Adaptively-Tuned Particle Swarm Optimization with Application to Spatial Design

A PSO and BBPSO details

In Section 2 we introduced PSO, BBPSO, and our adaptively tuned variants of both. Here we describe in detail several BBPSO variants as well as a class of neighborhood topologies called the ring topologies.

A.1 BBPSO

The standard BBPSO algorithm was introduced by Kennedy (2003) and updates from t to $t+1$ via equation (2). A commonly used variant of BBPSO also introduced by Kennedy (2003) is called BBPSOxp. In this variant, each coordinate of each particle has a 50% chance of updating according to (2) and a 50% chance of moving directly to that particle's personal best location on that coordinate. In other words

$$\theta_{ij}(t+1) = \begin{cases} N\left(\frac{p_{ij}(t)+g_{ij}(t)}{2}, s_{ij}^2(t)\right) & \text{with probability } 0.5 \\ p_{ij}(t) & \text{otherwise,} \end{cases} \tag{A.1}$$

where $s_{ij}(t) = |p_{ij}(t) - g_{ij}(t)|$.

A downside of both BBPSO and BBPSOxp is that any particle whose personal best is currently its group best location does not move due to the definition of the standard deviation term. Several methods have been proposed to overcome this; e.g. Hsieh and Lee (2010) and Zhang et al. (2011). Zhang et al. (2011) propose using mutation and crossover operations for the group best particle. To do this, each group best particle randomly selects three other distinct particles from the entire swarm, i_1, i_2, and i_3, and updates according to

$$\theta_{ij}(t+1) = p_{i_1 j}(t) + 0.5\{p_{i_2 j}(t) - p_{i_3 j}(t)\}. \tag{A.2}$$

This combines easily with BBPSOxp to create BBPSOxp-MC by updating group best particles according to (A.2) and all other particles according to (A.1), but it does not completely solve the problem in BBPSOxp-MC. A particle which finds a personal best that is the same as its group best on some but not all coordinates due to the "xp" component of BBPSOxp-MC will have a standard deviation of zero for those coordinates during the next iteration. This is easily overcome by using the "MC" operation for those coordinates as well. So both BBPSOxp-MC and BBPSO-MC can be encapsulated into the following general BBPSO algorithm. For each particle i, sample three other particles i_1, i_2, i_3 from the swarm, and for each coordinate j let $s_{ij}(t) = |p_{ij}(t) - g_{ij}(t)|$. Then the general updating equation is

$$\theta_{ij}(t+1) = \begin{cases} \begin{cases} N\left(\frac{p_{ij}(t)+g_{ij}(t)}{2}, s_{ij}^2(t)\right) & \text{with probability } \rho \\ p_{ij}(t) & \text{with probability } 1-\rho, \end{cases} & \text{if } s_{ij}^2(t) > 0 \\ p_{i_1 j}(t) + 0.5\{p_{i_2 j}(t) - p_{i_3 j}(t)\} & \text{otherwise.} \end{cases} \tag{A.3}$$

When $\rho = 0$ we have BBPSO-MC, and when $\rho = 0.5$ we have BBPSOxp-MC. These are the two BBPSO variants that we consider.

Similarly, we can create adaptively tuned versions of BBPSO-MC and BBPSOxp-MC by analogy with (3) and (A.3):

$$\theta_{ij}(t+1) = \begin{cases} \begin{cases} t_{df}\left(\frac{p_{ij}(t)+g_{ij}(t)}{2}, \sigma^2(t) s_{ij}^2(t)\right) & \text{with probability } \rho \\ p_{ij}(t) & \text{with probability } 1-\rho, \end{cases} & \text{if } s_{ij}^2(t) > 0 \\ p_{i_1 j}(t) + 0.5\{p_{i_2 j}(t) - p_{i_3 j}(t)\} & \text{otherwise.} \end{cases}$$

$$\log \sigma^2(t+1) = \log \sigma^2(t) + c \times \{R(t+1) - R^*\},$$

where $R(t) = \#\{i : Q(\boldsymbol{p}_i(t)) > Q(\boldsymbol{p}_i(t-1))\}/n$, and df, R^*, and c are defined in Section 2.1. Then AT-BBPSO-MC sets $\rho = 0$ and AT-BBPSOxp-MC sets $\rho = 0.5$.

