1.1) Muy fácil

1.2) Demasiado sencillo

1.3)

$$\mathcal{E}_{a-b} = \int_a^b \|x(t)\|^2 dt$$

$$P_{a-b} = \frac{1}{b-a}\int_a^b \|x(t)\|^2 dt = \frac{1}{b-a}\mathcal{E}_{a-b}$$

a) $$\mathcal{E}_\infty = \int_0^\infty e^{-4t} dt = \frac{1}{4}$$

$$P_\infty = \lim_{T\to\infty} \frac{1/4}{2T} = 0$$

$\lim_{T\to\infty} 2T$ ∞

b) $$\mathcal{E}_\infty = \int_{-\infty}^{\infty} \|e^{2jt} e^{j\pi/4}\|^2 dt = t\Big|_{-\infty}^{\infty} = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{\cancel{2T}}{\cancel{2T}} = 1$$

c)

$$\mathcal{E}_\infty = \int_{-\infty}^{\infty} \|\cos(t)\|^2 dt =$$

$$\lim_{T \to \infty} \frac{2T + \sin 2T}{2} =$$

$$\xrightarrow{} -\frac{1}{2} \leq \frac{\sin 2T}{2} \leq \frac{1}{2}$$

$$\lim_{T \to \infty} T + \frac{\sin 2T}{2}$$

$$\lim_{T \to \infty} T - \frac{1}{2} \leq \lim_{T \to \infty} T + \frac{\sin 2T}{2} \leq \lim_{T \to \infty} T + \frac{1}{2}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\infty \qquad \mathcal{E}_\infty = \infty \qquad \infty$$

$$P_\infty = \lim_{T \to \infty} \frac{2T + \sin 2T_1}{4T}$$

$$\xrightarrow{} -1 \leq \sin 2T \leq 1$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2} + \frac{\sin 2T}{4T}$$

$$\lim_{T \to \infty} \frac{1}{2} - \frac{1}{T} \leq \lim_{T \to \infty} \frac{1}{2} + \frac{\sin 2T}{4T} \leq \lim_{T \to \infty} \frac{1}{2} + \frac{1}{T}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\frac{1}{2} \qquad P_\infty = \frac{1}{2} \qquad \frac{1}{2}$$

1.6)

a) No es periodica debido a que la func. $u(t)$ corta la func. en $0<t$

1.9)

a) $x(t) = j e^{j10t}$

$= e^{\ln(j)} e^{j10t}$

$= e^{j(10t + \pi/2)}$

$\cos\left(10t + \frac{\pi}{2}\right) + i\sin\left(10t + \frac{\pi}{2}\right)$

$T = \frac{2\pi}{\|10\|} = \frac{\pi}{5}$

b) $x(t) = e^{-t} e^{jt}$

↳ decaemiento

$\therefore x(t)$ no es Periodica

1.10) $x(t) = 2\cos(10t+1) - \sin(4t-1)$

$$\begin{array}{cc|c} 10 & & 2 \\ 5 & & 5 \\ (1) & & \end{array} \qquad \begin{array}{c|c} 4 & 2 \\ 2 & 2 \\ (1) & \end{array}$$

$\omega_{común} = 2$ $\quad T_{común} = \dfrac{2\pi}{\|\omega_{común}\|}$

$= \pi$ //

1.13) $y(t) = \displaystyle\int_{-\infty}^{t} \delta(t+2) - \delta(t-2)\,dt$

$-2 \leq t \qquad t \geq 2$

$y(t) = u(t+2) - u(t-2)$

$$\mathcal{E}_\infty = \int_{-\infty}^{\infty} \|u(t+2) - u(t-2)\|^2 dt = \int_{-2}^{2} dt = 4$$

1.14)

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ -2 & 1 < t < 2 \end{cases}$$

Ni en pedo hago xD

1.17) $y(t) = x(\sin(t))$

a) Para $y(t_0) = x(\sin(t_0))$

$-1 \leq \sin(t_0) \leq 1$

Si $t = -n\pi, n \in \mathbb{N} \rightarrow y(-n\pi) = x(1)$

Futuro

$\therefore$ El sist. no es causal

b) $x_1(t) \rightarrow y_1(t) = x_1(\sin(t))$

$x_2(t) \rightarrow y_2(t) = x_2(\sin(t))$

$x_3(t) = ax_1 + bx_2$

$\rightarrow y_3(t) = ax_1(\sin(t)) + bx_2(\sin(t))$

$= ay_1(t) + by_2(t)$

$\therefore$ El sist. es lineal

1.19)

a) $y(t) = t^2 x(t-1)$

$y(t-t_0) = (t-t_0)^2 x(t-t_0-1)$

$\neq$

Si $x(t-t_0) \xrightarrow{S} t^2 x(t-t_0-1)$

$\therefore$ Sistema variable en el tiempo

Por su forma ya vez que es lineal

$\therefore$ Sistema lineal

1.20)

$$x_a(t) = e^{j2t} \to y_a(t) = e^{j3t}$$

$$x_b(t) = e^{-j2t} \to y_b(t) = e^{-j3t}$$

Sistema lineal

a) $$\cos(2t) = \frac{1}{2} e^{j2t} + \frac{1}{2} e^{-j2t}$$

$$x_3(t) = \frac{1}{2} x_a(t) + \frac{1}{2} x_b(t)$$

$$y_3(t) = \frac{1}{2} y_a(t) + \frac{1}{2} y_b(t)$$

$$= \frac{1}{2} \cos(3t)$$

b) $$\cos(2t-1) = \frac{1}{2}\left[e^{j(2t-1)} + e^{-j(2t-1)}\right]$$

$$= \frac{e^{-j}}{2} e^{2jt} + \frac{e^{j}}{2} e^{-2jt}$$

$$= \frac{e^{-j}}{2} x_a + \frac{e^{j}}{2} x_b$$

$$y_3 = \frac{e^{j}}{2} y_a + \frac{e^{j}}{2} y_b$$

$$= \cos(3t - 1)$$

1.21) Mi oito te voy a dibujar

1.23) Mi oito te voy a dibujar x2

1.25)

a) $T = \frac{2}{3}\pi$

b) $x(t) = e^{j\pi t} e^{j\pi}$ → ángulo inicial

$$T = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$$

c) $x(t) = \cos^2(2t - \pi/3)$

$$= \frac{1}{2}\cos(4t - 2\pi/3) + \frac{1}{2}$$

$$T = \frac{\cancel{2}\pi}{\cancel{4}_2} = \frac{\pi}{2}$$

1.27) Muy fácil y largo ya

1.30)

a) $x(t) \xrightarrow{t \to t-4} x(t-4) \xrightarrow{t \to t+4} x(t)$

$$y^{-1}(t) = x(t+4)$$

b) $x(t) \xrightarrow{\cos(\,)} \cos[x(t)] \xrightarrow{\cos^{-1}(\,)} x(t)$

$\cos^{-1}$ no recupera todo valor de $x(t)$

Solo es invertible para

$$-\frac{\pi}{2} \leq x(t) \leq \frac{\pi}{2}$$

1.32) Paja.

1.38) $\delta(2t) = \lim\limits_{\Delta \to 0} \delta_\Delta(2t)$

$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & \Delta \leq t \\ 0 & \text{o.c} \end{cases} \rightarrow \delta_\Delta(2t) = \begin{cases} \frac{1}{\Delta} & \frac{\Delta}{2} \leq t \\ 0 & \text{o.c} \end{cases}$$

2.8)

$$x(t) = (t+1)[u(t) - u(t-1)] + (2-t)[u(t-1) - u(t-2)] =$$

$$(t+1)u(t) + (\underbrace{-t-1+2-t}_{-2t+1})u(t-1) - (2-t)u(t-2)$$

$$(t+1)u(t) + (-2t+1)u(t-1) + (t-2)u(t-2)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)[\delta(t-\tau+2) + 2\delta(t-\tau+1)]\,d\tau$$

$$= x(t+2) + 2x(t+1)$$

$$= (t+3)u(t+2) + [-2(t+2)+1]u(t+1) + t\,u(t)$$

$$+ 2[(t+2)u(t+1) + (-2(t+1)+1)u(t) + (t-1)u(t-1)]$$

$$= (t+3)u(t+2) + u(t+1) - (3x+2)u(t) + (2t-2)u(t-1)$$

2.11)

$$y(t) = \int_{-\infty}^{\infty} \overset{\tau \le t-3}{u(t-\tau-3)}\, e^{-3\tau}\, \overset{\tau \ge 0}{u(\tau)}\, d\tau$$

