JIE MEI z5173405

Question5

a. Using $\log(a^b) = b\log a$ we obtain

$\mathbf{g(n)} = \log_2(n^{\log_2 n})^2 = 2 * (\log_2 n) * (\log_2 n) = 2 * (\log_2 n)^2 = \Theta((\log_2 n)^2)$ because f(n) and g(n) has same asymptotic growth rate.

b. We want show that $n^{10} = O(2^{\sqrt[10]{n}})$,that means that we have to show that $n^{10} < c\left(2^{\sqrt[10]{n}}\right)$ for some positive c and all sufficiently large n. But, since the log function is monotonically increasing, this will hold just in case

$$\log n^{10} < \log c + \log 2^{\sqrt[10]{n}}$$

Which holds just in case

$$10\log n < \log c + n^{\frac{1}{10}} * \log 2$$

We now take c = 1 show that

$$10\log n < n^{\frac{1}{10}} * \log 2$$

Which equivalent to showing that

$$\frac{10\log n}{n^{\frac{1}{10}} * \log 2} < 1$$

To this end we use the L'Hˆopital's to compute the limit

$$\lim_{n\to\infty} \frac{10\log n}{n^{\frac{1}{10}} * \log 2} = \lim_{n\to\infty} \frac{(10\times\log n)'}{\left(n^{\frac{1}{10}}\times\log 2\right)'} = \frac{\frac{10}{n}}{\frac{1}{10} * \frac{1}{n^{\frac{9}{10}}} * ln2} = 0$$

Because $\lim_{n\to\infty} \frac{10\log n}{n^{\frac{1}{10}} * \log 2} = 0$ then, for sufficiently large n we will have $\frac{10\log n}{n^{\frac{1}{10}} * \log 2} < 1$.

c. We want to show that not true for f(n) = O(g(n)) or g(n) = O(f(n)). $f(n) = n^{1+(-1)^n}$ we will log the f(n) and g(n) function show that

$$f(n) = \log n^{1+(-1)^n} = (1 + (-1)^n)\log n$$

$$= \log n + (-1)^n * \log n$$

$$g(n) = \log n$$

If n is even number, f(n)=2*log(n). If n is odd number, f(n) = 0. So, it is not true that for every two functions f(n) and g(n) either f(n) = O(g(n)) or g(n) = O(f(n)).