COMP 3540 Theory of Computation - Fall 2020

ASSIGNMENT 2

Name: KEERTHANA MADHAVAN

Student ID: [redacted]

"Students of the University of Windsor pursue all endeavours with honour and integrity, and will not tolerate or engage in academic or personal dishonesty."

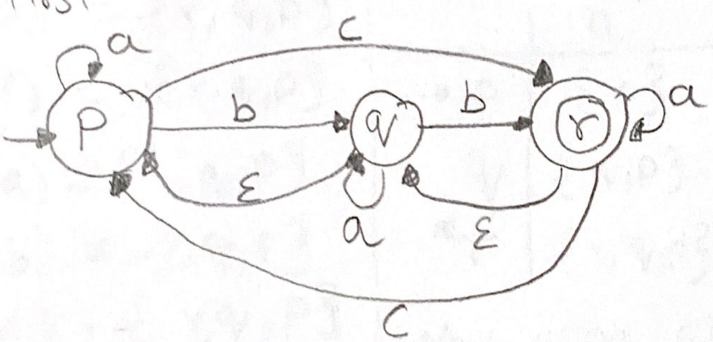
Questions start from second page.

1.) Given the following ε-NFA

	ε	a	b	c
→p	∅	{p}	{q}	{r}
q	{p}	{q}	{r}	∅
*r	{q}	{r}	∅	{p}

a.) Compute the ε-closure of each state.

→ First let's convert the table to ε-NFA Diagram



So, for ε-closure of each state, the closure for p is just {p}, for q is {p, q} and for r is {p, q, r}

	ε-Closure
→ p	{p}
q	{q, p}
*r	{r, q, p}

b.) Convert it DFA. Provide all the details and steps in the conversion.

First we have to convert the εNFA to NFA transition table. We will do this using this e-closure Formula $\hat{\delta}(q, x) = \varepsilon\text{closure}(\delta(\varepsilon\text{-closure}(q), x))$

$$\hat{\delta}(p,a) = \varepsilon\text{-closure}(\delta[\varepsilon\text{-closure}(p), a])$$
$$= \varepsilon\text{-closure}\ [p]$$
$$\{p\}$$

* we do this for each state a,b,c

$$\hat{\delta}(p,b) = \varepsilon\text{-closure}(\delta[\varepsilon\text{-closure}(p), b])$$
$$= \varepsilon\text{-closure}\ (q)$$
$$= \{q,p\}$$

$\delta(p,c) = \{r,q,p\}$

$\delta(q,a) = \{q,p\}$

$\delta(q,b) = \{r,q,p\}$

$\delta(q,c) = \{r,q,p\}$

$\delta(r,a) = \{r,q,p\}$

$\delta(r,b) = \{r,q,p\}$

$\delta(r,c) = \{r,q,p\}$

Now lets make the transition NFA table out these ε-closure.

NFA Transition Table

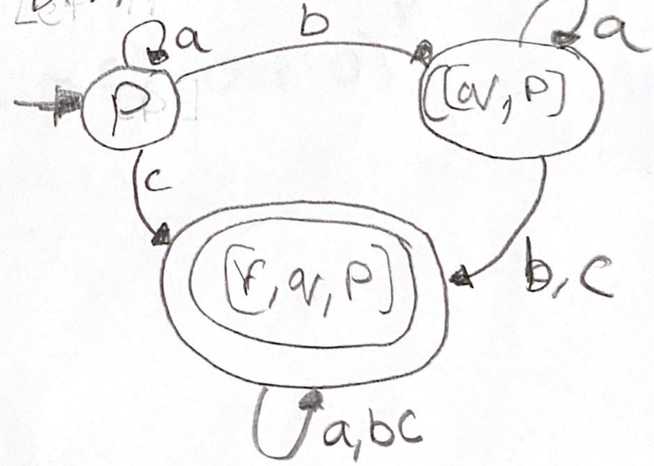
	a	b	c
→ p	{p}	{q,p}	{r,q,p}
q	{q,p}	{r,q,p}	{r,q,p}
*r	{r,q,p}	{r,q,p}	{r,q,p}

Now, using subset construction, we can create a DFA transition table. From that the DFA diagram can be built.

