COMP 3540 Theory of Computation- Fall 2020

Assignment 4.

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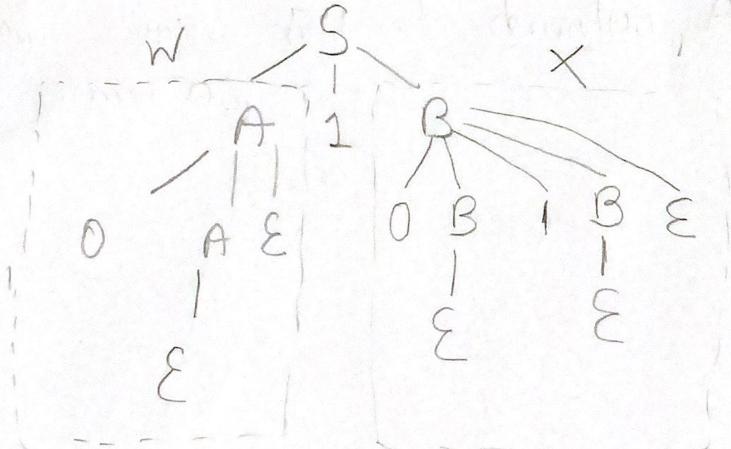
1.) Given the following grammar: $S \to A1B$

$A \to 0A \mid \varepsilon$

$B \to 0B \mid 1B \mid \varepsilon$

First, we can prove if there is a parse tree with the root labelled $S \to A1B$, $S \underset{lm}{\Rightarrow} A1B$ $2 = A1B$.

Basis height 1 where $S \to A\,1\,B$ is in the production tree below.



By unique leftmost derivation, A can be a string of 0 or more 0's as you see in parse tree W. B can be a string that consist of 0s, 1s, or be empty as you see in parse tree X. Each string will have unique left most derivation. Thus, they are each unambiguous.

Assume $S \Rightarrow_{lm}^{*} A1B$ with height h. From the tree above, we have height h.

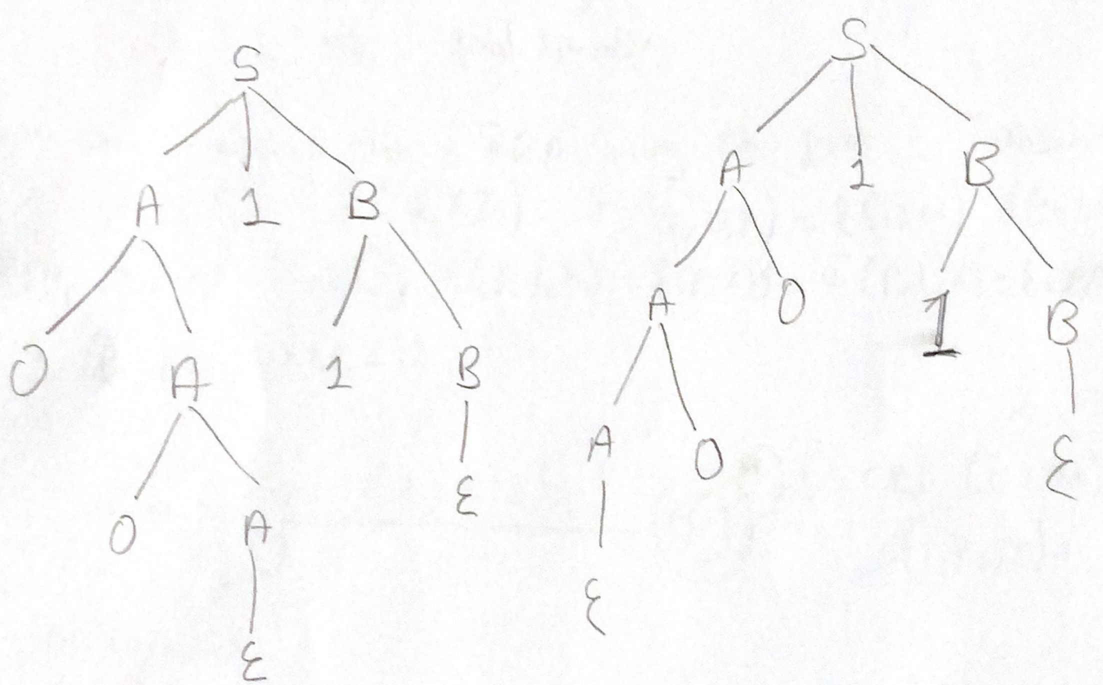
By Inductive Hypothesis, $X_i \Rightarrow_{lm}^{*} w_i$;

