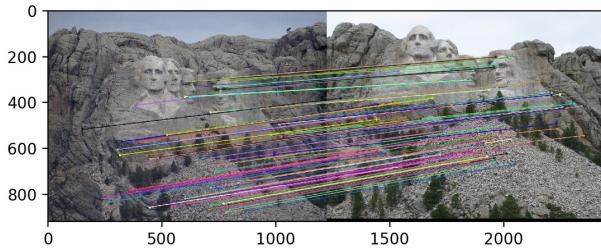


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## Project 3 / Camera Calibration and Fundamental Matrix Estimation with RANSAC

In this assignment, we focus on mapping the 3D world coordinates to image coordinates and estimating the translation for 2 images so that we achieve better accuracy in feature matching. The 3D world coordinates are mapped to image coordinates by estimating the camera projection matrix while the fundamental matrix relates the points in one image to epipolar lines in second image. We use RANSAC to estimate the fundamental matrix using features matched using Oriented FAST and Rotated BRIEF algorithm.



Example of features matched basis fundamental matrix

Following are the key steps:

1. Camera Projection Matrix Estimation
2. Fundamental Matrix Estimation
3. Fundamental Matrix Normalization
4. Estimation of Fundamental Matrix using RANSAC (for larger number of features)

We evaluate the Fundamental Matrix estimation for feature matching in 4 images provided. Additionally, we evaluate the Fundamental Matrix for images which have higher translation or higher proportion of incorrect feature matching by ORB. (Sacre Coeur, Statue of Liberty, Pantheon)

### Camera Projection Matrix Estimation

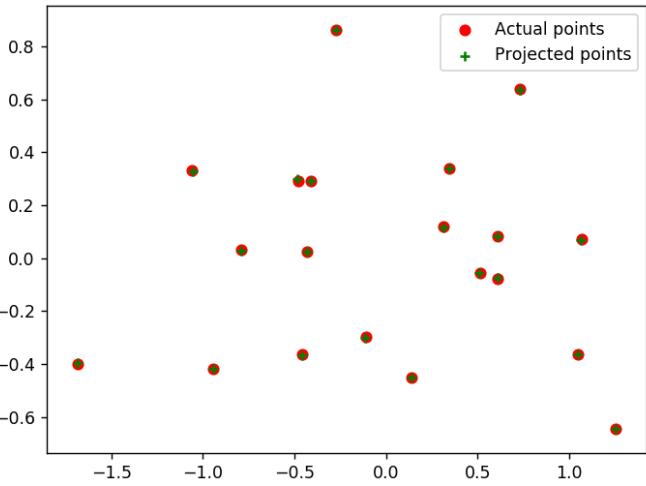
We utilize the following equation to model the relationship between 2D image coordinates and 3D camera coordinates. We set  $m34 = 1$  so that 0 is not the solution and then utilize linear least squares method(`np.linalg.lstsq()`) to solve for  $M$ .

#### Calculated Projection Matrix

```
The projection matrix is
[[ 0.76785834 -0.49384797 -0.02339781  0.00674445]
 [-0.0852134 -0.09146818 -0.90652332 -0.08775678]
 [ 0.18265016  0.29882917 -0.07419242  1.        ]]
The total residual is 0.044535
```

The total residual is very low which indicates that the formulation of camera projection matrix is correct. Moreover, the projected points are well aligned with the actual points.

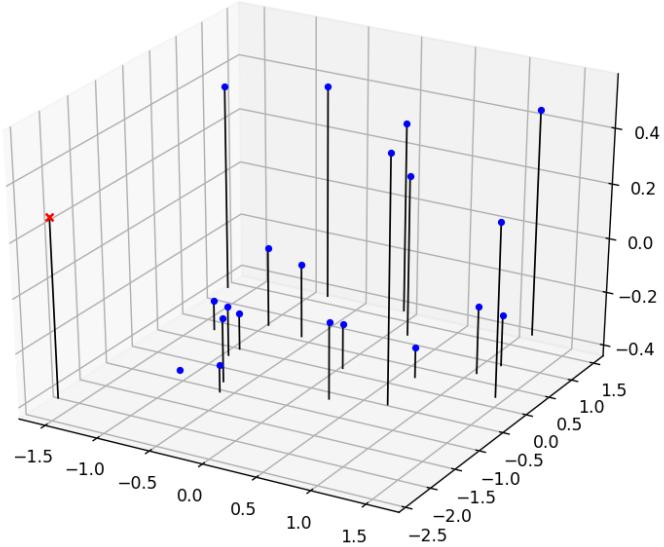
#### Actual vs Projected Points



After calculating the projection matrix ( $M$ ), we derive matrices  $Q$  &  $m_4$  and estimate the actual position of the camera using the relation:  
 $C = -Q^{-1}m_4$

## Camera Location

The estimated location of the camera is  $\langle -1.5126, -2.3517, 0.2827 \rangle$



## Fundamental Matrix Estimation (without normalization)

The fundamental matrix is computed using the following relationship between matches in images 1 and 2.  $f_{33}$  is set to 1 to avoid degenerate so and SVD is used to compute  $F$  ensuring that the rank of  $F = 2$