DFA Transition Table.

	a	b	c
→ [p]	[p]	[q,p]	[r,q,p]
[q,p]	[q,p]	[r,q,p]	[r,q,p]
*[r,q,p]	[r,q,p]	[r,q,p]	[r,q,p]

DFA



2.) Give Regular expressions for the following ~~statements~~ languages.

a.) The set of strings over alphabet $\{a,b,c\}$ containing atleast one $\underline{a}$ and at least one $\underline{b}$.

$$(a+b+c)^*(a(a+b+c)^*b + b(a+b+c)^*a)(a+b+c)^*$$

b.) The set of strings of 0's and 1's whose tenth symbol from right end is 1.

RE $\rightarrow$ $(0+1)^* 1 (0+1)^9$

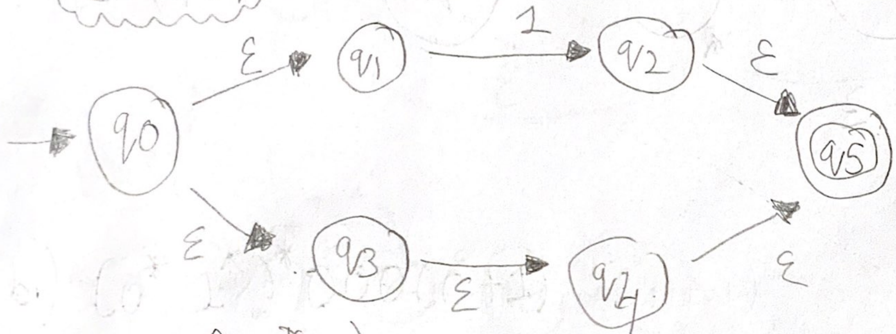
c.) The set of strings of 0's and 1's with at most one pair of consecutive 1's.

RE $\rightarrow$ $(0+10)^*(11+\varepsilon)(0+10)^*$

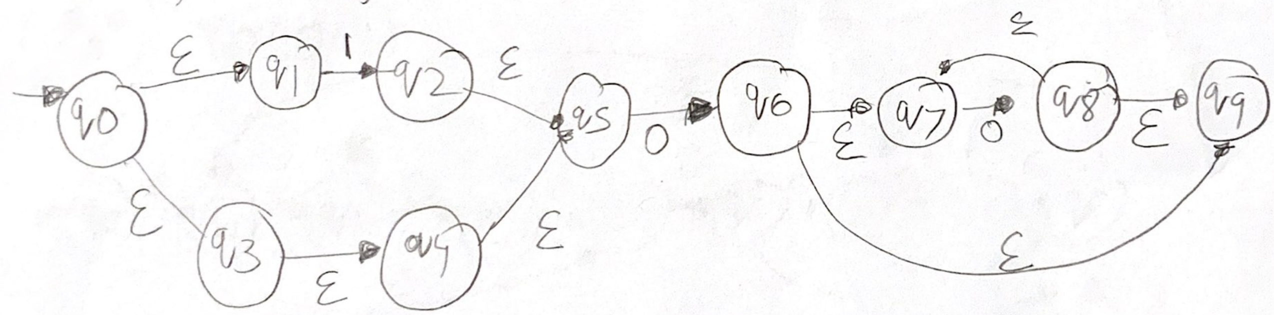
3.) Construct ε-NFAs for the following regular expressions?