B Comparing PSO and BBPSO algorithms

In order to compare AT-BBPSO to other PSO variants, we employ a subset of test functions used in Hsieh and Lee (2010). Each function is listed in Table 1 along with the global maximum and argmax, and the initialization range for the simulations. Further description of many of these functions can be found in Clerc (2010). For each function, we set $D = 20$ so the domain of each function is $\Re^{20}$. For each function, the PSO algorithms are initialized in a range that does not contain the true maximum.

Equation	ArgMax	Maximum	Initialization
$Q_1(\boldsymbol{\theta}) = -\sum_{i=1}^{D}\theta_i^2$	$\boldsymbol{\theta}^* = \mathbf{0}$	$Q_1(\boldsymbol{\theta}^*) = 0$	$(50, 100)^D$
$Q_2(\boldsymbol{\theta}) = -\sum_{i=1}^{D}\left(\sum_{j=1}^{i}\theta_j\right)^2$	$\boldsymbol{\theta}^* = \mathbf{0}$	$Q_2(\boldsymbol{\theta}^*) = 0$	$(50, 100)^D$
$Q_3(\boldsymbol{\theta}) = -\sum_{i=1}^{D-1}\left[100\{\theta_{i+1} + 1 - (\theta_i + 1)^2\} + \theta_i^2\right]$	$\boldsymbol{\theta}^* = \mathbf{0}$	$Q_3(\boldsymbol{\theta}^*) = 0$	$(15, 30)^D$
$Q_4(\boldsymbol{\theta}) = 9D - \sum_{i=1}^{D}\{\theta_i^2 - \cos(2\pi\theta_i) + 10\}$	$\boldsymbol{\theta}^* = \mathbf{0}$	$Q_4(\boldsymbol{\theta}^*) = 0$	$(2.56, 5.12)^D$
$Q_5(\boldsymbol{\theta}) = -\frac{1}{4000}\|\|\boldsymbol{\theta}\|\|^2 + \prod_{i=1}^{D}\cos\left(\frac{\theta_i}{\sqrt{i}}\right) - 1$	$\boldsymbol{\theta}^* = \mathbf{0}$	$Q_5(\boldsymbol{\theta}^*) = 0$	$(300, 600)^D$
$Q_6(\boldsymbol{\theta}) = 20\exp\left(-0.2\sqrt{\frac{1}{D}\|\|\boldsymbol{\theta}\|\|}\right)$			
$+\exp\left\{\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi\theta_i)\right\} - 20 - \exp(1)$	$\boldsymbol{\theta}^* = \mathbf{0}$	$Q_6(\boldsymbol{\theta}^*) = 0$	$(16, 32)^D$

Table 1: Test functions for evaluating PSO algorithms. The dimension of $\boldsymbol{\theta}$ is D and $||.||$ is the Euclidean norm: $||\boldsymbol{\theta}|| = \sqrt{\sum_{i=1}^{D}\theta_i^2}$.

