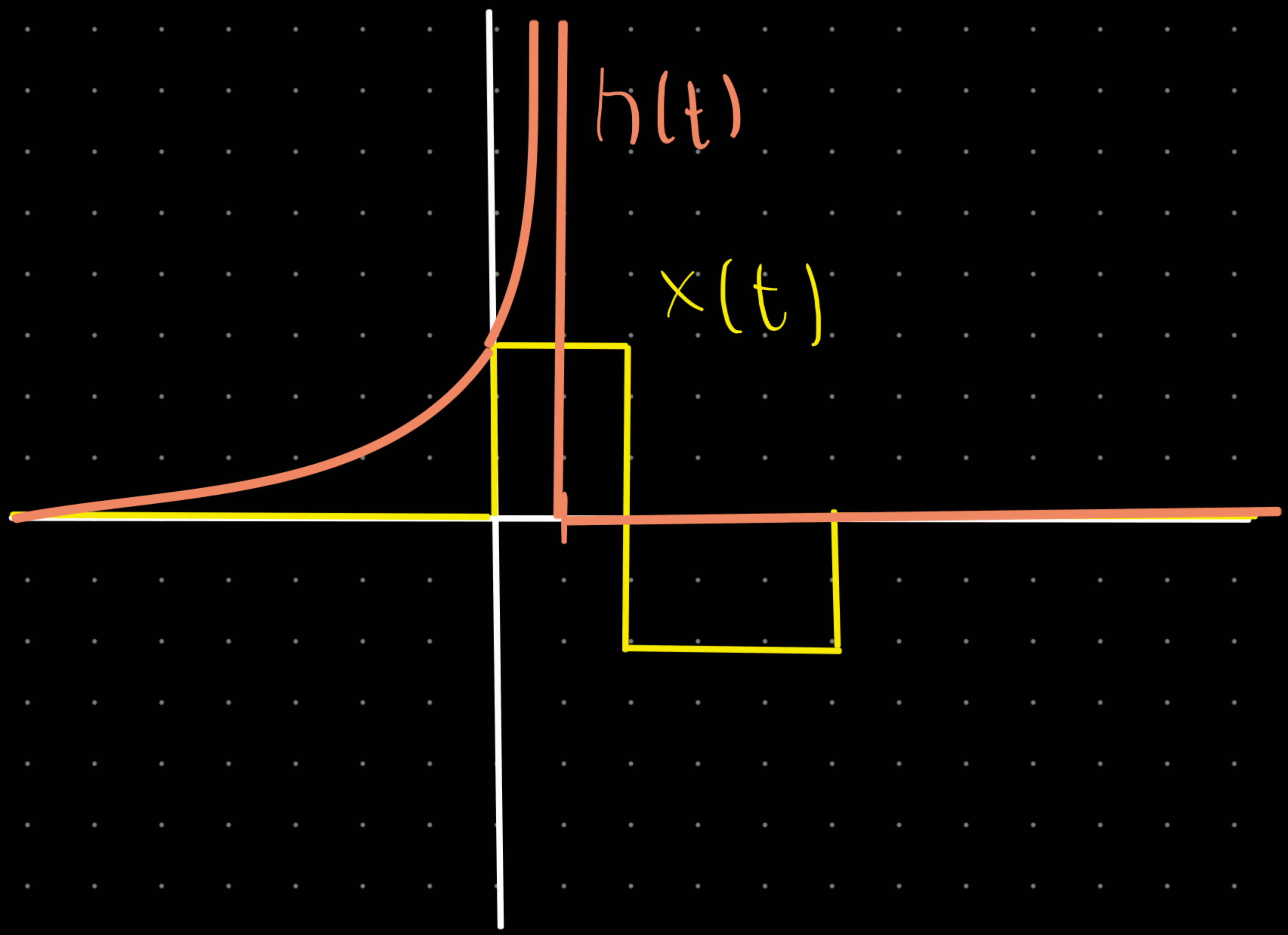
$$u(t)\, e^{-\alpha t} \int_0^t d\tau$$

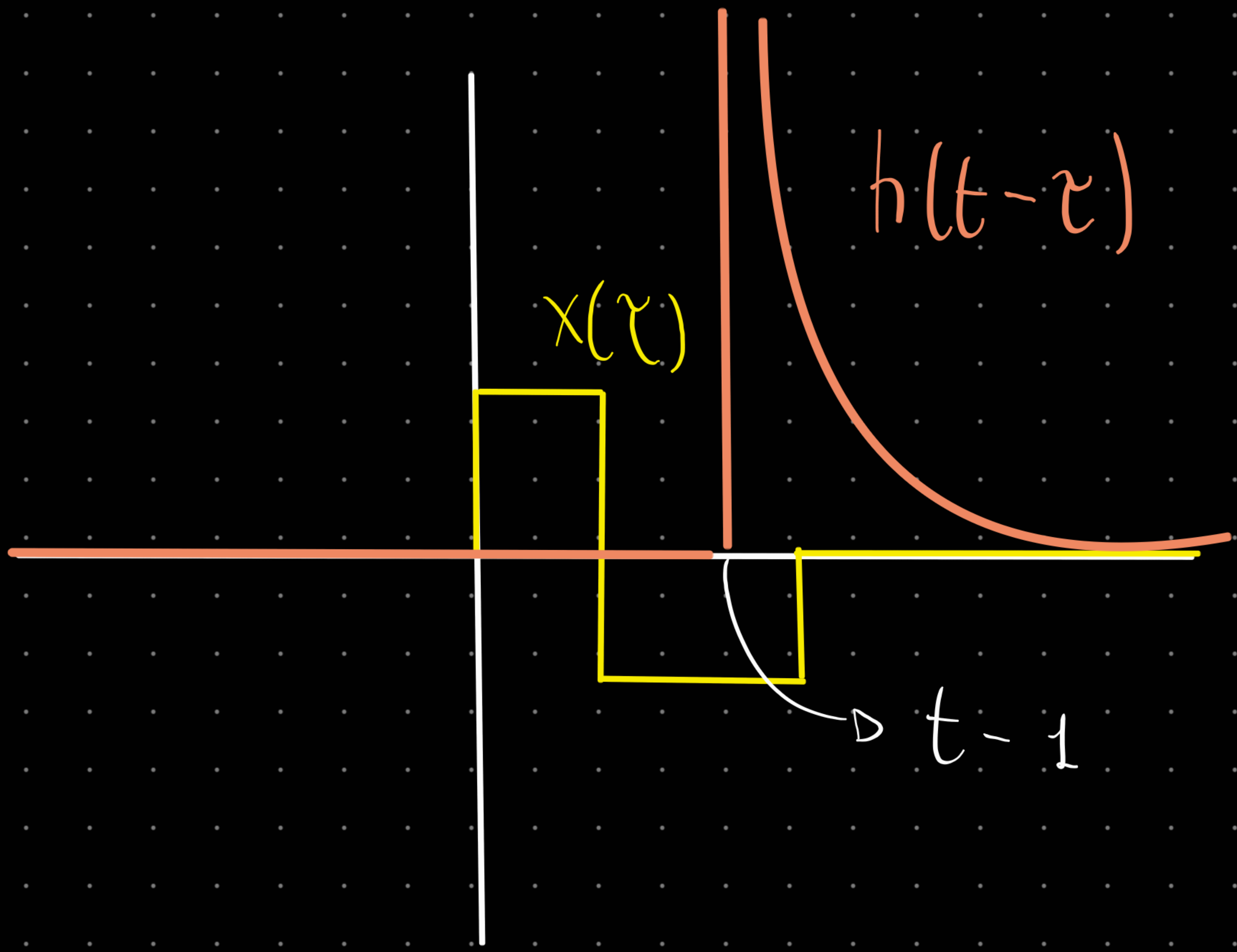
$$t\, e^{-\alpha t}\, u(t)$$



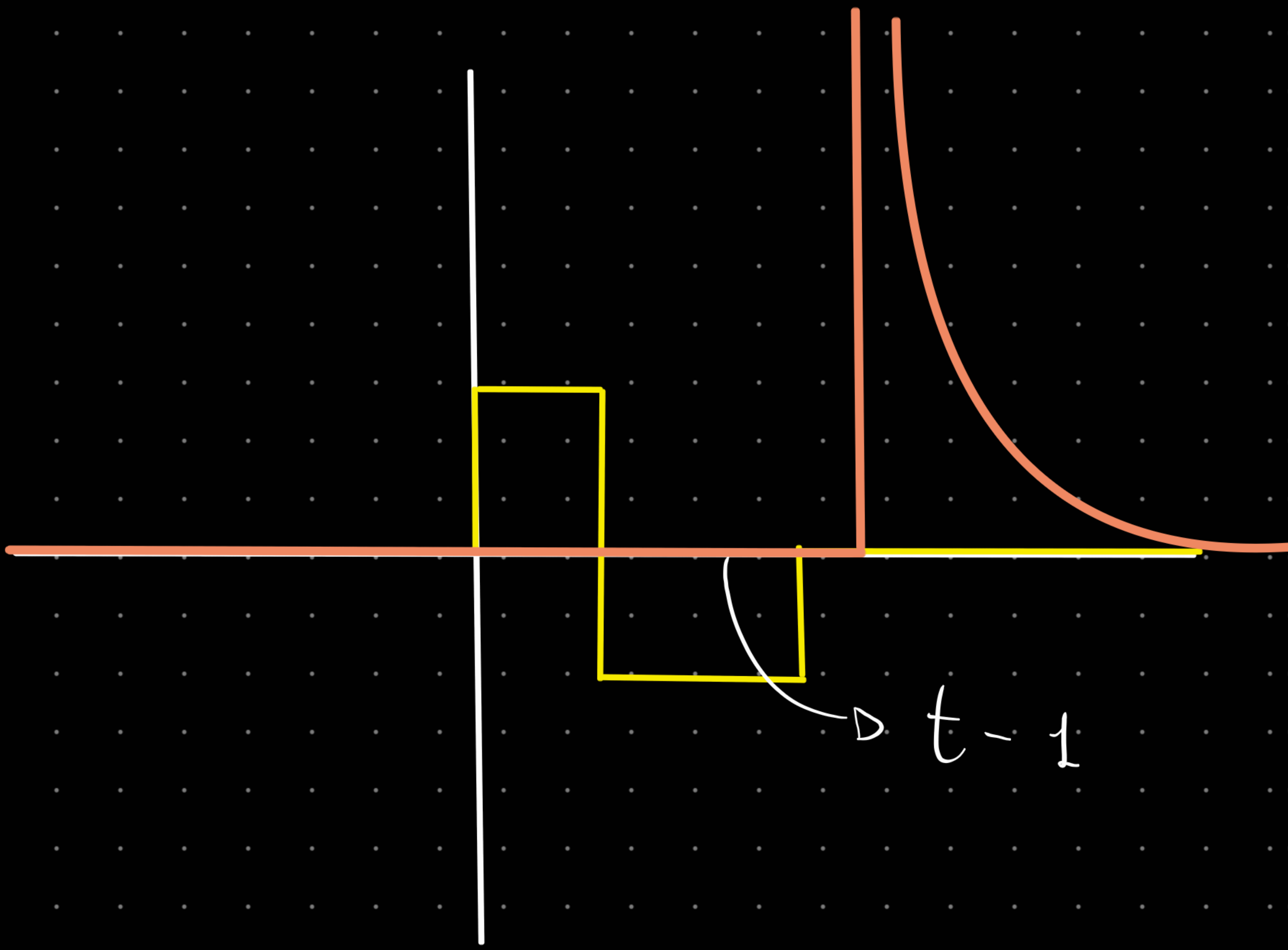
b)

$$y(t) = \int_{-\infty}^{\infty} \overbrace{[\underset{\tau>0}{u(\tau)} - 2\underset{\tau>2}{u(\tau-2)} + \underset{t>3}{u(\tau-5)}]}^{\text{desde}} \underset{e^{2t}e^{-2\tau}}{e^{2(t-\tau)}} \, \underset{\underbrace{\tau > t-1}_{\text{hasta}}}{u(1-t+\tau)} \, d\tau$$