a.) $(1+\varepsilon)(00^* 1)^* 0^*$

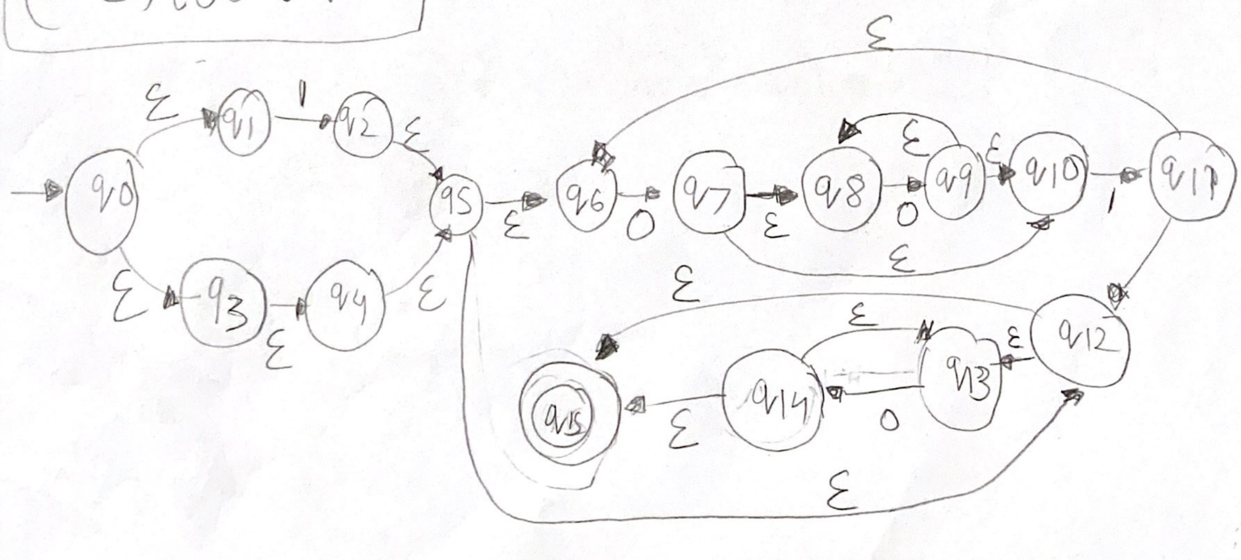
$(1+\varepsilon)$



$(1+\varepsilon)(00^* 1)$

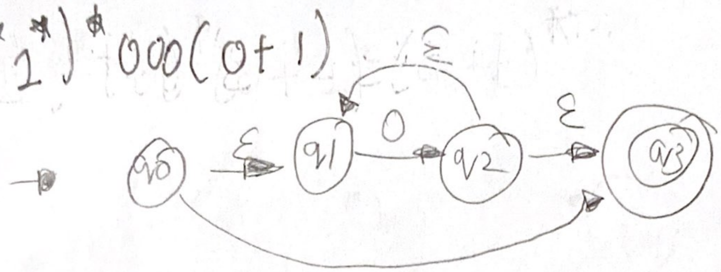


$\boxed{(1+\varepsilon)(00^* 1)^* 0^*}$