We use several PSO algorithms in the simulation study. The standard PSO algorithm uses the parameter values suggested by Blum and Li (2008) and Clerc and Kennedy (2002). The AT-BBPSO variants are implemented a wide variety of parameter values, but all have the scale parameter initialized at $\sigma(0) = 1$, and both set $c = 0.1$. The AT-PSO variants are initialized at $\omega(0) = 1$ and also set $c = 0.1$. In addition, each algorithm is implemented using each of three neighborhood structures — the global, ring-3, and ring-1 neighborhoods. Each algorithm was used to optimize each objective function for 500 iterations over 50 replications using 20 particles. Initializations were changed across replications but held constant across algorithms. The standard PSO, DI-PSO, and AT-PSO algorithms initialized their velocity terms using the same method as a function of the initial locations of the particles, which we will denote by $\boldsymbol{x}_i$ for particle i. Let $x_{max,j}$ be the maximum initial value of coordinate j of $\boldsymbol{x}_i$ for each particle i, and let $x_{min,j}$ be the corresponding minimum. Then let $d_{max} = max_j x_{max,j} - x_{min,j}$. Then we initialize the velocities with $v_{ij}(0) \overset{iid}{\sim} U(-d_{max}/2, d_{max}/2)$. Tables 2-7 contain the simulation results for objective functions 1-6 respectively (OF1, OF2, etc.). We use several measures to quantify how well each algorithm finds the global maximum. First, each table includes the mean and standard deviation of the absolute difference between the true global maximum and the algorithm's estimated global maximum across all 50 replications, denoted by Mean and SD. Second, each table includes a convergence criterion — the proportion of the replications that came within 0.01 of the true global maximum, denoted by $\widehat{p}$. Finally, $\widehat{t}$ denotes the median number of iterations until the algorithm reaches the convergence criterion. When $\hat{p} < 0.5$ then $\hat{t} = \infty$ since greater than 50% of the replications did not converge in the maximum number of iterations allowed. Then Mean, $\hat{p}$, and $\hat{t}$ can be thought of how close the algorithm gets to the global maximum on average, what proportion of the time it converges, and long it takes to converge respectively.

We highlight only some of the features of these tables. First and foremost, PSO, BBPSO-MC, and BBPSOxp-MC almost always do worse than their AT cousins. Adaptively tuning either the scale or inertia parameter leads to gains in all three of our measures, sometimes large. The comparison is starkest between BBPSO variants and AT-BBPSO variants, partially because BBPSO tends to be pretty bad but also because AT-BBPSO does very well. Second, for most non-AT algorithms the more restrictive neighborhoods appear to yield algorithms which do a better job of finding the global max. For the AT-PSO algorithms, it appears that the target improvement rate (R^*) and the neighborhood interact. When the rate is high a more restrictive neighborhood is preferable, while when the rate is low a less restrictive neighborhood is preferable. On the other hand, for the AT-BBPSO algorithms, the opposite appears to be true — when the target improvement rate is high, a less restrictive neighborhood is desirable and vice versa. In addition, the best AT-BBPSO algorithms often use the global neighborhood while for the other classes of algorithms their best versions typically use the ring-1 or ring-3 neighborhood.

For the DI-PSO algorithms often there is a parameter-neighborhood combination that does well, typically from setting $\alpha = 200$ (20% of the 500 iterations) and $\beta = 1$ and using either the ring-1 or ring-3 neighborhood. One of these combinations typically does the best of all the DI-PSO algorithms but they can sometimes still do much worse than the best alternatives (e.g., for OF2). In the AT-BBPSO algorithms, it is not always clear what the best parameter settings are, but good default values appear to be $df = 3$ or 5 and $R^* = 0.3$ or 0.5. When these values are good, they often lead to the best performing PSO algorithms we consider. However, it is not always clear whether to use AT-BBPSO-MC or AT-BBPSOxp-MC. For some objective functions, e.g. OF4, the xp version is consistently better than the non-xp alternative, but the opposite is true for others, e.g. OF2.. AT-PSO is also often very competitive with $R^* = 0.3$ or $R^* = 0.5$, though again sometimes different parameter settings also appear to work well.

