COMP 3540 THEORY OF COMPUTATION - FALL2020

Assignment 5

Pledge:

"As a student of the University of Windsor, I pledge to pursue all endeavours with honour and integrity, and will not tolerate or engage in academic or personal dishonesty. I confirm that I have not received ~~and will not tolerate or~~ any unauthorized assistance in preparing for or writing this assignment. I acknowledge that a mark of 0 may be assigned for copied work."

+ KEERTHANA MADHAVAN + [redacted]

1.) Given two languages $L_1 = \{a^n b^{2n} c^m \mid n, m \geq 0\}$

and $L_2 = \{a^n b^m c^{2m} \mid n, m \geq 0\}$

a.) show that each of these is context free (by giving grammars for each of them.

The language L_1 is defined as follows:

$L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$

Define the context free grammar that generates the language L_1 as follows:

$S \rightarrow AC \mid \varepsilon$

$A \rightarrow aAbb \mid \varepsilon$

$C \rightarrow cC \mid \varepsilon$

The language L_1 has context free grammar. Therefore, the language L_1 is a CFL.

The language L_2 is defined as follows:

$L_2 = \{a^n b^m c^{2m} \mid n, m \geq 0\}$

Define the context free grammar that generates the language L_2 as follows:

$S \rightarrow AB \mid \varepsilon$

$A \rightarrow Aa \mid \varepsilon$

$B \rightarrow bBcc \mid \varepsilon$

The language L_2 has context free grammar.

Therefore, the language L_2 is a CFL.

Thus, each of the language has context free grammar an are CFLs because $L_1 \cap L_2 = \{a^n b^{2n} c^{4n} \mid n \geq 0\}$

1.b.) Find $L = L_1 \cap L_2$. Is L a CFL? Give a proof for your answer.

We can use the theorem statement from the book → that CFL's are not closed under intersection.

$$L_1 \cap L_2 = \{a^n b^{2n} c^m \mid n, m \geq 0\}$$

$$w = a^m b^{2m} c^{4m}$$

By using, Pumping lemma, we can say the language is not context free grammar.

Therefore, language L is not context free.

Grammar:

2.) $S \to AB \mid BC$, use CYK

$A \to BA \mid a$

$B \to CC \mid b$

$C \to AB \mid a$

a.) **baaab**

input string $w = baaab$.

First step

$X_{11} = \{B\}$ $X_{12} = \{AC\}$ $X_{33} = \{A, C\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

b a a a b

Second step

$X_{12} = \{S, A\}$ $X_{23} = \{C, S\}$ $X_{34} = \{B\}$ $X_{45} = \{S, C\}$

$X_{11} = \{B\}$ $X_{22} = \{A, C\}$ $X_{33} = \{A, C\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

b a a a b

Third step

$X_{13} = \{\}$ $X_{24} = \{C, S\}$ $X_{35} = \{B\}$

$X_{12} = \{S, A\}$ $X_{23} = \{A, C, S\}$ $X_{34} = \{B\}$ $X_{45} = \{S, C\}$

$X_{11} = \{B\}$ $X_{22} = \{A, C\}$ $X_{33} = \{A, C\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

b a a a b

Fourth step:

$X_{14} = \{A, C, S\}$ $X_{25} = \{C, S\}$ $X_{35} = \{B\}$

$X_{13} = \{\}$ $X_{24} = \{A, C, S\}$ $X_{34} = \{B\}$ $X_{45} = \{S, C\}$

$X_{12} = \{S, A\}$ $X_{23} = \{B\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

$X_{11} = \{B\}$ $X_{22} = \{A, C\}$ $X_{33} = \{A, C\}$

b a a a b

The Final table.

$X_{15} = \{C, S\}$

$X_{14} = \{A, C, S\}$ $X_{25} = \{C, S\}$

$X_{13} = \{\}$ $X_{24} = \{A, C, S\}$ $X_{35} = \{B\}$

$X_{12} = \{S, A\}$ $X_{23} = \{B\}$ $X_{34} = \{B\}$ $X_{45} = \{S, C\}$

$X_{11} = \{B\}$ $X_{22} = \{A, C\}$ $X_{33} = \{A, C\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

b a a a b

The string baaab is accepted by the CYK algorith in L(G).

b.) aabab

First step

$X_{11} = \{A C\}$ $X_{22} = \{A, C\}$ $X_{33} = \{B\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

a a b a b

second step.