Then $S \Rightarrow_{lm} A1B \Rightarrow_{lm}^{*} w_1 1B \Rightarrow_{lm}^{*} w_1 1 w_2$ where w_1 is represented by $A \Rightarrow 0A \mid \varepsilon$ and w_2 is $B \Rightarrow 0B \mid 1B \mid \varepsilon$

Therefore, the parse tree for S is unambiguous.

b.) Find a grammar for the same language that is ambiguous. Show how the grammar you obtained is ambiguous.

Let's take an example string $w = 0011$.
So we will have two parse trees. Therefore the grammar is ambiguous.



∴ The grammar is ambiguous.

2.) Convert the following PDA $P = (\{q, p\}, \{0,1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$ to a CFG.

transition   start stack symbol

PDA $\quad P = (\{q, p\}, \{0,1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$.

Start state.   final accepting states.

$\{q, p\}$ are the finite set of states

$\{0,1\}$ are the input alphaphets/symbols.

$\{Z_0, X\}$ set of stack symbols.

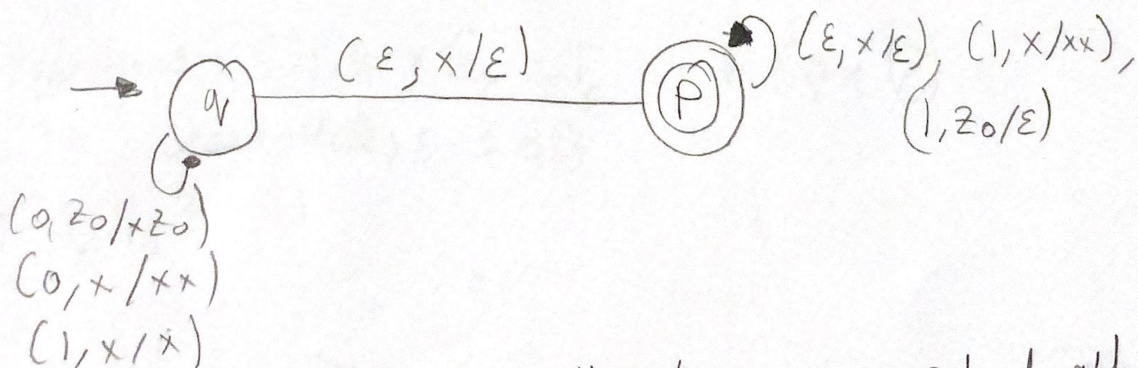
$\{p\}$ is the final symbols.

We can construct the PDA using the given Transitions.

$\delta(q, 0, Z_0) = \{(q, XZ_0)\}$ $\quad \delta(q, 0, X) = \{(q, XX)\}$ $\quad \delta(q, 1, X) = \{(q, X)\}$

$\delta(q, \varepsilon, X) = \{(p, \varepsilon)\}$, $\quad \delta(p, \varepsilon, X) = \{(p, \varepsilon)\}$ $\quad \delta(p, 1, X) = \{(p, XX)\}$

$\delta(p, 1, Z_0) = \{(p, \varepsilon)\}$.



The PDA defines the language: set of all strings that start with 0.

The Context-Free Grammar is

$$S \rightarrow 0A$$
$$A \rightarrow 0A \mid 1A \mid \varepsilon$$

~~Q4) Find the CNF of the following CFGs, Show all steps.~~

~~Q5)~~

3.) Design PDAs that accept, by empty stack, the following CFGs.

a.) $\{0^n 1^m 2^{2(n+m)} \mid n \geq 0, m \geq 0\}$

For every 0 and 1, we read in the first part of the string the PDA pushed X onto the stack, the we can pop single X for every 2 read at the end of string.