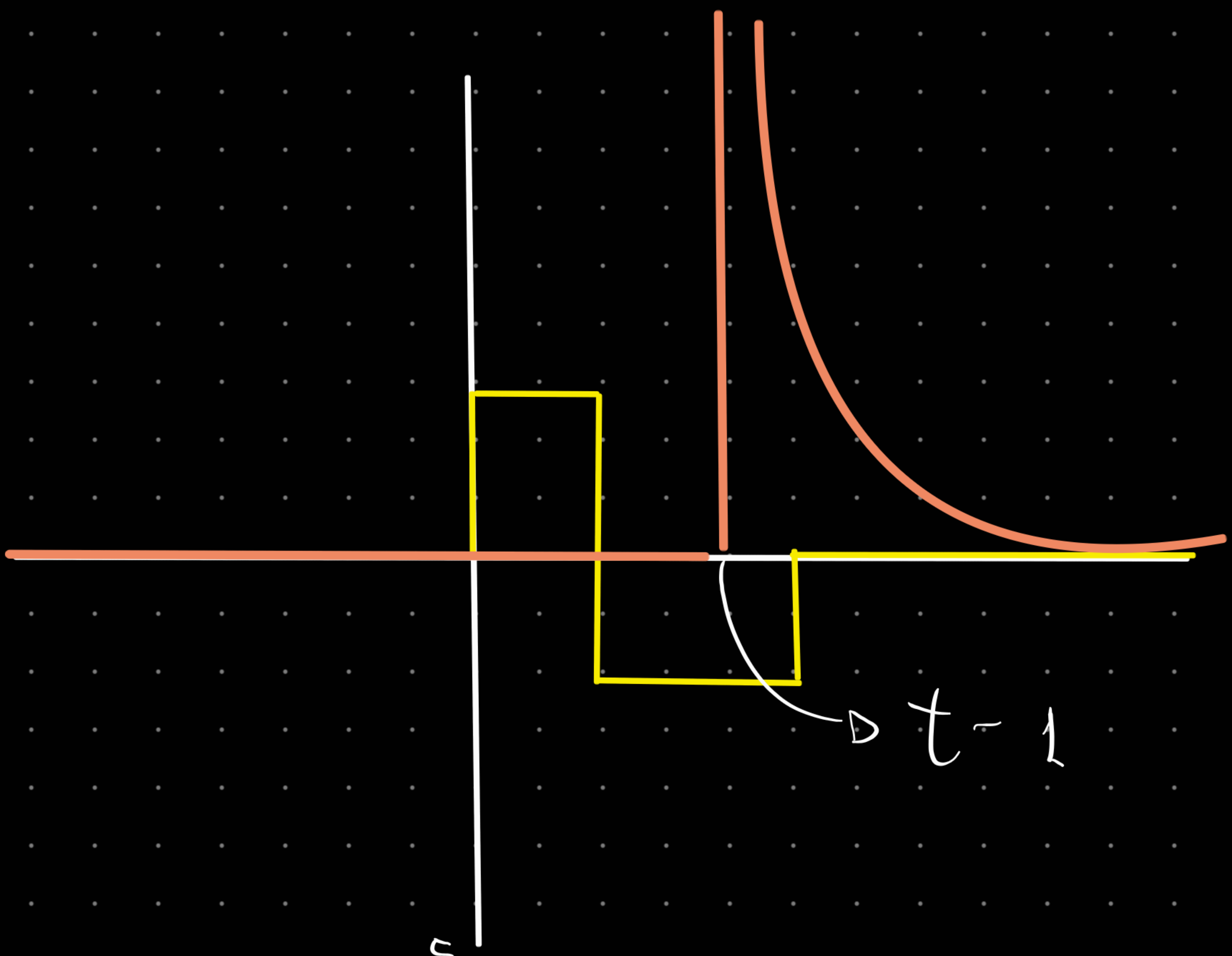
Para $t-1 \geq 5 \rightarrow t \geq 6$



$y(t) = 0$

Para $2 \leq t-1 \leq 5$

$$3 \leq t \leq 6$$

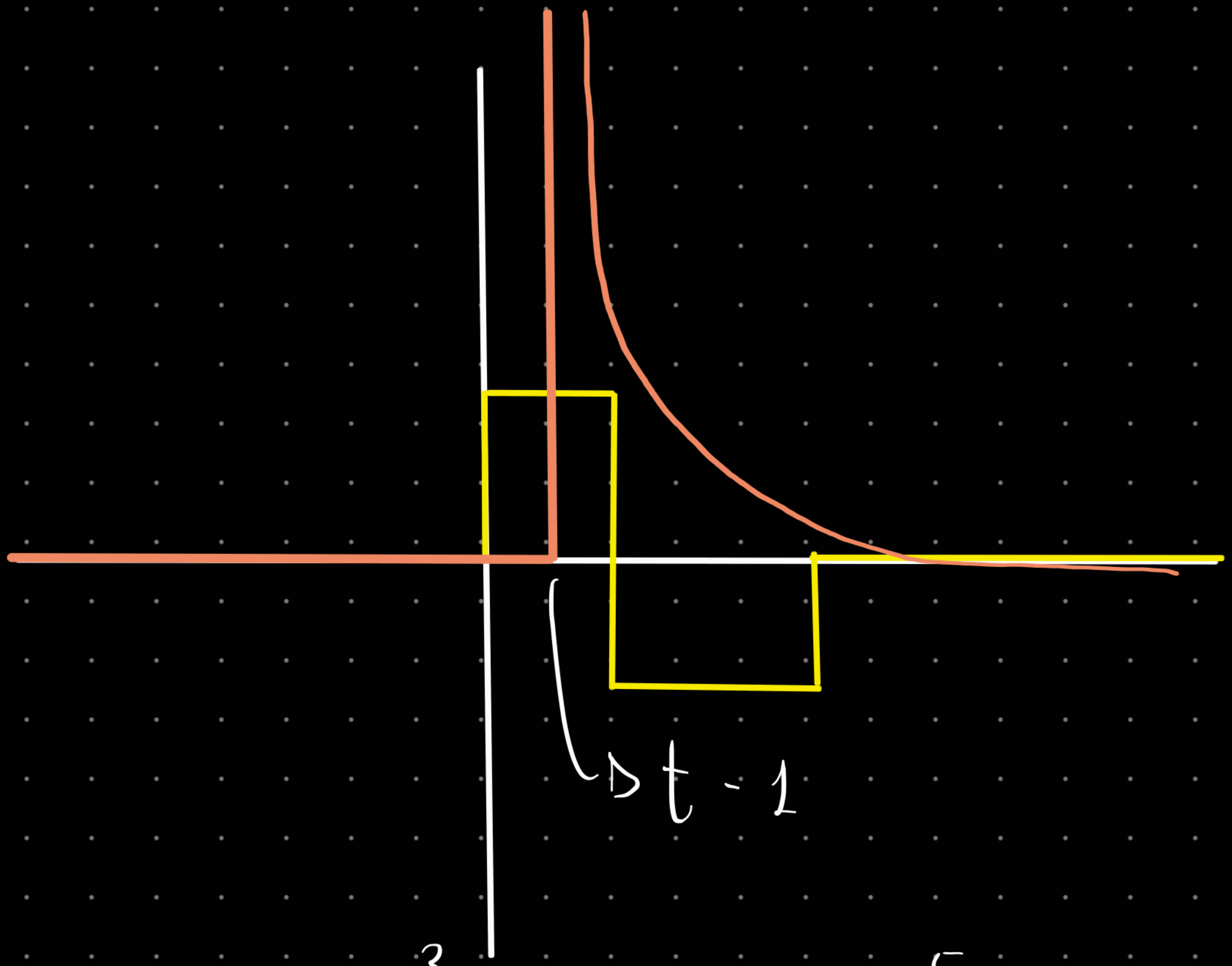


$$y(t) = -\int_{t-1}^{5} e^{2(t-\tau)}\,d\tau =$$

$$= \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right]$$

Para $0 \leq t-1 \leq 2$

$1 \leq t \leq 3$



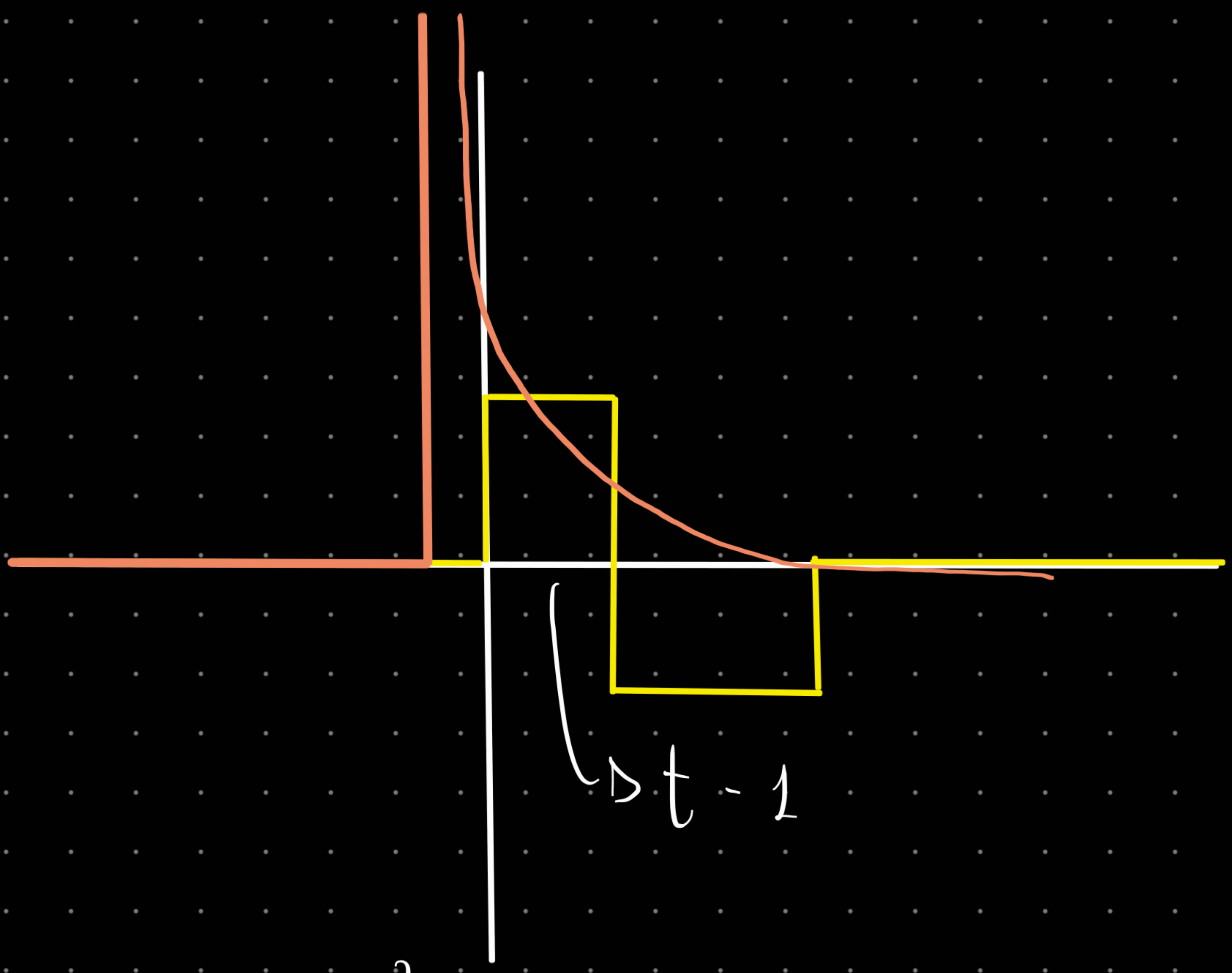
$$y(t) = \int_{t-1}^{2} e^{2(t-\tau)} d\tau - \int_{2}^{5} e^{2(t-\tau)} d\tau$$