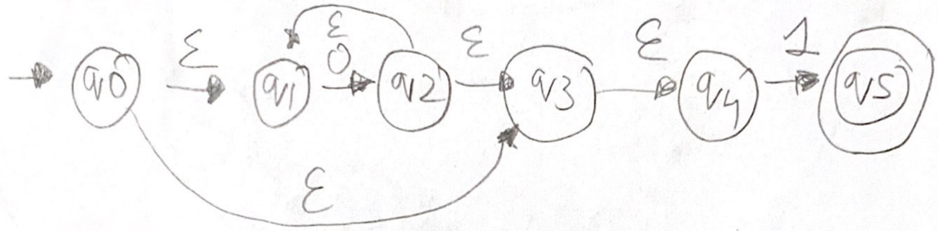


3b.) $(0^*1^*)^*000(0+1)$

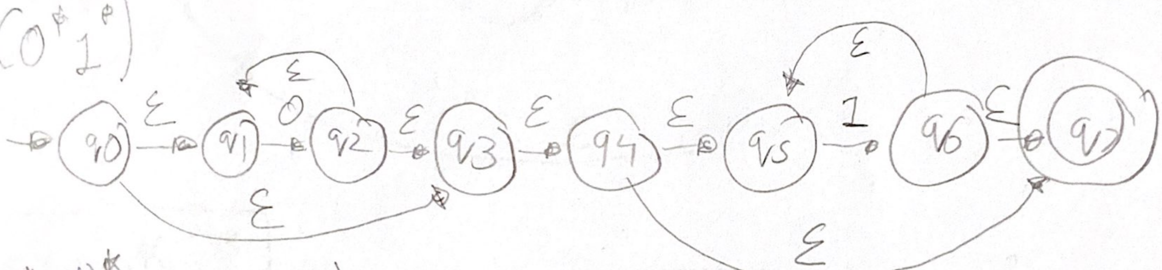
$\underline{0^*}$



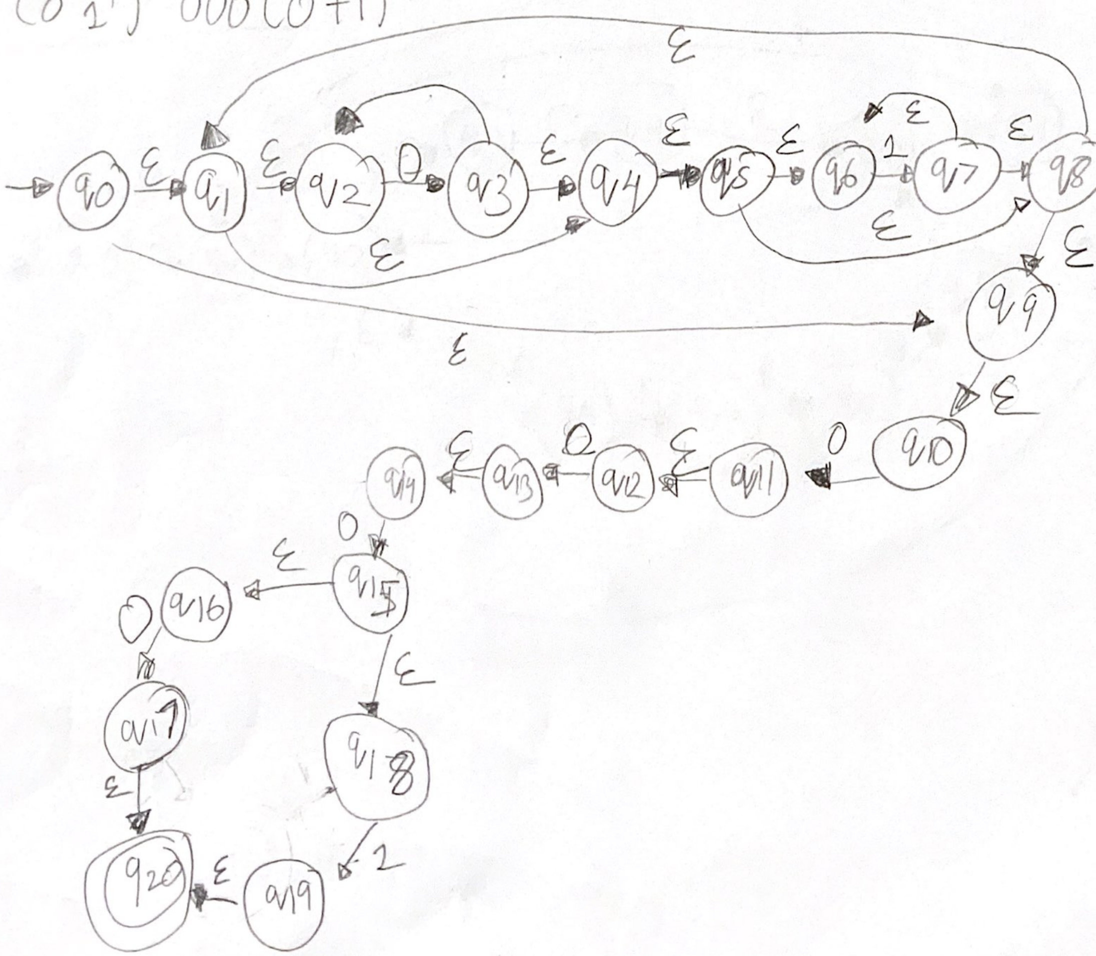
$\underline{0^*1}$



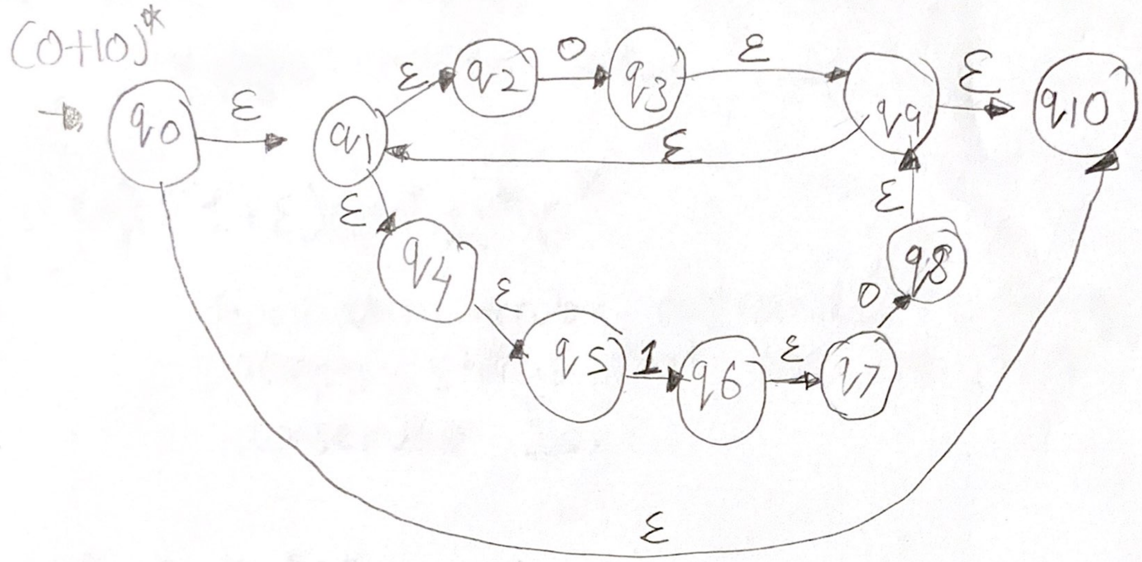
(0^*1^*)