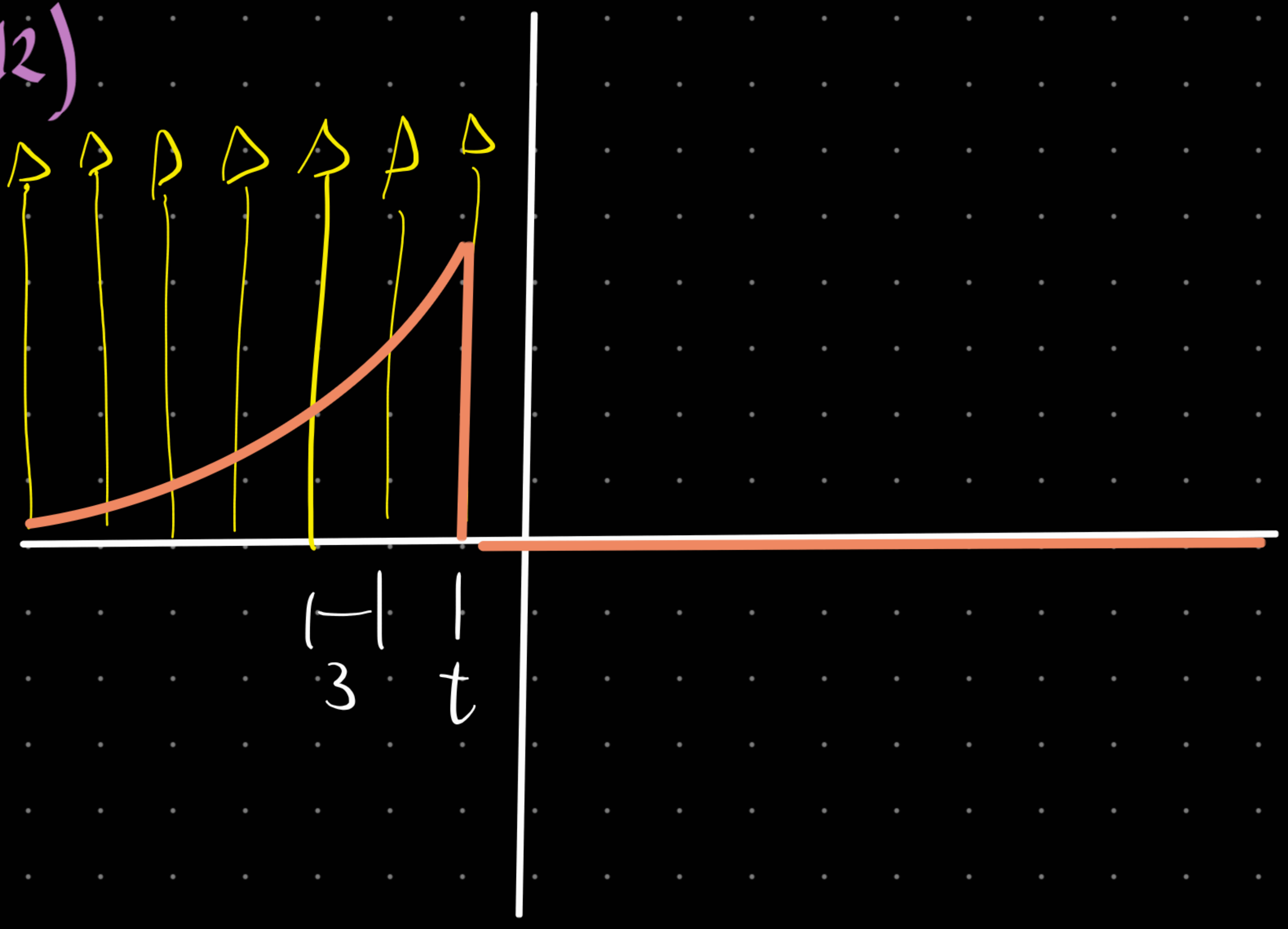
$$\int_{-\infty}^{\infty} \overset{\tau \le t-5}{u(t-\tau-5)}\, e^{-3\tau}\, \overset{\tau \ge 0}{u(\tau)}\, d\tau$$

$$= u(t)\left[u(t-3)\int_{0}^{t-3} e^{-3\tau}\, d\tau - u(t-5)\int_{0}^{t-5} e^{-3\tau}\, d\tau\right]$$

$$\frac{1}{3}\left\{u(t-3)\left[1-e^{-3(t-3)}\right] + u(t-5)\left[e^{-3(t-5)}-1\right]\right\}$$

$$g(x) = \frac{\partial x}{\partial t} * h = \frac{\partial y}{\partial t}$$

2-12)



$$y(t) = \dots e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) \dots$$

$$e^{-t}\left(e^{0} + e^{-3} + e^{-6} + \dots\right)$$

$$\sum_{k=0}^{\infty} e^{-3k} = \frac{e^{0}}{1-e^{-3}}$$

2.14)

a) $$\int_{-\infty}^{\infty} \left| e^{-t} \cancelto{1}{e^{2jt}} u(t) \right| dt$$

$u(t) \to t \geq 0$

$$\int_{0}^{\infty} \left| e^{-t} \right| dt = 1 \to \text{Estable}$$

b) $$\int_{-\infty}^{\infty} \left| e^{-t} \cos(2t) u(t) \right| dt$$

$\to -1 \leq \cos(2t) \leq 1$

$$0 \leq \underbrace{\int_{0}^{\infty} \left| e^{-t} \right| \left| \cos(2t) \right| dt}_{\to I} \leq \int_{0}^{\infty} e^{-t} dt = 1$$

$0 \leq I \leq 1 \to$ Estable

2.17)

a) $y' + 4y = x \qquad y(0) = 0$

$$e^{4\int dt} = e^{4t}$$

$$y(t) = \frac{\int x(t) e^{4t} dt}{e^{4t}} - \left.\frac{\int x(t) e^{4t} dt}{e^{4t}}\right|_{t \to 0}$$

$$\int x(t) e^{4t} dt = \frac{e^{3t(1+j)}}{3+3j}$$

$$3t + 3jt - 4t$$

$$y(t) = \frac{e^{3(1+j)t}}{3(1+j)\, e^{4t}} - \frac{1}{3(1+j)\, e^{4t}}$$

$$y(t) = \frac{1-j}{6}\left[e^{(-1+3j)t} - e^{-4t}\right]$$

b) La misma cagada que el a

2.20)

a) $$\int_{-\infty}^{\infty} \overset{\to\delta(t)}{u_0(t)}\cos(t)\,d\tau = \cos(0) = 1$$

b) $$\int_{0}^{5} \sin(2\pi t)\,\overset{\to t=-3}{\delta(t+3)}\,dt = 0$$

c) $$\int_{-5}^{5} u_1\overset{\to \tau=1}{(1-\tau)}\cos(2\pi\tau)\,d\tau =$$

$$\int_{-\infty}^{\infty} u_1(t-\tau)\cos(2\pi\tau)\left[u(\tau+5)-u(\tau-5)\right]d\tau\,\Big|_{t\to1} =$$

$$u_1(t) * \left[\cos(2\pi t)\left(u(t+5)-u(t-5)\right)\right]\Big|_{t\to1} =$$

$$\frac{\partial}{\partial t}\left[\cos(2\pi t)\left(u(t+5)-u(t-5)\right)\right]\Big|_{t\to1} =$$

$$-2\pi\sin(2\pi t)\left(u(t+5)-u(t-5)\right)] + \left[\cos(2\pi t)\overset{0}{\cancel{\left(\delta(t+5)-\delta(t-5)\right)}}\right]\Big|_{t\to1} =$$

