Fundamentals of Mathematics

Course Code: 23B31MA111

Lecture-1

CO1

Module: Sets, Relations and Functions

Contents to be Covered

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References

1. Mathematics Textbook for Class XI, NCERT, 2019.

Introduction

- Concepts of set theory is being used in almost every branch of mathematics.

- Sets are used to define the concepts of relations and functions. The study of geometry, sequences, probability, etc. requires the knowledge of sets.

Sets and their Representations

A set is a well-defined collection of objects

(i) Even natural numbers less than 10, i.e., 2, 4, 6, 8

(ii) The rivers of India

(iii) The vowels in the English alphabet, namely, *a*, *e*, *i*, *o*, *u*

(iv) Prime factors of 210, *viz*. 2,3,5 and 7

(v) The solution of the equation: $x^2 - 5x + 6 = 0$, *viz*, *2 and 3*

We note that each of the above example is a well defined collection

Note:

- The collection of tall persons is not a set.
- The collection of good batsman of India is not a set.

We give below a few more examples of sets used particularly in mathematics, viz.

N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

$\mathbf{Z}^+$: the set of positive integers

$\mathbf{Q}^+$: the set of positive rational numbers, and

$\mathbf{R}^+$: the set of positive real numbers.

The symbols for the special sets given above will be referred to throughout this text.

Representation of Sets

There are two methods of representing a set :

(i) Roster or tabular form

(ii) Set-builder form.

In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}. Some more examples of representing a set in roster form are given below :

(a) The set of all natural numbers which divide 42 is {1, 2, 3, 6, 7, 14, 21, 42}.

- We note that in roster form, the *order in which the elements are listed is immaterial*.

  Thus, the above set can also be represented as

  $\{1, 3, 7, 21, 2, 6, 14, 42\}$.

- It may also be noted that while writing the set in roster form an element is *not generally repeated*, i.e., all the elements are taken as distinct. For example, the set of letters forming the word 'SCHOOL' is { S, C, H, O, L} or {H, O, L, C, S}. Here,

  the order of listing elements has no relevance.

Set-builder form i.e., $\{x : P(x) \text{ is true}\}$

Ex: $A = \{x : x \epsilon N, x < 4\} = \{1,2,3\}$

(ii) In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set $\{a, e, i, o, u\}$, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write

$V = \{x : x \text{ is a vowel in English alphabet}\}$

$A = \{x : x$ is a natural number and $3 < x < 10\}$ is read as "the set of all x such that x is a natural number and x lies between 3 and 10. Hence, the numbers 4, 5, 6, 7, 8 and 9 are the elements of the set A.

Example 1 Write the solution set of the equation $x^2 + x - 2 = 0$ in roster form.

Example 2 Write the set $\{x : x$ is a positive integer and $x^2 < 40\}$ in the roster form.

Example 3 Write the set $A = \{1, 4, 9, 16, 25, \ldots\}$ in set-builder form.

Example 4 Write the set $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\}$ in the set-builder form.

Example 5 Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form :

(i)	{P, R, I, N, C, A, L}	(a) $\{ x : x$ is a positive integer and is a divisor of $18\}$
(ii)	$\{ 0 \}$	(b) $\{ x : x$ is an integer and $x^2 - 9 = 0\}$
(iii)	$\{1, 2, 3, 6, 9, 18\}$	(c) $\{x : x$ is an integer and $x + 1 = 1\}$
(iv)	$\{3, -3\}$	(d) $\{x : x$ is a letter of the word PRINCIPAL$\}$

Solution The given equation can be written as

$$(x-1)\ (x+2)=0, \text{ i. e., } \quad x=1,-2$$

Therefore, the solution set of the given equation can be written in roster form as $\{1, -2\}$.

Solution The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is $\{1, 2, 3, 4, 5, 6\}$.

Solution We may write the set A as

$$A = \{x : x \text{ is the square of a natural number}\}$$

Alternatively, we can write

$$A = \{x : x = n^2, \text{ where } n \in \mathbf{N}\}$$

Solution We see that each member in the given set has the numerator one less than the denominator. Also, the numerator begin from 1 and do not exceed 6. Hence, in the set-builder form the given set is

$$\left\{x : x = \frac{n}{n+1}, \text{ where } n \text{ is a natural number and } 1 \leq n \leq 6\right\}$$

Solution Since in (d), there are 9 letters in the word PRINCIPAL and two letters P and I are repeated, so (i) matches (d). Similarly, (ii) matches (c) as $x + 1 = 1$ implies $x = 0$. Also, 1, 2 ,3, 6, 9, 18 are all divisors of 18 and so (iii) matches (a). Finally, $x^2 - 9 = 0$ implies $x = 3, -3$ and so (iv) matches (b).

