MBA 204: Operations – Professor David Robb

"SAMPLE FINAL" SOLUTIONS

Important: This "Sample Final" contains quantitative practice problems. The actual final will contain qualitative questions as well. The quantitative problems on the actual exam may be less (or more) difficult than the ones here. You are encouraged to work through the problems before viewing the solutions. (Frankly, you will get much less out of working through these problems if you don't do this.)

Solutions to these practice problems will be posted to bCourses at the start of Saturday's review session. The review session will go over the solutions to Problems 1, 2, 3, and 4.

Instructions:

- For each question that asks for a written response, write your answer in the box provided using clear, legible, normal size writing (don't try to squeeze in a lot of text by writing small; such text will be ignored). (So that you can put the final answer you want graded in the provided box, it is suggested that you use a pencil rather than a pen.) Please show all of your work. In particular, show your work in coming to the answer in a box in the area below the question.
- You may not consult anyone other than the professor on this exam.
- The exam is closed notes and closed book.
- A Normal Probability Table is provided on bCourses.
- One page of equations is provided on bCourses.

Your name: ______________________________

Your cohort: ______________________________

1. Three Queens. You are the chief buyer for housewares at a large department store. Unfortunately, your store faces stiff competition from specialty retailers that that carry imported cooking and dining articles. To meet this competitive challenge, you have reorganized your housewares department to create a "store within a store" that has the same ambiance as your competitors'. To kick off your concept, you plan a one-month promotion that features a sale on several special items, including an imported Three Queens baking dish. (The Three Queens baking dish leaves a "crown" image on the top of the cake.) These baking dishes must be ordered several months in advance. Any leftover inventory at the end of the promotion will be sold to a discount chain at a reduced price.

You have collected some pricing and cost data, listed in the following table, to help you decide how many of the Three Queens baking dishes to order. You predict total demand for the imported baking dish will be normally distributed with mean of 980 and standard deviation of 354. Leftover dishes will be sold to the discount chain for $10.

Three Queens Baking Dish	
Selling price	$40.00
Purchase price	$16.00
Shipping cost*	$ 3.00
Handling cost**	$ 0.80
Warehouse surcharge***	$ 1.10
Total cost	$20.90

*Variable cost to ship one dish.
**Estimate of variable cost for the receiving process for one dish (uncrating, cleaning, and transporting the dish to the housewares department).
***Allocation of fixed overhead expenses in the shipping and receiving department.

(a) Suppose 1,200 imported Three Queens baking dishes are ordered. What is the service level (i.e., the probability that the department store has enough baking dishes to meet all of the demand)?
$\boxed{0.7324}$

Note that 1200 units corresponds to a z-value of z=(1200-980)/354=0.6214. From the Normal Table, this corresponds to a probability of 0.7324, meaning that

Prob(Demand≤1200)=0.7324.

(b) How many Three Queens baking dishes should you purchase? *1139 dishes*

If we stock an additional unit and sell it, we gain

$$G=\$40.00-\$16.00-\$3.00-\$0.80=\$20.20$$

(the warehouse surcharge is irrelevant because it is fixed overhead cost that is independent of the number of units that we stock). If we stock an additional unit and don't sell it, we lose

$$L=\$16.00+\$3.00+\$0.80 - \$10.00=\$9.80.$$

The critical fractile is

$$G/(G+L)=\$20.20/(\$20.20+\$9.80)=0.6733.$$

From the Normal Table, this corresponds to a z-value of z=0.45. Therefore, you should purchase

$$\begin{aligned} Q &= \text{mean} + z \times \text{standard deviation} \\ &= 980 + 0.45 \times 354 \\ &= 1139 \text{ (rounded to nearest integer)} \end{aligned}$$

baking dishes. (You would also receive full credit if you applied the round up rule and obtained 1140.)