$$y(t) = \frac{1}{2}\left(-e^{2t-4} + e^{2} + e^{2t-10} - e^{2t-4}\right)$$

$$= \frac{1}{2}\left[e^{2} - 2e^{2(t-2)} + e^{2(t-5)}\right]$$

Para $t - 1 \leq 0$

$t \leq 1$

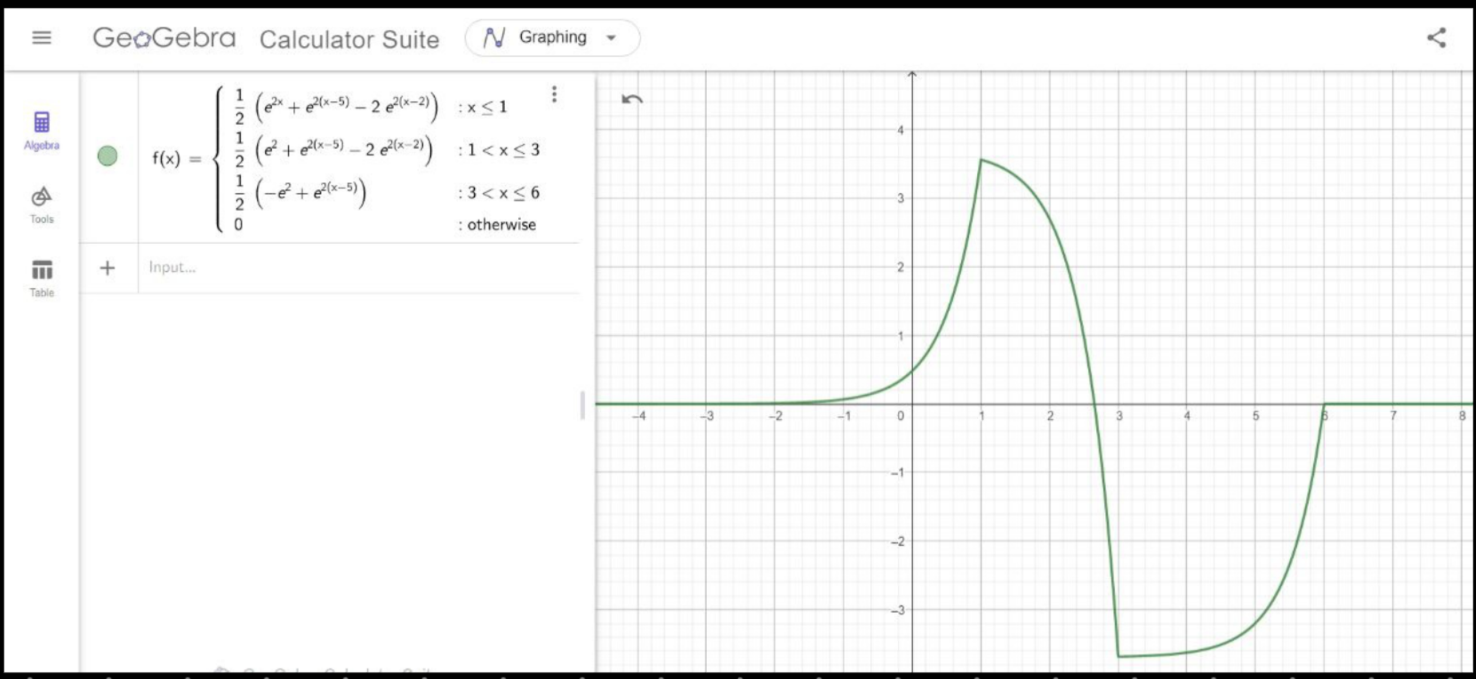


$$y(t) = \int_0^2 e^{2(t-\tau)} d\tau - \int_2^5 e^{2(t-\tau)} d\tau$$

$$y(t) = \frac{1}{2}\left(-e^{2t-4} + e^{2t} + e^{2t-10} - e^{2t-4}\right)$$

$$\frac{1}{2}\left[e^{2t} + e^{2(t-5)} - 2e^{2(t-2)}\right]$$

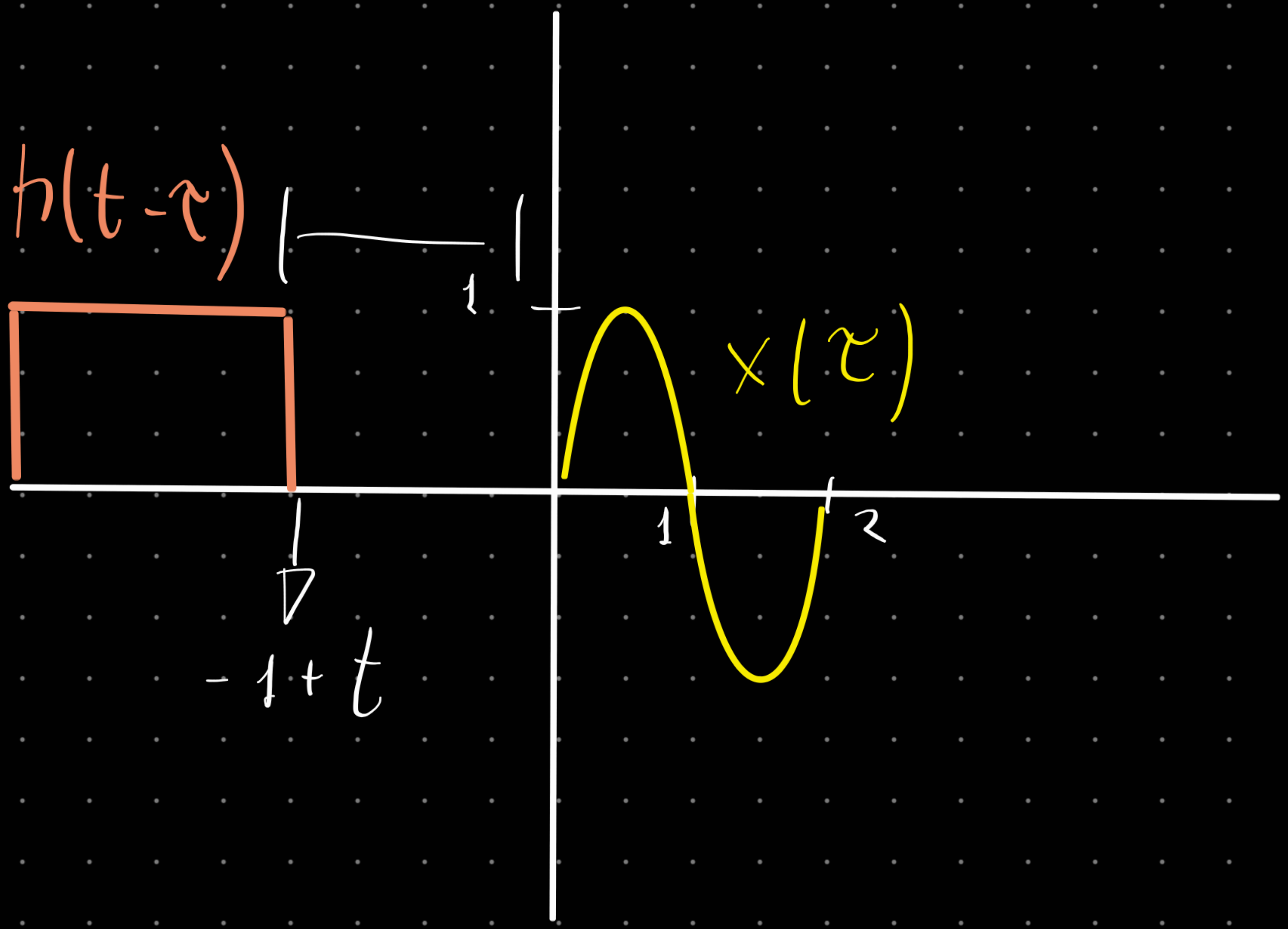
$$y(t) = \begin{cases} t < 1, & \frac{1}{2}\left[e^{2t} + e^{2(t-5)} - 2e^{2(t-2)}\right] \\ 1 \le t \le 3, & \frac{1}{2}\left[e^{2} - 2e^{2(t-2)} + e^{2(t-5)}\right] \\ 3 \le t \le 6, & \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right] \\ 6 \le t, & 0 \end{cases}$$



c) $x(t) = \sin \pi t\,[u(t) - u(t-2)]$

$$h(t) = 2[u(t-1) - u(t-3)]$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)$$