0

2.42)

$$y(t) = \int_{-0,5}^{0,5} e^{jw_0(t-\tau)} d\tau =$$

$$= \frac{1}{w_0 j}\left(e^{jw_0(t+0,5)} - e^{jw_0(t-0,5)}\right)$$

$$= \frac{e^{jw_0 t}}{w_0 j}\left(e^{0,5jw_0} - e^{-0,5jw_0}\right)$$

$$y(t) = \frac{2e^{jw_0 t}}{w_0} \sin\left(\frac{w_0}{2}\right)$$

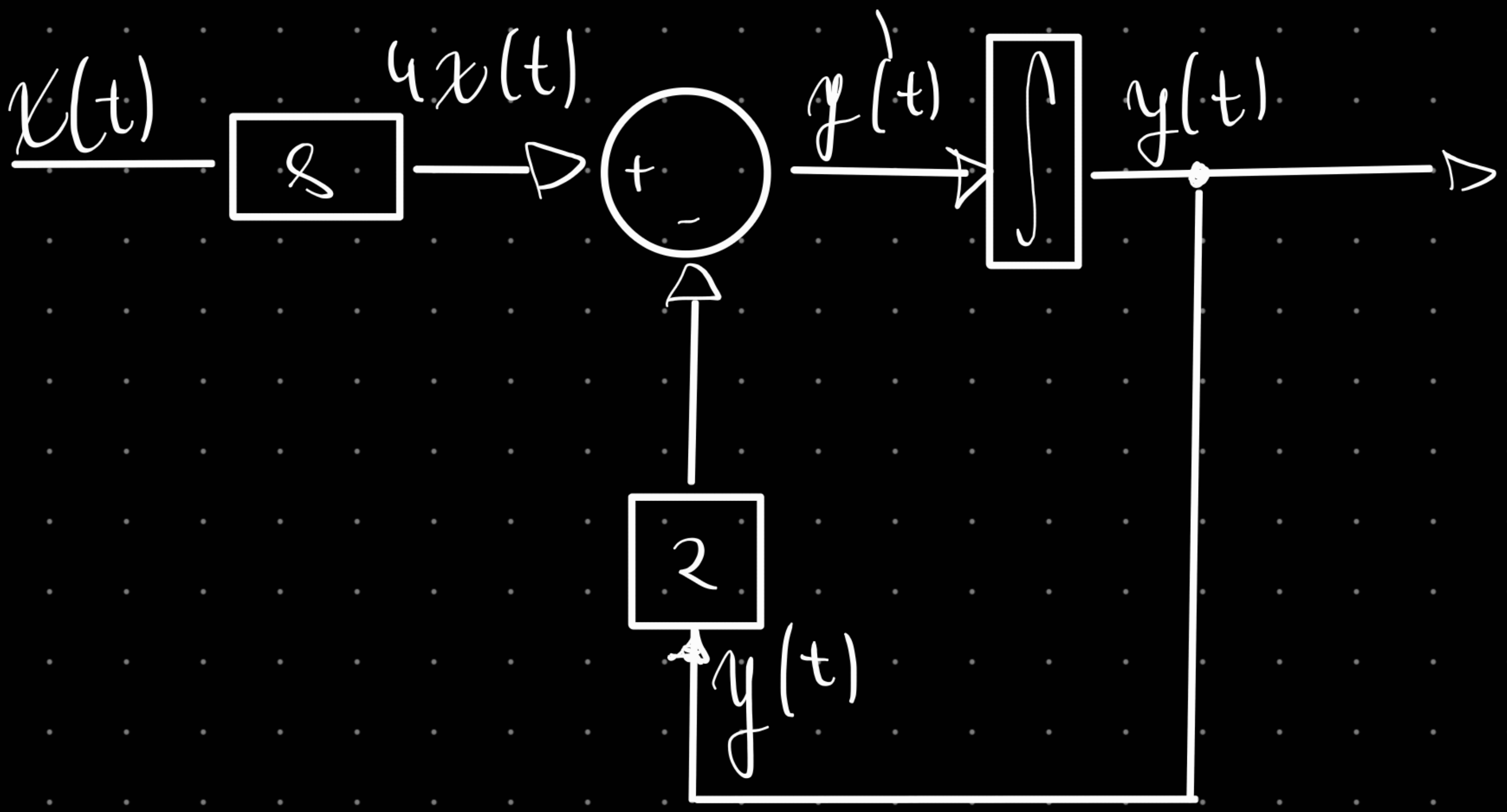
$$y(0) = \frac{2}{w_0} \sin\left(\frac{w_0}{2}\right) = 0$$

$$\frac{w_0}{2} = n\pi$$

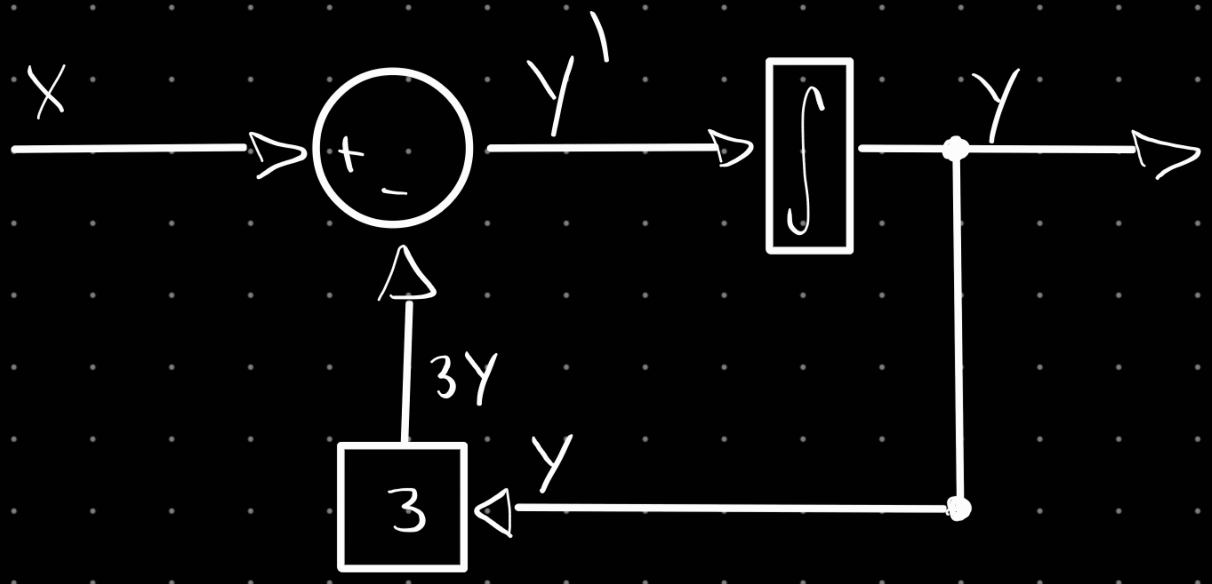
$$w_0 = 2n\pi$$

2.39)