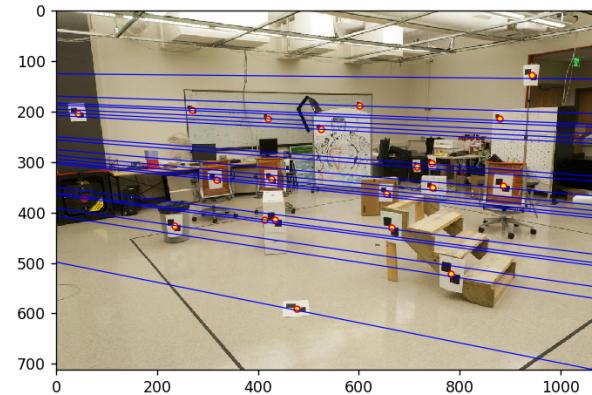
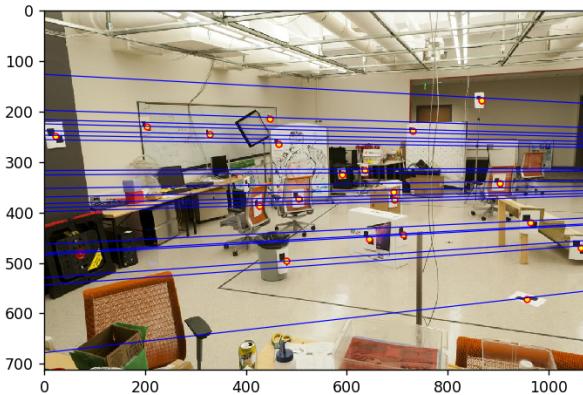
$$(f_{11}uu' + f_{12}vu' + f_{13}u' + f_{21}uv' + f_{22}vv' + f_{23}v' + f_{31}u + f_{32}v + f_{33}) = 0$$

## Calculated Fundamental Matrix

-0.000000	0.000008	-0.001886
0.000009	0.000001	0.017233
-0.000907	-0.026423	0.999500

## Epipolar Lines

Since, we haven't normalized the fundamental matrix, few points are not aligned with the lines (top-right, points towards the bottom)



## Fundamental Matrix Estimation (with normalization)

In this, we calculate the translation matrices  $T_b$  and  $T_a$  for normalizing points in the 2 images and subsequently adjust the fundamental matrix that it can operate on original images.

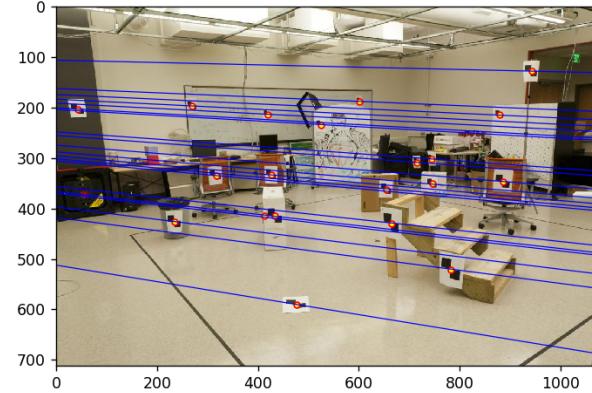
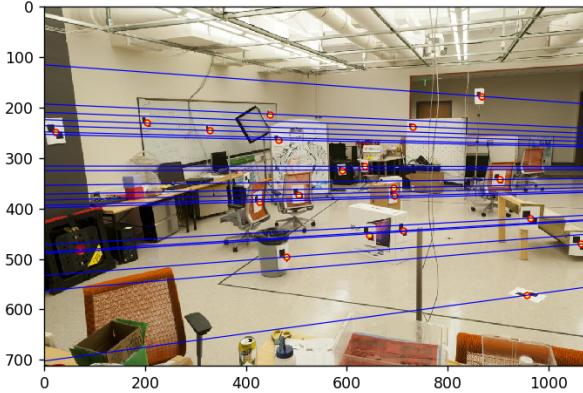
$$F_{orig} = T_b^T * F_{norm} * T_a$$

### Calculated Fundamental Matrix

-0.000000	0.000003	-0.000695
0.000002	-0.000000	0.005598
-0.000041	-0.007691	0.178588

### Epipolar Lines

After normalization, the lines pass through all of the points and there is very minor shift in some points.



There are multiple ways to determine the scale parameter(s) for normalizing the coordinates corresponding to features in the images to be mat

$$\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -c_u \\ 0 & 1 & -c_v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

In this case, we estimate the scale using the inverse of standard deviation of distances of coordinates from the origin(0,0) (after making the mean coordinates zero)

## Estimation of Fundamental Matrix using RANSAC

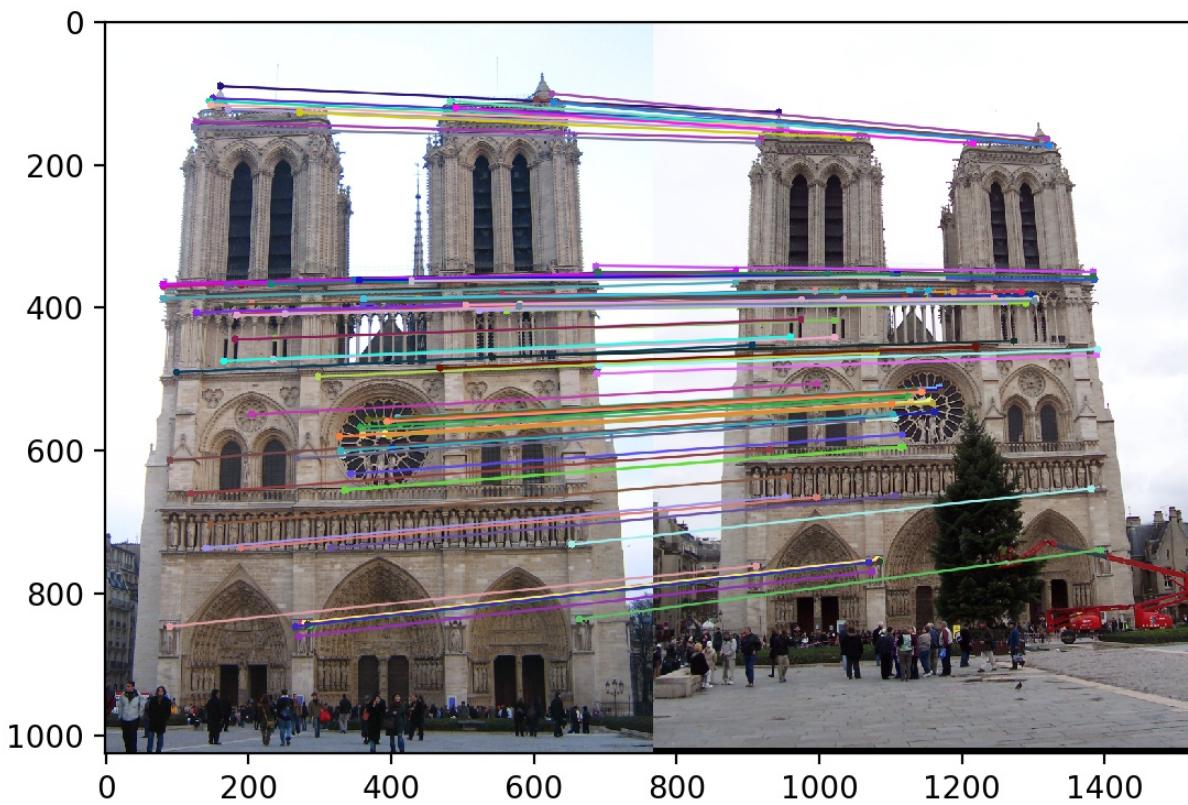
We utilize the 8 point algorithm for computing the fundamental matrix. I used an **adaptive approach** for deciding the **number of simulations**