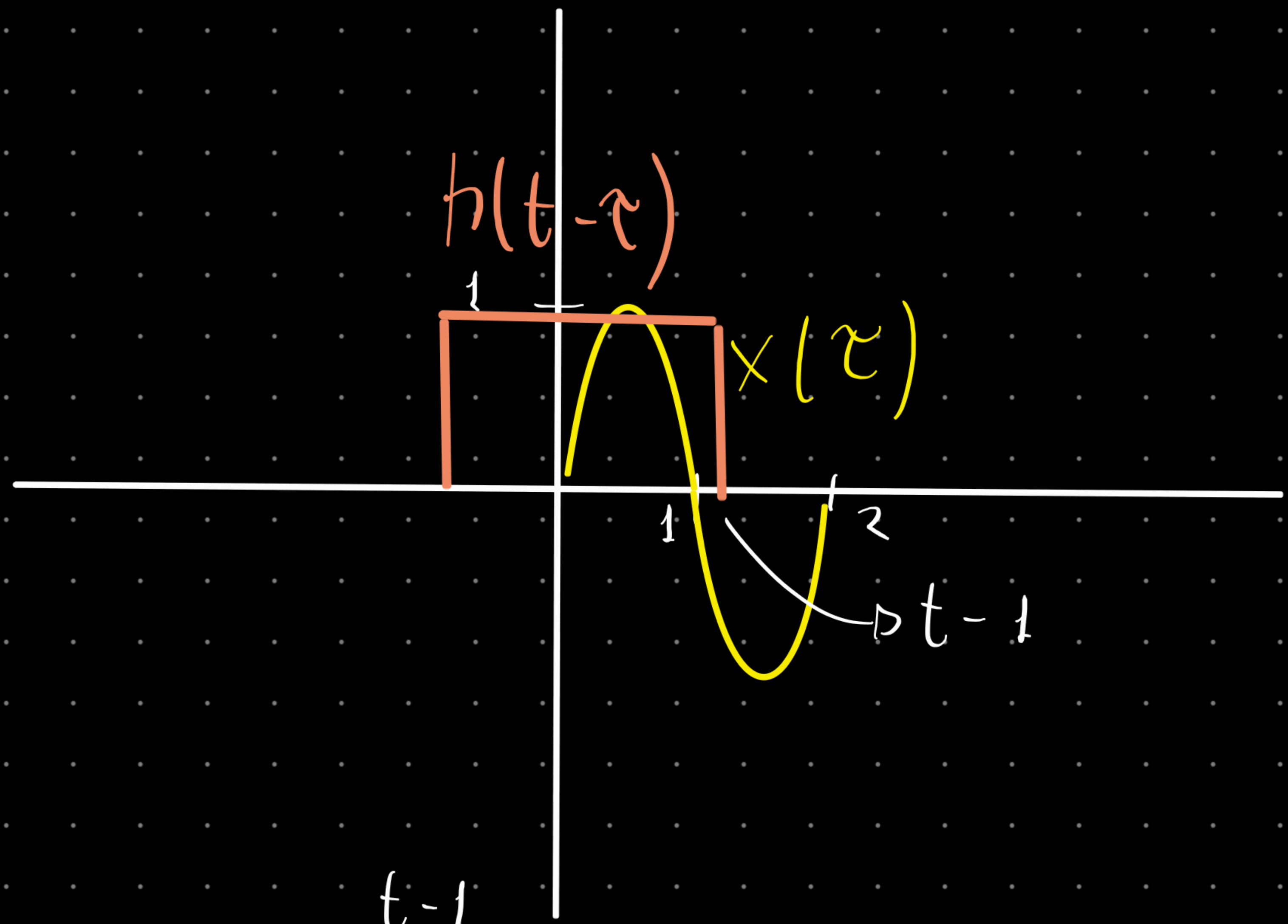


Para $-1+t < 0 \rightarrow t < 1$

$$y(t) = 0$$

Para $0 < t-1 < 2$

$1 < t < 2$

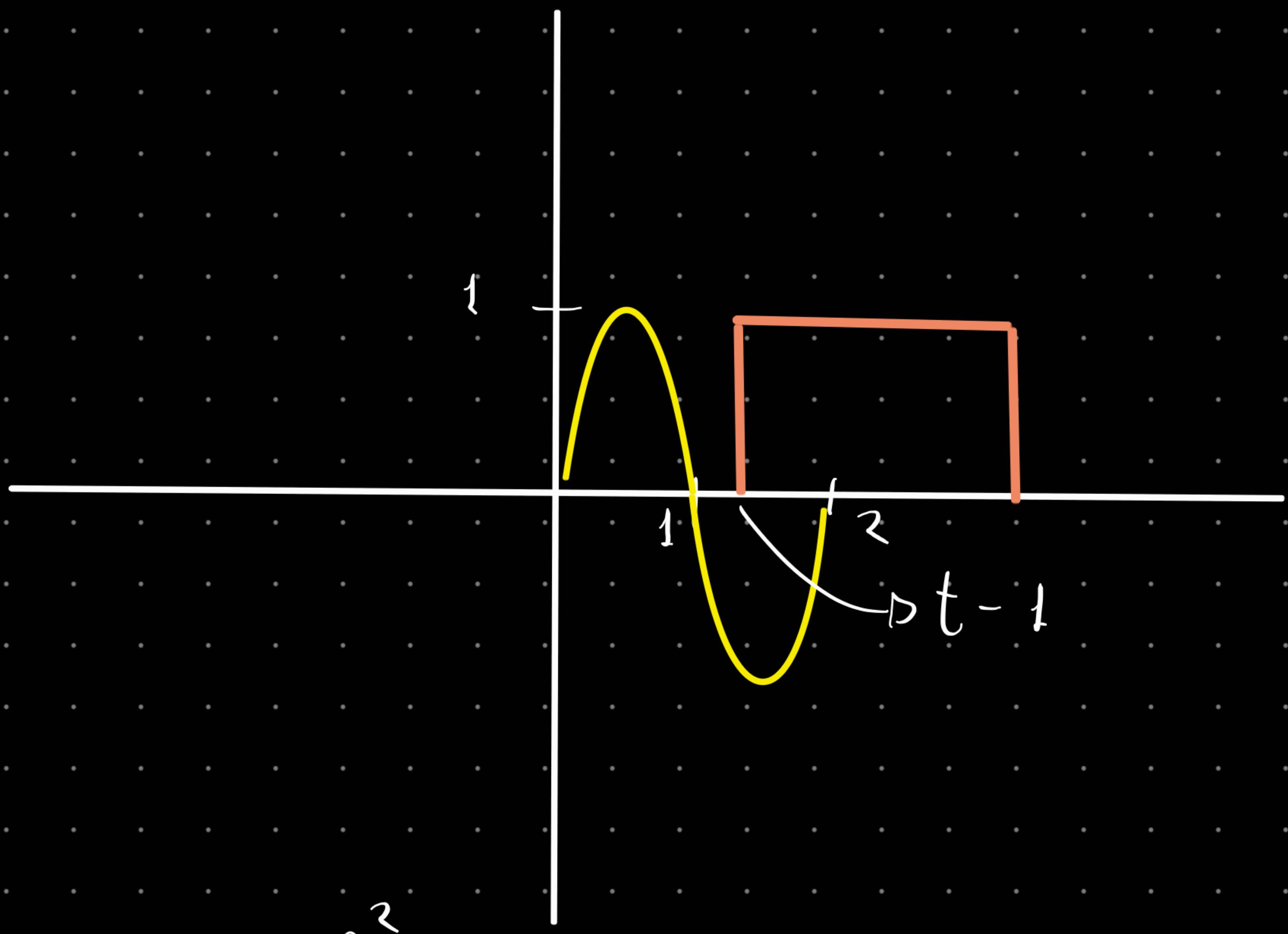


$$y(t) = \int_0^{t-1} \sin(\pi\tau)\,d\tau$$

$$= \frac{2}{\pi}\left[\cos(\pi t) + 1\right]$$

Para $2 < t-1 < 4$

$3 < t < 5$



$$y(t) = 2\int_{t-1}^{2} \sin(\pi \tau)\, d\tau$$

$$y(t) = \frac{-2\left[\cos(\pi t) + 1\right]}{\pi}$$

Para $5 < t$

$y(t) = 0$

$$y(t) = \begin{cases} \frac{2}{\pi}\left[\cos(\pi t) + 1\right], & 1 < t < 3 \\ -\frac{2}{\pi}\left[\cos(\pi t) + 1\right], & 3 < t < 5 \\ 0, & \text{o.c} \end{cases}$$

$f12(x) = \mathrm{Si}(x \leq 0, 0, x\, e^{-a x})$

$= \begin{cases} 0 & : x \leq 0 \\ x\, e^{-1.1 x} & : \text{en caso contrario} \end{cases}$

$f3(x) = \begin{cases} \frac{2}{\pi}\,(\cos(\pi\, x) + 1) & : 1 < x < 3 \\ -\frac{2}{\pi}\,(\cos(\pi\, x) + 1) & : 3 < x < 5 \\ 0 & : \text{en caso contrario} \end{cases}$

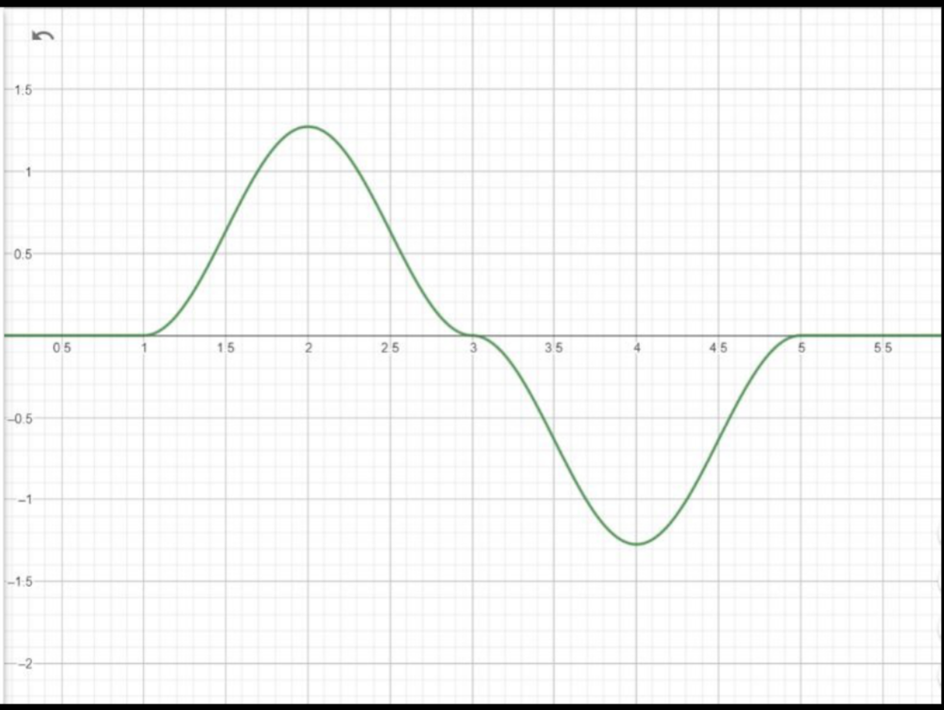
$f4(x) = \begin{cases} -x^2 + x + \frac{1}{4} & : -0.5 < x < 0.5 \\ x^2 - 3\,x + \frac{7}{3} & : 0.5 < x < 1.5 \end{cases}$

$f41(x) = f4(x + 2)$

$= \begin{cases} -(x+2)^2 + x + 2 + \frac{1}{4} & : -\frac{1}{2} < x + 2 \wedge x + 2 < \frac{1}{2} \\ (x+2)^2 - 3\,(x+2) + \frac{7}{3} & : \left(-\frac{1}{2} \geq x + 2 \vee x + 2 \geq \frac{1}{2}\right) \end{cases}$