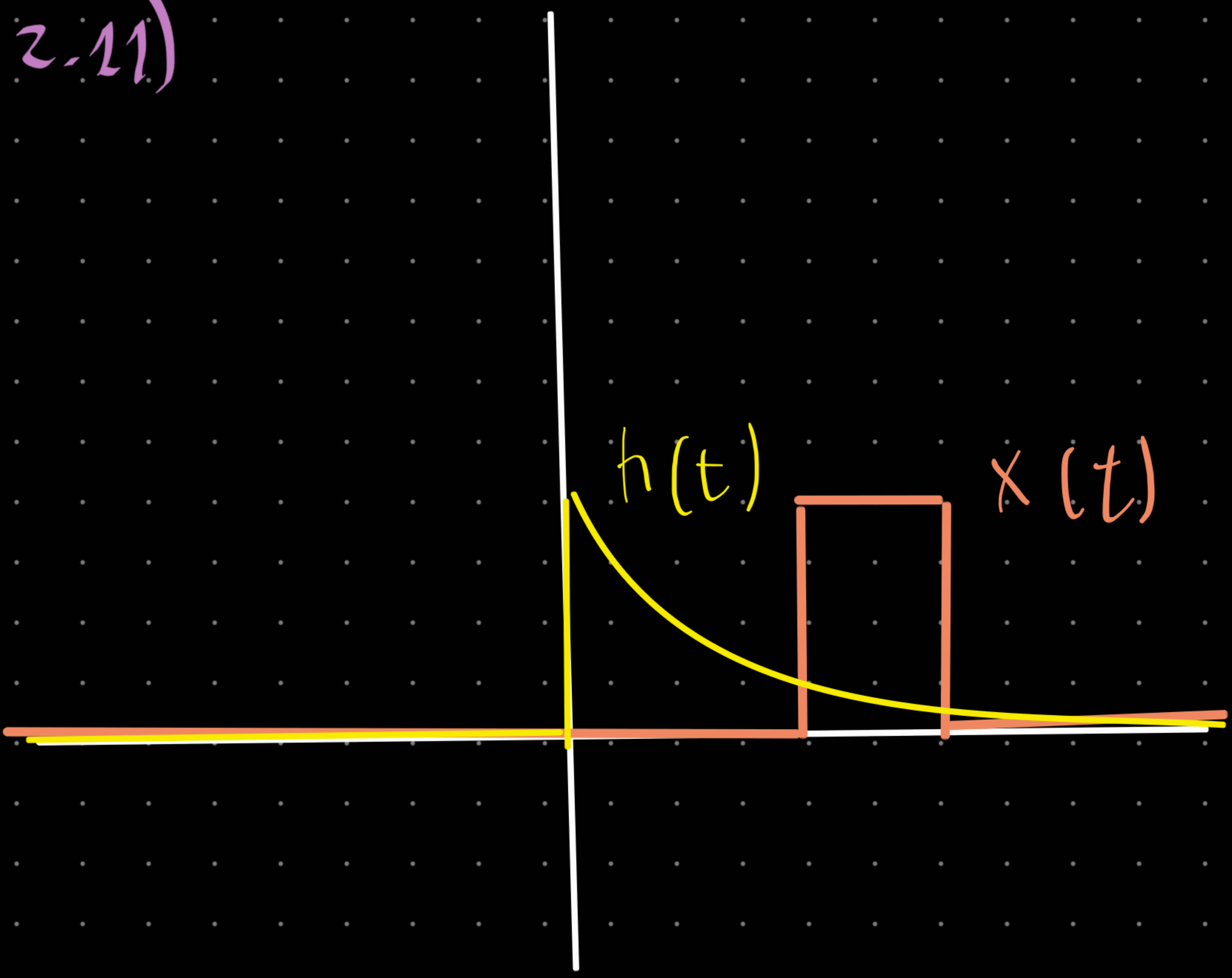
$$y' = -2y + 8x$$

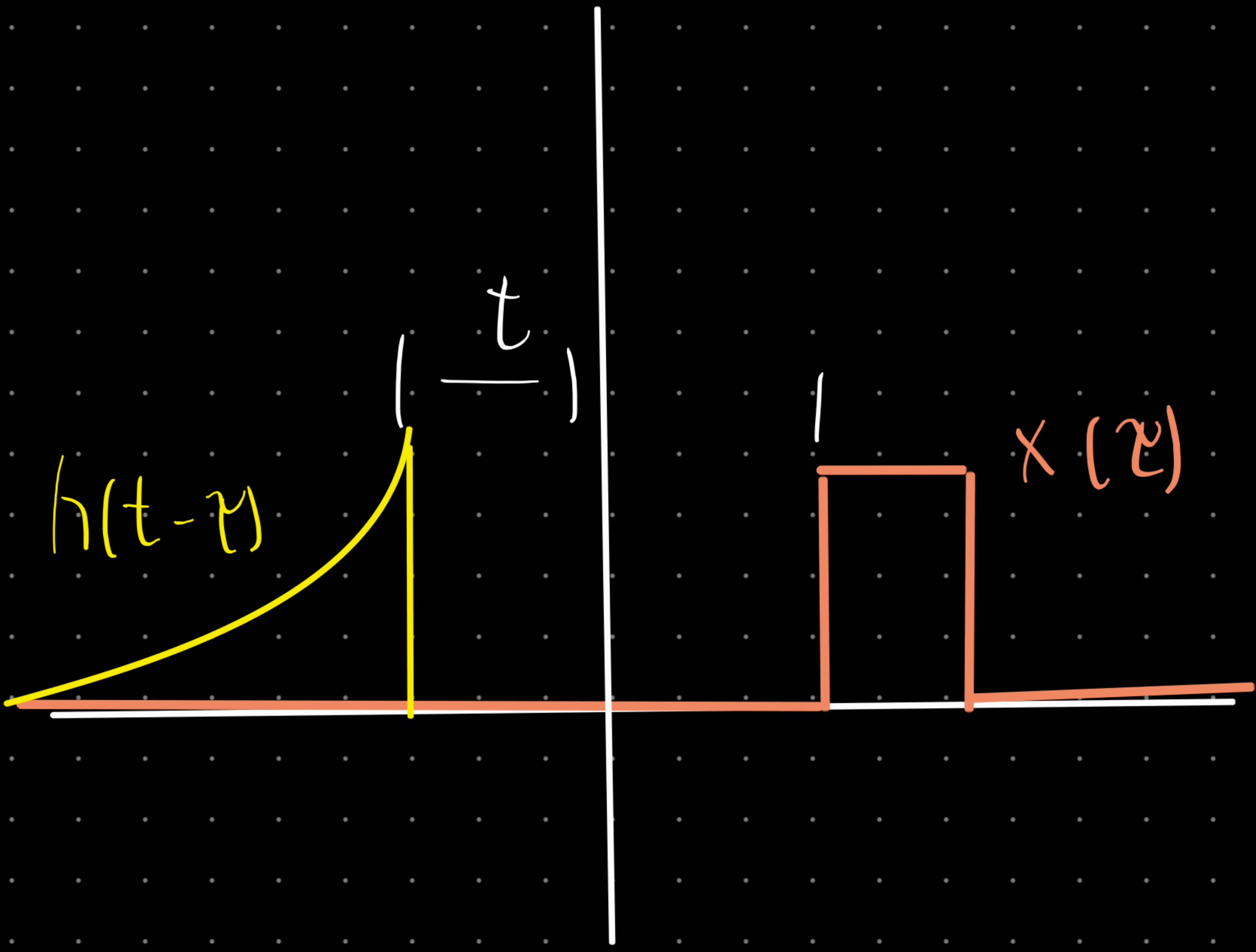


$$y' = x - 3y$$



2.11)

