Simpson Aerospace: Solution to Homework 2

Objective: Solution to Homework 2 for Orbit Determination Course

TO: Statistical Orbit Determination Class
CC:
REF:

Date: 8 Feb 2019
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SUMMARY:

Demonstrate understanding of orbital mechanics necessary to complete orbit determination course. In problem 1, position and velocity are converted between osculating elements and sub-satellite points. In problem 2, the receiver measurements confirm the node location varies over time. In problem 3 the equations of motion are numerically integrated for a GLONASS satellite for one day.

OVERVIEW:

Problem 1

Given the following position and velocity of a satellite expressed in a non-rotating geocentric coordinate system:

	Position (m)	Velocity (m/s)
X	7088580.789	-10.20544809
Y	-64.326	-522.85385193
Z	920.514	7482.075141

a) Determine the six orbital elements (a, e, i, Ω, ω, M_0)

a	e	i	Ω	ω	M_0
7091555 m	0.0013	94.00 deg	0.00 deg	-288.74 deg	-71.11 deg

b) Assuming X_0 is given and two-body motion, predict position and velocity at $t = 3{,}000$ sec. Determine flight path angle at this time.

	Position (m)	Velocity (m/s)
X	-7090459	253.3242
Y	17321	522.0966
Z	-247858	-7471.2

The flight path angle is $\phi = \text{atan}(e \sin f \,/ 1 + e \cos f)$ is $\phi = -0.0203°$.

c) Determine the latitude and longitude of the subsatellite point for $t = 3{,}000$ sec if α_G at $t = 0$ is 0. Assume the Z axis of the nonrotating system is coincident with the z axis of the rotating system.

t (min)	ϕ(deg)	λ(deg)	h(m)
50	-2.002	167.326	716673.70

Problem 2

Orbit of CRISTA-SPAS-2: Cryogenic Infrared Spectrometers and Telescopes for the Atmosphere

The joint venture of DLR and NASA, the small free-flying satellite contains three telescopes, four spectrometers, and a GPS receiver on-board. It is deployed from the shuttle Discovery on STS-85 in August 1997. Using on-board navigation, the receiver measurements are processed in an Earth-centered, Earth-fixed coordinate system.

August 18, 1997

GPS-T (hrs:min:sec)	00:00:0.000000	00:00:03.000000
x	3325396.441	3309747.175
y	5472597.483	5485240.159
z	-2057129.050	-2048664.333

August 19, 1997

GPS-T (hrs:min:sec)	00:00:0.000000	00:00:03.000000
x	4389882.255	4402505.030
y	-4444406.953	-4428002.728
z	-2508462.520	-2515303.456

a) Demonstrate that the node location is not fixed in space and determine an approximate rate of node change (degrees/day) from these positions. Compare the node rate with the value predicted by

$$\dot{\Omega} = -\frac{3}{2}J_2 \frac{n}{(1-e^2)^2}\left(\frac{a_e}{a}\right)^2 \cos i$$

The node rate from calculating the node location for the three second interval during August 18 and August 19 provides an ascending node rate of $\frac{d\Omega}{dt} = 4.5073$ deg/day. The secular perturbation predicts a change of $\frac{d\Omega}{dt} = -10.5038$ deg/day. The secular perturbation fails to adequately approximate the node rate because of the strength of the periodic variations.

b) Determine the inclination of CRISTA-SPAS-2 during the first 3-sec interval and the last 3-sec interval.

The inclination, i, during the first 3-sec interval is $i = 29.5694°$ and during the last 3-sec interval is $i = 29.1904°$. This is determined by finding the unit vector orthogonal to the orbital plane that the two position vectors in each interval sweep out. Having found the unit vector orthogonal to the plane, the dot product rule is used to determine the inclination of the plane.

Comment: The position vectors determined by GPS in this case are influenced at the 100-meter level by Selective Ability, but the error does not significantly affect this problem.