TYPES OF SETS

The Empty Set

A set which does not contain any element is called the empty set or the null set or the void set.

Examples:

(i) Let A = $\{x : 1 < x < 2, x$ is a natural number$\}$. Then A is the empty set, because there is no natural number between 1 and 2.

(ii) B = $\{x : x^2 - 2 = 0$ and x is rational number$\}$. Then B is the empty set because the equation $x^2 - 2 = 0$ is not satisfied by any rational value of x.

(iii) C = $\{x : x$ is an even prime number greater than 2$\}$. Then C is the empty set, because 2 is the only even prime number.

(iv) D = $\{ x : x^2 = 4, x$ is odd $\}$. Then D is the empty set, because the equation $x^2 = 4$ is not satisfied by any odd value of x.

Finite and Infinite Sets

- ***Definition 1:*** ***A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.***

- ***Definition 2:*** By number of elements of a set S, we mean the *number of distinct elements of the set called as cardinality of the set* and we denote it by *n(S)*.

If n(S) is a natural number, then S is non-empty finite set.

Consider some examples :

(i) Let W be the set of the days of the week. Then W is finite.

(ii) Let S be the set of solutions of the equation $x^2 - 16 = 0$. Then S is finite.

(iii) Let G be the set of points on a line. Then G is infinite.

Ques: State which of the following sets are finite or infinite :

(i) $\{x : x \in \text{N and } (x - 1)(x - 2) = 0\}$

(ii) $\{x : x \in \text{N and } x^2 = 4\}$

(iii) $\{x : x \in \text{N and } 2x - 1 = 0\}$

(iv) $\{x : x \in \text{N and } x \text{ is prime}\}$

Solution:

(i) Given set = $\{1, 2\}$. Hence, it is finite.

(ii) Given set = $\{2\}$. Hence, it is finite.

(iii) Given set = ϕ. Hence, it is finite.

(iv) The given set is the set of all prime numbers and since set of prime numbers is infinite. Hence the given set is infinite

Remark: In the roster form, it is not possible to write all the elements of an infinite set within braces { } because the numbers of elements of such a set is not finite.

So, we represent some infinite set in the roster form by writing a few elements which clearly indicate the structure of the set followed (or preceded) by three dots.

For example, {1, 2, 3 . . .} is the set of natural numbers, {1, 3, 5, 7, . . .} is the set of odd natural numbers, {. . .,–3, –2, –1, 0,1, 2 ,3, . . .} is the set of integers.

However, *all infinite sets cannot be described in the roster form*. *For example*, the set of real numbers cannot be described in this form, because the elements of this set do not follow any particular pattern.

Equal Sets

- ***Two sets A and B are said to be equal if they have exactly the same elements i.e.,*** if every element of A is also an element of B and if every element of B is also an element of A ***and we write A = B.***

Otherwise, the sets are said to be unequal and we write A ≠B.

Examples:

(i) Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.

(ii) Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and also these are less than 6.

➢ A set does not change if one or more elements of the set are repeated.

For example, the sets A = {1, 2, 3} and B = {2, 2, 1, 3, 3} are equal, since each element of A is in B and vice-versa. That is why we generally do not repeat any element in describing a set.

Ex: Find the pairs of equal sets, if any, give reasons:

$A = \{0\}$, $B = \{x : x > 15 \text{ and } x < 5\}$,

$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$,

$E = \{x : x \text{ is an integral positive root of the equation } x^2 - 2x - 15 = 0\}$.

Solution Since $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that, $A \neq B$, $A \neq C$, $A \neq D$, $A \neq E$.

Since $B = \phi$ but none of the other sets are empty. Therefore $B \neq C$, $B \neq D$ and $B \neq E$. Also $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.

Since $E = \{5\}$, $C = E$. Further, $D = \{-5, 5\}$ and $E = \{5\}$, we find that, $D \neq E$. Thus, the only pair of equal sets is C and E.

Thank You..