The DI-PSO and AT-PSO algorithms are similar conceptually, but often yield very different results. DI-PSO deterministically reduces the inertia parameter over time in the same manner given a fixed set of parameter values (α and β), while AT-PSO dynamically adjusts the inertia parameter to hit a target improvement rate. Figure B.1 plots the inertia over time for the DI-PSO algorithm with $\alpha = 200$ and $\beta = 1$, and observed inertia over time for one replication of the AT-PSO algorithm with target rate $R^* = 0.5$ and ring-1 neighborhood for OF1 and one replication for OF6. All three algorithms have an initial inertia of $\omega(0) = 1$. While DI-PSO smoothly decreases its inertia with a slowly decreasing rate, AT-PSO very quickly drops its inertia for OF1 to about 0.55 then bounces around around near that point. It also jumps up above 1 initially, imploring the particles to cast a wider net in search of higher value areas of the search space. This is pretty typical behavior for the inertia parameter of AT-PSO — it tends to bounce around a level which is approximately the average over time of the DI-PSO's inertia, though lower values of R^* will result in higher inertias. In this way, AT-PSO alternates periods of exploration (relatively high inertia) and periods of exploitation (relatively low inertia). The main exception to this pattern is when AT-PSO converges around a local maximum. In this case, inertia plummets to zero as the particles settle down. This is precisely what happens for OF6 in Figure B.1, though in this case the maximum is not global — Table 7 indicates that ring-1 AT-PSO with $R^* = 0.5$ never converged to the global max. In optimization problems with multiple local optima, both AT-PSO and AT-BBPSO variants can exhibit this behavior and prematurely converge to a local optima, so they may not be advantageous for those problems.

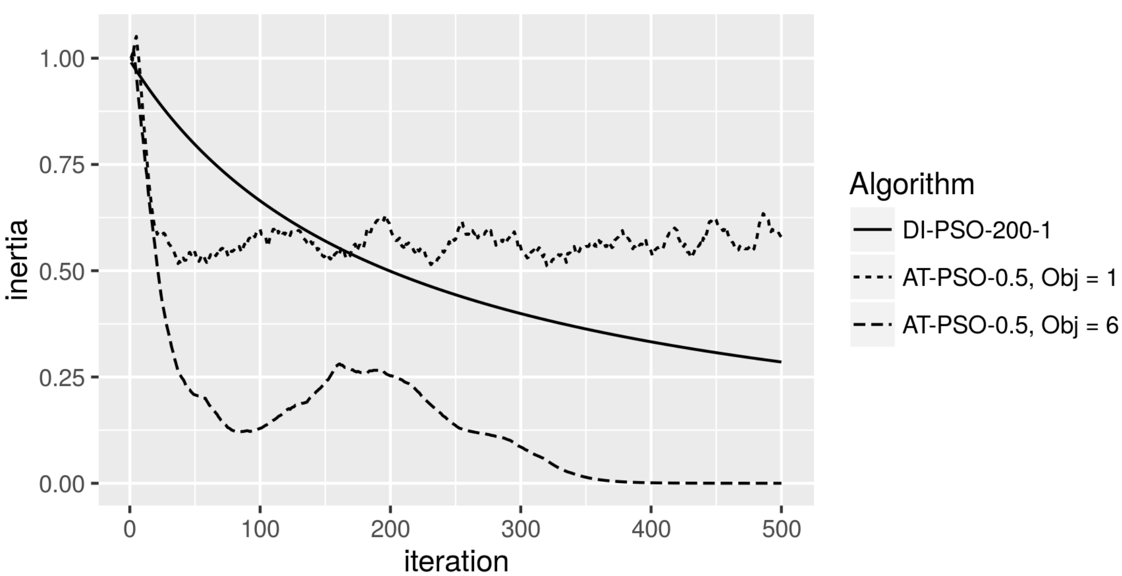

Figure B.1: Inertia over time for the DI-PSO algorithm with $\alpha = 200$ and $\beta = 1$, and for one replication of the AT-PSO-0.5 algorithm for each of OFs 1 and 6.

Based on these simulations, our default recommendation is to use AT-BBPSO-MC or AT-BBPSOxp-MC with $R^* = 0.3$ or 0.5 and $df = 3$ or $df = 5$ along with the global neighborhood. These algorithms will not always be the best of the PSO algorithms, but they will often be very good. AT-PSO with $R^* = 0.3$ or $R^* = 0.5$ with a restrictive neighborhood topology such as ring-3 also tends to be a very good choice, though perhaps less consistent than the AT-BBPSO variants. Default PSO also performs rather well and is a good baseline algorithm to use for comparisons.