Problem 3

GLONASS: Russia's answer for American GPS

Given a set of initial conditions for a high-altitude GLONASS satellite, numerically integrate the equations of motion for one day.

a) Assuming the satellite is influenced by J_2 only, derive the equations of motion in non-rotation coordinates. Assume the nonrotating Z axis coincides with the Earth-fixed z axis.

$$\ddot{\bar{r}} = \nabla U = -\left(\frac{\mu}{r^3}\right)\bar{r} + \bar{f}_{NS}$$

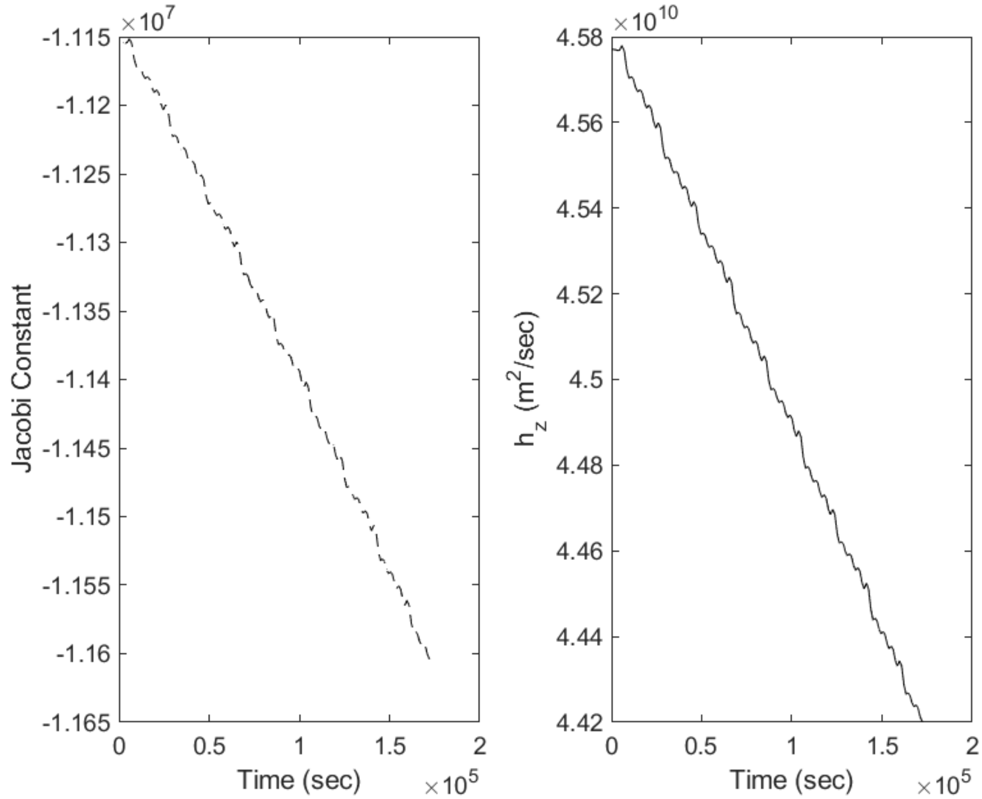
$$\bar{f}_{NS} = T_{xyz}^{XYZ} T_{r\phi\lambda}^{xyz} \nabla U'$$

$$U' = -\frac{\mu a_e^2}{r^3} J_2 P_2(\sin\phi)$$

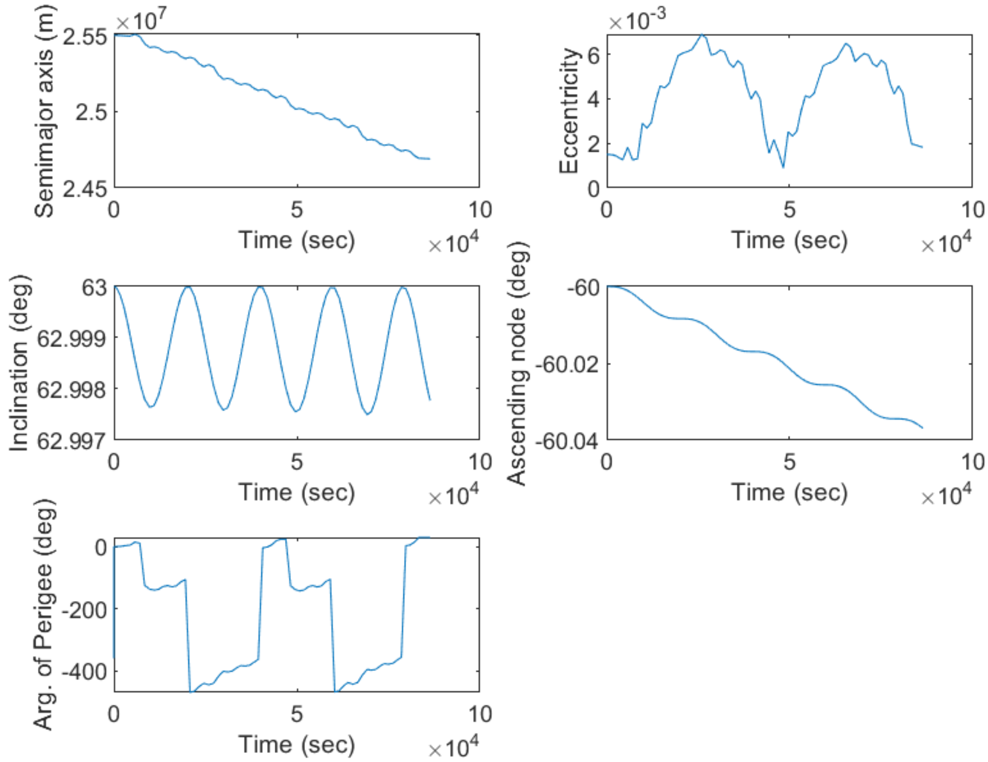
$$\nabla U' = \frac{\partial U'}{\partial r}\bar{u}_r + \frac{1}{r}\left(\frac{\partial U'}{\partial \phi}\right)\bar{u}_\phi + \left(\frac{1}{r\cos\phi}\right)\left(\frac{\partial U'}{\partial \lambda}\right)\bar{u}_\lambda$$