(c) [optional challenge problem; may be more difficult than what is on the final exam] You are concerned about customer service. In particular, you feel that there is a loss of goodwill of $8 for every customer that wants to purchase the Three Queens dish but is unable to do so due to a stockout. Now how many Three Queens baking dishes should you purchase? [Note to sample-test takers: The (subtle) issue of how to compensate for a goodwill cost was addressed in a practice problem in the Newsvendor (Session 9) Slide Pack Appendix. A solution for how to deal with goodwill cost was provided in the reading "The Critical Fractile method." In the review session, the GSI will describe a similar but slightly simpler approach than that given in the reading. Make sure you work through the practice problem and do the reading before asking questions in the review session about this issue. If you have *diligently* worked through those and still have questions, then you can raise them in the review session.]

1210 dishes

The imposition of the goodwill cost has no impact on the loss we incur if we stock an additional unit and don't sell it; we continue to have L=\$9.80. However, now if we stock an additional unit and sell it, we avoid incurring the goodwill cost of \$8, so the gain

$$G = \$40.00-\$16.00-\$3.00-\$0.80-(-\$8.00)=\$28.20.$$

The critical fractile is G/(G+L)=\$28.20/(\$28.20+\$9.80)=0.7421. From the Normal Table, this corresponds to a z-value of z=0.65. Therefore, you should purchase

$$\begin{aligned} Q &= \text{mean} + z \times \text{standard deviation} \\ &= 980 + 0.65 \times 354 \\ &= 1210 \text{ (rounded to nearest integer)} \end{aligned}$$

baking dishes.

2. Littlefield Labs. Consider a Littlefield Labs simulation with "stationary" demand with the average daily demand is 12.0 orders and the standard deviation of daily demand is 3.2 orders. Further, demand across subsequent days was uncorrelated (i.e., there were no systematic trends in daily demand). Suppose the supplier required exactly nine days to ship any quantity of raw materials to Littlefield Labs. In setting the reorder point, what is the relevant demand distribution?

The distribution of demand over the nine day lead time.

More precisely, what is the mean and standard deviation of this demand distribution?

mean = $9 \times 12.0 = 108.0$ *orders*

standard deviation = $\sqrt{9} \times 3.2 = 9.6$ *orders*

[optional challenge problem; may be more difficult than what is on the final exam] If demand across subsequent days was positively correlated, would the standard deviation of this demand distribution be smaller or larger than what you just calculated? *larger*

3. Alps Rail. Alps Rail provides passenger rail service between Vienna, Austria and Zurich, Switzerland. To receive the discounted price of €83 (eighty-three euros), passengers must purchase their ticket three weeks in advance (this helps to distinguish between leisure travelers, who tend to book early, and business travelers, who value the flexibility of booking late). The full-price fare is €135. Alps estimates that the demand from leisure travelers could fill the whole train while the demand from business travelers is distributed normally with a mean of 70 and a standard deviation of 25. The Vienna-Zurich train has 117 seats.

(a) What booking limit should Alps set for discount-price seats? $\boxed{\textit{54 seats}}$

If we allocate a seat to the high-fare class and we sell it, then (relative to the case where we did not allocate the room to the full-fare class) we gain

$$G = €135 - €83 = €52.$$

If we allocate a seat to the high-fare class and we do not sell it, then (relative to the case where we did not allocate the seat to the full-fare class), we lose

$$L = €83.$$

Therefore, the optimal protection level satisfies

$$Prob\left(full\ price\ demand \le protection\ level\right) = \frac{G}{G+L} = \frac{52}{52+83} = 0.3852.$$

From the Normal Table, this corresponds to a z-value of z=-0.29. Therefore, the protection level should be

$$\begin{aligned} protection\ level &= mean + z \times standard\ deviation \\ &= 70 - 0.29 \times 25 \\ &= 63 \text{ (rounded to nearest integer)} \end{aligned}$$

Therefore, the booking limit should be 117-63=54 seats.

