4.1) $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

a)

$t-1 \geq 0 \rightarrow t > 1$

$$X_a(j\omega) = \int_{-\infty}^{\infty} e^{-2(t-1)} u(t-1)\, e^{-j\omega t}\, dt$$

$$= \int_{1}^{\infty} e^{-2(t-1)} e^{-j\omega t}\, dt$$

$$= \left. \frac{i e^{-2t+2}}{w e^{wt} - 2 i e^{iwt}} \right|_{1}^{\infty}$$

$$= \left. \frac{1}{e^{t(2+j\omega)}} \frac{j e^{2}}{(w-2j)} \right|_{1}^{\infty}$$

$$- \frac{j}{e^{j\omega}(\omega - 2j)} = \frac{e^{-j\omega}}{\omega j + 2}$$

b) $X_b(j\omega) = \int_{-\infty}^{\infty} e^{-2|t-1|} e^{-j\omega t}\, dt$

$t-1 \geq 0$

$t \geq 1$

$$= \int_{-\infty}^{1} e^{2(t-1)} e^{-j\omega t}\, dt + \int_{1}^{\infty} e^{-2(t-1)} e^{-j\omega t}\, dt$$

$$\frac{e^{2t}}{e^{j\omega t}} \cdot \frac{j e^{-2}}{\omega + 2j}\Bigg|_{-\infty}^{1} + \frac{e^{-j\omega}}{\omega j + 2}$$

$$\frac{\cancel{e^{2}}}{e^{j\omega}} \frac{j \cancel{e^{-2}}}{(\omega + 2j)\frac{j}{j}} + //$$

$$\frac{e^{-j\omega}}{\omega j - 2}$$

4.9)

a) $$\int_{-\infty}^{\infty} [\delta(t+1) + \delta(t-1)] e^{-j\omega t} \, dt$$

$$\frac{2}{2}\left(e^{j\omega} + e^{-j\omega}\right) = 2\cos\omega$$

b) $$\frac{\partial}{\partial t}\left[u(-2-t) + u(t-2)\right]$$

$$-\delta(t+2) + \delta(t-2)$$

$$\Big\downarrow \mathcal{F}$$

$$-e^{j2\omega} + e^{-j2\omega}$$

$$\frac{2j}{2j} - 1\left(e^{j2\omega} - e^{-j2\omega}\right)$$

$$-2j\sin(2\omega)$$

4.3)

a) $\sin\left(2\pi t + \frac{\pi}{4}\right) \to T = \frac{2\pi}{2\pi} = 1$

$$\frac{e^{j\pi/4}}{2j}\left(e^{2\pi t j} - e^{-2\pi t j}\right)$$

$$\mathcal{F}\{f_{\text{periodica}}(t)\} = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \omega_k)$$

$$\frac{\cancel{2}\pi e^{j\pi/4}}{\cancel{2}j}\left[\delta(\omega - 2\pi) - \delta(\omega + 2\pi)\right]$$

b) $1 + \cos(6\pi t + \pi/8)$ $\to \omega_0 = 6\pi$

$$1 e^{j6\pi \cdot 0 t} + \frac{e^{j\pi/8}}{2}\left(e^{j6\pi t} + e^{-j6\pi t}\right)$$

$$X(j\omega) = \pi\left\{2\delta(\omega) + e^{j\pi/8}\left[\delta(\omega + 6\pi) + \delta(\omega - 6\pi)\right]\right\}$$

4.5)

$$X(jw) = |X(jw)| e^{j \measuredangle [X(jw)]}$$

$$\frac{1}{\cancel{2}\pi} \int_{-\infty}^{\infty} \cancel{2} [\underset{w>-3}{u(w+3)} - \underset{w>3}{u(w-3)}] e^{j(-\frac{3}{2}w+\pi)} e^{jwt} \, dw$$

$$\frac{1}{\pi} \int_{-3}^{3} e^{j(\frac{3}{2}w+\pi+wt)} \, dt$$

$$-\frac{4}{\pi(2t-3)} \sin(3t - 9/2)$$

4.6)

a) $X_1(j\omega) = X(-j\omega)e^{j\omega} + X(-j\omega)e^{-j\omega}$

$= 2\cos(\omega)\,X(-j\omega)$

b) $x_2(t) = x[3(t-2)] = x(3t)\Big|_{t \to t-2}$

$X_2(j\omega) = \frac{1}{3}X\left(\frac{j\omega}{3}\right)\Big|_{t \to t-2}$

$= \frac{e^{-j2}}{3}X\left(\frac{j\omega}{3}\right)$

c) $x_3(t) = \frac{\partial^2}{\partial t^2}x(t)\Big|_{t \to t-1}$

$= -\omega^2 e^{-j\omega}X(j\omega)$

4.7)

i) Si $x(t)$ es Real $\longleftrightarrow$ $X(j\omega) = X^*(-j\omega)$

a) $u(-\omega) - u(-\omega-2) \neq X_1(j\omega)$

No Real

No par

No impar

b) $\cos(2\omega)\sin\left(\frac{\omega}{2}\right)$

$$\begin{cases} \cos(2\omega)\sqrt{\frac{1-\cos\omega}{2}} \\ -\cos(2\omega)\sqrt{\frac{1-\cos\omega}{2}} \end{cases}$$

$$\frac{1}{2}\left[\sin\left(\frac{5}{2}\omega\right) + \sin\left(\frac{3}{2}\omega\right)\right]$$

