Adaptively Tuned Particle Swarm Optimization for Spatial Design

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Overview of the Talk

1. What is particle swarm optimization (PSO)?
   (Blum and Li, 2008; Clerc, 2010, 2012)
2. New adaptively-tuned PSO algorithms.
3. Using (adaptively-tuned) PSO for spatial design.
4. Example adding to an existing monitoring network.

Particle Swarm Optimization — Intuition

Mimic animal flocking behavior.
(Animation Here)

Particle Swarm Optimization

Goal: minimize some objective function $Q(\boldsymbol{\theta}) : \mathbb{R}^D \to \mathbb{R}$.

Populate Θ with n particles. Define particle i in period k by:

- a location $\boldsymbol{\theta}_i(k) \in \mathbb{R}^D$;
- a velocity $\boldsymbol{v}_i(k) \in \mathbb{R}^D$;
- a personal best location $\boldsymbol{p}_i(k) \in \mathbb{R}^D$;
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Basic PSO: update particle i from k to $k+1$ via:

- For $j = 1, 2, \ldots, D$:

$$\begin{aligned} v_{ij}(k+1) &= \omega v_{ij}(k) + \mathrm{U}(0, \phi_1) \times \{p_{ij}(k) - \theta_{ij}(k)\} \\ &\qquad + \mathrm{U}(0, \phi_2) \times \{g_{ij}(k) - \theta_{ij}(k)\} \\ &= \text{inertia} + \text{cognitive} + \text{social}, \\ \theta_{ij}(k+1) &= \theta_{ij}(k) + v_{ij}(k+1), \end{aligned}$$

- Then update personal and group best locations.

PSO — Parameters

Inertia parameter: ω.

- Controls the particle's tendency to keep moving in the same direction.

Cognitive correction factor: ϕ_1.

- Controls the particle's tendency to move toward its personal best.

Social correction factor: ϕ_2.

- Controls the particle's tendency to move toward its neighborhood best.

Default choices:

- $\omega = 0.7298$, $\phi_1 = \phi_2 = 1.496$ (Clerc and Kennedy, 2002).
- $\omega = 1/(2\ln 2) \approx 0.721$, $\phi_1 = \phi_2 = 1/2 + \ln 2 \approx 1.193$ (Clerc, 2006).

PSO — Neighborhood Topologies

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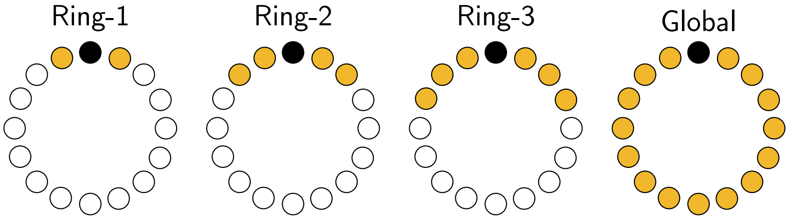
Easy to visualize example: Ring-k neighborhood topology.



Each particle is informed by k neighbors to the left and k to the right.

Stochastic Star Topology, and Other Bells and Whistles

We use the stochastic star neighborhood topology (Miranda et al., 2008).

- Each particle informs itself and m random particles.
  $\rightarrow$ sampled with replacement once during initialization.
- On average each particle is informed by m particles.
- A small number of particles will be informed by many particles.

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Many variants available (Clerc, 2012), (Simpson et al., 2017, appendix).

- Handling search space constraints.
- Coordinate free velocity updates.
- Parallelization.
- Asynchronous updates.
- Redraw neighborhoods.

Bare Bones PSO (BBPSO)

Developed by Kennedy (2003).

Strips out the velocity term:

$$\theta_{ij}(k+1) \sim \mathrm{N}\left(\frac{p_{ij}(k)+g_{ij}(k)}{2}, |p_{ij}(k)-g_{ij}(k)|^2\right).$$

Mimics the behavior of standard PSO.

Easier to analyze, but tends to perform worse.

Adaptively Tuned BBPSO

Add flexibility to the scale parameter:

$$\theta_{ij}(k+1) \sim \mathrm{T}_{df}\left(\frac{p_{ij}(k)+g_{ij}(k)}{2}, \sigma^2(k)|p_{ij}(k)-g_{ij}(k)|^2\right).$$

with e.g. $df = 1$ by default.

- Larger $\sigma^2(k)$: more exploration.
- Smaller $\sigma^2(k)$: more exploitation.

How to choose $\sigma^2(k)$'s progression?