b) During the integration, compute the Jacobi constant and the Z component of the angular momentum. Are these quantities constant?

No, these quantities are not constant. As should be expected not only from the variation of kinematic parameters but from the application of perturbations to the acceleration. See the plots for the Jacobi constant and h_z quantities of the satellite over two days.



c) Plot the six orbital elements as a function of time.



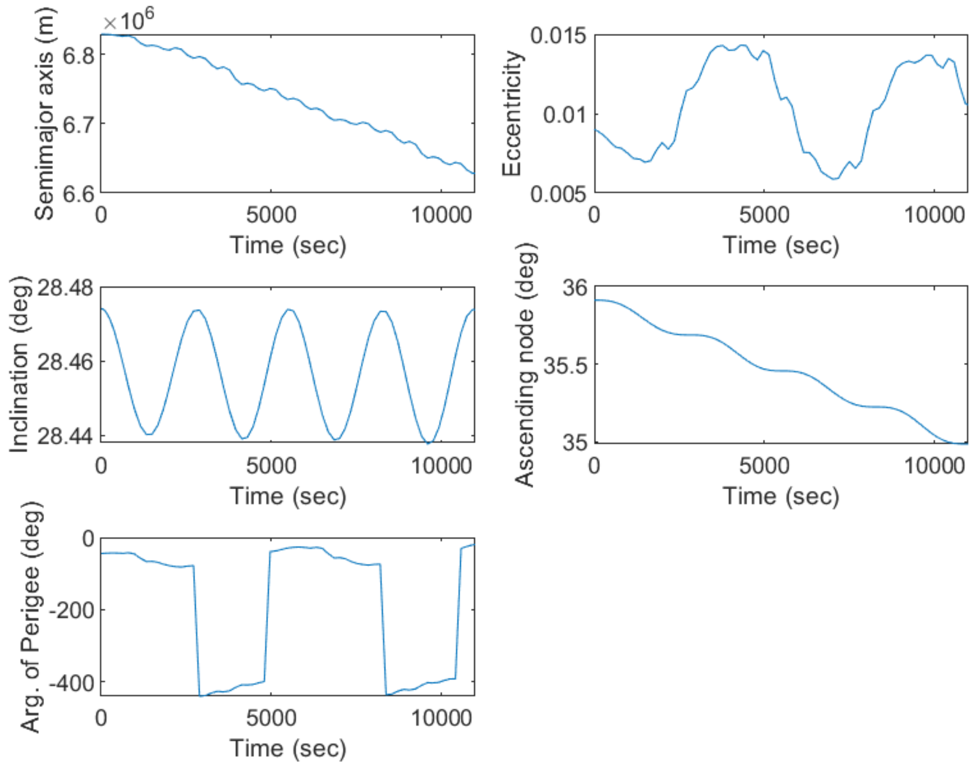
See Appendix A for expanded versions of the plots.

d) Identify features similar to and different from Fig. 2.3.5

When we maintain the same period of about 11,000 seconds; long-term periodic and secular variations in eccentricity, e, semimajor axis, a, inclination, i, and right angular ascending node, Ω are similar to the features in Fig. 2.3.5. There is no short-term periodic variation that can be recognized in $e, a, i,$ or Ω for the GLONASS satellite. For the argument of perigee, ω, there is some small secular variation but there is no obvious short-term nor long-term periodic variation.