(b) [optional challenge problem; may be more difficult than what is on the final exam] Due to intense competition, Alps anticipates that it will have to cut both its discount-price and full-price fare by 20%. For simplicity, assume that with this change, the demand characteristics of the leisure and business segments will be unchanged.

Circle the best answer:

i. Under this change, the optimal booking limit will increase.

ii. Under this change, the optimal booking limit will decrease.

iii. Under this change, the optimal booking limit will not change.

iv. There is insufficient information to answer the question of the impact of this change on the optimal booking limit.

If you circled *iv,* state what additional information you would need to answer the question of the impact of this change on the optimal booking limit.

Please show your work in coming to your answer to question (b) below.

The effect of reducing both prices by 20% is to reduce G and L by 20%.

More explicitly,

$G = 80\% \times €135 - 80\% \times €83 = 80\% \times €52$

$L = 80\% \times €83.$

Therefore the critical fractile

$$\frac{G}{G+L} = \frac{80\% \times 52}{80\% \times 52 + 80\% \times 83} = \frac{52}{52+83} = 0.3852$$

is unchanged. Consequently, the optimal protection level and booking limit are unchanged.

(c) [optional challenge problem; may be more difficult than what is on the final exam] As in part (b) assume that:

- Due to intense competition, Alps anticipates that it will have to cut both its discount-price and full-price fare by 20%.
- With this change, the demand characteristics of the leisure segment will be unchanged.

However, suppose that a more sophisticated study of the competitive market reveals that with this change in pricing and competition, the mean demand from business travelers will decrease (although the standard deviation will remain unchanged).

Circle the best answer:

i. Under this change, the optimal booking limit will increase.

ii. Under this change, the optimal booking limit will decrease.

iii. Under this change, the optimal booking limit will not change.

iv. There is insufficient information to answer the question of the impact of this change on the optimal booking limit.

If you circled *iv,* state what additional information you would need to answer the question of the impact of this change on the optimal booking limit.

Please show your work in coming to your answer to question (c) below.

As noted in part (b), the critical fractile is unchanged. However, the optimal protection level satisfies

$$Prob\left(full\ price\ demand \leq protection\ level\right) = \frac{G}{G+L} = 0.3852.$$

Because full-price demand has shifted downward (graphically, the bell curve has shifted to the left), the optimal protection level will decrease, which means that the optimal booking limit will increase.

4. Pompelmo Frizzante. You are a distributor of specialty non-alcoholic frozen Italian beverages, including bottles of Pompelmo Frizzante. Weekly demand is for 6 cases of Pompelo Frizzante. Assume demand occurs steadily over the 52 weeks of the year. You purchase the bottles directly from the supplier in Italy, and sell to retailers in the United States. Your annual cost of capital is 25 percent, which also includes all other inventory-related costs. Below are data on the costs of shipping and handling, as well as on the retail price. These costs include the usual ordering and handling costs, plus the cost of chilling/freezing and a variable component that depends on the number of cases in inventory.

- Price you charge your retailer customers per case: $57
- Price the Italian supplier charges per case: $40
- Shipping cost from supplier in Italy (for any size of shipment): $290
- Cost of labor to place and process an order: $10
- Cost of labor to pack a case for shipment when it is sold to a retailer: $2/case
- Variable cost for chilling/freezing: $3.50/case/week

(a) What is the weekly holding cost for one case of Pompelmo? *$3.69/case/week*

The cost of labor to pack a case for shipment when it is sold is irrelevant. The financial cost of holding a case of wine is

the weekly holding cost 0.48%/week × $40 = $0.19/week, and
the variable refrigeration cost is $3.50/case/week,

so the total holding cost is

$0.19+$3.50=$3.69.

(b) How many cases should you purchase each time you order?

31 cases

The fixed cost of placing an order (i.e., the setup cost) is $290+$10=$300. The Economic Order Quantity is

$$\sqrt{\frac{2 \times \text{demand} \times \text{setup cost}}{\text{holding cost}}} = \sqrt{\frac{2 \times 6 \times 300}{3.69}} = 31.23,$$

and rounding to the nearest integer yields 31 cases per order.