- Estimate the fundamental matrix using a sample of points matched using ORB (sample 8 points)
- Estimate the number of simulations assuming an initial outlier percentage (generally kept around 80-90%) and the probability that at least one of the simulations has all inliers correct
- Compute the inliers using a threshold ( $\epsilon_i \leq \epsilon$ ) on the following equation  $m_b^T F^* m_a \leq \epsilon$
- If the percentage of inlier is higher than the maximum inlier count before current iteration, the store the inliers and sort them in increasing order of errors( $\epsilon_i$ )
- Update the simulation count basis the new outlier percentage if the new outlier percentage is lesser than previous outlier percentage
- Continue until the current simulation count is less than the required simulation count to ensure 'p' probability

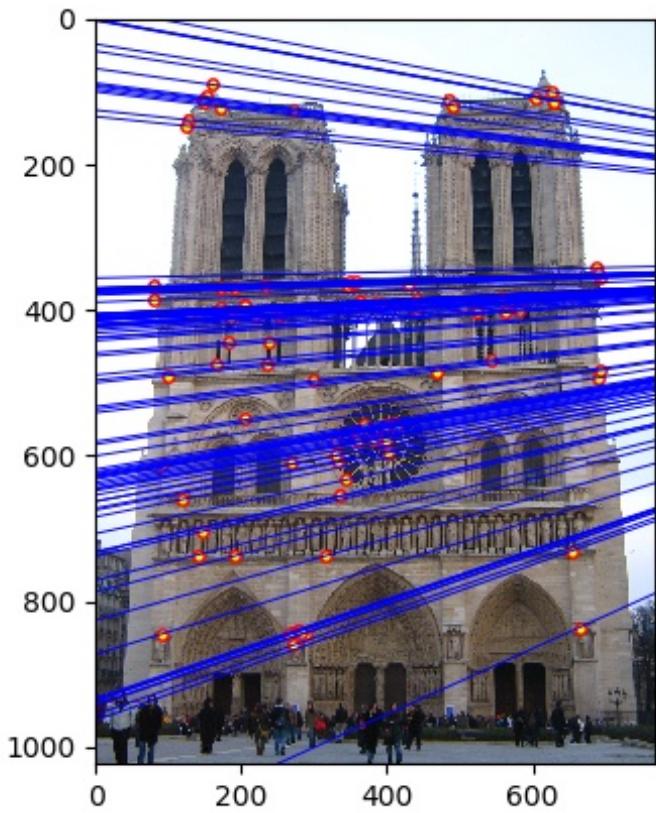
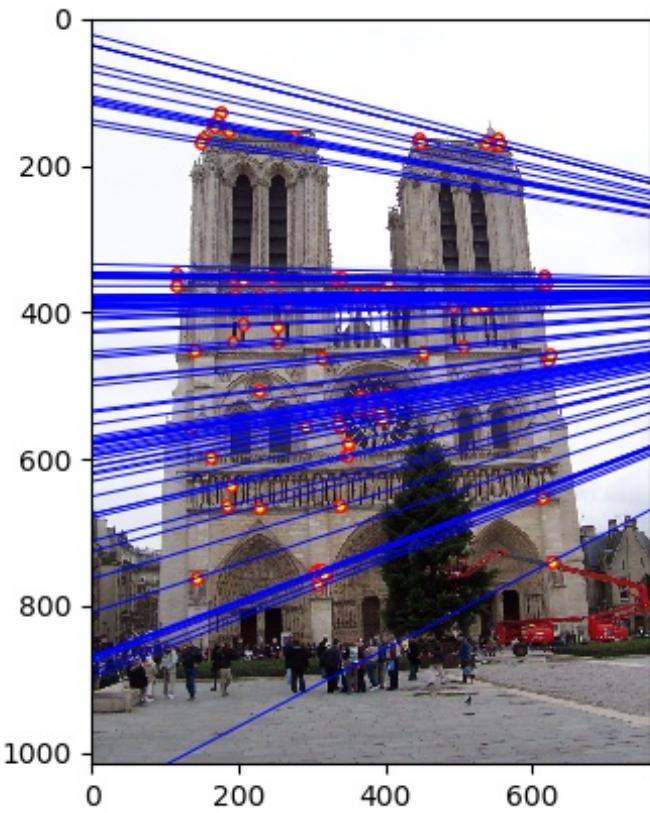
## Evaluation of Images provided

### Notre Dame

Total number of inliers = 556 (43%). Threshold = 0.007. Here, we show the best 100 matches and almost all matches seem to be correct.