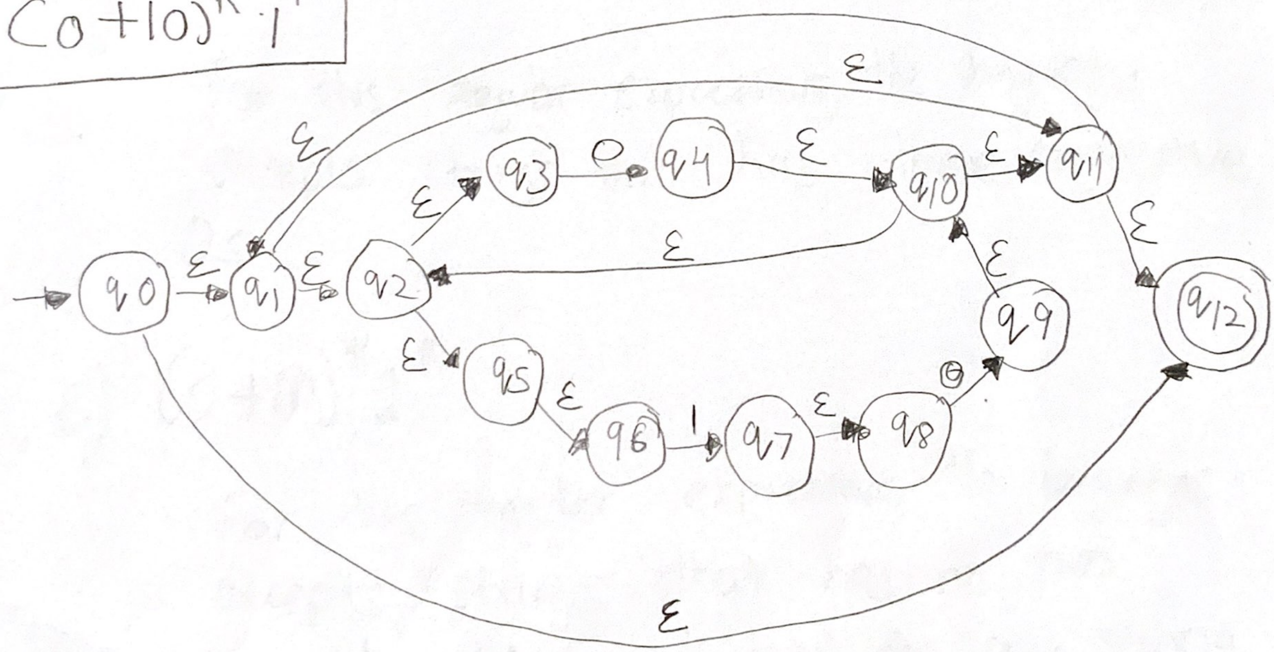
$(0^*1^*)^*000(0+1)$



3c.) $(0+10)^* 1^*$



$\boxed{(0+10)^* \cdot 1^*}$



4.) Give set former set definitions for the languages of the regular expressions of question 3.

a.) $(1+\varepsilon)(00^*1)^*0^*$

For this regular expressions, the language accepts strings that has no two consecutive 1s.

b.) $(0^*1^*)^*\underline{000}(0+1)$

For this regular expressions, the language accepts strings that has three consecutive 0s.

c.) $(0+10)^*1^*$

For this sregular expressions, the languages accepts strings that has no two consecutive 1s, and not for any string that may with 1s at the end.

5.) Consider the following DFA

a.) Give all regular expressions for $R_{ij}^{(0)}$, $R_{ij}^{(1)}$, $R_{ij}^{(2)}$, for all i & j

Simplify the expressions as much as you can.

