

Concept Check #2: Queueing

Instructions for this and all subsequent concept checks:

- Submit the assignment as a quiz in bCourses; uploaded PDFs will not be graded unless the associated quiz has been completed.
- To receive partial credit for incorrect numeric answers, please upload a PDF of your work, which can be handwritten or typed. The GSI Team must be able to clearly follow and understand your work to award partial credit.
- If you make any assumptions, state them clearly.
- If you complete the assignment with a partner, use the first question in the bCourses quiz to list your partner. Only one partner should submit. If you and your partner are in different cohorts, you must submit by the earlier deadline.
- Prior to the due date, discuss the assignment with no one other than your partner.
- There is no time limit and you will have a single attempt to complete the “quiz,” so work through the entire assignment prior to submission.

Question 1. The Right to Vote (and not to wait) (10 points)

In the recent presidential elections in the US, very long wait times have been witnessed at precincts (voting stations) in states that ultimately decided the election (Florida in 2000 and Ohio in 2004).

In Philadelphia as well, some voters complained about the long lines in some precincts, with most complaints coming from precinct A. In 2004, the average number of votes arriving at Precinct A was 35 per hour, and the arrivals of voters were random with inter-arrival times that had a coefficient of variation of 1.0 ($CV_a=1.0$). Philadelphia had deployed 1 voting machine in Precinct A. Suppose that each voter spent on average 100 seconds in the voting booth (this is the time needed to cast his/her vote using a voting machine), with a standard deviation of 120 seconds. How long on average did a voter have to wait in line at precinct A in 2004 *before* entering in a booth to cast his/her vote?

Question 2. Convenience store (20 points)

You run a convenience store at a beach resort with a single check-out and no employees except yourself. For several hours a day during the peak summer period (which lasts several weeks) customers arrive at your check-out at an average rate of 48 per hour. The arrival process is Poisson with $CV_a=1$. Your utilization rate is 0.8, and the standard deviation of your service time distribution is 2 minutes.

- (a) What is the average time (in minutes) it takes you to process a customer at the checkout? (Hint: consider the utilisation and customer arrival rate) (4 points)
- (b) What is the average length of the checkout queue? (8 points)
- (c) List **two** practical steps you could take to reduce the waiting time during this period. (8 points)

Question 3. Government Department (20 points)

A government department has outsourced its call center operations to *Ultra*, a firm that dedicates 7 customer service representatives (CSRs) to the department during office hours. Calls arrive randomly, with an average inter-arrival time of 45 seconds (i.e., 0.75 minutes), and are directed to the CSRs, who are pooled. Historical records show the average and standard deviation of the processing time (not including waiting) are both 5.0 minutes.

- (a) What is the utilization of the CSRs? (3 points)
- (b) What is the average time (in minutes) a customer waits before being served? (6 points)
- (c) What is the average number of customers waiting to be served? (3 points)
- (d) The department is considering outsourcing to a different provider, *Alpha*, which would dedicate more staff to the task than *Ultra* does, but the productivity of each CSR is correspondingly lower – indeed, the ratio of the average processing times of *Alpha* and *Ultra* CSRs is the **same** as the ratio of the number of staff members at *Alpha* compared to *Ultra*.
 - i. Would the *Alpha* contract average customer waiting time be lower, the same, or higher than for the *Ultra* contract? Explain your answer. (4 points)
 - ii. Assuming the department pays any provider the same amount per customer served, state **two** other criteria (one related to time) that the department should consider before making the decision to switch or not. (4 points)

Shortly after you finish this Concept Check, you should work through at least a subset of questions 11 to 14 in *Practice Problems for Parts I and II of the Course*, which is on bCourses. The exam will include questions that are more difficult than those in the keep-up-with-the-main-ideas Concept Checks. You should anticipate that at least some exam questions will be on par with the more challenging practice problems that are provided.