PDA as 6-Tuple $(Q, \Sigma, \Gamma, \delta, q_1, F)$ where

→ $Q = \{q_1, q_2, \ldots q_5\}$

→ $\Sigma = \{0, 1, 2\}$

→ $\Gamma = \{X, \$\}$

transitions $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$

q_1 is the start state; $F = \{q_6\}$

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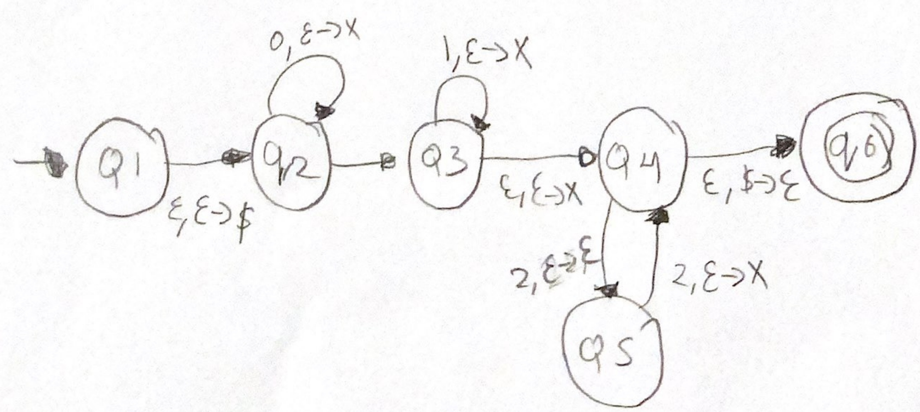


3b.) $\{0^n 1^m \mid n \le m \le 2n\}$

For every 0 read in from start of the string, the PDA would push X onto the stack, every 1 read in the end of the string, PDA would push X onto the stack.

PDA as 6-tuple $(Q, \Sigma, \Gamma, \delta, q, F)$

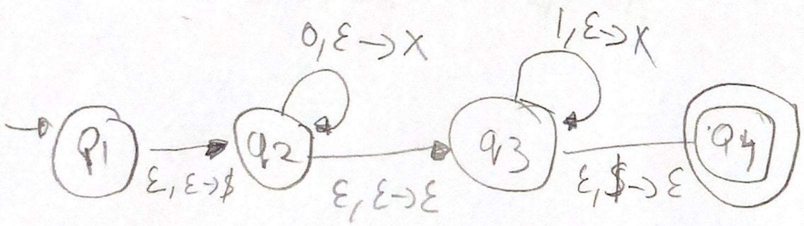
Q is $\{q_1, q_2, \ldots q_4\}$

$\Sigma = \{0,1\}$

$F = \{x, \$\}$

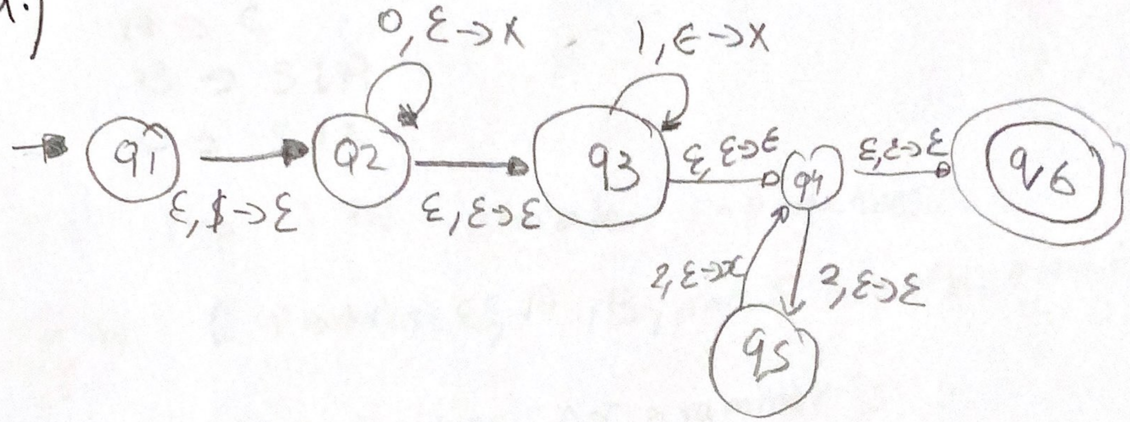
Transition $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$