(c) Given your answer in (b), how many times a year do you order? $\boxed{\textit{10.06 orders/year}}$

What is the annual cost associated with placing these orders? $\boxed{\textit{\$3,018 or \$15,498}}$

Annual demand is 6 cases/week × 52 weeks/year = 312 cases/year. You order $\frac{\textit{312 cases / year}}{\textit{31 cases / order}} = \textit{10.06 orders / year}$*. Therefore, you incurs an annual fixed order cost of 10.06 × \$300=\$3018. (The answer immediately above is sufficient to receive full credit. However, you could have also said that the variable purchasing cost is \$40 / case × 6 cases / week × 52 weeks=\$12,480. Thus, the total annual cost associated with ordering is \$12,480+\$3,018=\$15,498.)*

(d) Given your answer in (b), how many dollars per year do you spend to hold Pompelo Frizzante in inventory (including the cost of capital)? $\boxed{\textit{\$2,974}}$

Millennium's average inventory is 31/2 = 15.5 cases. The annual holding cost is 15.5 cases × \$3.69/case/week × 52 weeks/year = \$2,974.

(e) [optional challenge problem; may be more difficult than what is on the final exam] Considering the analysis you have done in parts (a)-(d) and the economics of the product, would you recommend continuing to sell the Pompelmo Frizzante product? Hint: First calculate the revenue generated by the product; second calculate the total cost incurred with the product. (You may want to come back to this problem.)

From parts (c)-(d), the annual cost of procurement and inventory is \$15,498+\$2,974=\$18,472. For every case that you ship to a retailer you receive \$57 less the packing cost of \$2, so after this cost, the net revenue is \$55. On an annual basis, the net revenue is \$55×312 cases/year=\$17,160, which is less than the \$18,472 in procurement and inventory costs. Considering just the factors included in this analysis, it would be prudent to consider dropping the Pompelmo Frizzante product.

5. Le Petite Boulangerie. Le Petite Boulangerie is a large chain of gourmet bagel-and-sandwich shops. At each shop, the morning begins with a large production run of bagels. After this production run, the ovens shift over to baking bread for the lunchtime sandwich crowd. The founder of the chain designed the shops to offer a high level of service, promising, for example, that 80% of the time each shop would have enough fresh bagels to satisfy all customer demand. She designed the ovens to have just enough capacity to meet this promise. Bagels are sold for \$1.80. It costs approximately \$0.40 in materials and labor to make a bagel. In a typical shop, daily demand for fresh bagels is normally distributed with mean 55 and standard deviation 20. The shops close at 3 p.m., and bagels not sold by the end of the day are sold the next day as "day-old" bagels for \$0.60. About one-half of the day-old bagels are sold; the remainder are just thrown away.

(a) What is the size of the ovens (measured in terms of the number of bagels that can be produced in the morning production run)? $\boxed{\textit{72 bagels}}$

We know that Prob(Demand≤Oven Capacity)=0.80. From the Normal Table, this probability corresponds to a z-value of z=0.85. Therefore,

Oven Capacity = mean + z × standard deviation
= 55 + 0.85 × 20
= 72.

(b) If you didn't face the oven constraint, how many fresh bagels would you make in the morning production run? $\boxed{\textit{85 bagels}}$

If we stock an additional unit and sell it, we gain
G=\$1.80-\$0.40=\$1.40.
If we stock an additional unit and don't sell it, we lose
L=\$0.40-(50%×\$0.60+50%×\$0)=\$0.10.
The critical fractile is
G/(G+L)=\$1.40/(\$1.40+\$0.10)=0.933.
From the Normal Table, this corresponds to a z-value of z=1.50. Therefore, you should make
Q = mean + z × standard deviation
= 55 + 1.50 × 20
= 85 bagels.