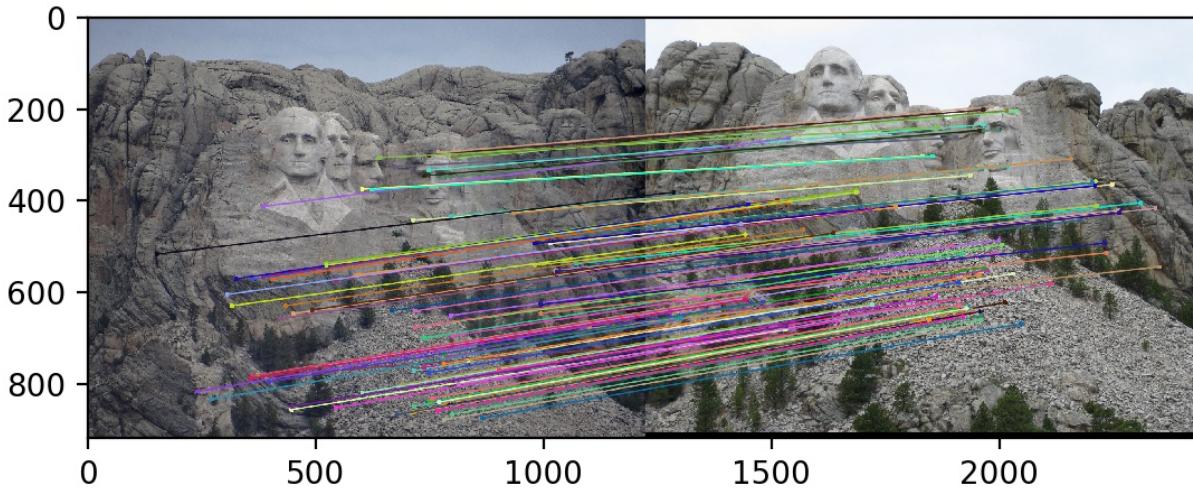
Epipolar Lines



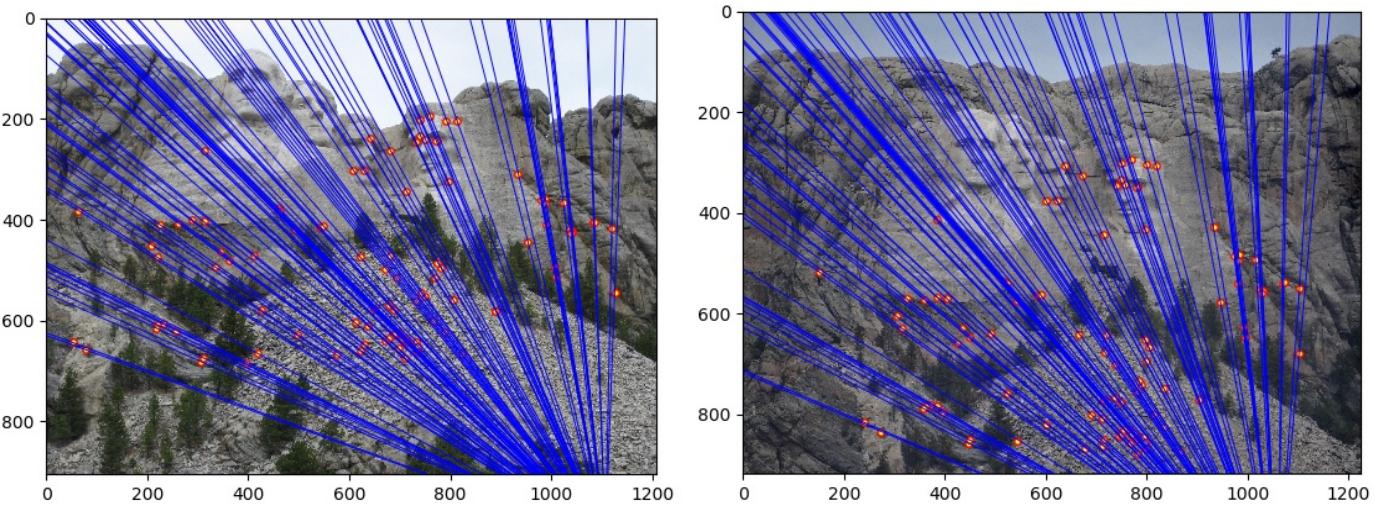
■

### Mount Rushmore

Total number of inliers = 290. Threshold = 0.009. Number of simulations = 7000. Here, we show the best 100 matches and 1 match seems to be incorrect.



