Concept Check #2: Queuing

Question 1. The Right to Vote (and not to wait) (10 points)

In the recent presidential elections in the US, very long wait times have been witnessed at precincts (voting stations) in states that ultimately decided the election (Florida in 2000 and Ohio in 2004).

In Philadelphia as well, some voters complained about the long lines in some precincts, with most complaints coming from precinct A. In 2004, the average number of votes arriving at Precinct A was 35 per hour, and the arrivals of voters were random with inter-arrival times that had a coefficient of variation of 1.0 (CVa=1.0). Philadelphia had deployed 1 voting machine in Precinct A. Suppose that each voter spent on average 100 seconds in the voting booth (this is the time needed to cast his/her vote using a voting machine), with a standard deviation of 120 seconds. How long on average did a voter have to wait in line at precinct A in 2004 before entering in a booth to cast his/her vote?

$$\text{avg. time in queue } T_q = \left(\frac{\text{service time}}{m}\right) \times \left(\frac{\text{utilization}^{\sqrt{2(m+1)}-1}}{1-\text{utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

We can use the M/G/1 queue model because the arrivals are random (Markovian arrivals), the service times are general (G for General), and there's only one server (the voting machine; 1)

Given:
Arrival rate = 35 voters per hour
Average service time = 100 seconds per voter
Standard deviation of service time = 120 seconds
m = 1 server
Coefficient of variation of arrival times (CVa) = 1.0 (which is typical for a Poisson process)
Coefficient of variation of service times (CVp) is calculated from the standard deviation and the mean service time.

Conversion and Calculations
Average Service Time = 100/3600 = 0.028 hours
St Dev of Service Time = 120/3600 = 0.033 hours
Service Rate = 1/.028 = 35.714/hour
CVp = 120 / 100 = 1.2
Utilization = 35 / 35.714 = 0.98

(.028)*(.98/(1-.98)) = 1.372
1.372*((1+1.44)/2) =1.674 hours = 100.44 minutes

On average, a voter **had to wait approximately 1.674 hours** in line at Precinct A in 2004 before entering the booth to cast their vote.

Question 2. Convenience store (20 points)
You run a convenience store at a beach resort with a single check-out and no employees except yourself. For several hours a day during the peak summer period (which lasts several weeks) customers arrive at your check-out at an average rate of 48 per hour. The arrival process is Poisson with CVa=1. Your utilization rate is 0.8, and the standard deviation of your service time distribution is 2 minutes.

(a) What is the average time (in minutes) it takes you to process a customer at the checkout? (Hint: consider the utilization and customer arrival rate)
Utilization = .8
St dev = 2 minutes = .033 hours
CVa = 1
CVp = .033/48 = 0.001
Arrival rate = 48 customers/hour
The average time it takes to process a customer at the **checkout is 1.0 minute**.

(b) What is the average length of the checkout queue?
The average waiting time in the queue is **approximately 8.0 minutes**, and the average length of the checkout queue is **approximately 6.4 customers**

(c) List two practical steps you could take to reduce the waiting time during this period.
One step is to streamline the checkout process by implementing faster checkout methods to reduce the average service time. Another step is to implement a “virtual queue” where people can register and then get up based on their call.

Question 3. Government Department (20 points)
A government department has outsourced its call center operations to Ultra, a firm that dedicates 7 customer service representatives (CSRs) to the department during office hours. Calls arrive randomly, with an average inter-arrival time of 45 seconds (i.e., 0.75 minutes), and are directed to the CSRs, who are pooled. Historical records show the average and standard deviation of the processing time (not including waiting) are both 5.0 minutes.

Number of customer service representatives (CSRs) = 7
Average inter-arrival time = 45 seconds = 0.75 minutes
Average processing time = 5.0 minutes
Standard deviation of processing time = 5.0 minutes
CVa = 1
CVp = 1

(a) What is the utilization of the CSRs?
(1/.75)/(7*(⅕)) = **0.952**

(b) What is the average time (in minutes) a customer waits before being served?

$$\text{avg. time in queue } T_q = \left(\frac{\text{service time}}{m}\right) \times \left(\frac{\text{utilization}^{\sqrt{2(m+1)}-1}}{1-\text{utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

(5/7)*((.952^(sqrt(16)-1))/(1-.952))*(1) =

5/7 = 0.714
.952^3 = 0.863
1 - .952 = 0.048
.714*(0.863/0.048) = **12.837 minutes**

(c) What is the average number of customers waiting to be served?
*(1/.75)**(12.837) = 17.116

(d) The department is considering outsourcing to a different provider, Alpha, which would dedicate more staff to the task than Ultra does, but the productivity of each CSR is correspondingly lower – indeed, the ratio of the average processing times of Alpha and Ultra CSRs is the same as the ratio of the number of staff members at Alpha compared to Ultra.

i. Would the Alpha contract average customer waiting time be lower, the same, or higher than for the Ultra contract? Explain your answer.

If Alpha has significantly more staff members than Ultra (substantially increasing c), the lower ρ resulting from increased capacity could lead to lower average customer waiting times compared to the Ultra contract.

However, if Alpha's CSRs are considerably less productive (higher average processing time), it might offset some of the benefits of having more staff members. So my guess would be that the customer waiting time may be the same if not higher

ii. Assuming the department pays any provider the same amount per customer served, state two other criteria (one related to time) that the department should consider before making the decision to switch or not.

Operational Efficiency and Response Time: The department should assess the operational efficiency of both providers in handling customer inquiries and resolving issues. A provider that can streamline its processes, minimize call handling time, and respond to customer needs promptly can enhance the overall customer experience. A shorter response time ensures that customers receive assistance quickly, leading to higher satisfaction and potentially reducing the number of repeat calls.

Workforce Management and Staffing Flexibility: The ability to effectively manage and allocate staff resources is crucial in call center operations. The department should consider which provider offers better workforce management tools and practices. A provider that can optimize staffing levels in real-time, adjust schedules, and allocate resources efficiently can minimize customer waiting times and maximize CSR

productivity. This operational flexibility is essential for meeting fluctuating call volumes efficiently.