Q_1 is start state, $F = \{q_4\}$

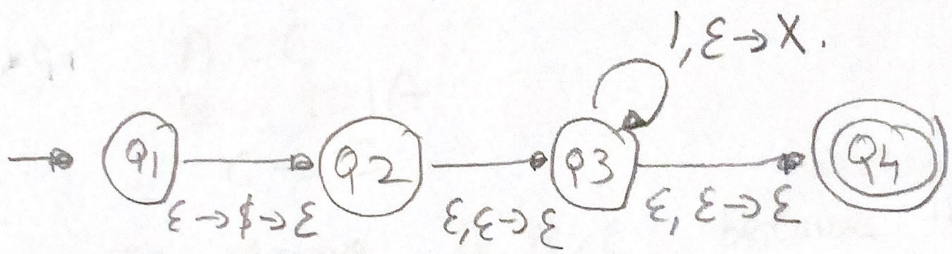


3.C

a.)


0, ε → x
1, ε → x
q1
q2
q3
q4
q5
q6
ε, $ → ε
ε, ε → ε
ε, ε → ε
ε, ε → ε
2, ε → x
3, ε → ε

b.)


1, ε → X.
q1
q2
q3
q4
ε → $ → ε
ε, ε → ε
ε, ε → ε

Q4. Find the CNF of the following CFGs - Show all steps.

a.) $S \to 0A0 \mid 1B1 \mid BB$
$A \to C$
$B \to S \mid A$
$C \to S \mid \varepsilon$.

The first step is to eliminate ε-productions.
Since C produces ε, A, B, and S can also produce ε.
So after eliminations, our grammar is

$S \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid B \mid BB$
$A \to C$
$B \to S \mid A$
$C \to S$

Then we can remove A, B, C because they all just produce S again:

$S \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB$
$A \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB$
$B \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB$
$C \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB$

Now we can eliminate useless symbols like C because its not reachable. We can also remove A and B because it is just equal to S.
our grammar becomes $S \to 00 \mid 0S0 \mid 1S1 \mid SS$

Chomsky Normal Form.

To make the grammar into CNF, we can create $A \to 0$, $B \to 1$ variables. The grammar is then divided into two productions of length 3 (C and D variable).

Final gramar: $S \to AA \mid AC \mid BD \mid SS$
$A \to 0$
$B \to 1$
$C \to SA$
$D \to SB$

4 b.)

$S \to aAa \mid bBb \mid \varepsilon;$

$A \to C \mid a$

$B \to C \mid b$

$C \to CDE \mid \varepsilon$

$D \to A \mid B \mid ab$

First step is to eliminate ε-productions or transitions. We can look at all symbols and identity nullable.

C is nullable because it has direct transition to ε,

A, B is nullable because they have unit transition to nullable symbol.

After eliminating ε and adding productions, the resulting grammar is as follows:

$S \to aAa \mid bBb \mid aa \mid bb$

$A \to C \mid a$

$B \to C \mid b$

$C \to CDE \mid DE$

$D \to A \mid B \mid ab$

Second step is to eliminate all unit productions; replace A, B, D. The resulting grammar is as follows

$S \to aAa \mid bBb \mid aa \mid bb$

$A \to CDE \mid DE \mid a$

$B \to CDE \mid DE \mid b$

$C \to CDE \mid DE$

$D \to CDE \mid DE \mid a \mid b \mid ab$

Third step, we can eliminate useless symbols. Since E does not exist, we can remove all productions that have E in it. The resulting grammar.