$f42(x) = f4(x - 2)$

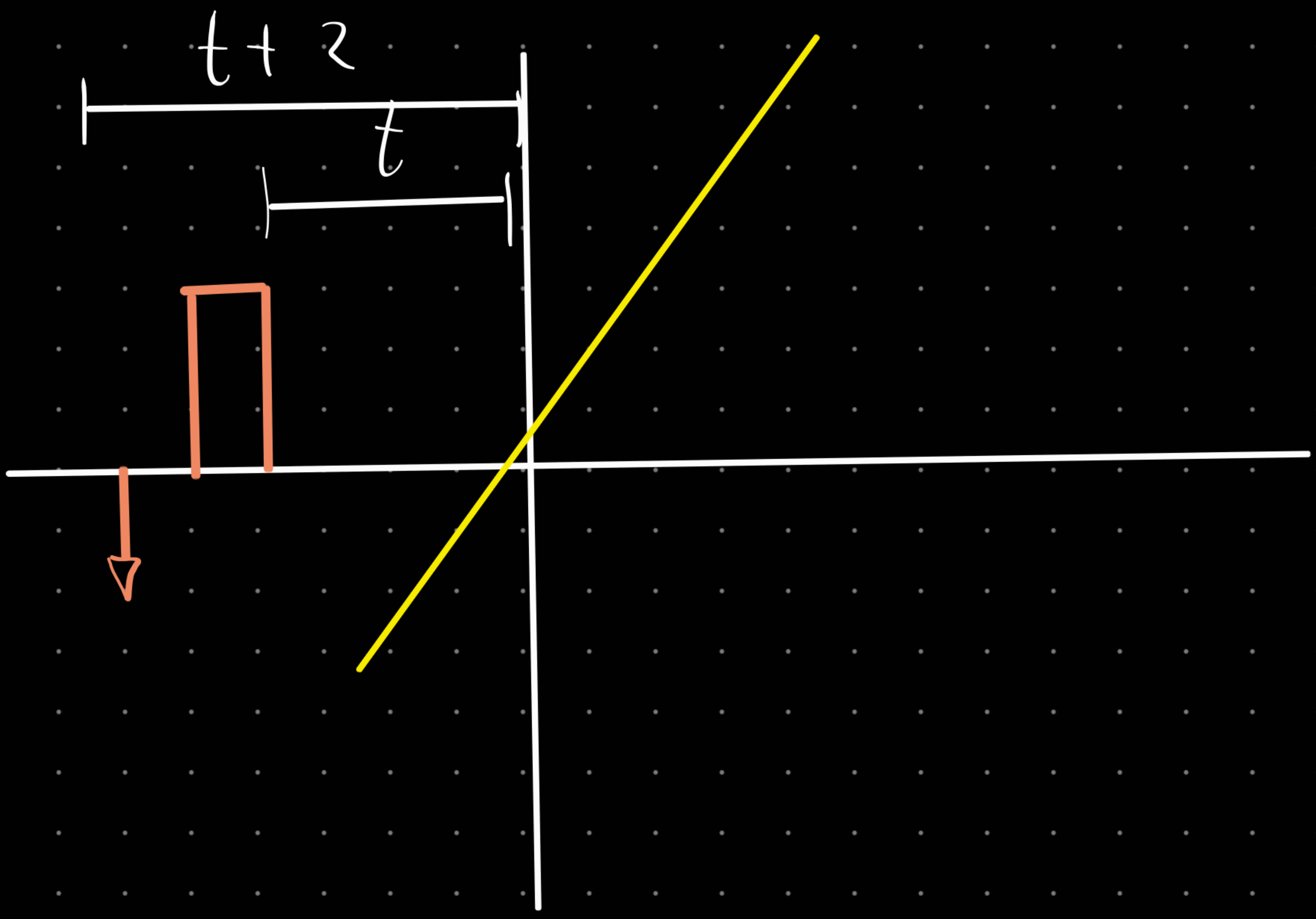
$= \begin{cases} -(x-2)^2 + x - 2 + \frac{1}{4} & : -\frac{1}{2} < x - 2 \wedge x - 2 < \frac{1}{2} \\ (x-2)^2 - 3\,(x-2) + \frac{7}{3} & : \left(-\frac{1}{2} \geq x - 2 \vee x - 2 \geq \frac{1}{2}\right) \end{cases}$



d) $x(t) = at + b$

$$h(t) = \frac{4}{3}\left[u(t) - u(t-1)\right] - \frac{1}{3}\delta(t-2)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

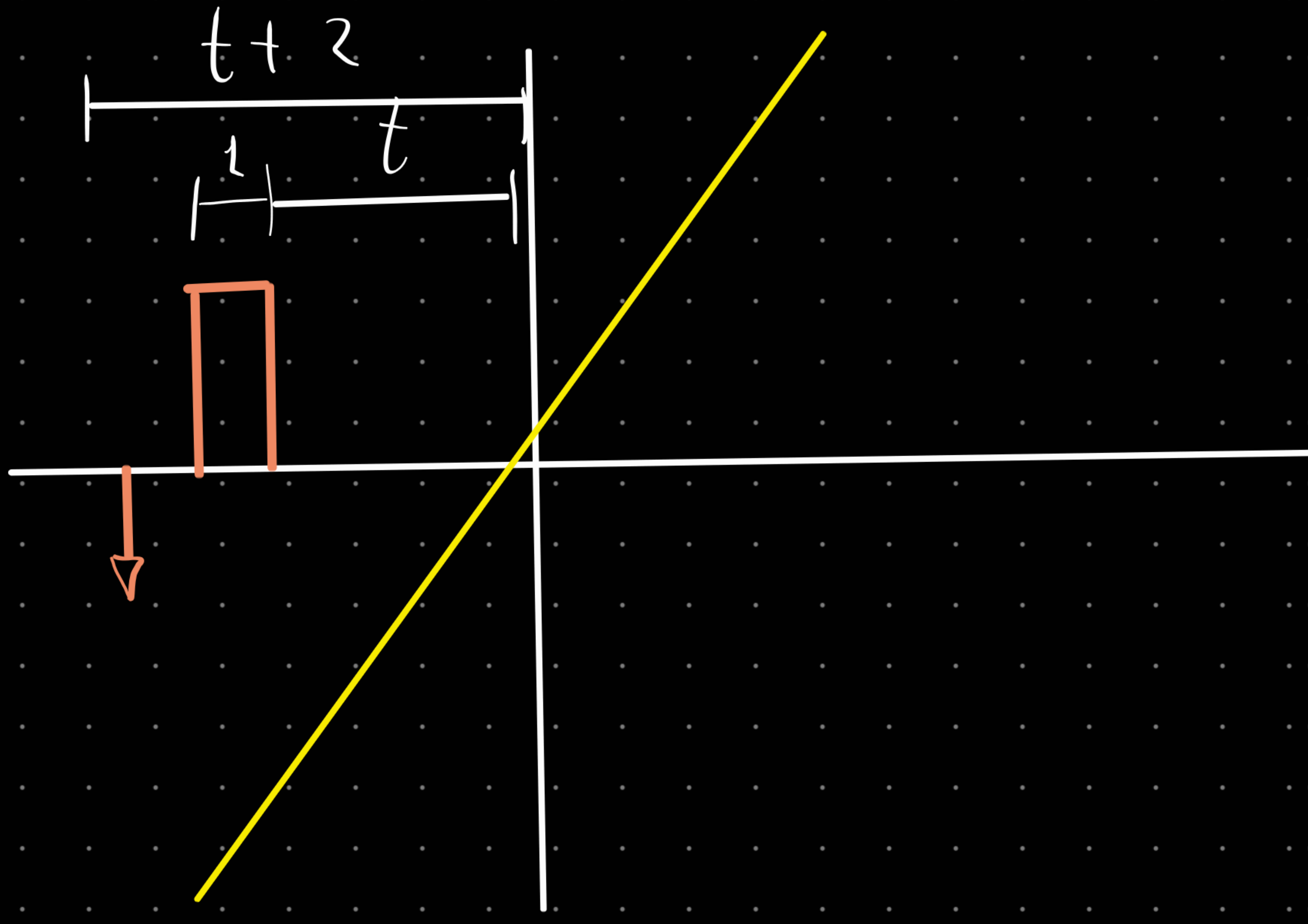


Analizando primero $\delta(t-\tau-2)$

$$y_1(t) = -\frac{1}{3}\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau-2)\,d\tau \rightarrow \tau = t-2$$

$$y_1(t) = -\frac{1}{3}\left[a(t-2) + b\right]$$

Analizando ahora la func. recto



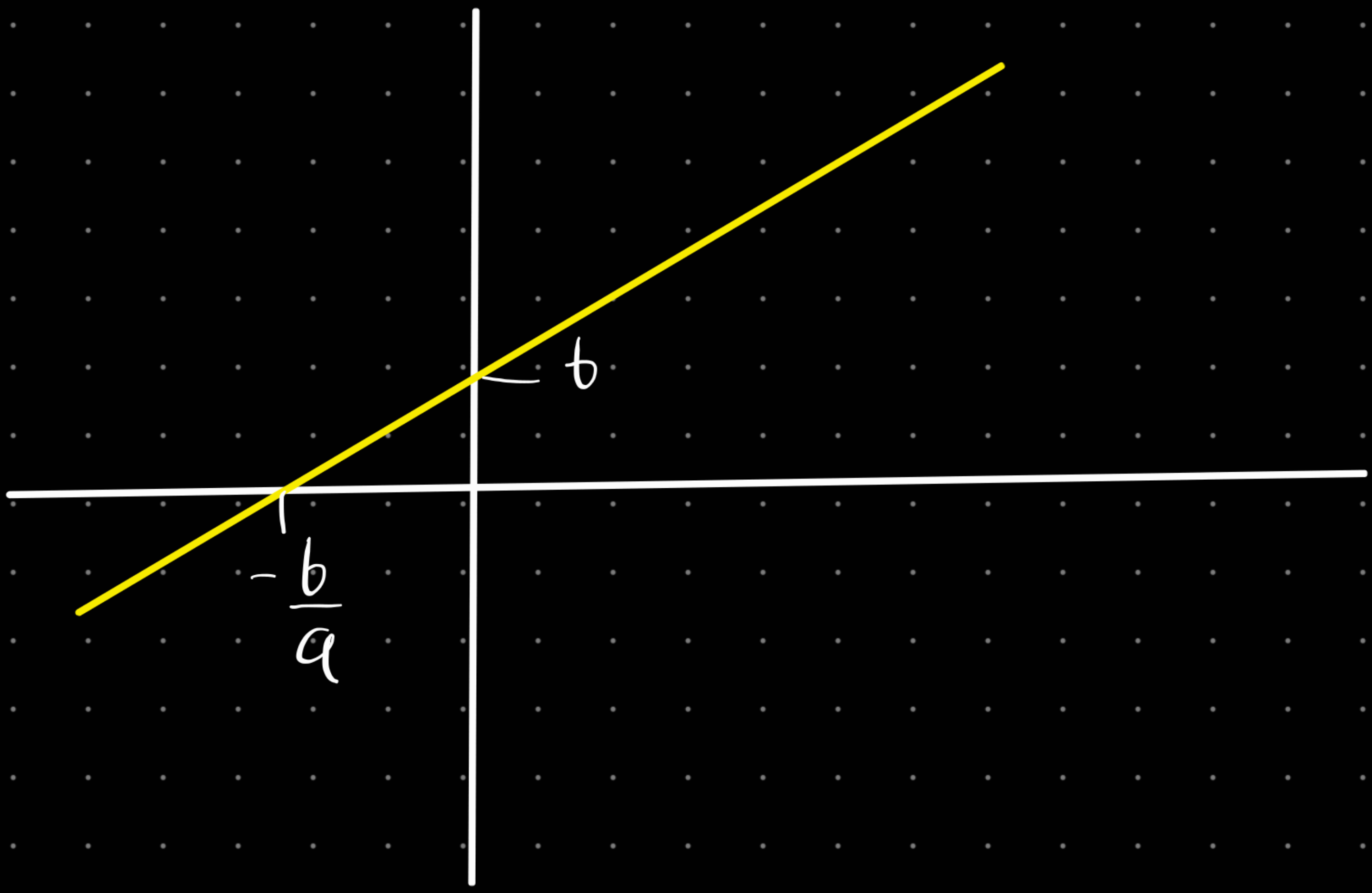
$$y_2(t) = \int_{t-1}^{t} \frac{4}{3}(a\tau + b)\,d\tau$$

$$= \frac{4}{3}\left\{\frac{a}{2}\left[t^2 - (t-1)^2\right] + b\left[t - (t-1)\right]\right\}$$