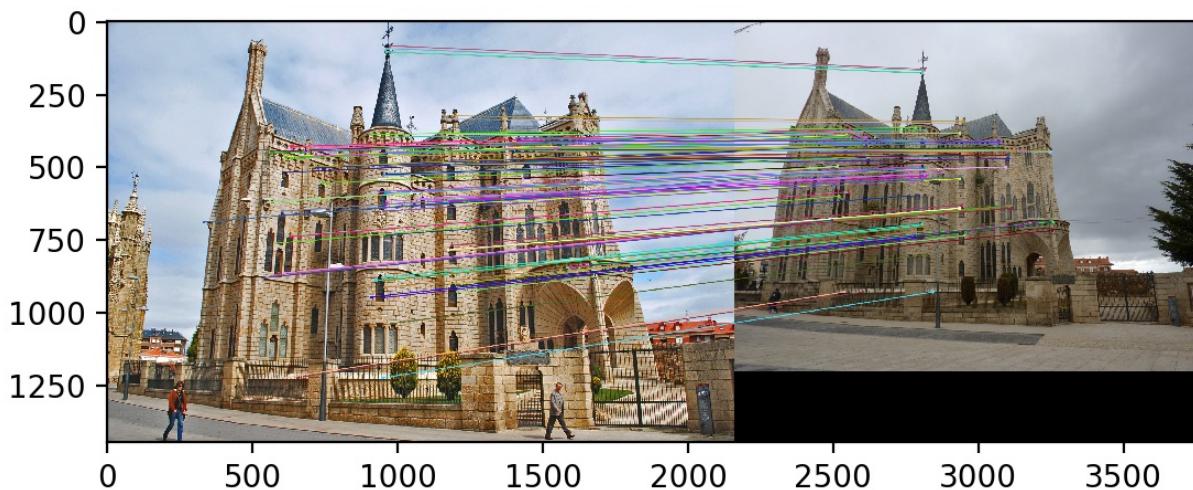
Epipolar Lines



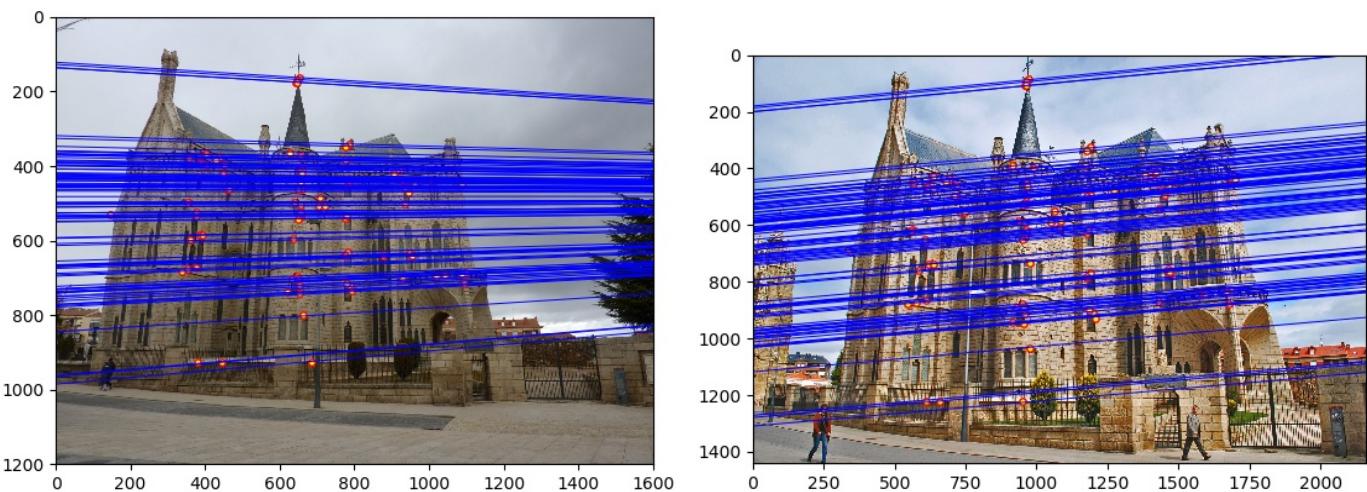
■

Episcopal Gaudi

Total number of inliers = 370 (36%). Threshold = 0.009. Here, we show the best 100 matches and almost all matches seem to be correct.



