Kalman Filters and Localization

November 1, 2022

Localization

Common Sensors

- GPS: Uses time-of-flight to several satellites to estimate x/y/z position
- Gyroscope: Electromechanical device for determining the direction of gravity
- Accelerometer: Measures the angular acceleration
- Magnetometer: Uses a magnet to determine magnetic North
- IMU: Integrates gyroscope, accelerometer, magnetometer to determine location.

The actual IMU and GPS algorithms are outside the scope of this presentation.

Kalman Filter

Many techniques exist for filtering the data. The provided MATLAB code uses the standard kalman filter.
$\bar{u}_k = u_k - \delta u_k + w_k$
where u_k is a signal, w_k is an additive noise matrix, and δu_k is a slowly varying measurement bias:
$\delta u_k = \delta u_{k-1} + w_k^*$
and w_k^* is an additive noise matrix with a specific covariance matrix, derived from the data. Each datapoint is then determined by another iterative function: $u_{k+1} = f(u_k)$

$$u_k = u_{k-1} - \delta(\delta u_{k-2} + w_k^*) + w_k$$

Our gyroscope has a known error of .01 degrees latitude per second, skewing North due to the angular momentum of the earth. In meters near the equator, .01 degree of latitude is approximately equal to 111 meters [1].

[1] USNA

Task 1: Error Growth

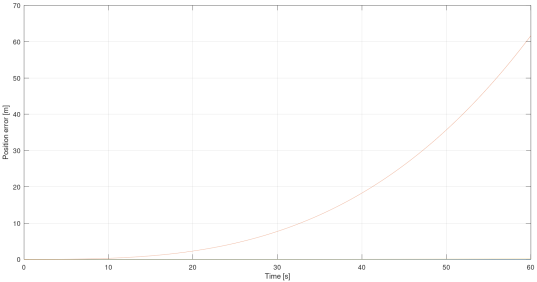


Figure: Error Growth over time for unaided IMU.

We can see that, as expected, the error rate grows over time without bound.

Error Bias

This error arises from the uncertainty of the accelerometer since the acceleratain, a in the x, z directions is initialized as:

$$a_x = 0, a_z = -g$$

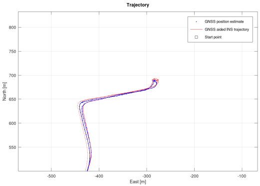
$$\delta a = \begin{bmatrix} \delta a_x \\ \delta a_z \end{bmatrix} = \begin{bmatrix} -g\delta a \\ 0 \end{bmatrix}$$

Integrating over the iterative function from the previous, we know that

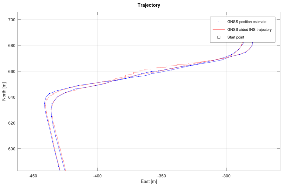
$$\delta a(t) = \delta u_0 + \int_0^t err_0 \cdot dt$$

where the err_0 is the bias due to the gyroscope's assumption that gravity is truly straight down relative to the angular momentum of the planet. Because it isn't and because the earth spins east, our gyroscope is biased North as per the right hand rule meaning that we would expect a larger error in Umeå than Stockholm or Lund.

Task2 : Location



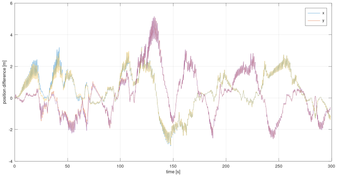
(a) With Outage RMS Error = 1.65



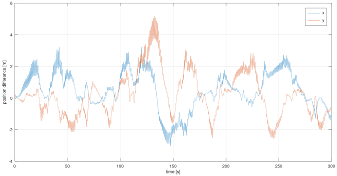
(b) Without Outage RMS Error = 1.56

Figure: Location over time for aided IMU. The error is too small for these plots to be of much use, even when zoomed in.

Task2 : Difference



(a) With Outage RMS Error = 1.65



(b) Without Outage RMS Error = 1.56

Figure: Error over time for aided IMU. Using IMU + Accelerometer data.

Task3 : Non- Holonomic

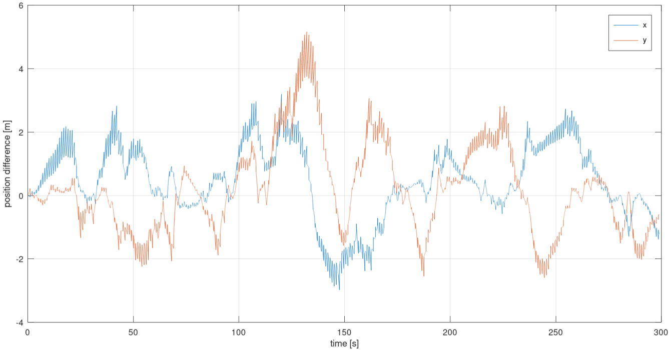


Figure: Error over time for aided IMU. RMS = 1.84. This adds non-holonomic data in which instantaneous acceleration is experienced, but displacement does not occur (for example, while turning).

Task4 : Speed data

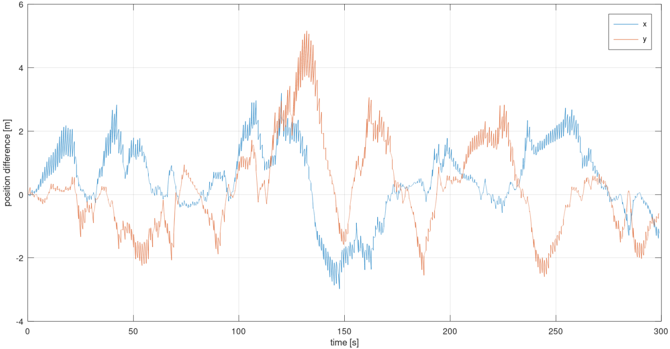


Figure: Error over time for aided IMU. RMS = 1.86. This adds speed data and removes the non-holonomic data.