$$y(t) = y_1(t) + y_2(t)$$

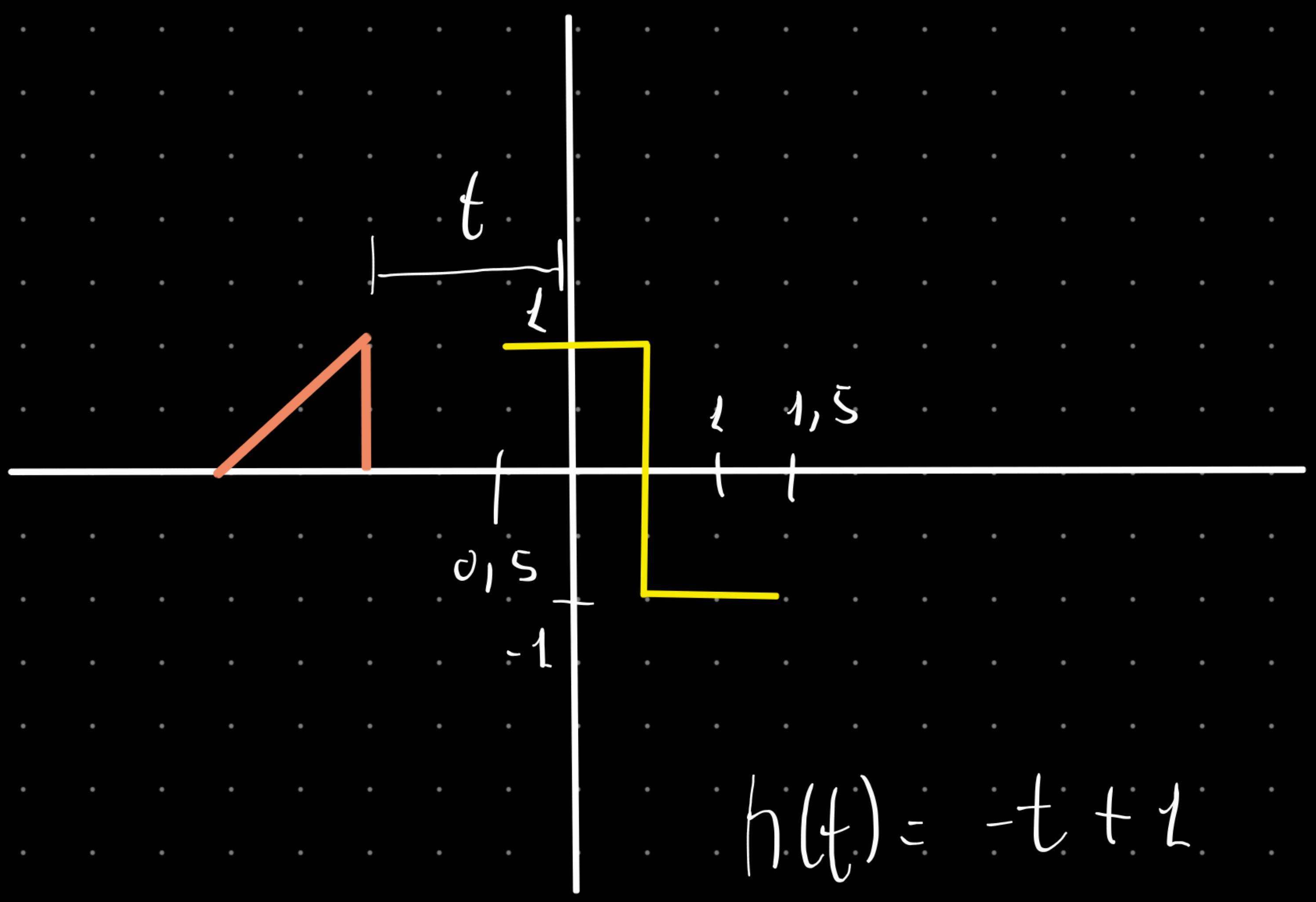
$$y(t) = \frac{4}{3}\left[\frac{a}{2}(2t-1) + b\right] - \frac{1}{3}\left[a(t-2) + b\right]$$

$$y(t) = at + b = x(t)$$



b
$-\frac{b}{a}$

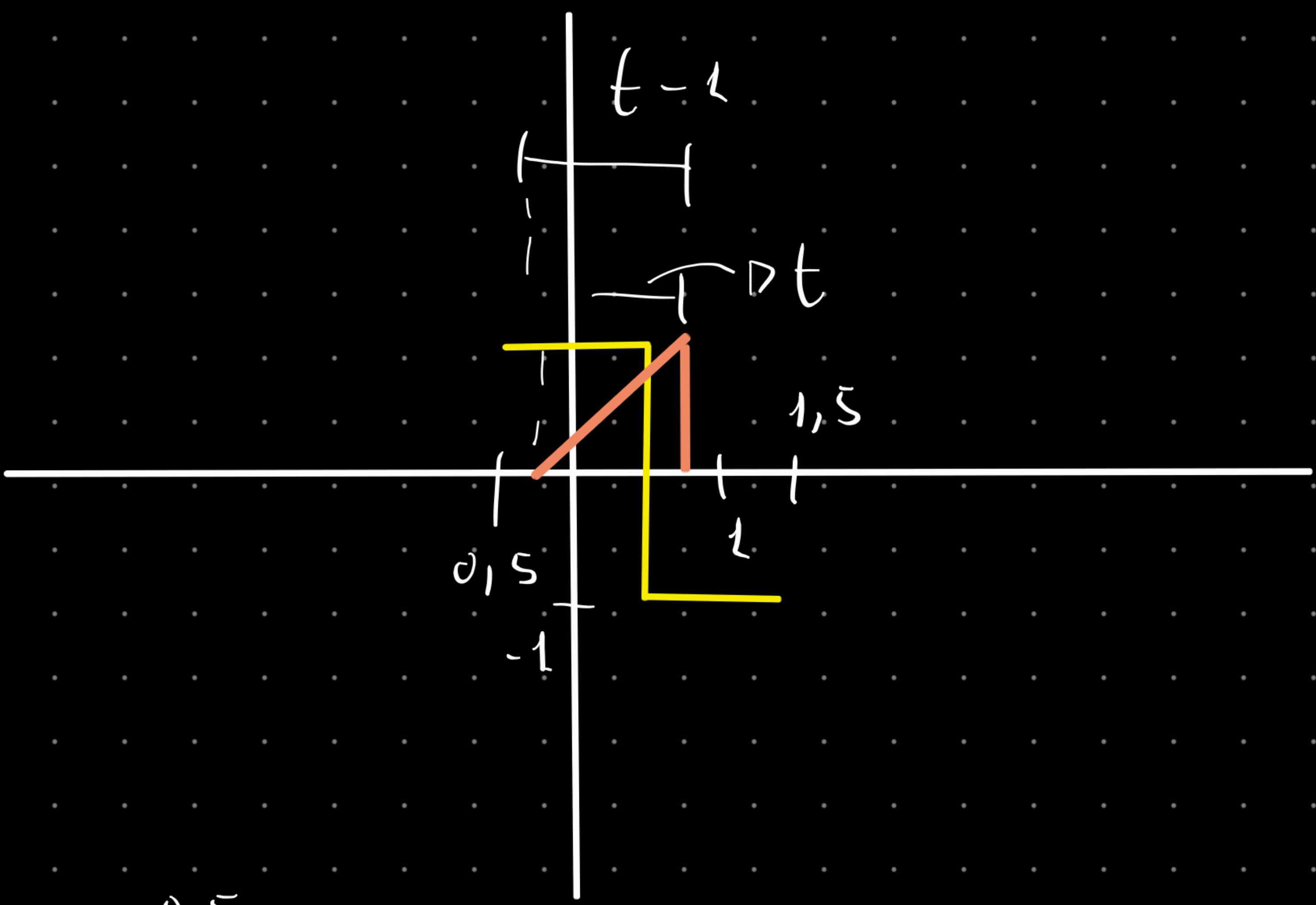
e) Para funciones periódicas basta con analizar un periodo y definir y(t) como periódica