6. **Raycott**. *Raycott* is a U.S. company that imports a raw material (with landed cost of $12 per *kg*) in full container loads from Fiji. The demand is very consistent at 3000 units per month. *Raycott* waits until inventory drops to a reorder point (established using a safety factor, k=2.0), and then places an order of size 7000*kg* (a full-container load) of product from Fiji, paying for it upon arrival. The leadtime is assumed to be normally distributed with mean and standard deviation of 1.5 and 0.6 months, respectively. The inventory holding cost per year is 30%. The average inventory under this system is equal to the safety stock (SS) plus Q/2.

(a) Using the above terminology and the Reorder Point Model from the course, what is *Raycott*'s total cost (in $) of holding inventory for this product for a year?

Average Inventory in units is $\frac{Q}{2}+k\sigma_x=\frac{Q}{2}+k\sqrt{(E[L])\operatorname{var}[D]+(E[D])^2\operatorname{var}[L]}=\frac{Q}{2}+kD\sigma_L$

Cost of holding this per year is $\left(\frac{Q}{2}+kD\sigma_L\right)ic=\left(\frac{7000}{2}+2(3000)(0.6)\right)(0.3)(12)$

$=(3500+3600)(3.6)=\$25,560$ per year

(b) The Fijian supplier has just told *Raycott* they would need to pay for the inventory *at the time the order was placed*. How much will this add to the annual cost of holding inventory?

*The additional cost will be the average kg of inventory "on order", i.e., D*E[L], multiplied by the cost of holding on kg of inventory for a year, iC, or (DE[L]ic=(3000)(1.5)(0.3)(12)=(4500*3.6)=$16,200 per year*

(c) *Raycott* has just read that various factors including congestion on the U.S. West Coast have led to the on-time reliability of ocean shipping falling to a record low. What decision parameter(s) should *Raycott* change? What managerial decisions might they also consider as a result?

The mean and/or standard deviation of L have increased, and this should result in an increase in the reorder point. Raycott might need additional storage, or could switch to another port (e.g., through Savannah Georgia), or another supplier in another country, or another freight mechanism (such as air), or an alternate product

7**. Three-Stage Production Process.** Consider a process consisting of three sequential stations.

Station	Processing time (min./unit)	Number of Workers
A	50	20
B	30	10
C	80	30

The processing times, given above, reflect the time that it takes for one worker at a station to complete one unit. For example, at Station A, it takes one worker 50 minutes to complete the processing done at Station A. The production process is run 24 hours a day and 7 days a week, with a staff of workers working three 8-hour shifts each day. Assume that the system has been up and running for a considerable period of time (i.e., ignore start-up and shut-down effects).

(a) What is the bottleneck? $\boxed{\textit{Station B}}$

What is the process capacity (stated in units/hour)? $\boxed{\textit{20 units/hour}}$

First, we compute the capacity of each station.

Station A capacity = 20/50 units per minute =0.40 units per minute.

Station B capacity = 10/30 units per minute =0.333 units per minute.

Station C capacity = 30/80 units per minute =0.375 units per minute.

Station B has the lowest capacity; process capacity therefore is 0.333 units per minute, which is equal to 20 units per hour.

(b) Suppose you increase the number of workers at the bottleneck station by 10%. What is the new process capacity (stated in units/hour)? $\boxed{\textit{22 units/hour}}$

With 11 workers, the new

Station B capacity = 11/30 units per minute =0.367 units per minute.

Because Station A capacity and Station C capacity are greater than the new Station B capacity, Station B continues to be the bottleneck. The new process capacity is 0.367 units per minute, which is equal to 22 units per hour (10% greater than the original process capacity).