When the period is extended to a full 24 hours; the long-term and short-term periodic and secular variational behavior is the same as in Fig. 2.3.5. A longer period is required for the orbital elements to display similar behavior because of the larger semimajor axis, closeness to the critical inclinations, and small argument of perigee.

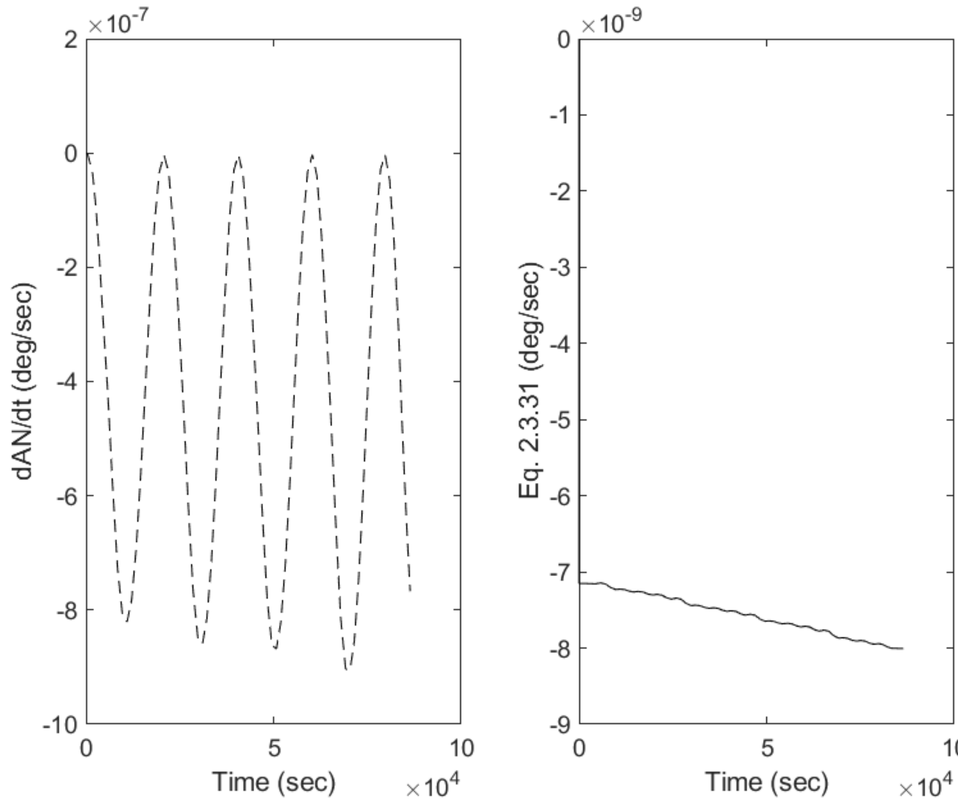
See the behavior from Fig. 2.3.5 reproduced in the plots below.



e) Compare the node rate predicted by

$$\dot{\Omega} = -\frac{3}{2} J_2 \, \frac{n}{(1 - e^2)^2} \left(\frac{a_e}{a}\right)^2 \cos i$$

with a value estimated from (c).