$$h(t) = -t + 1$$

$$h(t-\tau) = -(t-\tau) + 1$$
$$= -t + \tau + 1$$

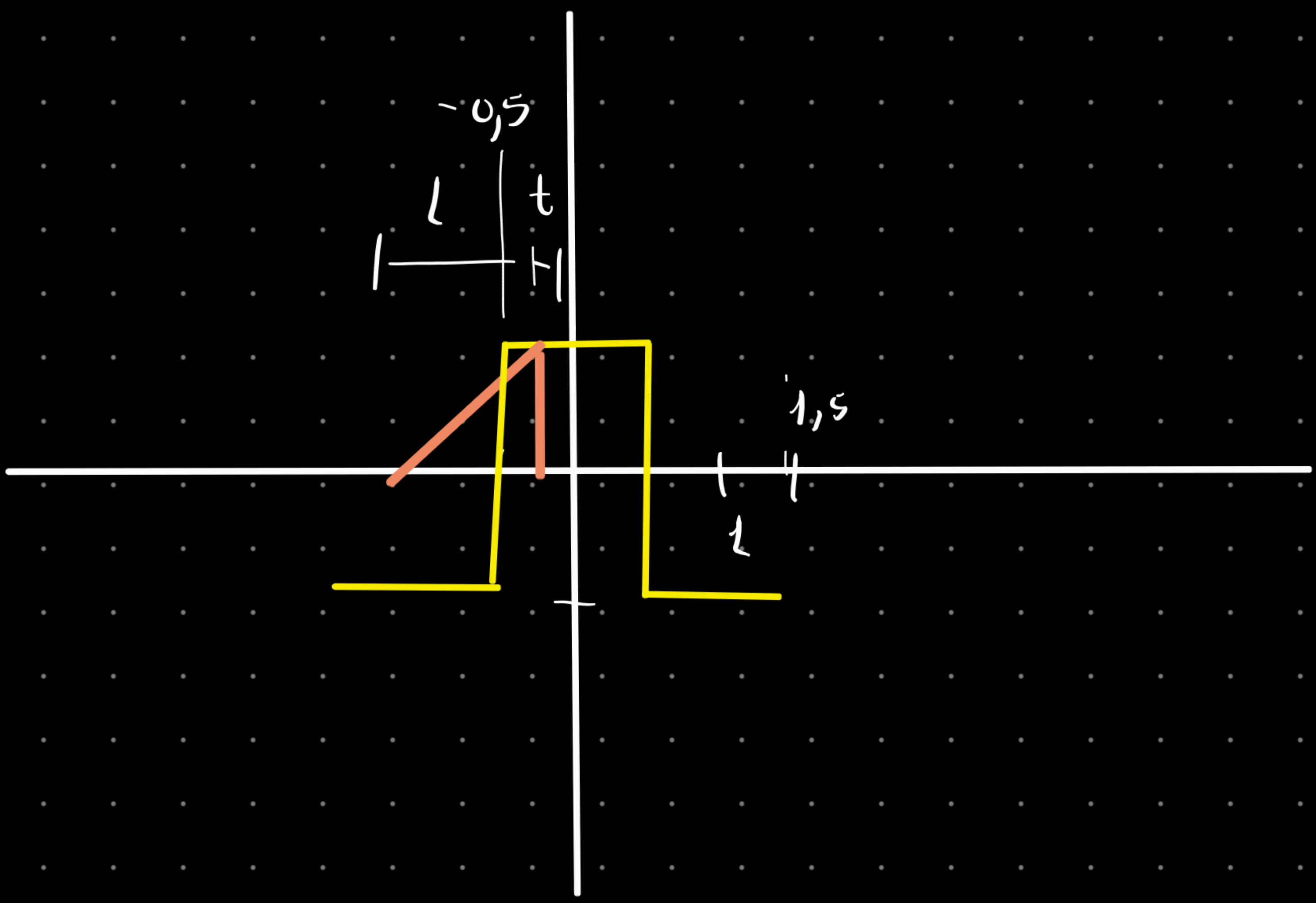
Para $\frac{1}{2} < t < 1{,}5$



$$\int_{t-1}^{0,5} (-t+\tau+1)\,d\tau - \int_{0,5}^{t} (-t+\tau+1)\,d\tau$$

$$= t^2 - 3t + \frac{7}{4}$$

$$-\frac{1}{2} < t < \frac{1}{2}$$

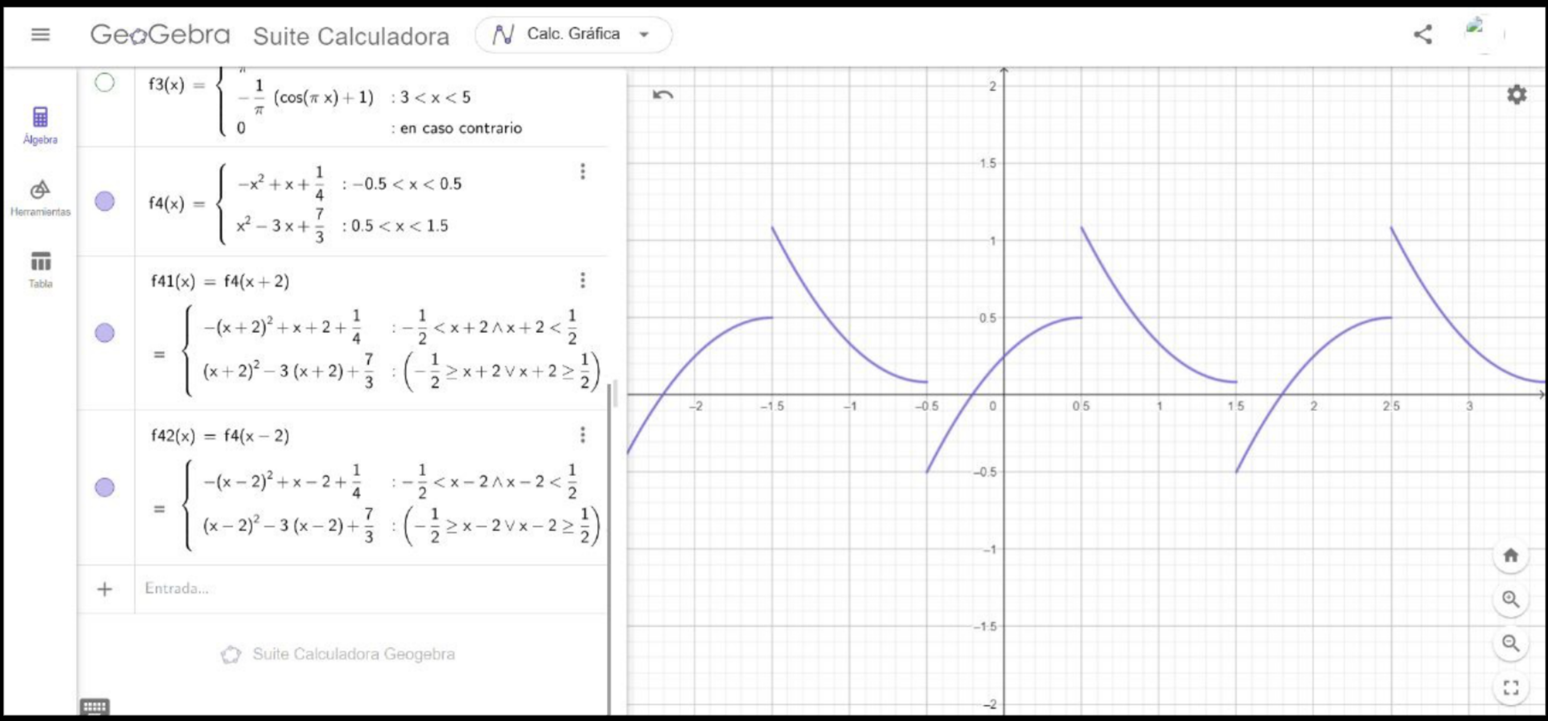


$$\int_{-0,5}^{t} (-t + \tau + 1)\,d\tau \quad - \int_{t-1}^{0,5} (-t + \tau + 1)\,d\tau =$$

$$-t^2 + t + \frac{1}{4}$$

$$T = 2$$

$$\forall(n): n \in \mathbb{Z} \,/\, y(t) = \begin{cases} -t^2 + t + \frac{1}{4}, & -\frac{1}{2} + 2n < t < \frac{1}{2} + 2n \\ t^2 - 3t + \frac{7}{4}, & \frac{1}{2} + 2n < t < \frac{3}{2} + 2n \end{cases}$$



2.33) $$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$y(0^-) = 0$$

$$\mu(t) = e^{\int 2dt} = e^{2t}$$

$$y'e^{2t} + 2e^{2t}y = e^{2t}x$$

$$\frac{d\left(ye^{2t}\right)}{dt} = e^{2t}x$$

$$\int d\left(ye^{2t}\right) = \int e^{2t}x\,dt$$

$$y(t) = \frac{1}{e^{2t}}\left(\int e^{2t}x(t)\,dt + c\right)$$

$$c = -\int e^{2t}x(t)\,dt\,(0)$$

$$y(t) = \frac{1}{e^{2t}}\left[\int e^{2t}x(t)\,dt - \int e^{2t}x(t)\,dt\,(0)\right]$$

i) $x(t) = e^{3t}u(t)$

$$\int e^{2t} \cdot e^{3t} u(t)\, dt = \frac{1}{5} e^{5t} \quad t \geq 0$$

$$c = -\frac{1}{5} e^{5.0} u(0) = -\frac{1}{5} \quad t \geq 0$$

$$y(t) = \begin{cases} \frac{1}{5e^{2t}}\left(e^{3t} - 1\right) & t \geq 0 \\ 0 & \text{O.C.} \end{cases}$$

$$y(t) = \frac{u(t)}{5e^{2t}}\left(e^{3t} - 1\right)$$

ii) $x(t) = e^{2t} u(t)$

$$\int e^{2t} \cdot e^{2t} u(t)\, dt = \frac{1}{4} e^{4t} \quad t \geq 0$$

$$c = -\frac{1}{4} e^{4 \cdot 0} u(0) = -\frac{1}{4} \quad t \geq 0$$

$$y(t) = \begin{cases} \frac{1}{4e^{2t}} \left(e^{4t} - 1 \right) & t \geq 0 \\ 0 & \text{O.C.} \end{cases}$$

$$y(t) = \frac{u(t)}{4e^{2t}} \left(e^{4t} - 1 \right)$$

iii)

$y' + 2y = x$ es un sistema LTI

Ya que $x_3(t) = a\,x_1(t) + b\,x_2(t)$, o sea, C.L. de x_1 y x_2, por propiedad de los sistemas lineales sabemos que $y_3 = a y_1 + b y_2$

$$y_3(t) = \frac{1}{e^{2t}}\left[\int e^{2t}(ax_1 + bx_2)\,dt - \int e^{2t}(ax_1+bx_2)\,dt\,(0)\right]$$

$$y_3(t) = \frac{a}{e^{2t}}\left[\int e^{2t}x_1(t)\,dt - \int e^{2t}x_1(t)\,dt\,(0)\right] + \frac{a}{e^{2t}}\left[\int e^{2t}x_2(t)\,dt - \int e^{2t}x_2(t)\,dt\,(0)\right]$$

$$y_3(t) = a y_1(t) + b y_2(t)$$

$$y_3(t) = u(t)\left[\frac{a}{5e^{2t}}\left(e^{5t} - 1\right) + \frac{b}{4e^{2t}}\left(e^{5t} - 1\right)\right]$$

iv) A diferencia del caso anterior, las func x_1 y x_2 no comparten $u(t)$, pero de todos modos:

$$y(t) = \frac{1}{e^{2t}}\left[\int e^{2t} x(t)\,dt - \int e^{2t} x(t)\,dt\,(0)\right]$$

Simplemente se distribuiría por las propiedades de linealidad de la integral, o sea

$$y_3(t) = \frac{1}{e^{2t}}\left[\int e^{2t}(ax_1+bx_2)\,dt - \int e^{2t}(ax_1+bx_2)\,dt\,(0)\right]$$

$$y_3(t) = \frac{a}{e^{2t}}\left[\int e^{2t} x_1(t)\,dt - \int e^{2t} x_1(t)\,dt\,(0)\right] + \frac{a}{e^{2t}}\left[\int e^{2t} x_2(t)\,dt - \int e^{2t} x_2(t)\,dt\,(0)\right]$$

$$y_3(t) = a y_1(t) + b y_2(t)$$

2.45) a)

i) $x(t) \longrightarrow \boxed{\text{Sistema LTI}} \longrightarrow y(t)$

$-$

$x(t-h) \longrightarrow \boxed{\text{Sistema LTI}} \longrightarrow y(t-h)$

$=$

$x(t)-x(t-h) \longrightarrow \boxed{\text{Sistema LTI}} \longrightarrow y(t)-y(t-h)$

... $\%\, h$

$=$

$\dfrac{x(t)-x(t-h)}{h} \longrightarrow \boxed{\text{Sistema LTI}} \longrightarrow \dfrac{y(t)-y(t-h)}{h}$

... $\lim\limits_{h \to 0} (\quad)$

$=$

$\underbrace{\lim\limits_{h \to 0} \dfrac{x(t)-x(t-h)}{h}}_{x'(t)} \longrightarrow \boxed{\text{Sistema LTI}} \longrightarrow \underbrace{\lim\limits_{h \to 0} \dfrac{y(t)-y(t-h)}{h}}_{y'(t)}$

$=$

$x'(t) \longrightarrow \boxed{\text{Sistema LTI}} \longrightarrow y'(t)$

ii)

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$\frac{dy(t)}{dt} = \frac{d}{dt}\left[h(t) * x(t)\right]$$

$$y'(t) = \frac{d}{dt}\left[\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau\right]$$

$\to$ constante para t

$$y'(t) = \int_{-\infty}^{\infty} h(\tau)\, \frac{d}{dt}\left[x(t-\tau)\right] d\tau$$

$$y'(t) = \int_{-\infty}^{\infty} h(\tau)\, x'(t-\tau)\, d\tau$$

$$y'(t) = h(t) * x'(t)$$

iii)

$$x(t) \longrightarrow \boxed{u_1(t)} \longrightarrow y_{aux}(t)$$

$$y_{aux}(t) = x(t) * u_1(t)$$

$$y_{aux}(t) \longrightarrow \boxed{h(t)} \longrightarrow s(t)$$

$$s(t) = y_{aux}(t) * h(t)$$

$$s(t) = x(t) * u_1(t) * h(t)$$

P/ $y(t) = x(t) * h(t)$

$$s(t) = u_1(t) * y(t)$$

P/ $w(t) * u_1(t) = w'(t)$

$$s(t) = y'(t)$$

b)

i) $x(t) * h'(t)$

$$x(t) * u_1(t) * h(t)$$

$$x'(t) * h(t) = y'(t)$$

ii)

a)

$$\int_{-\infty}^{t} x(\tau)\,d\tau = \int_{-\infty}^{\infty} x(\tau)\,u(t-\tau)\,d\tau = x(t) * u(t)$$

1

$$x(t) * u(t) * h'(t) =$$

$$x(t) * u(t) * u_1(t) * h(t) =$$

3

P/ $u(t) = u_{-1}(t)$

$$x(t) * \delta(t) * h(t) =$$

$$x(t) * h(t) =$$

$$y(t)$$

b) Partiendo desde la la ec (3) de la parte a)

$$x(t) * u(t) * u_1(t) * h(t) =$$

$$[x(t) * u_1(t)] * [h(t) * u(t)]$$

Usando la ec (1) de la parte a) $\wedge$ $w(t) * u_1(t) = w'(t)$

$$x'(t) * \int_{-\infty}^{t} h(\tau)\,d\tau$$

Y sabemos que la ec (3) concluye siendo igual a $y(t)$

$\therefore$

$$x'(t) * \int_{-\infty}^{t} h(\tau)\,d\tau = y(t)$$

c) $y(t) = x(t) * h(t) \ldots \mathcal{L}\{\ \}$

$$Y(s) = X(s) \cdot H(s)$$

$$h(t) = \mathcal{L}^{-1}\left\{ \frac{\mathcal{L}\{x(t)\}}{\mathcal{L}\{y(t)\}} \right\}$$