(c) [optional challenge problem; may be more difficult than what is on the final exam] Return to the original problem description, where the number of workers at each station is as given in the table. Suppose you wanted to increase the capacity of this original system by 20%. How many additional workers do you need? *4 more workers*

To increase the capacity of the system by 20%, each station needs to produce at a rate of (at least) 0.40 units per minute (equivalent to 24 units per hour). Station A's capacity is already sufficient, so no workers need to be added at Station A. Let W_B denote the number of workers at Station B, and W_C denote the number of workers at Station C. We need to have sufficient workers so that

Station B's capacity = $W_B \times (1/30)$ units per minute ≥ 0.40,

or, equivalently,

$$W_B \geq 30 \times 0.40 = 12;$$

therefore, we need 2 more workers at Station B.

Similarly, we need

$W_C \times (1/80)$ units per minute ≥ 0.40,

or, equivalently,

$$W_C \geq 80 \times 0.40 = 32;$$

therefore, we need 2 more workers at Station C. In total we would need 2+2=4 more workers.

(d) [optional challenge problem; may be more difficult than what is on the final exam] How would your answer to part (c) change if the current workers were cross-trained and so could be reassigned (e.g., workers at Station A could be assigned to Station B)?

The answer would not change if the current workers could be reassigned, because all the workers at Station A are needed to stay there if Station A's capacity is to be sufficient.

***8.* Pacific Videogames.** Pacific Videogames, a small videogame rental store in Berkeley, is open 24 hours a day, and – due to its proximity to the campus – experiences customers arriving around the clock. A recent analysis done by the store manager indicates that there are 30 customers arriving every hour, with a standard deviation of interarrival times of 2 minutes. This arrival pattern is consistent and is independent of the time of the day. The check-out is currently operated by one employee, who needs on average 1.7 minutes to check out a customer. The standard deviation of this check-out time is 3 minutes, primarily as a result of customers taking home different numbers of videogames.

(a) If you assume that every customer rents at least one videogame (i.e. has to go to the check-out), what is the average time a customer has to wait in line before getting served by the check-out employee, not including the actual check-out time (within 1 minute)? *19.8 minutes*

We know that the average interarrival time = 60 minutes/30 customers = 2 minutes. Using a = 2, p = 1.7, and the values for the standard deviations of a and p (2 and 3, respectively), we can calculate that the average wait time

$$\textit{average time in queue } T_q = \left(\frac{\textit{service time}}{m}\right) \times \left(\frac{\textit{utilization}^{\sqrt{2(m+1)}-1}}{1-\textit{utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

$$= 19.8 \textit{ minutes.}$$

(b) If there are no customers requiring check-out, the employee is sorting returned videogames, of which there are always plenty waiting to be sorted. How many videogames can the employee sort over an 8 hour shift (assume no breaks), if it takes exactly 1.5 minutes to sort a single videogame?
48 videogames

First we calculate the utilization of the employee, which is equal to (1/a) / (1/p) = (1/2) / (1/1.7) = 85%. On an 8-hour shift, which is equal to 480 minutes, the employee is free 480 × 0.15 = 72 minutes. Therefore, the employee can sort 72/1.5 = 48 videogames on an 8-hour shift.

(c) What is the average number of customers who are *waiting in line* to be served (exclude any customers being served) at the check-out desk? $\boxed{\textit{9.9 customers}}$

The flow rate = 1/a = 0.5. Thus, the average number of customers in line waiting I_q *= flow rate* × T_q *= 0.5* × *19.8 = 9.9 customers.*

(d) Now assume that 10% of the customers do not rent a videogame at all and therefore do not have to go through check-out (and that the standard deviation of the interarrival time is unchanged). What is the average time a customer has to wait in line before getting served by the check-out employee, not including the actual check-out time? $\boxed{\textit{10.9 minutes}}$

Now only 30 × *0.9 = 27 customers arrive at the counter, so the average interarrival time a = 60/27 minutes. Using the original values for p and the standard deviations, the new average wait time* T_q *= 10.9 minutes.*

For more problems covering the material in the first half of the course, please see the "Optional Sample Mid-Course Exam."