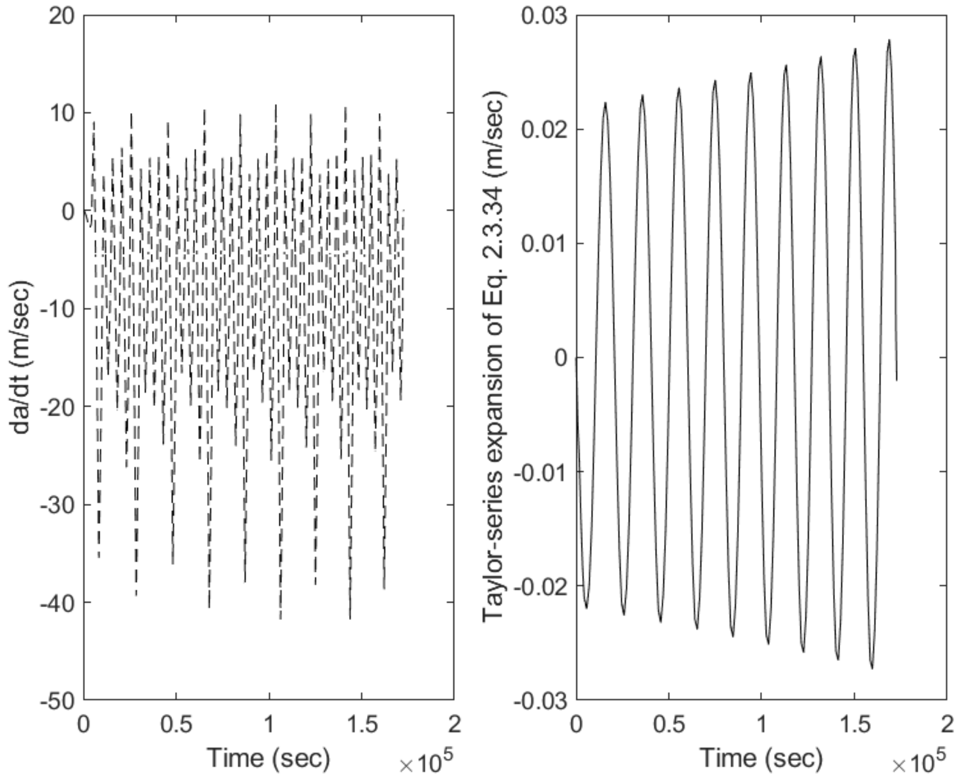


The secular rate from Eq. 2.3.31 will not successfully capture the amplitude of the changes in Ω. Note the two orders of magnitude difference in the rate plotted.

f) Compare the amplitude of the semimajor axis periodic term with

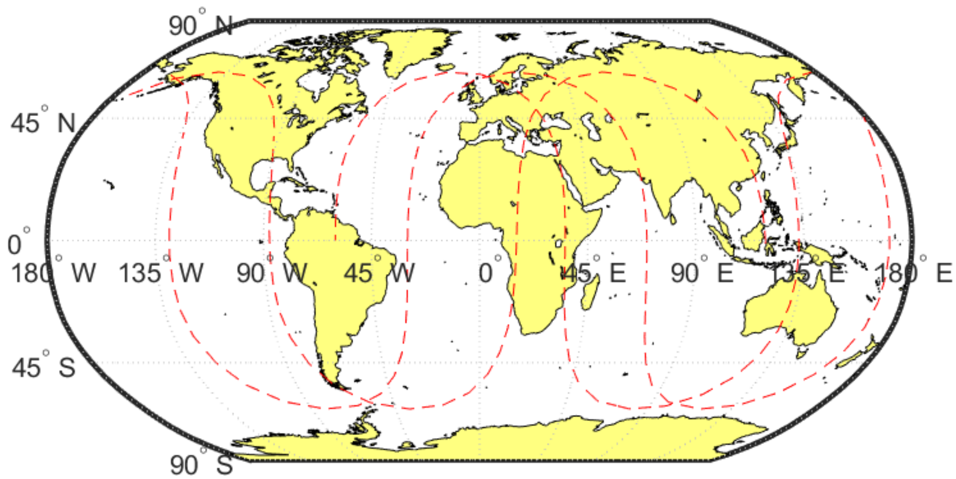
$$a(t) = \bar{a} + 3\bar{n}\bar{a}J_2\left(\frac{a_e}{\bar{a}}\right)^2 \sin^2 \frac{\bar{\imath}(\cos(2\omega + 2M))}{2\dot{\omega}_s + 2\dot{M}_s}$$

We compare the amplitude by expanding the periodic term using first order Taylor-series expansion.



The Taylor-series expansion of Eq. 2.3.34 fails to capture the magnitude of variation by an order of magnitude. The actual semimajor axis rate calculated is centered around -10 m/sec whereas the analytical rate is centered around 0 m/sec.