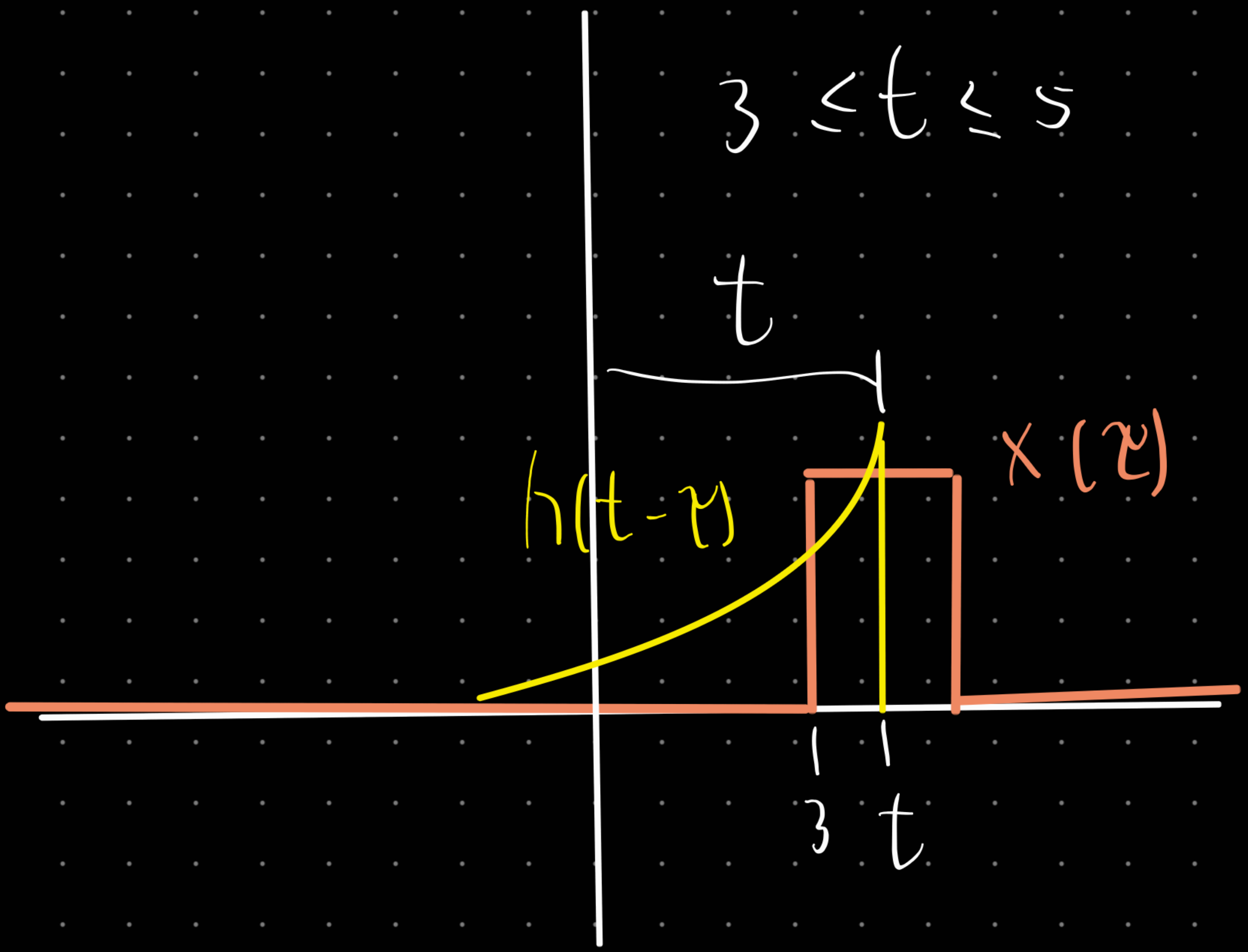




$t \leq 3$

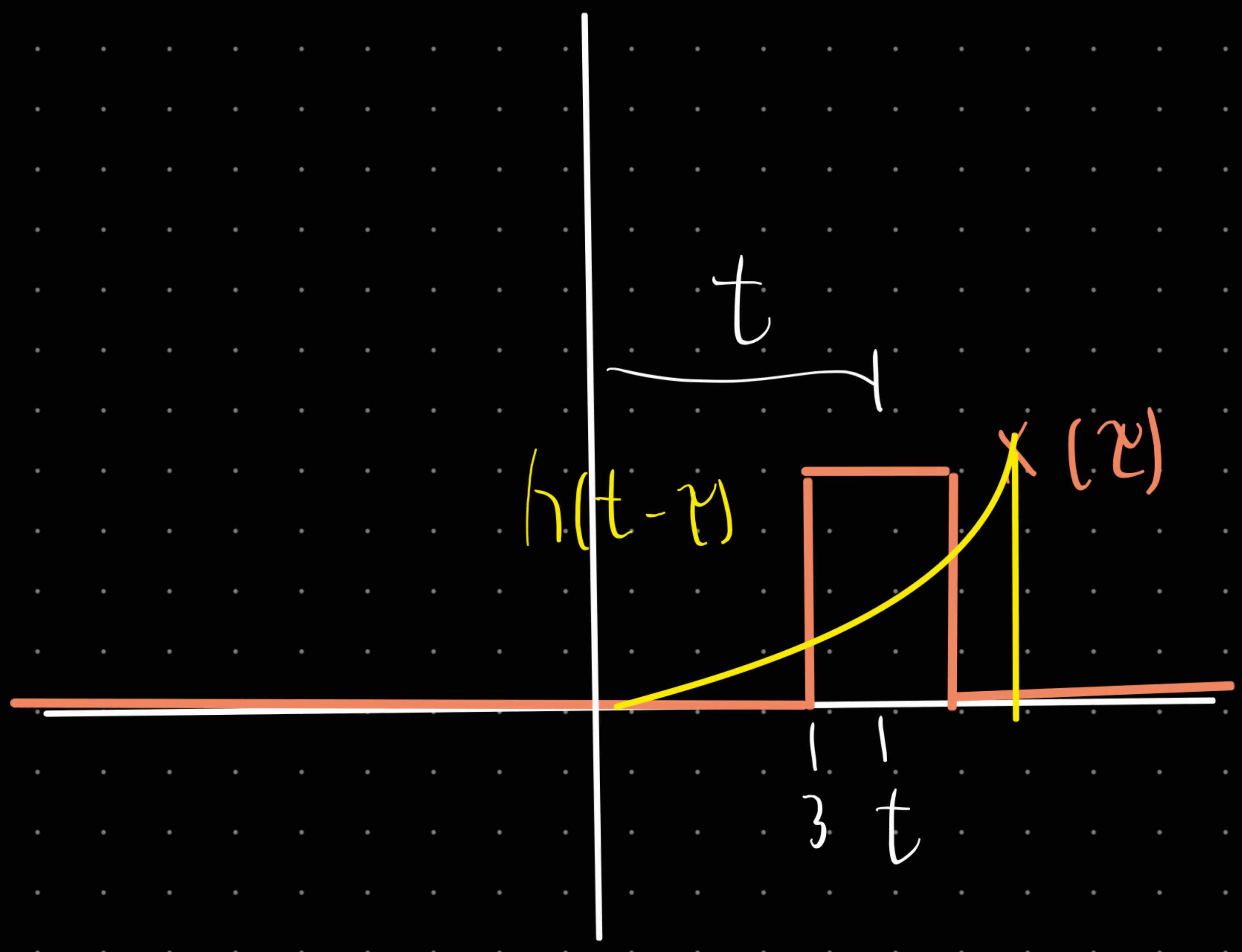
$y(t) = 0$

$3 \leq t \leq 5$



$$\int_3^t 1 \cdot e^{-3(t-\tau)}\, d\tau =$$

$t \geq 5$



$$\int_3^5 e^{-3(t-\tau)}\,d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$t \geq \tau$

$\tau \geq 3$ $\quad t-\tau \geq 0$ $\quad \tau \geq 5$ $\quad u(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} u(\tau-3)\, e^{-3(t-\tau)}\, u(t-\tau)\, d\tau - \int_{-\infty}^{\infty} u(\tau-5)\, e^{-3(t-\tau)}\, d\tau$$

$$= \cancel{u(t)} \left[u(t-3) \int_{3}^{t} e^{-3(t-\tau)}\, d\tau - u(t-5) \int_{5}^{t} e^{-3(t-\tau)}\, d\tau \right]$$

$$x(t) \xrightarrow{S} y(t)$$

$$x'(t) \xrightarrow{S} y'(t)$$

$t-\tau-2=0 \rightarrow \tau = t-2$

$$\int_{-\infty}^{\infty} f(\tau)\, \delta(t-\tau-2)\, d\tau = f(t-2)$$

$t = 8$

$t-8=0$

$$\int_{-5}^{5} \cos(t)\, \delta(t-8)\, dt = 0$$

$$\delta(t-3)-\delta(t-5)$$

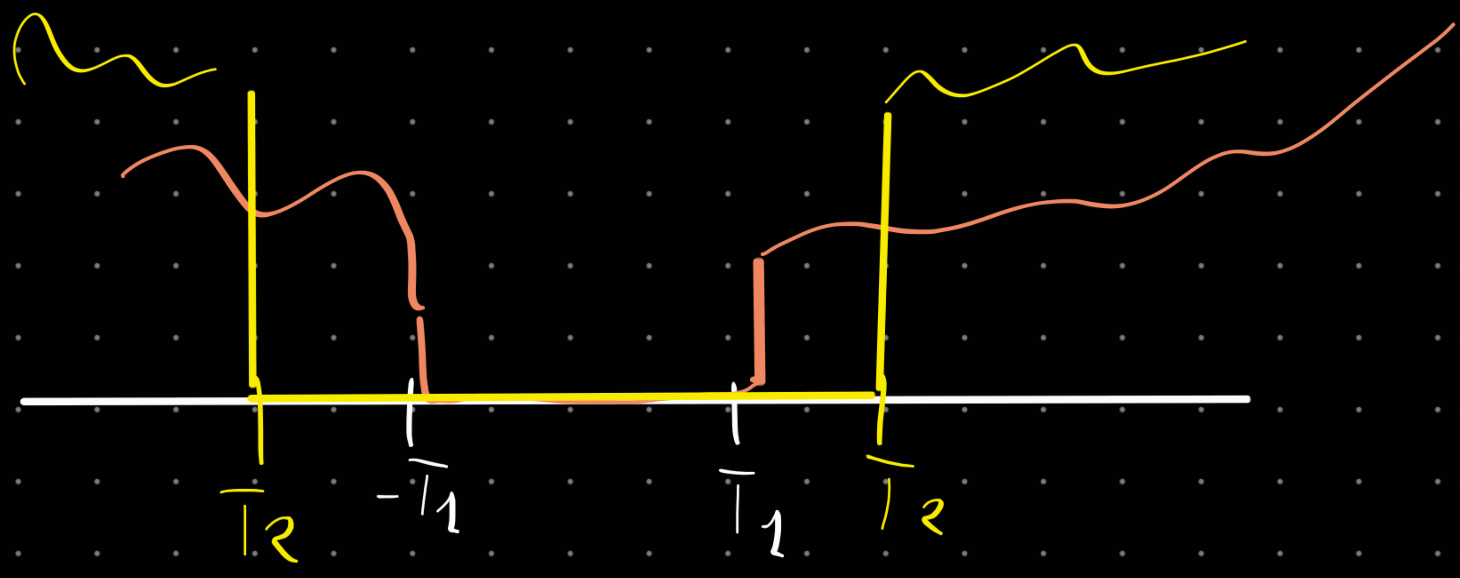
$$=\int_{-\infty}^{\infty}\delta(\tau-3)\underbrace{e^{-3(t-\tau)}u(t-\tau)}_{x(\tau)\to x(3)}d\tau-\int_{-\infty}^{\infty}\delta(\tau-5)e^{-3(t-\tau)}u(t-\tau)d\tau$$

