3.1)

3.1. Una señal periódica continua $x(t)$ es de valor real y tiene un periodo fundamental de $T = 8$. Los coeficientes de la serie de Fourier diferentes de cero para $x(t)$ son

$$a_1 = a_{-1} = 2,\ a_3 = a_{-3}^* = 4j.$$

Exprese $x(t)$ en la forma

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{4}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\omega_k = k\omega_0$$

$$2\cos\theta = e^{j\theta} + e^{-j\theta}$$

$$= \dots + a_{-3} e^{-3j\omega_0 t} + \cancel{a_{-2} e^{-2j\omega_0 t}}$$

$$+ a_{-1} e^{-j\omega_0 t} + \cancel{a_0} + a_1 e^{j\omega t} + a_1 e^{2j\omega_0 t}$$

$$+ \cancel{a_2 e^{2j\omega_0 t}} + a_3 e^{3j\omega_0 t} + \dots$$

$$4j \cdot 2j \sin(3\omega_0 t) + 2 \cdot 2 \cos(\omega_0 t)$$

$$-8\sin(3\omega_0 t) + 4\cos(\omega_0 t)$$

$$8\cos\left(3\omega_0 t + \frac{\pi}{2}\right) + 4\cos(\omega_0 t)$$

→ Bode

Condiciones Dirichlet

4.4)

4.4. Use la ecuación de síntesis (4.8) de la transformada de Fourier para determinar las transformadas inversas de Fourier de

(a) $X_1(j\omega) = 2\pi\,\delta(\omega) + \pi\,\delta(\omega - 4\pi) + \pi\,\delta(\omega + 4\pi)$

a) $$x(t) = \frac{1}{\cancel{2\pi}}\left[\cancel{2\pi}\int_{-\infty}^{\infty}\delta(\omega)e^{j\omega t}\,d\omega + \cancel{\pi}\int_{-\infty}^{\infty}\delta(\omega-4\pi)e^{j\omega t}\,d\omega + \cancel{\pi}\int_{-\infty}^{\infty}\delta(\omega+4\pi)e^{j\omega t}\,d\omega\right]$$

$$1e^{j0} + \frac{1}{2}e^{-j4\pi t} + \frac{1}{2}e^{j4\pi t}$$

$$x(t) = 1 + \cos(4\pi t)$$

b)

(b) $X_2(j\omega) = \begin{cases} 2, & 0 \leq \omega \leq 2 \\ -2, & -2 \leq \omega \leq 0 \\ 0, & |\omega| > 2 \end{cases}$

$$x(t) = \frac{1}{2\pi}\left[\int_{-2}^{0} -\cancel{2}e^{j\omega t}\,d\omega + \int_{0}^{2} \cancel{2}e^{j\omega t}\,d\omega\right]$$

$$= \frac{1}{\pi j t}\left[-e^{j\omega t}\Big|_{-2}^{0} + e^{j\omega t}\Big|_{0}^{2}\right]$$

$$= \frac{1}{\pi j t}\left[-1 + e^{-2jt} + e^{j2t} - 1\right]$$

$$\frac{1}{\pi j t}\left[-2 + 2\cos(2t)\right]$$

$$\frac{4}{\pi j t}\sin^2(4t)$$

4.2)

a) $$\int_{-\infty}^{\infty} \delta(t+1) e^{-j\omega t} dt$$ –

$$+ \int_{-\infty}^{\infty} \delta(t-1) e^{-j\omega t} dt$$

$$e^{+j\omega} + e^{-j\omega}$$

$$2\cos(\omega)$$

b) $$\frac{\partial}{\partial t}\left\{u(-2-t) + u(t-2)\right\}$$

$$-\delta(-t-2) + \delta(t-2)$$

$$\int_{-\infty}^{\infty} \left[-\underbrace{\delta(-t-2)}_{\to t=-2} + \underbrace{\delta(t-2)}_{\to t=2}\right] e^{-j\omega t} dt$$

$$-e^{2j\omega} + e^{-2j\omega} = 2j\sin(-2\omega)$$

$$= -2j\sin(2\omega)$$

4.6)

a) $x_1(t) = x(1-t) + x(-1-t)$

$x[-(t-1)] + x[-(t+1)]$

$\hookrightarrow t_0 = 1 \qquad \hookrightarrow t_0 = -1$

$$X(-wj)\Big|_{t \to t-1} + X(-wj)\Big|_{t \to t+1}$$

$$e^{-jw} X(-wj) + e^{jw} X(-wj)$$

$$2\cos(w)\, X(-jw)$$

b)

$x_2(t) = x(3t-6)$

$x[3(t-2)]$

$$\frac{1}{\|3\|} X\left(\frac{jw}{3}\right)\Bigg|_{t \to t-2}$$

$$\frac{1}{3} X\left(\frac{jw}{3}\right) e^{-2wj}$$

c) $x_3(t) = \frac{\partial^2}{\partial t^2}\left[x(t-1)\right]$