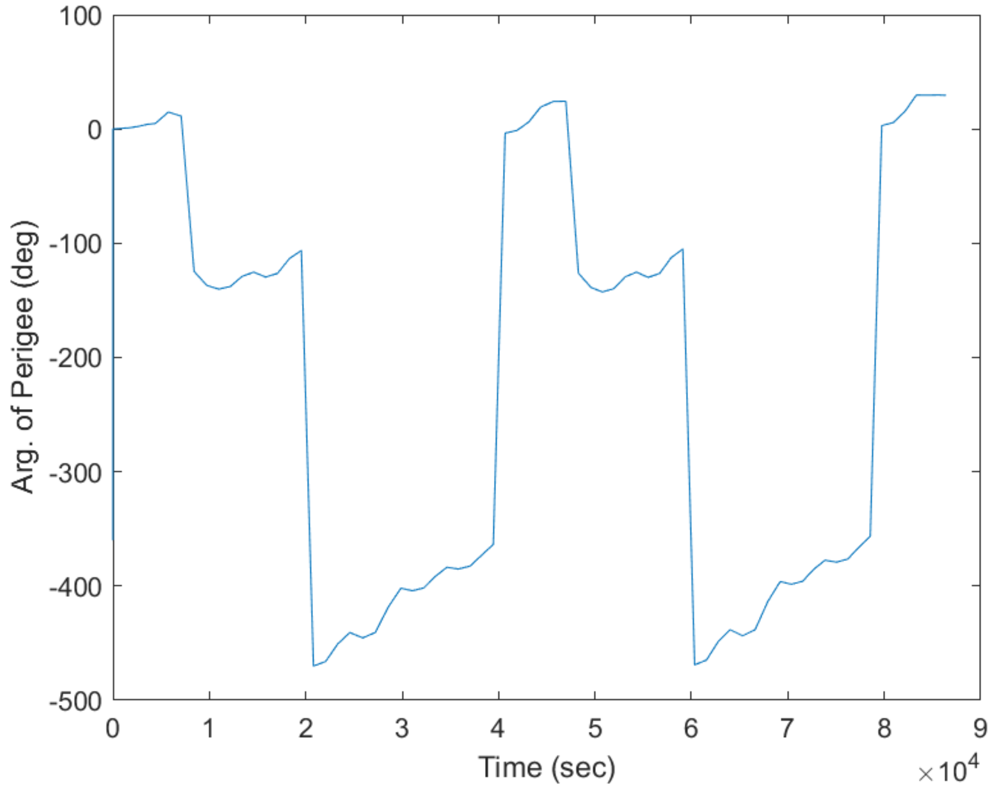
g) Plot the ground track. Does the ground track repeat after one day?
No, the ground track does not repeat. See the ground track plot for two days from the initial state provided.



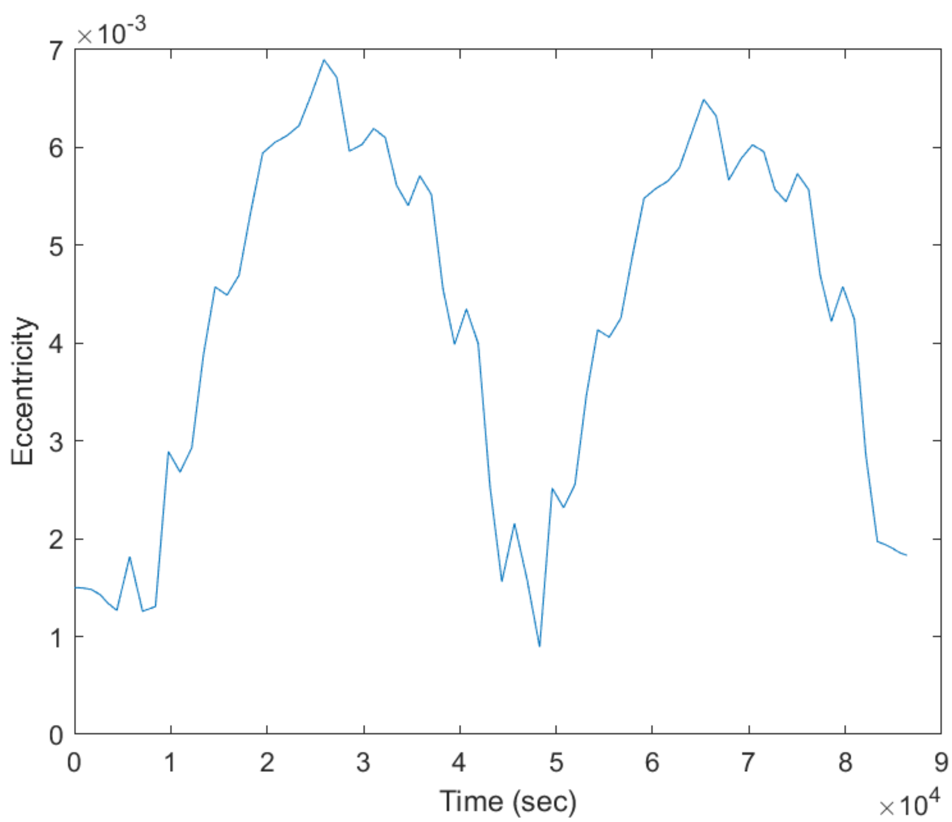
a	e	i	Ω	ω	M_0
25500.0 km	0.0015	63 deg	-60 deg	0 deg	0 deg