$X_{12} = \{B\}$ $X_{23} = \{S, C\}$ $X_{34} = \{S, A\}$ $X_{45} = \{S, C\}$

$X_{11} = \{A, C\}$ $X_{22} = \{A, C\}$ $X_{33} = \{B\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

a a b a b

Third step

$X_{13} = \{B\}$ $X_{24} = \{B\}$ $X_{35} = \{C, S\}$

$X_{12} = \{B\}$ $X_{23} = \{S, C\}$ $X_{34} = \{S, A\}$ $X_{45} = \{S, C\}$

$X_{11} = \{A, C\}$ $X_{22} = \{A, C\}$ $X_{33} = \{B\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

a a b a b

Fourth Step

$X_{14} = \{S, C, A\}$ $X_{25} = \{B\}$

$X_{13} = \{B\}$ $X_{24} = \{B\}$ $X_{35} = \{C, S\}$

$X_{12} = \{B\}$ $X_{23} = \{S, C\}$ $X_{34} = \{S, A\}$ $X_{45} = \{S, C\}$

$X_{11} = \{A, C\}$ $X_{22} = \{A, C\}$ $X_{33} = \{B\}$ $X_{44} = \{A, C\}$ $X_{55} = \{B\}$

a a b a b

The Final Table

$X_{15} = \{C, S\}$				
$X_{14} = \{S, C, A\}$	$X_{25} = \{B\}$			
$X_{13} = \{B\}$	$X_{24} = \{B\}$	$X_{35} = \{C, S\}$		
$X_{12} = \{B\}$	$X_{23} = \{S, C\}$	$X_{34} = \{S, A\}$	$X_{45} = \{S, C\}$	
$X_{11} = \{A, C\}$	$X_{22} = \{A, C\}$	$X_{33} = \{B\}$	$X_{44} = \{A, C\}$	$X_{55} = \{B\}$
a	a	b	a	b

∴ The string aabab is accepted by the CYK algorithm in $L(G)$.

3.) Consider Language $L = \{ ww^R \mid w$ is any string of 0's and 1's $\}$.

a.) Design a one-tape, TM that accepts strings of L. Your TM should accept by "Final state", and should also halt when when the input strings is not in L. Make sure you show what the final state(s) is/are.

The language L, should accept palindromes the first character should be the same as last character and etc, until the middle, character of the string length is reached.

If the input is empty, we accept the string because it is in our language $w = \varepsilon$.

In any case, the string doest not match, the machine stops. We move onto second character, and repeat until every character is crossed off.

TM = $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$, $\Sigma = \{0, 1, X, Y\}$, $\Gamma = \{0, 1, X, Y, B\}$, q_0, B, $F = \{q_6\}$

Here is the transition table...

state	0	1	X	Y	B
q_0	(q_1, X, R)	(q_2, X, R)	–	–	(q_6, B, R)
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	–	–	(q_3, B, L)
q_2	$(q_2, 0, R)$	$(q_2, 1, R)$	–	–	(q_4, B, L)
q_3	(q_5, Y, L)	–	–	–	–
q_4	–	(q_5, Y, L)	–	–	–
q_5	$(q_5, 0, L)$	$(q_5, 1, L)$	(q_0, X, R)	–	–
q_6	–	–	–	–	–

Symbol(s)

q_0 is the final state.

b.) Convert your TM to a TM that accepts by halting.

State	0	1	X	Y	B
q_0	(q_1, X, R)	(q_2, X, R)	(s, X, R)	(s, Y, R)	(q_6, B, R)
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	(s, X, R)	(s, Y, R)	(q_3, B, L)
q_2	$(q_2, 0, R)$	$(q_2, 1, R)$	(s, X, R)	(s, Y, R)	(q_4, B, L)
q_3	(q_5, Y, L)	$(s, 1, R)$	(s, X, R)	(s, Y, R)	(s, B, R)
q_4	$(s, 0, R)$	(q_5, Y, L)	(s, X, R)	(s, Y, R)	(s, B, R)
q_5	$(q_5, 0, L)$	$(q_5, 1, L)$	(q_0, X, R)	(s, Y, R)	(s, B, R)
q_6	–	–	–	=	–
s	$(s, 0, R)$	$(s, 1, R)$	(s, X, R)	(s, Y, R)	(s, B, R)

4) Consider a very simple. computational problem: adding 1 to a positive integer n.

a.) Design a one-tape TM that ass 1 to n; n is input as binary string.