$S \to aAa \mid bBb \mid aa \mid ab$

$A \to a$

$B \to b$

Finally we can put into CNF form. Since chomsky normal form has to be in this form $A \to a$, $A \to BC$. We can replace aa with AA and bb with BB, and we can combine terminals and variables. The resulting grammar is as follows:

Final CNF Gramar:

$$S \to [aA]A \mid [bB]B \mid AA \mid BB$$
$$A \to a$$
$$B \to b$$

$$[aA] \to AA$$
$$[bB] \to BB$$

Q5.) Prove that the following languages are not context free

a.) $L = \{a^i b^n c^n \mid i \leq 2n\}$

We can prove this using pumping lemma.

Lets assume that language L is not context-free language.

Let the pumping Length be $n = 2$

$i \leq 2(n) = i \leq 2(2) = i \leq = 4$

Now, $L = \{a^4 b^2 c^2\}$

$L = \{aaaabbcc\}$, satisfying language.

We can now divide the language into 5 parts for string uvxyz

$L = \{\underline{a}\,\underline{aaa}\,\underline{b}\,\underline{b}\,\underline{cc}\}$

$x = b, |x| = 1$

$y = b, |y| = 1$

$u = a, |u| = 1$ $v = aaa, |v| = 3$ $z = cc, |z| = 2$.

The string length is 7. We can analyze using case to Check if it is context free language or not. Our assumption is that Language L is context-free.

Case 1: $|vxy| \leq n$

Total length of $|vxy|$ is $3 + 1 + 1 = 5$

$= |vxy| \leq n$

$= 5 \leq 7$ True

Case 2: $|vy| > 0$

Total length of $|vy|$ is $3 + 1 = 4$

$= |vy| > 0$

$= 4 > 0$ True

Case 3: $L = uv^ixy^iz$ where $i > 0$ for different values of i

$i = 1 \Rightarrow$

$L = a(aaa)^1 b(b)^1 cc = a^4 b^2 c^2 \in L$

$i = 2 \Rightarrow$

$L = a(aaa)^2 b(b)^2 cc$

$aaaaaaa\ bbbcc = a^7 b^3 c^2 \notin L$

since $7 \nless 2(4) = 7 \nless 4$

case 3 fails.

Since case 3 fails for different values of i, our assumption fails. So the language L is not context-free.

Thus, $L\ \{a^i b^n c^n \mid i < 2n\}$ is not context-free.

5b.) $L = \{ww^Rw \mid w$ is a string of a's and b's$\}$.

We can prove this using pumping lemma.

Let's assume that language L is context-free language.

Let the pumping length be $w = abb$, $w^R = bba$, $w = abb$

Where $L = \{abbbbaabb\}$, based on the given condition.

Now we can split the string in 5 parts.

Str = $\underline{ab}_{u}$ $\underline{b}_{v}$ $\underline{bba}_{x}$ $\underline{a}_{y}$ $\underline{bb}_{z}$ string length is $\underline{9}$ a^3b^6

$n = 9$

$u = ab$, $v = b$, $x = bba$, $y = a$, $z = bb$.

Now we can check three cases of pumping Lemma

Case (i): $|vxy| \leq n$

Total length of $|vxy|$ is $1+3+1 = 5$

$5 \leq 9$, True.

Case (ii): $|vy| > 0$

Total length of $|vy|$ is $1+1 = 2$

$2 > 0$ True

Case (iii): check $uv^ixy^iz \in L$ for different values of i.

$i = 1$, $ab(b)^1 bba(a)^1 bb$ = $abbbbaabb$ $= a^3b^6 \in L$

$i = 2 \Rightarrow ab(b)^2 bba(a)^2 bb$ = $abbbbbaaabb$ $= a^4b^7 \notin L$

$i = 3 \Rightarrow ab(b)^3 bba(a)^3 bb$

$L = \underbrace{abbbb}_{w}\underbrace{bba}_{w^R}aaabb$, failed string. $a^5b^8 \notin L$

Since w and w^R is different for same string, case (iii) fails.

Therefore, since case iii fails, our assumption is not Right.

Thus, $L\{ww^Rw \mid w$ is a string a's and b's$\}$ is not context-free.