APPENDIX A: PROBLEM 3 PLOTS

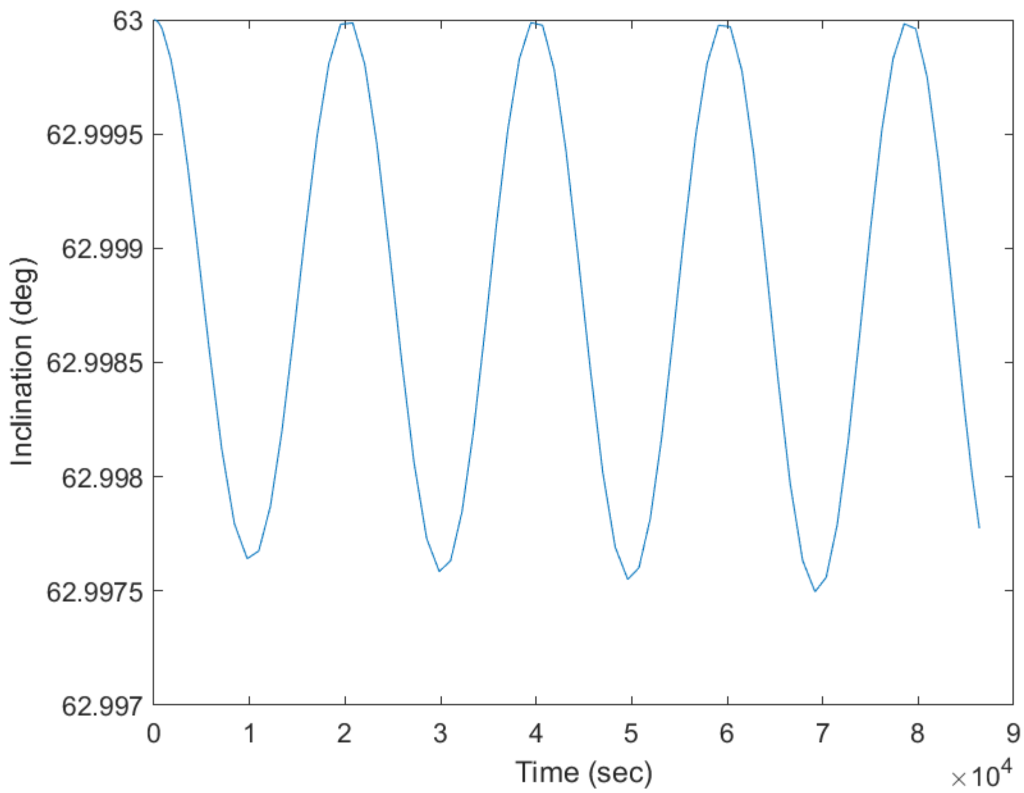
Orbital elements over time



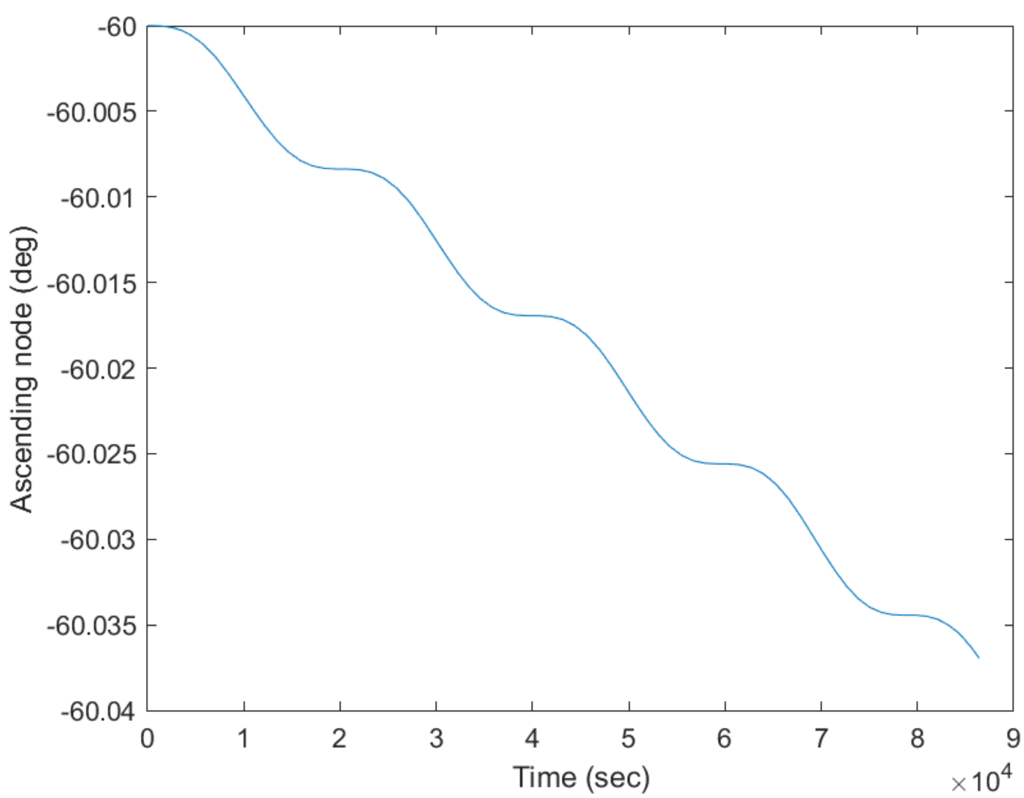
100
0
-100
-200
-300
-400
-500
Arg. of Perigee (deg)
0
1
2
3
4
5
6
7
8
9
Time (sec)
×10^4