Basis, By K=0

$$R_{11}^{(0)} = \varepsilon + 0$$
$$R_{12}^{(0)} = 1$$
$$R_{21}^{(0)} = 0$$
$$R_{22}^{(0)} = \varepsilon + 1$$

By K = 1

$$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)}$$
$$= (\varepsilon+0) + (\varepsilon+0)(\varepsilon+0)^*(\varepsilon+0)$$
$$= 0^*$$

$$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)}$$
$$= 1 + (\varepsilon+0)(\varepsilon+0)^* 1$$
$$= 0^* 1$$

$$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)}$$
$$= 0 + 0(\varepsilon+0)^*(\varepsilon+0)$$
$$= 0^{n+}$$

$$R_{22}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)}$$
$$= (\varepsilon+1) + 0(\varepsilon+0)^* 1$$
$$= (\varepsilon+1) + 0^{n+1}$$

By K=2

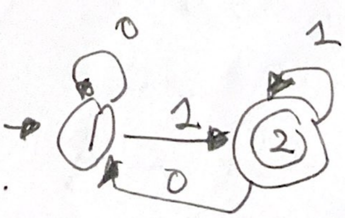
$$R_{11}^{(2)} = R_{11}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{21}^{(1)}$$
$$= 0^* + 0^*(\varepsilon+1+0^{n+1})0^{n+}$$

$$R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)}$$
$$= 0^*1 + 0^* 1(\varepsilon+1+0^{n+}1)^*(\varepsilon+1+0^{n+}1)$$
$$= 0^*1 + 0^*1(1+0^{n+2})^*(\varepsilon+1+0^{n+1})$$
$$= 0^*(1+1(1+0^{n+1})^*(\varepsilon+1+0^n+1))$$
$$= 0^*1(1+0^n+1)$$
$$= 0^*1(1+00^*1)$$

$$R_{21}^{(2)} = R_{21}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{21}^{(1)}$$
$$= 0^{n++}((\varepsilon+1)+0^n+1)((\varepsilon+1))^* R_{21}^{(1)}$$
$$= (0+((\varepsilon+1)+0^{n+1})((\varepsilon+1)+0^{n+1})^*0)0^*$$
$$= ((\varepsilon+1)+0^{n+1})((\varepsilon+1)+0^{n+1})^*00^*$$
$$= ((\varepsilon+1)+0^{n+1})^*00^*$$
$$= (1+0^{n+1})^*00^*$$
$$= (1+00^*1)^*00^*$$

$$R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)}$$
$$= ((\varepsilon+1)+0^{n+1}) + ((\varepsilon+1)+0^{n+1})((\varepsilon+1)+0^{n+1})^*((\varepsilon+1)+0^{n+1})$$
$$= 0^{n+1} + ((\varepsilon+1)+0^{n+1})((\varepsilon+1)+0^{n+1})^*((\varepsilon+1)+0^{n+1})$$
$$= ((\varepsilon+1)+0^n+1)^*$$
$$= (1+0^{n+1})^*$$

b.) Give a Regular expression for the DFA.



Final Regular Expression from $R_{12}^{(2)} = 0^*1(1+00^*1)$

6.) Suppose a, b, c are regular expressions. Prove or disapprove the following statements.

a.) $(a^*b)a^* = a^*(ba^*)$

(Direct Proof)

Let $E1 = (a^*b)a^*$

$E2 = (a^*(ba^*)$

$L(E1) = L((a^*b)a^*)$

$= L(a^*b)L(a^*)$ Induction 2 $L(E1E2) = L(E1)L(E2)$

$= L(a^*)L(b)L(a^*)$ Induction 2

$= L(a^*)L(ba^*)$ Induction 2 $L(E1)L(E2) = L(E1E2)$

$= L((a^*)(ba^*))$ Induction 2

$= L(E2)$

LHS = RHS, so the statement is correct

b.) $(\varepsilon + a+b)^*\varepsilon = \varepsilon(a+b)^*$

LHS $= (\varepsilon + a+b)^*\varepsilon$

$= (\varepsilon^*(a+b)^*)^*\varepsilon$

$= ((a+b)^*)^*\varepsilon$

$= (a+b)^*\varepsilon$

$= \varepsilon(a+b)^*$

LHS = RHS, so the statement is correct

c.) $a(b+c) = ab + ca$

Disproving this statement because concatenation is not commutative.

LHS $= a(b+c)$

$= ab + ac$

$\neq ab + ca$

hence, $ac = ca$ //

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