OF1	Global nbhd				Ring-3 nbhd				Ring-1 nbhd			
Algorithm	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$
PSO	4015.81	3124.33	0.00	∞	0.00	0.01	0.98	362.00	0.01	0.01	0.78	488.50
BBPSO-MC	84517.26	8944.33	0.00	∞	83674.80	9956.73	0.00	∞	84784.51	9451.47	0.00	∞
BBPSOxp-MC	84104.44	8192.44	0.00	∞	82952.29	9655.82	0.00	∞	85886.72	7182.36	0.00	∞
AT-BBPSO-MC												
$df = 1$, $R^* = 0.1$	1.59	0.47	0.00	∞	2.08	0.73	0.00	∞	2.10	0.89	0.00	∞
$df = 1$, $R^* = 0.3$	0.00	0.00	1.00	445.00	0.01	0.00	0.96	467.00	0.01	0.01	0.54	499.50
$df = 1$, $R^* = 0.5$	0.00	0.00	1.00	289.00	0.00	0.00	1.00	309.50	0.00	0.00	1.00	348.00
$df = 1$, $R^* = 0.7$	0.00	0.00	1.00	223.00	0.00	0.00	1.00	256.00	0.00	0.00	1.00	329.00
$df = 3$, $R^* = 0.1$	6.92	2.41	0.00	∞	6.92	1.79	0.00	∞	5.10	2.02	0.00	∞
$df = 3$, $R^* = 0.3$	0.01	0.00	0.22	∞	0.02	0.01	0.06	∞	0.03	0.01	0.00	∞
$df = 3$, $R^* = 0.5$	0.00	0.00	1.00	331.50	0.00	0.00	1.00	347.50	0.00	0.00	1.00	387.50
$df = 3$, $R^* = 0.7$	0.00	0.00	1.00	259.50	0.00	0.00	1.00	288.50	0.00	0.00	1.00	400.50
$df = 5$, $R^* = 0.1$	11.50	2.93	0.00	∞	11.72	2.53	0.00	∞	8.14	2.55	0.00	∞
$df = 5$, $R^* = 0.3$	0.03	0.01	0.00	∞	0.04	0.01	0.00	∞	0.06	0.02	0.00	∞
$df = 5$, $R^* = 0.5$	0.00	0.00	1.00	356.50	0.00	0.00	1.00	368.50	0.00	0.00	1.00	415.50
$df = 5$, $R^* = 0.7$	0.00	0.00	1.00	288.50	0.00	0.00	1.00	326.50	222.69	288.53	0.02	∞
$df = \infty$, $R^* = 0.1$	42.86	10.31	0.00	∞	38.44	8.12	0.00	∞	20.28	8.59	0.00	∞
$df = \infty$, $R^* = 0.3$	0.14	0.05	0.00	∞	0.15	0.03	0.00	∞	0.25	0.08	0.00	∞
$df = \infty$, $R^* = 0.5$	0.00	0.00	1.00	415.00	0.00	0.00	1.00	433.00	0.01	0.00	0.94	489.00
$df = \infty$, $R^* = 0.7$	0.00	0.00	1.00	352.00	0.00	0.00	0.98	441.50	16949.57	3141.36	0.00	∞
AT-BBPSOxp-MC												
$df = 1$, $R^* = 0.1$	5.71	1.58	0.00	∞	6.00	1.76	0.00	∞	3.91	1.75	0.00	∞
$df = 1$, $R^* = 0.3$	0.02	0.01	0.00	∞	0.04	0.01	0.00	∞	0.05	0.02	0.00	∞
$df = 1$, $R^* = 0.5$	0.00	0.00	1.00	358.00	0.00	0.00	1.00	381.00	0.00	0.00	1.00	419.50
$df = 1$, $R^* = 0.7$	0.00	0.00	1.00	291.00	0.00	0.00	1.00	326.00	0.00	0.01	0.98	432.50
$df = 3$, $R^* = 0.1$	17.42	4.87	0.00	∞	16.91	4.41	0.00	∞	8.64	3.12	0.00	∞
$df = 3$, $R^* = 0.3$	0.10	0.03	0.00	∞	0.12	0.04	0.00	∞	0.15	0.05	0.00	∞