$\to \tau=3$ $\qquad \to \tau=5$

$$=e^{-3(t-3)}u(t-3)-e^{-3(t-5)}u(t-5)$$

$$\tau+0,5\geq 0$$

$$\tau\geq -0,5$$



$$-T_1 \geq t \qquad T \leq t$$

$$x(t) = f(t)\left[u(-T_1 - t) + u(t - T_1)\right]$$

$$h(t) = g(t)\left[u(-T_2 - t) + u(t - T_2)\right]$$

$$-T_1 - \tau_1 \geq 0 \rightarrow -T_1 \geq \tau_1$$

$$\int f(\tau)\left[u(T_1 + \tau) + u(\tau - T_1)\right] g(t-\tau)\left[u(\underbrace{-T_2 - t + \tau}) + u(t - \tau - T_2)\right]$$

$$-T_1 - \tau \geq 0 \wedge \tau - T_1 \geq 0$$

$$+$$

$$-T_2 - t + \tau \geq 0 \wedge t - \tau - T_2 \geq 0$$

$$-T_1 \geq \tau \geq T_2 + t \wedge t - T_2 \geq \tau \geq T_1$$

$$-T_1 \geq T_2 + t \wedge t - T_2 \geq T_1$$

$$-T_1 - T_2 \geq t \geq T_2 + T_1$$

$$t \geq |T_1 + T_2|$$

$$f(t) = \cos(4t+1) + \sin(2t-\pi)$$

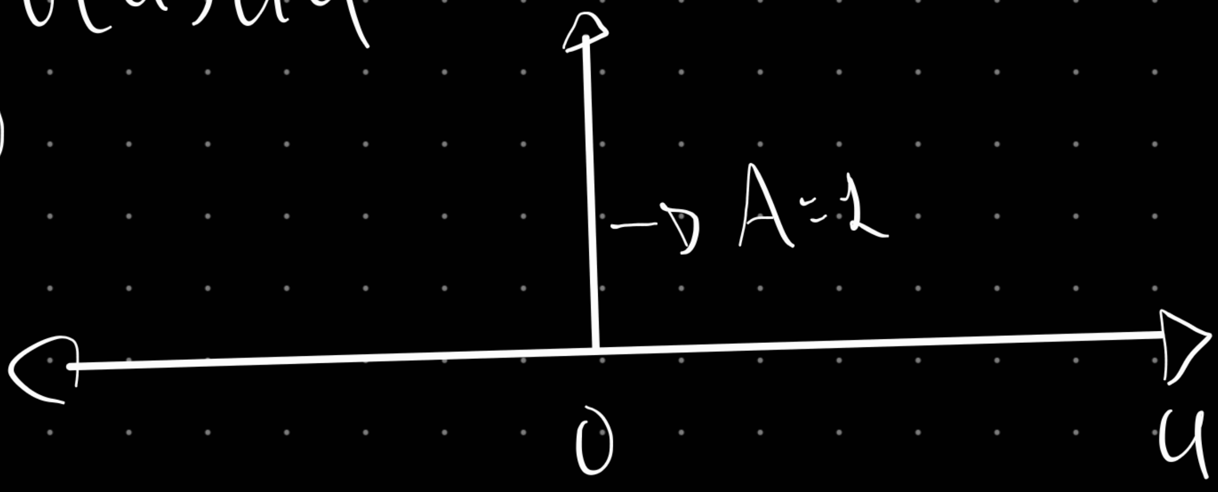
$$MCM(1,2) = \omega_f = 2$$

$$\int_{-\infty}^{t-s} \delta(\tau)\,d\tau = u(t-s)$$

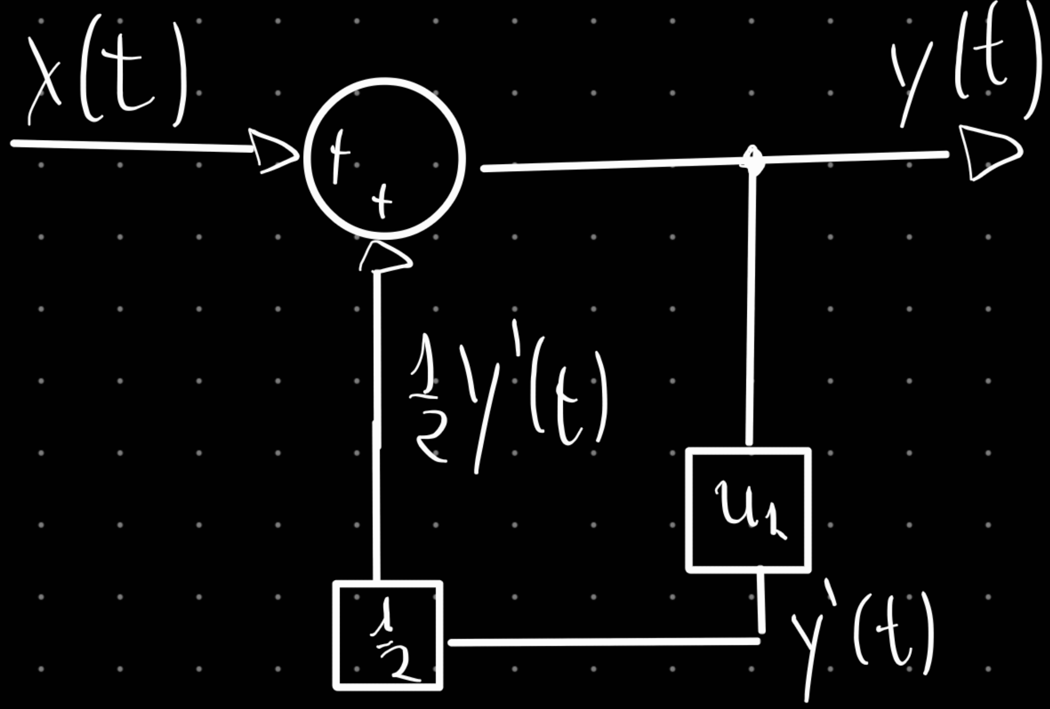
$$\int_{-\infty}^{t} \delta(t-\tau)\,d\tau =$$

$$-\int_{\infty}^{0} \delta(u)\,du \qquad \begin{aligned} u &= t-\tau \\ du &= -d\tau \end{aligned}$$