P/ $x(t) = e^{-5t}u(t) \wedge y(t) = \sin w_0 t$

$$\mathcal{L}\{e^{-5t}u(t)\} = \frac{e^{-0t}}{s+5} = \frac{1}{s+5}$$

$$\mathcal{L}\{\sin w_0 t\} = \frac{w_0}{s^2+w_0^2}$$

$$H(s) = \frac{\left[\frac{1}{s+5}\right]}{\left[\frac{w_0}{s^2+w_0^2}\right]} = \underbrace{\frac{1}{w_0}\frac{s^2}{s+5} + \frac{w_0}{s+5}}_{\text{No conocidos}}$$

:/

c) $x(t) = e^{-5t} u(t)$

$$x'(t) = -5e^{-5t} u(t) + \cancel{e^{-5t}}\delta(t)$$ no contribuye

$$x'(t) = -5e^{-5t} u(t) + \delta(t)$$

Si $x(t) \xrightarrow{h(t)} y(t)$

Entonces $x'(t) \xrightarrow{h(t)} y'(t)$

$$-5e^{-5t}u(t) + \delta(t) \longrightarrow \underbrace{\overbrace{-5e^{-5t}u(t)}^{-5x(t)} * h(t)}_{y(t)} + \delta(t) * h(t)$$

$$// \qquad // \longrightarrow -5y(t) + h(t) = y'(t)$$

$$h(t) = y'(t) + 5y(t)$$

$$= \frac{d}{dt}\left[\sin w_0 t\right] + 5 \sin w_0 t$$

$$= w_0 \cos w_0 t + 5 \sin w_0 t$$

d)

i) $s(t) = h(t) * u(t)$

$$= \int_{\infty}^{t} h(\tau)\,d\tau$$

Partiendo desde la la ec ③ del b)

$$y(t) = x(t) * \overbrace{u(t)} * u_1(t) * \overbrace{h(t)}$$

$$= \overbrace{x(t)} * \overbrace{u_1(t)} * s(t)$$

$$= x'(t) * s(t)$$

$$= \int_{-\infty}^{\infty} x'(\tau)\, s(t-\tau)\,d\tau$$

ii) $x(t) = x(t) * \delta(t)$

$$\delta(t) = u_0(t) = \overbrace{u_{-1}(t)}^{u(t)} * u_1(t)$$

$$x(t) = \overbrace{x(t)} * u(t) * \overbrace{u_1(t)}$$

$$= x'(t) * u(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\,d\tau$$

e) P/

$$s(t) = (e^{-3t} - 2e^{-2t} + 1)\,u(t)$$

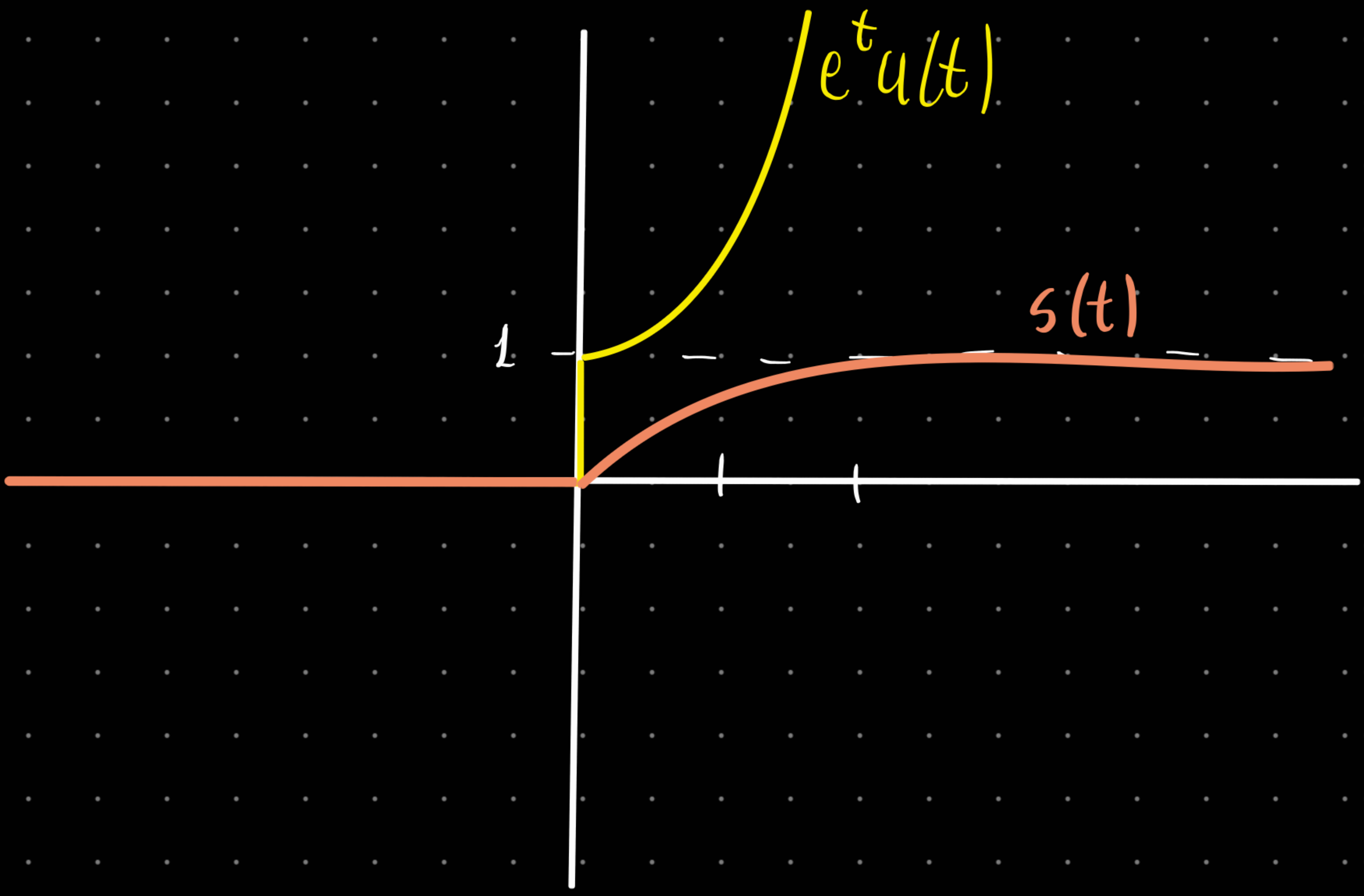
$$x(t) = e^{t} u(t)$$

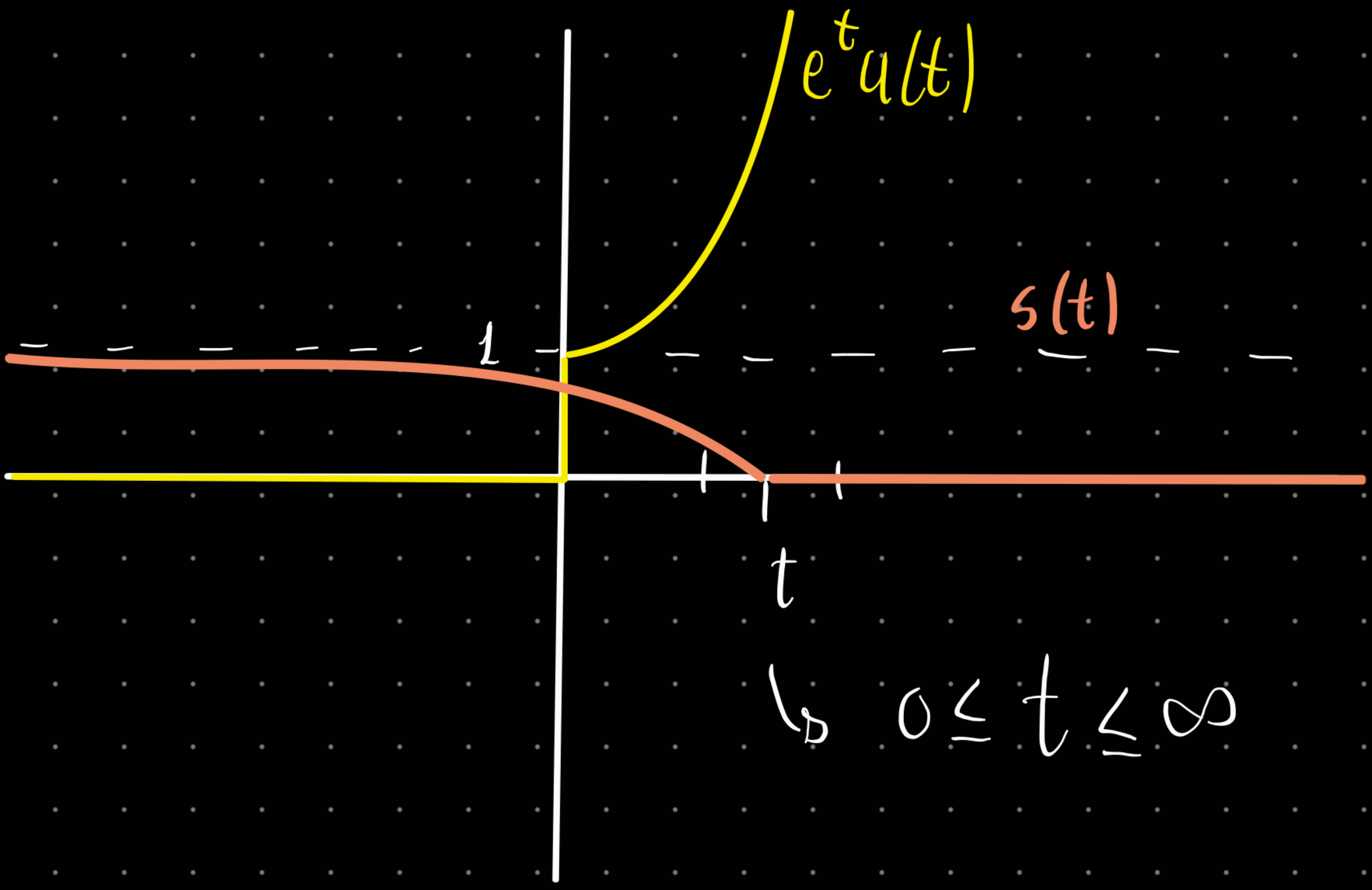
$$x'(t) = e^{t} u(t) + \delta(t)$$

$$y(t) = x'(t) * s(t)$$

$$y(t) = \left[e^{t} u(t) + \delta(t)\right] * (e^{-3t} - 2e^{-2t} + 1)\,u(t)$$

$$y(t) = e^{t} u(t) * (e^{-3t} - 2e^{-2t} + 1)\,u(t) + s(t)$$





$\hookrightarrow\ 0 \leq t \leq \infty$

$$y(t) = \int_0^{\infty} e^{\tau} u(\tau)\left[e^{-3(t-\tau)} - 2e^{-2(t-\tau)} + 1\right] u(t-\tau)\, d\tau$$

$\tau \geq 0 \quad \wedge \quad t - \tau \geq 0$

$\tau \geq 0 \ \wedge \ t \geq \tau$

$0 \leq \tau \leq t$

$$y(t) = \int_0^{t} e^{\tau}\left[e^{-3(t-\tau)} - 2e^{-2(t-\tau)} + 1\right] d\tau + s(t)$$

$$y(t) = \left[\frac{7}{12} e^{t} - \frac{4}{3} e^{-2t} + \frac{3}{4} e^{-3t}\right] u(t)$$

f) No entran func discretas en el temario