$\downarrow$

$X_2$ Real e Impar $\rightarrow$ $x_2$ imag e impar

c) $X_3 = \dfrac{e^{j4w} - 1}{2w}$

$$X^*(-jw) = \frac{e^{j4w-1}}{-2w}$$

che forza

4.8) a) $x(t) = \begin{cases} 0 & t < -1/2 \\ t+\frac{1}{2} & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 1 & \frac{1}{2} < t \end{cases}$

$$y(t) = \frac{\partial x}{\partial t} = \begin{cases} 0 & t < -1/2 \\ 1 & -1/2 \leq t \leq 1/2 \\ 0 & \frac{1}{2} < t \end{cases}$$

$$x(t) = \int_{-\infty}^{t} y(\lambda)\, d\lambda$$

$$Y(jw) = \frac{2\sin w/2}{w}$$

$$Y(0) = 1$$

$$X(jw) = \frac{1}{jw} Y(jw) + \pi Y(jw)\delta(w)$$

$$X(jw) = \frac{2\sin w/2}{jw^2} + \pi\delta(w)$$

b) Mis Bolas

4.9) $x(t) = \begin{cases} 0 & |t| > 1 \\ (t+1)/2 & -1 \le t \le 1 \end{cases}$

$$\frac{\partial x}{\partial t} = y(t) = \begin{cases} 0 & |t| > 1 \\ \frac{1}{2} & -1 \le t \le 1 \end{cases}$$

$$x(t) = \int_{-\infty}^{t} y(\tau)\, d\tau$$

$$Y(j\omega) = \frac{\sin \omega/2}{j\omega} \qquad Y(0) = 1/2$$

$$\frac{\sin \omega/2}{j\omega^2} + \frac{\pi}{2}\delta(\omega)$$

([illegible]
[illegible])

4.10) $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$

$$X(jw) = j\frac{\partial}{\partial w}\mathcal{F}\left\{\left(\frac{\sin t}{\pi t}\right)^2\right\}$$

$$X(jw) = j\frac{\partial}{\partial w}\left[\mathcal{F}\left\{\frac{\sin t}{\pi t}\right\} * \mathcal{F}\left\{\frac{\sin t}{\pi t}\right\}\right]$$

$$= \frac{j}{2\pi}\frac{\partial}{\partial w}\left[\begin{cases} 0, & |t|>1 \\ 1, & -1 \le t \le 1 \end{cases} * \begin{cases} 0, & |t|>1 \\ 1, & -1 \le t \le 1 \end{cases}\right]$$

Para $-2 < w < 0$

$$F * F \int_{-1}^{w} dw = w+1$$

Para $0 < w < 2$

$$F * F = \int_{w}^{1} d\tau = -w+1$$

Para o.c $\quad F * F = 0$

$$X(jw) = \frac{j}{2\pi} \frac{\partial}{\partial w}\left[ \begin{cases} w+1 & -2 \leq w \leq 0 \\ -w+1 & 0 < w \leq 2 \\ 0 & \text{o.c.} \end{cases} \right]$$

$$X(jw) = \frac{1}{2\pi} \begin{cases} 1 & -2 \leq w \leq 0 \\ -1 & 0 < w \leq 2 \\ 0 & \text{o.c.} \end{cases}$$

4.12) Si $e^{-|t|} \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{2}{1+w^2}$

$$t e^{-|t|} \overset{\mathcal{F}}{\longleftrightarrow} j \frac{\partial}{\partial w} \frac{2}{1+w^2}$$

$$t e^{-|t|} \overset{\mathcal{F}}{\longleftrightarrow} -\frac{4jw}{(1+w^2)^2}$$

$$\frac{4t}{(1+t^2)^2} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi t e^{-|t|}$$

4.14)

$$\mathcal{F}^{-1}\{X + j\omega X\} = x(t) + j\frac{\partial x}{\partial t}$$

$$\mathcal{F}\{A e^{-2t} u(t)\} = \frac{A}{2+j\omega} = (1+j\omega) X$$

$$X = \frac{A}{x+1} - \frac{A}{x+2}$$

$$x(t) = \left[A e^{-t} - 6 e^{-2t}\right] u(t)$$

$$= \int_{-\infty}^{\infty} |x|^2 dt = 1$$

$$\frac{A^2}{12} = 1 \rightarrow A = 2\sqrt{3}$$

4.15) $\mathcal{E}v\{x\} = |t| e^{-|t|}$

$2|t| e^{-|t|} u(t)$

dibujen en sus cabezas y tiene sentido

4.16)