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Analogy with adaptively tuned random walk Metropolis.
(Andrieu and Thoms, 2008)

Adaptively Tuned BBPSO — $\sigma^2(k)$'s progression

Define the improvement rate of the swarm in period k:

$$R(k) = \text{proportion of particles that improved on their personal best last period.}$$

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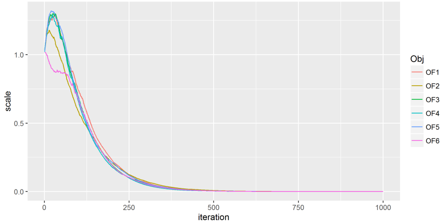
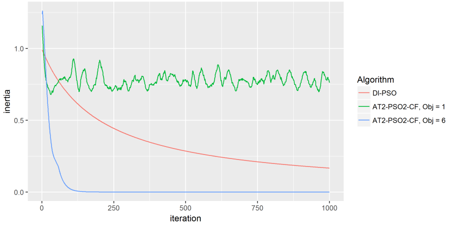
Similar PSO algorithm in spirit: Zhang et al. (2003).

- ω is constant while ϕ_1 and ϕ_2 vary across time *and particle*.
- Harder to have intuition for choosing how ϕ_1 and ϕ_2 adapt.

Example progressions of $\sigma^2(k)$ (left) and $\omega(k)$ (right):





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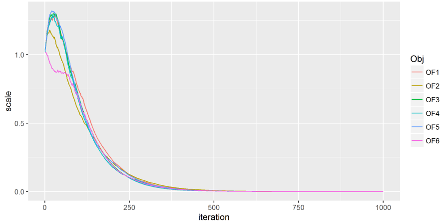

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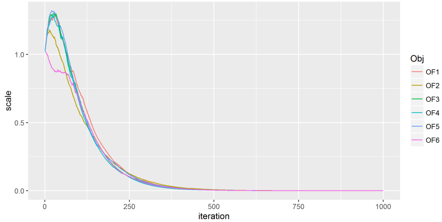

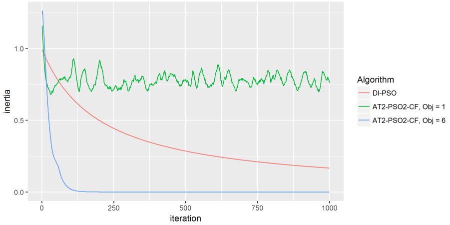

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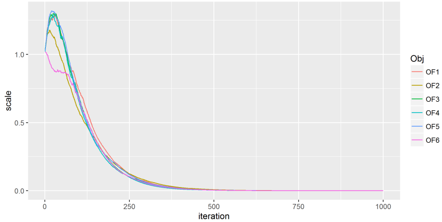

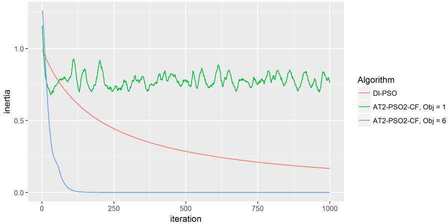

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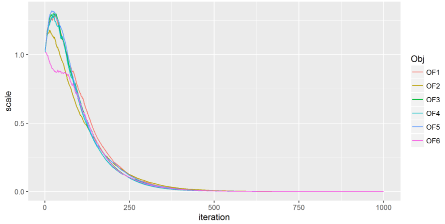

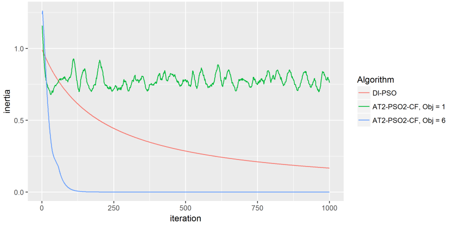

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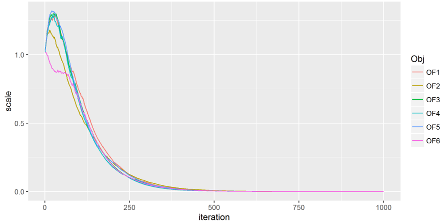

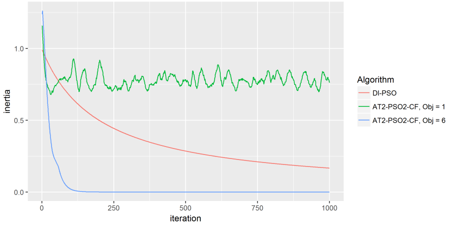

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- AT-PSO's inertia crashes to zero when it converges (may be premature local convergence).

Comparing AT-PSO/BBPSO to PSO/BBPSO

Tuning $\omega(k)/\sigma^2(k)$ allows the swarm to adjust the exploration / exploitation tradeoff based on local conditions.

- This has a tendency to speed up convergence.
- ...but convergence may be premature in multi-modal problems.

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Thank you!

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