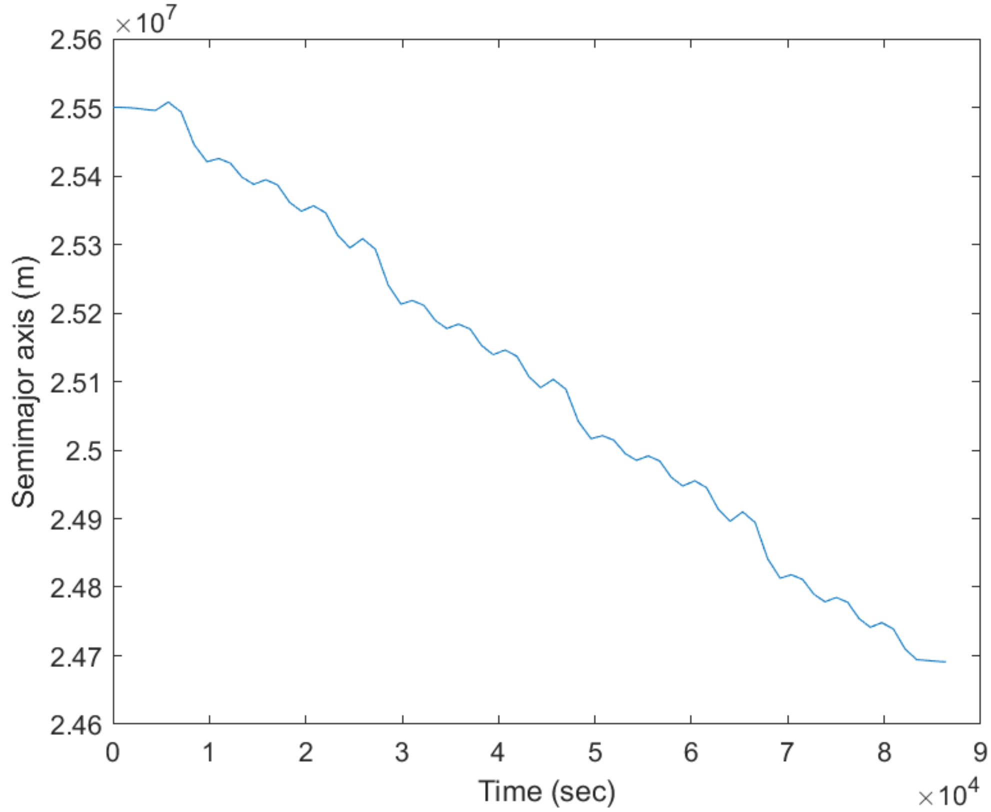
×10^-3
7
6
5
4
3
2
1
0
Eccentricity
0
1
2
3
4
5
6
7
8
9
Time (sec)
×10^4



63
62.9995
62.999
62.9985
62.998
62.9975
62.997
Inclination (deg)
0
1
2
3
4
5
6
7
8
9
Time (sec)
×10^4



-60
-60.005
-60.01
-60.015
-60.02
-60.025
-60.03
-60.035
-60.04
Ascending node (deg)
0
1
2
3
4
5
6
7
8
9
Time (sec)
×10^4



×10^7
2.56
2.55
2.54
2.53
2.52
2.51
2.5
2.49
2.48
2.47
2.46
Semimajor axis (m)
0
1
2
3
4
5
6
7
8
9
Time (sec)
×10^4