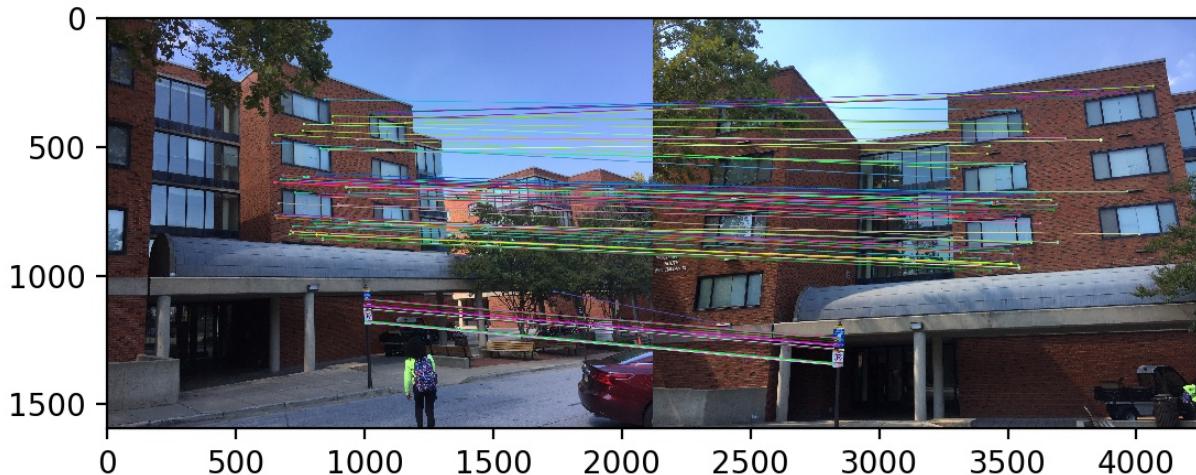
Epipolar Lines



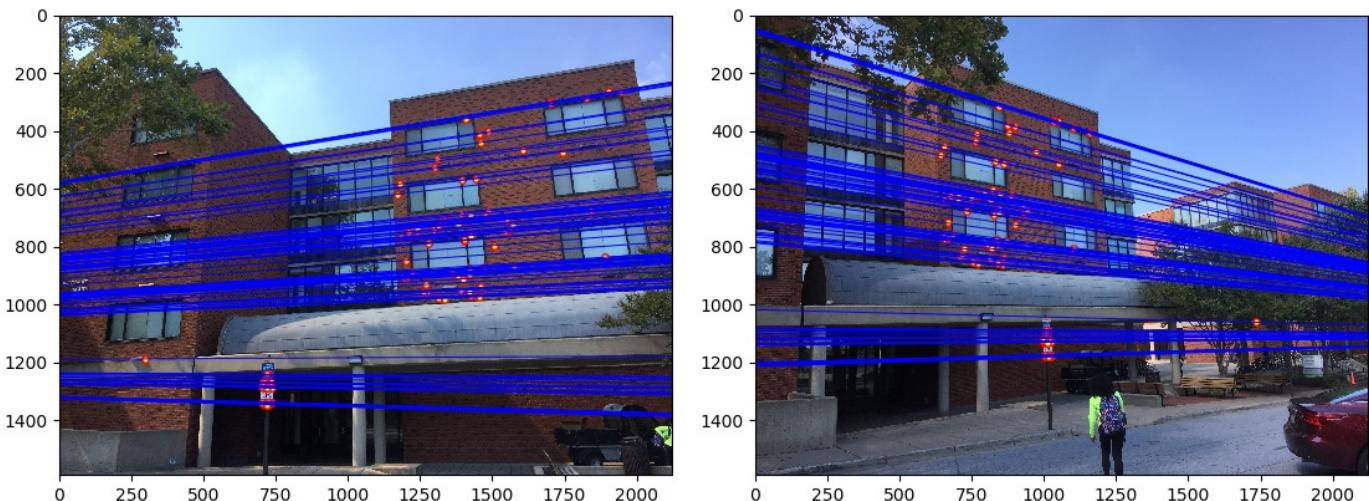
■

Woodruff

Total number of inliers = 340 (30%). Threshold = 0.007. Here, we show the best 100 matches and 1 match on the bottom side of the image seen be incorrect.

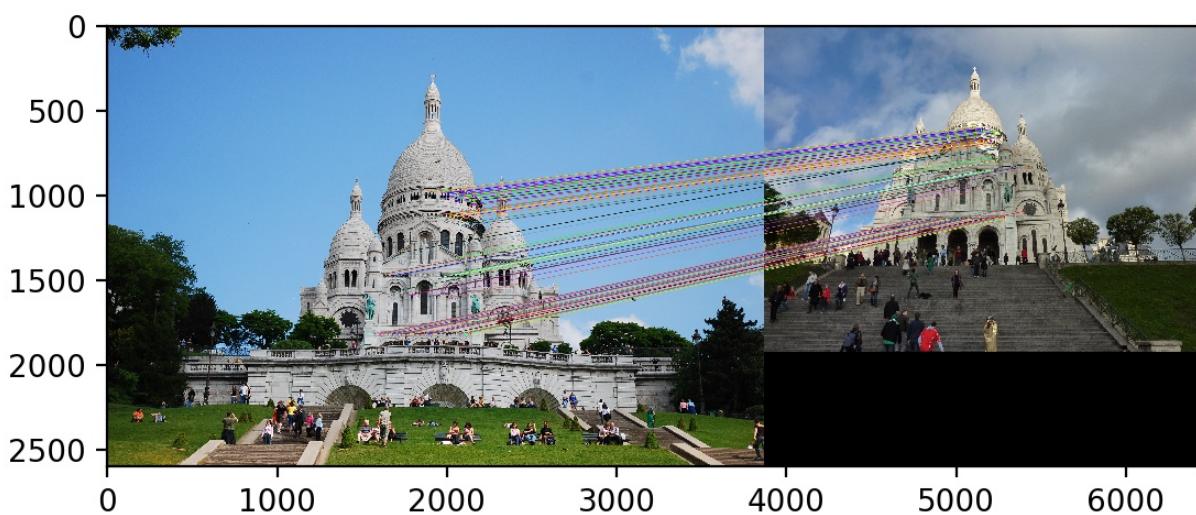


Epipolar Lines

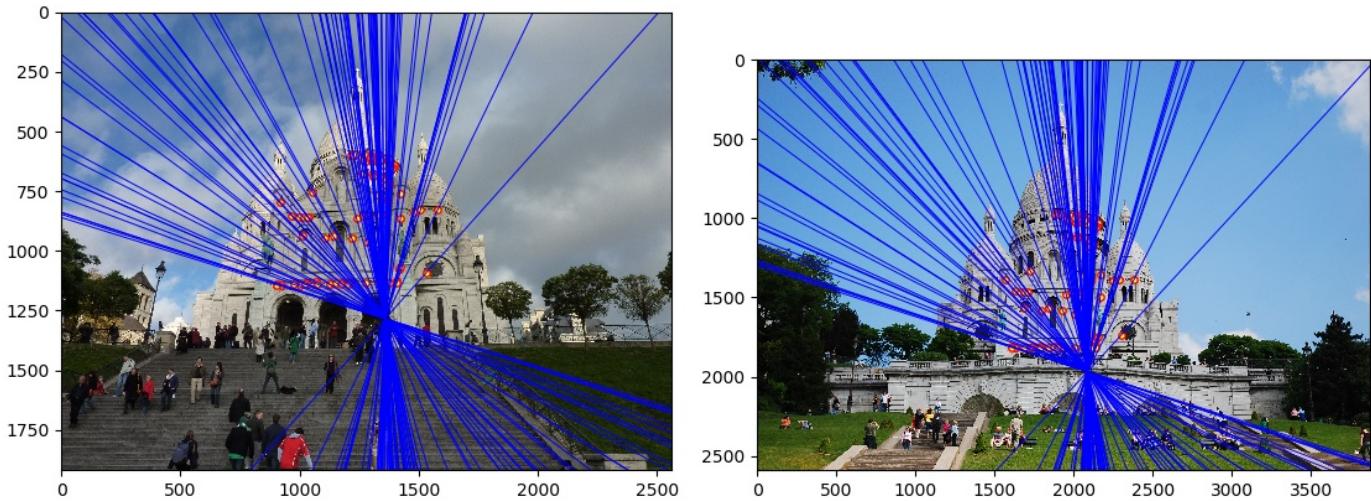


Sacre Coeur

Total number of inliers = 243 (40%). Threshold = 0.003. Here, we show the best 100 matches and almost all the matches seem to be correct.