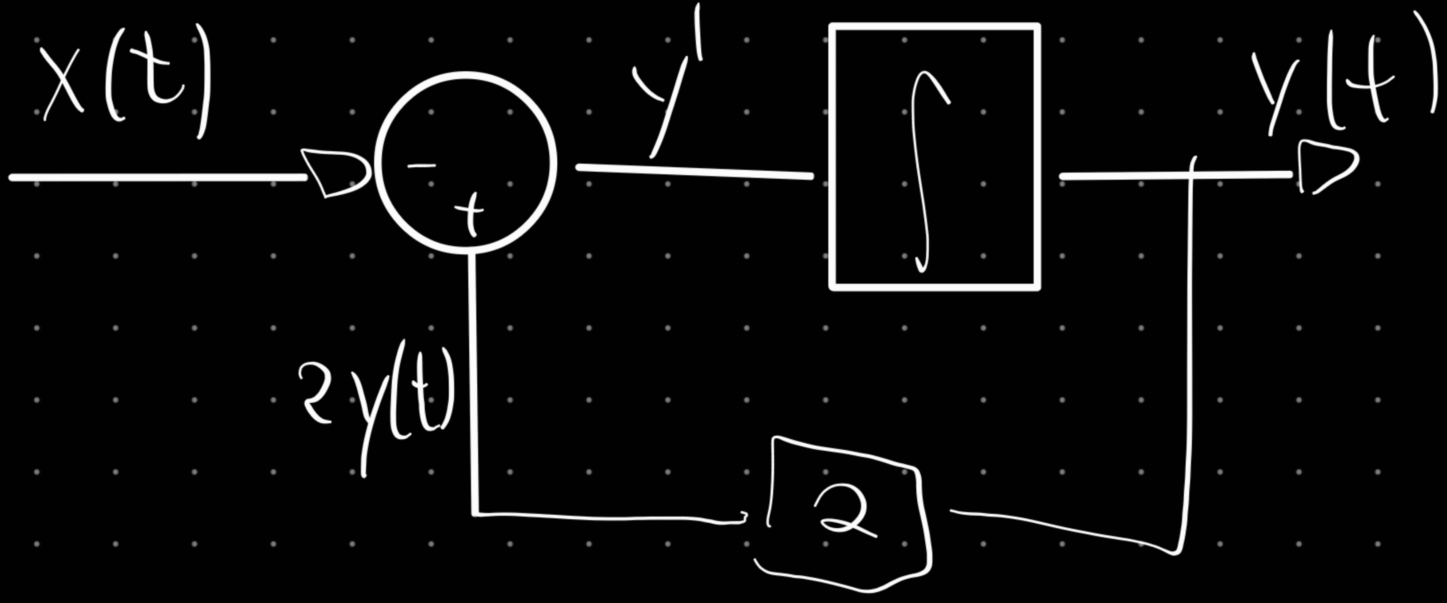
$$\int_{0}^{\infty} \delta(u)\,du = 1$$

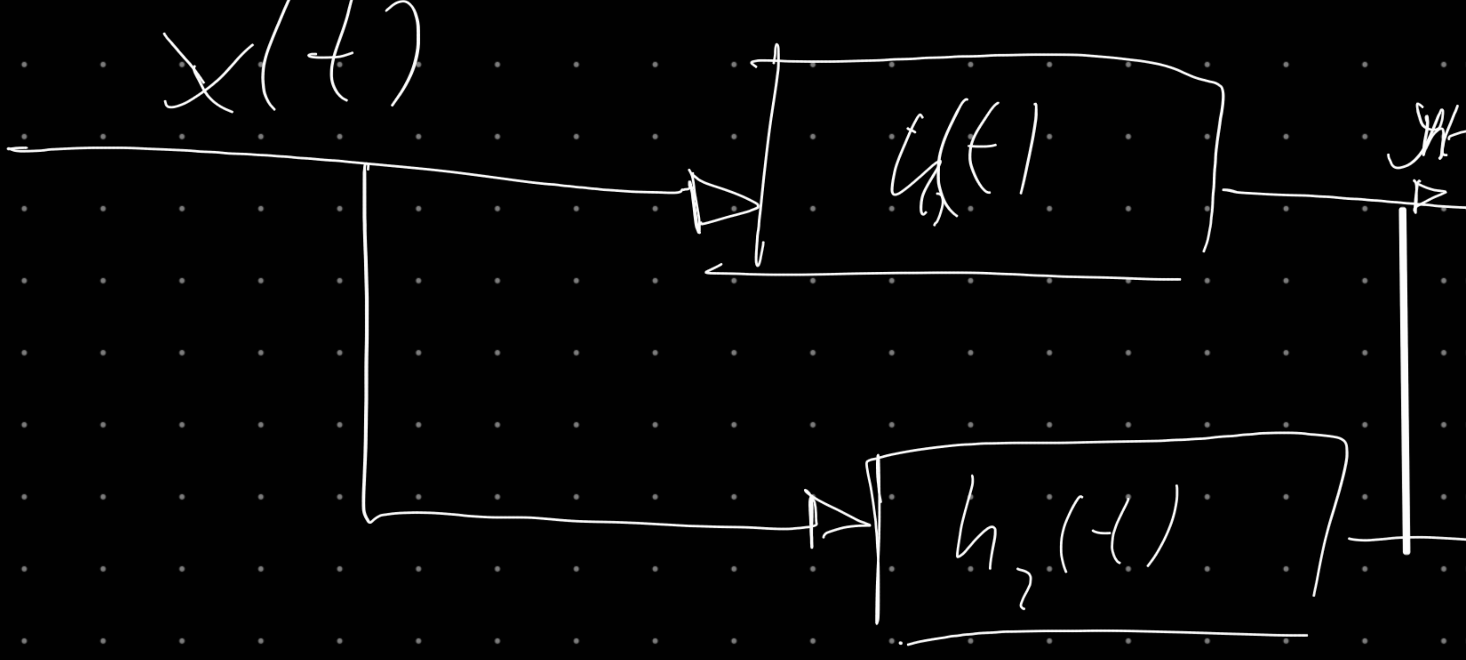


$$y(t) = \frac{1}{2} y'(t) + x(t)$$



$$y'(t) = 2y(t) - 2x(t)$$





$$y(t) = x(t-2) + x(2-t)$$

$$x(t) \xrightarrow{S} x(t-2) + x(2-t)$$

$$x(0) \longrightarrow x(-2) + x(2)$$

Τιμή μημηία

$$x(t) \xrightarrow{S} x(\sin(t))$$

$$x(n) \longrightarrow x(\sin n)$$

$$x(0)$$

$$x(t-t_0) \longrightarrow y(t-t_0)$$

$$x(t) \longrightarrow \cos(3t)\, x(t)$$

$$x(t-t_0) \longrightarrow \cos(3t)\, x(t-t_0)$$

$$\ne$$

$$y(t-t_0) = \cos(3(t-t_0))\, x(t-t_0)$$

$$-B \leq x(t) \leq B$$

$y(t) \rightarrow$ Estable

$$-2B \leq x(t-2) + x(2-t) \leq 2B$$

$$-\frac{1}{B} \leq \frac{1}{x(t)} \leq \frac{1}{B}$$

$$B \rightarrow 0$$

$$-\infty \leq \frac{1}{x(t)} \leq \infty$$

$$\frac{t^2}{2} + \infty$$

$$y(t) = X(2t) - X(-\infty)$$

si $x(t) = t$

$x(t) \longrightarrow \cos(x(t-2)) \xrightarrow{\cos^{-1}(x(t+2))} x(t)$

$T_{\cos/\sin} = \frac{2\pi}{w_0}$ $T_{\sec/t} = \frac{\pi}{w_0}$

$\frac{2\pi}{4} =$

b) $e^{j(\pi t-1)} = e^{j\pi t} e^{-j} \longrightarrow$ no cambia

$[\cos(\pi t) + i\sin(\pi t)]e^{-j}$

$\overset{\pi}{(1)} \; \overset{\pi}{(1)} \; | \; \pi$

$\cos(8t+8) + \sin(9t-\pi)$

$T = MCM(T_1; T_2) = \frac{2\pi}{w}$

$w = MCM(w_1; w_2)$

8	36	2
4	18	2
2	9	

$\left.\begin{matrix}2\\2\end{matrix}\right\} 4$

$$x_1 \to y_1$$

$$x_2 \to y_2$$

$$x_3 = a x_1 + b x_2$$

$$x_3 \to y_3 = a y_1 + b y_2$$

$$y(t) = \cos[x(t)]$$

$$x_1(t) \to y_1(t) = \cos[x_1(t)]$$

$$x_2(t) \to y_2(t) = \cos[x_2(t)]$$

$$x_3(t) = a x_1(t) + b x_2(t)$$

$$y_3(t) = a y_1(t) + b y_2(t)$$

$$\underset{\to \text{obj}}{y_3(t)} = a\cos(x_1(t)) + b\cos(x_2(t))$$

$$x_3(t) \to y_3(t) = \cos(x_3(t))$$

No Lineal.

$$= \cos(a x_1(t) + b x_2(t))$$

$$= \cos(a x_1)\cos(b x_2) - \sin \ldots$$

invariable en el tiempo.

