

## Question 4

- a. According to the question, set sequence  $A = \langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle$ . We need to calculate the  $A * A$ .

For  $A = \langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle$  the associated polynomial is  $A(x) = 1 + x^{k+1}$ ;

thus the convolution of  $A$  with itself is the sequence of the coefficients of the polynomial  $A^2(x) = 1 + 2 * x^{k+1} + x^{2k+2}$ . So the  $\langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle * \langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle = \langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 2, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle$

$$\langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle = \langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 2, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle$$

- b. Since  $A = \langle 1, \underbrace{0, 0, 0, \dots, 0}_k, 1 \rangle$ , the corresponding polynomial is  $P_A(x) = 1 + x^{k+1}$  and

$$\begin{aligned} DFT(A) &= \langle P_A(\omega_{k+2}^0), P_A(\omega_{k+2}^1), \dots, P_A(\omega_{k+2}^{k+1}) \rangle \\ &= \langle 1 + \omega_{k+2}^{0*(k+1)}, 1 + \omega_{k+2}^{1*(k+1)}, \dots, 1 + \omega_{k+2}^{(k+1)*(k+1)} \rangle \\ &= \langle 2, 1 + \omega_{k+2}^{k+1}, 1 + \omega_{k+2}^{2*(k+1)}, \dots, 1 + \omega_{k+2}^{(k+1)^2} \rangle \end{aligned}$$