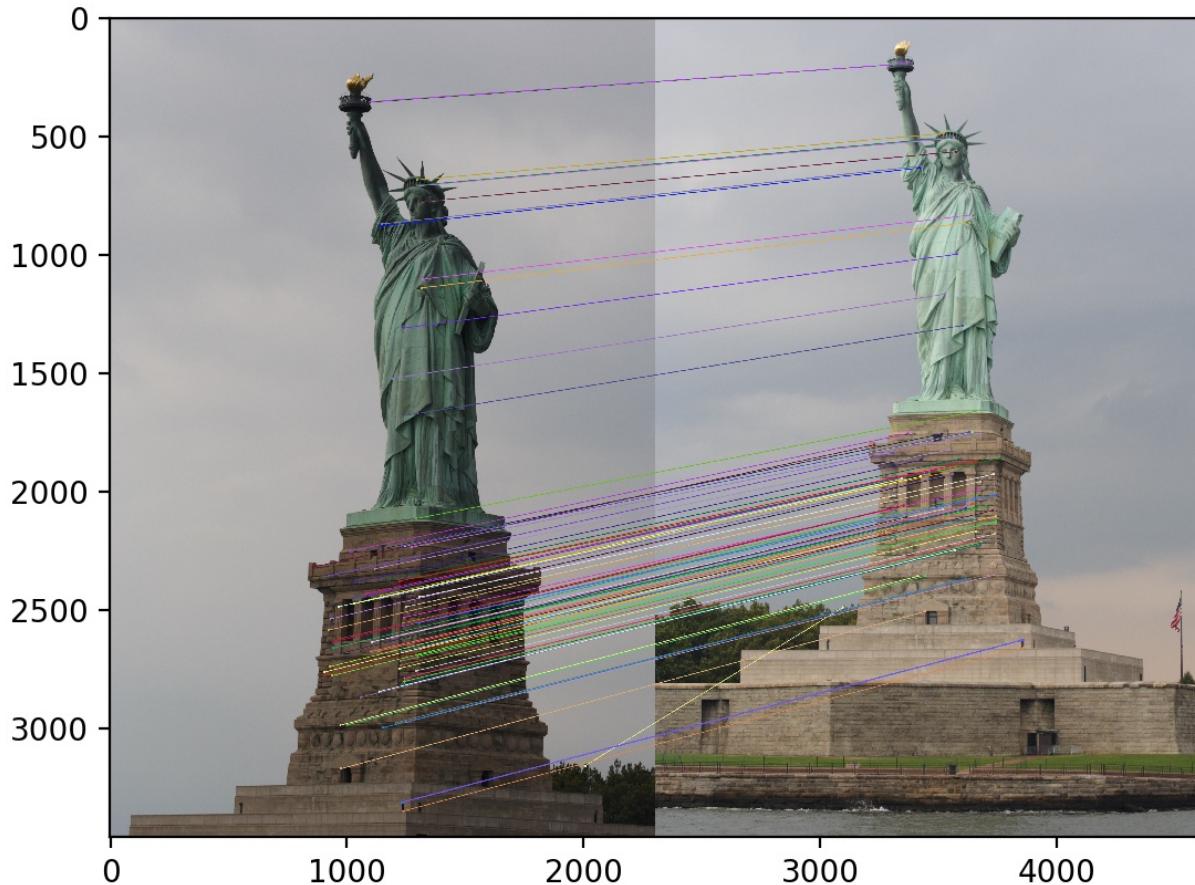
## Epipolar Lines



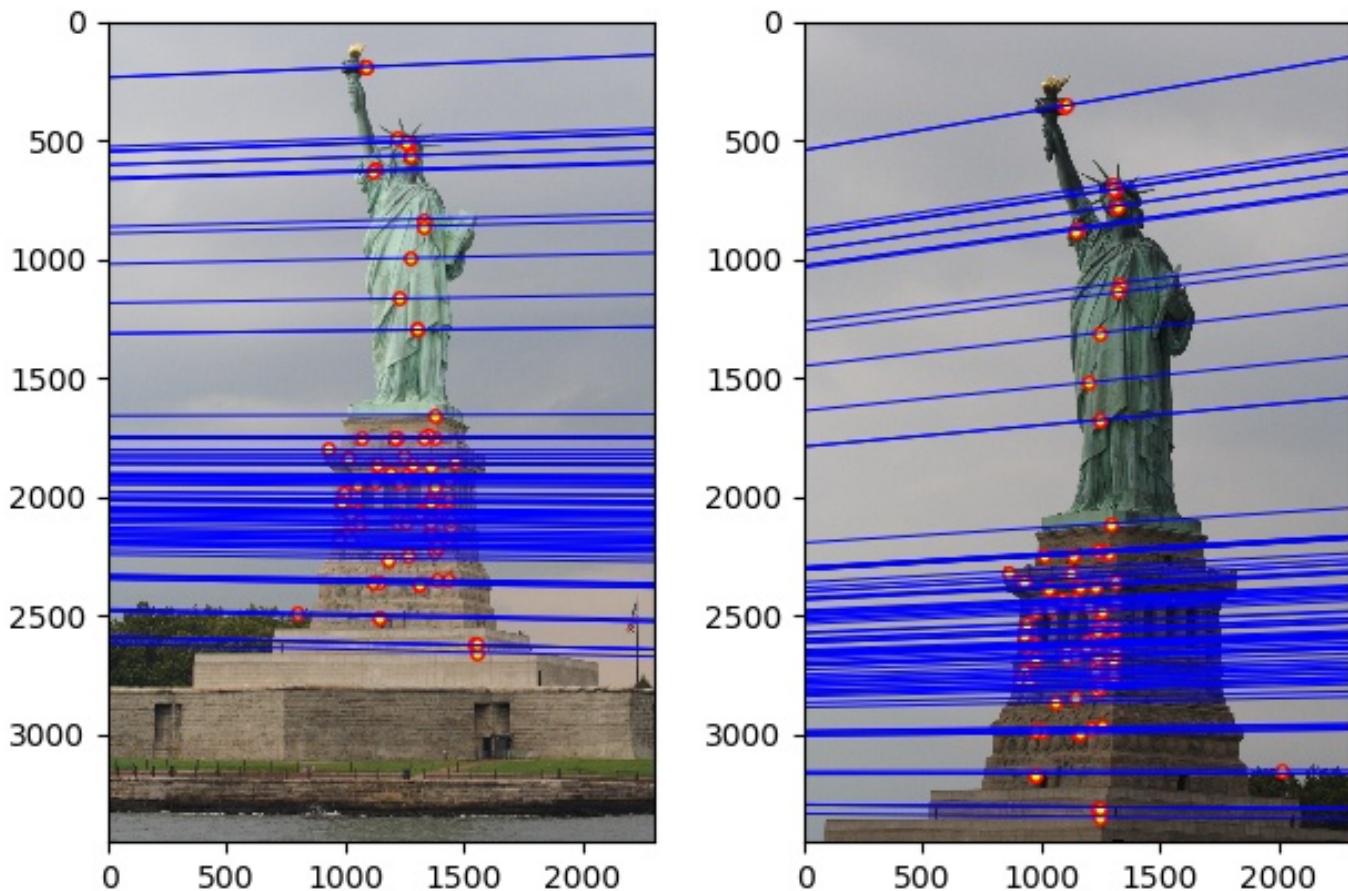
■

## Statue of Liberty

Total number of inliers = 156 (18%). Threshold = 0.003. Here, we show the best 100 matches and almost all the matches seem to be correct incl the matching of the trees behind the statue



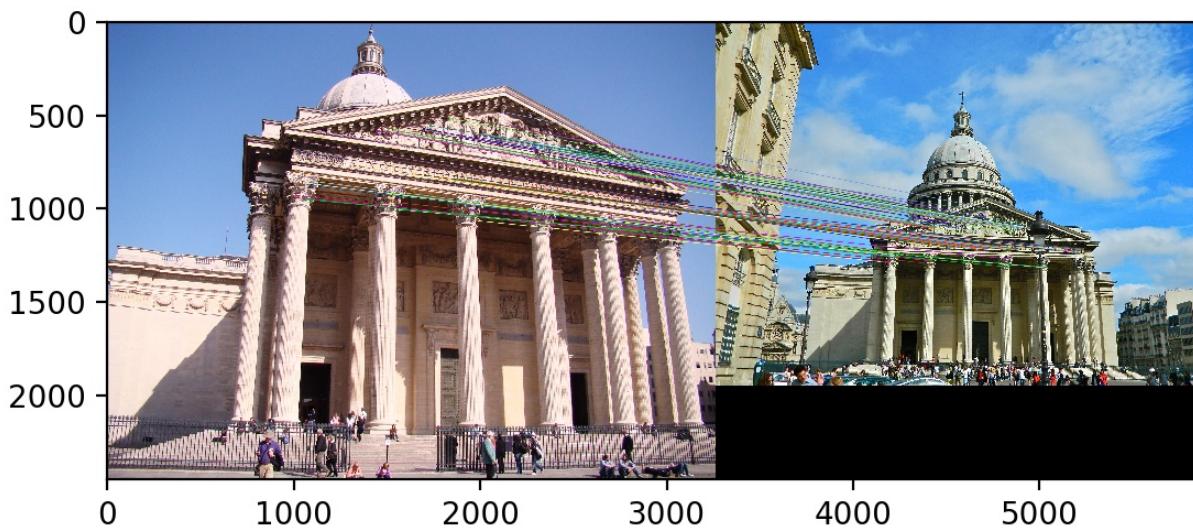
## Epipolar Lines



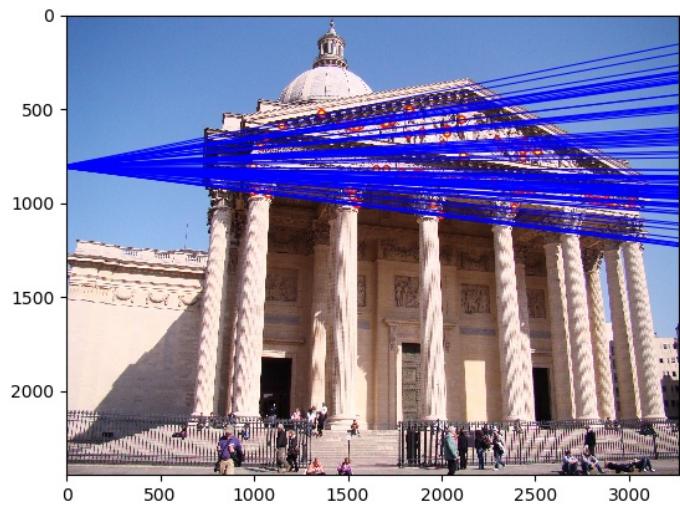
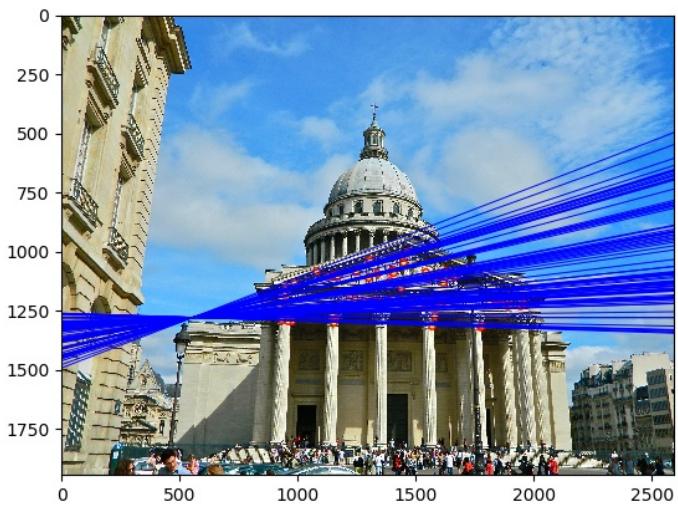
■

## Pantheon

Total number of inliers = 684 (34%). Threshold = 0.009. Here, we show the best 100 matches and almost all the matches seem to be correct. The matches rely extensively on the intricate geometry on the top of Pantheon



Epipolar Lines



■

## Conclusion

The fundamental matrix gives pretty good results in almost all images while adaptively determining the number of samples in RANSAC leads to consistent results in feature matching.