$$x(t) \longrightarrow y(t) = \cos[x(t)]$$

$$x(t-t_0) \longrightarrow y(t-t_0) = \cos[x(t-t_0)]$$

$$x(t) \longrightarrow y(t) = -t\cos[x(t)]$$

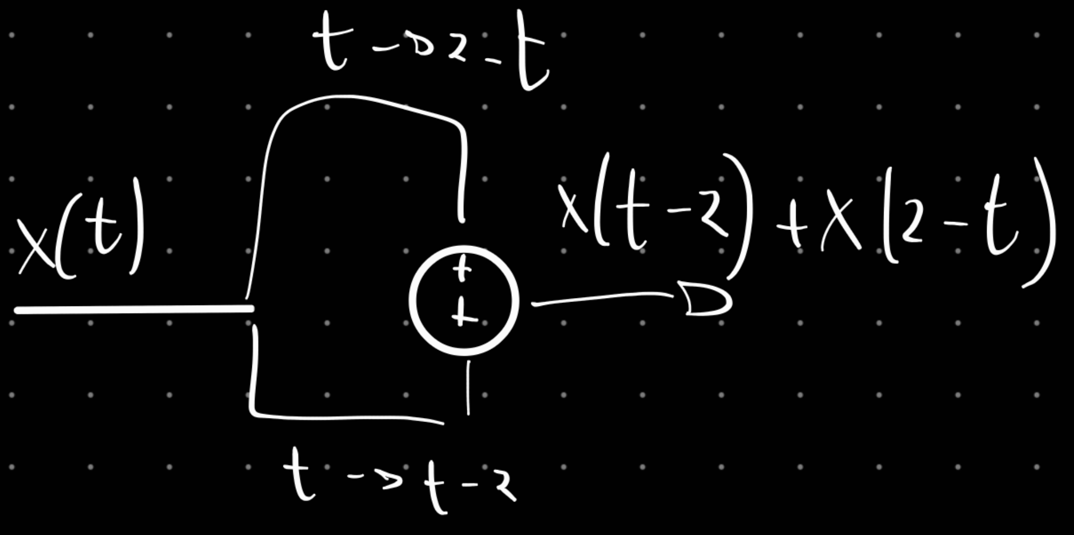
$$y(t-t_0) = -(t-t_0)\cos[x(t-t_0)]$$

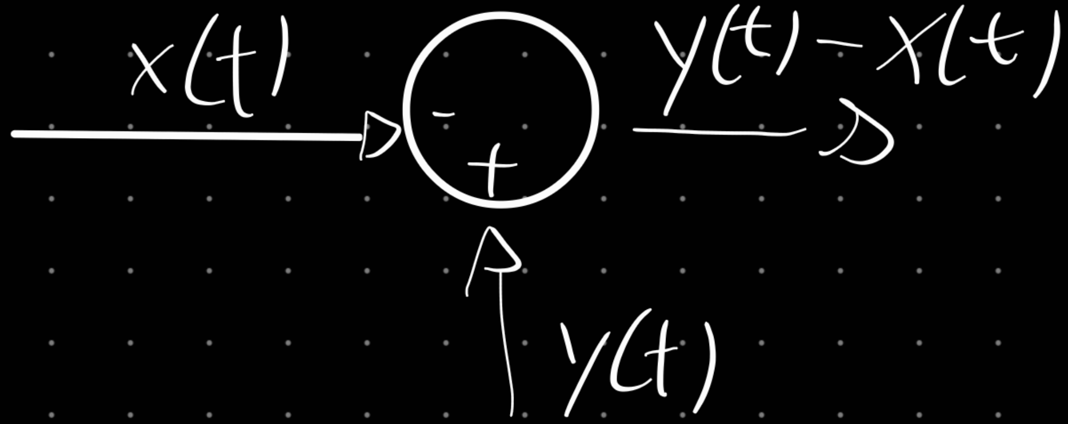
$$x(t-t_0) \longrightarrow y(x(t-t_0)) = -t\cos[x(t-t_0)]$$

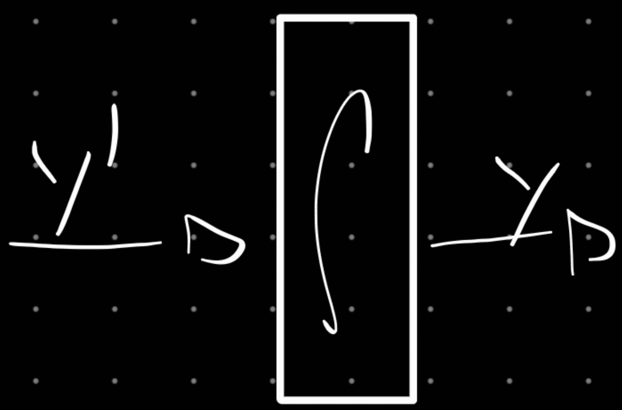
$$x(t) \longrightarrow y(t) = x(t-3)$$

$$x(t) \xrightarrow{t \to t-3} x(t-3) \xrightarrow{t \to t+3} x(t)$$

$$x(t) \longrightarrow x(t-2) + x(2-t) \longrightarrow$$





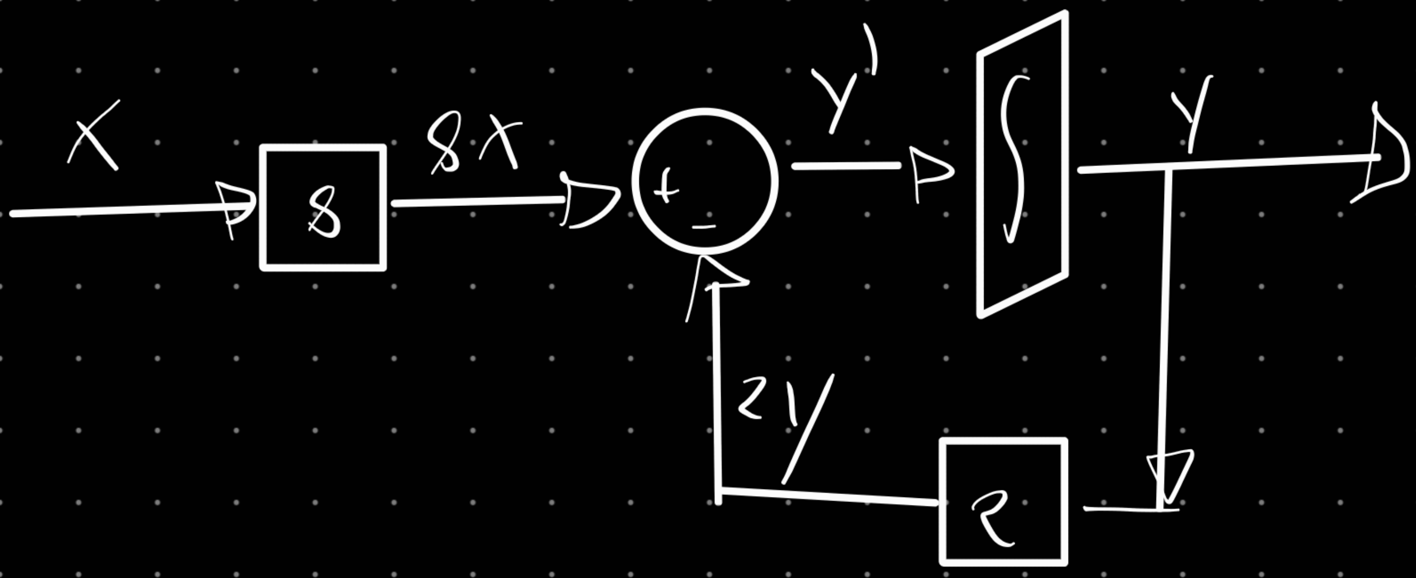


$$y' + p(t)y = x(t)$$

$$u(t) = e^{\int p(t)\,dt}$$

$$y = \frac{1}{u(t)}\left(\int u(t)\,x(t)\right)$$

a) $y' = 8x - 2y$



$$y' = 8x - 2y$$

$$y' + 2y = 8x$$

$$y' + p(t)y = x(t)$$

$$z(t) = e^{\int p(t)\,dt}$$

$$y = \frac{1}{z(t)}\left(\int z(t)\,x(t)\,dt\right)$$

$$y' + 2y = x(t)$$

$$z(t) = e^{\int 2\,dt} = e^{2t}$$

$$y = \frac{1}{e^{2t}}\left(\int e^{2t}\,x(t)\,dt\right)$$

$$\int e^{2t} e^{3t} u(t)\,dt$$

$$y(t) = \frac{1}{e^{2t}}\left(\frac{1}{5} e^{5t} u(t) + c\right)$$

$$y(0) = 0 = \frac{1}{e^{2\cdot 0}}\left(\frac{1}{5}\overbrace{e^{5\cdot 0}}^{1} u(0) + C\right)$$

$$C = -1$$

$$\int f(t)\,\delta(t-\alpha) = f(\alpha)$$

$$t \geq \alpha$$

$$\int f(t)\,u(t-\alpha)\,dt = u(t-\alpha)\int_{\alpha}^{\infty} f(t)$$

$$\to \tau = t - b$$

$$t - \tau - b = 0$$

$$\int \underbrace{f(\tau)\,u(\tau-a)}^{x(\tau)}\,\delta(t-\tau-b)\,d\tau$$

$$f(\tau)\,u(\tau-a)\Big|_{\tau \to t-b}$$

$$f(t-b)\,u(t-b-a)$$