The tape initially contains a \$ followed by N.

First we scan the end of the input string. If the tape see a 0, it will change the character to 1. If it's 1, change to 0 and move on to next digit. In the end, the tape will reach a 0 or \$. So, if the tape reaches a 0, change to 1 and stop. If it is \$ at the beginning, change to 1 and stop.

$TM = (Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1, \$\}, \Gamma = \{0, 1, \$, B\}, q_0, B, F = \{\})$

The transition table:

state	0	1	\$	B
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	$(q_0, \$, R)$	(q_1, B, L)
q_1	$(q_2, 1, R)$	$(q_1, 0, L)$	$(q_2, 1, R)$	–
q_2	–	–	–	–

b.) \$ 1111

$q_0\$1111 \vdash \$q_01111 \vdash \$1q_0111 \vdash \$11q_011 \vdash \$111q_01$

$\vdash \$1111q_0B \vdash 111q_11B \vdash \$11q_110B \vdash \$1q_1100B \vdash q_1\$000B$

$\vdash 1q_20000B$ . . . Here the TM halts at $q_2 0$ because there is no possible move.

4b.) \$1000

$q_0\$1000 \vdash \$q_0 1000 \vdash \$1q_0 000 \vdash \$10q_0 00 \vdash \$100q_0 0$

$\vdash \$1000 q_0 B \vdash \$100 q_1 0B \vdash \$1001 q_1 B, \ldots$ TM here halts at $q_2 B$ because there is no possible moves.

4c.) Convert your TM to a TM that accepts by final State.

$TM = (Q = \{q_0, q_1, q_2, S\}, \Sigma\{0,1,\$\}, \Gamma=\{0,1,\$,B\}, q_0, B, F=\{S\})$

State	0	1	\$	B
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	$(q_0, \$, R)$	(q_1, B, L)
q_1	$(q_2, 1, R)$	$(q_2, 0, L)$	$(q_2, 1, R)$	(S, B, R)
q_2	$(S, 0, R)$	$(S, 1, R)$	$(S, \$, R)$	(S, B, R)
S	-	-	-	-

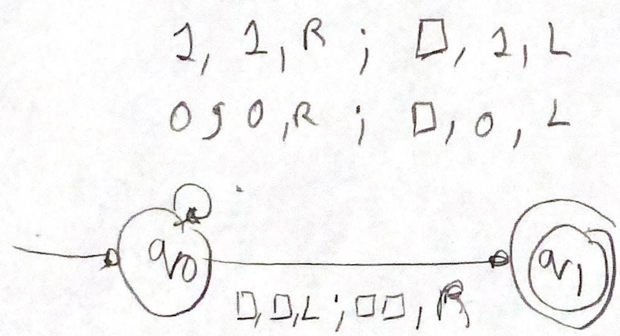
q5.) Design Multitrack TM that finds the reversal W^R | where w is a binary string of 0's and 1's.

Turing Machine will have two tracks (input and output). So tape 1 is read as input. If track 1 is at 0 move right, when on track 2 replace with 0 and move left.
So, If track 1 points to 1, move right, when track 2 replace it with 1 and move left. Finally, if track 1 points to B, halt the TM.

$TM = (Q = \{q_0, q_1\}, \Sigma = \{0,1\}, \Gamma = \{0,1,B\}, q_0, B, F = \{\})$

states	0	1	B
q_0	$\{(q_0,0,R),(q_0,1,L)\}$	$\{(q_0,1,R),(q_0,1,L)\}$	$\{(q_1,B,L),(q_1,B,R)\}$
q_1	-	-	-

TM



When moving lef to right on track 1, each symbol will have reverse on track 2. Track 1 move right, Track 2 moves left.

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