$df = 3$, $R^* = 0.5$	0.00	0.00	1.00	412.00	0.00	0.00	1.00	431.50	0.01	0.00	0.92	470.50
$df = 3$, $R^* = 0.7$	0.00	0.00	1.00	350.50	0.00	0.01	0.98	438.00	4193.16	2013.59	0.00	∞
$df = 5$, $R^* = 0.1$	28.39	6.37	0.00	∞	28.42	7.58	0.00	∞	13.22	5.27	0.00	∞
$df = 5$, $R^* = 0.3$	0.19	0.05	0.00	∞	0.22	0.07	0.00	∞	0.26	0.10	0.00	∞
$df = 5$, $R^* = 0.5$	0.00	0.00	1.00	440.00	0.00	0.00	1.00	466.00	0.03	0.02	0.04	∞
$df = 5$, $R^* = 0.7$	0.00	0.00	1.00	405.50	505.43	755.11	0.02	∞	17674.52	4505.53	0.00	∞
$df = \infty$, $R^* = 0.1$	65.18	14.92	0.00	∞	60.90	13.82	0.00	∞	26.77	10.11	0.00	∞
$df = \infty$, $R^* = 0.3$	0.50	0.11	0.00	∞	0.66	0.14	0.00	∞	0.82	0.25	0.00	∞
$df = \infty$, $R^* = 0.5$	0.01	0.00	0.82	492.00	0.03	0.01	0.02	∞	0.85	0.75	0.00	∞
$df = \infty$, $R^* = 0.7$	0.19	0.33	0.04	∞	17369.34	3631.71	0.00	∞	35172.80	4377.92	0.00	∞
DI-PSO												
$\alpha = 50$, $\beta = 1$	6703.90	3993.99	0.00	∞	543.02	683.48	0.00	∞	51.63	95.71	0.00	∞
$\alpha = 50$, $\beta = 2$	7823.89	4787.27	0.00	∞	1821.42	1508.50	0.00	∞	499.64	441.33	0.00	∞
$\alpha = 50$, $\beta = 4$	15373.54	6906.92	0.00	∞	5572.40	3137.25	0.00	∞	3705.93	2324.41	0.00	∞
$\alpha = 100$, $\beta = 1$	3928.51	3855.22	0.00	∞	236.79	733.83	0.00	∞	0.99	4.03	0.20	∞
$\alpha = 100$, $\beta = 2$	7618.89	4250.92	0.00	∞	1206.09	1393.42	0.00	∞	84.99	137.32	0.00	∞
$\alpha = 100$, $\beta = 4$	13044.34	6195.43	0.00	∞	4218.13	2881.34	0.00	∞	1745.64	1532.00	0.00	∞
$\alpha = 200$, $\beta = 1$	2321.68	2088.02	0.00	∞	178.09	618.06	0.02	∞	0.00	0.00	1.00	383.50
$\alpha = 200$, $\beta = 2$	6427.62	3741.12	0.00	∞	305.62	523.71	0.00	∞	5.45	12.55	0.00	∞
$\alpha = 200$, $\beta = 4$	12004.75	5708.12	0.00	∞	2236.94	1729.58	0.00	∞	475.93	503.87	0.00	∞
AT-PSO												
$R^* = 0.1$	111.02	237.73	0.00	∞	211.79	889.18	0.00	∞	12223.23	9590.30	0.00	∞
$R^* = 0.3$	54.27	252.08	0.04	∞	0.00	0.00	1.00	347.00	0.00	0.01	0.88	457.00
$R^* = 0.5$	329.77	586.04	0.00	∞	0.00	0.00	1.00	274.50	0.00	0.00	1.00	303.50
$R^* = 0.7$	2131.94	2209.74	0.00	∞	0.04	0.21	0.92	381.50	0.00	0.00	1.00	341.50

Table 2: Simulation results for OF1. See text for description.

OF2	Global nbhd				Ring-3 nbhd				Ring-1 nbhd			
Algorithm	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$
PSO	10915.05	17201.47	0.00	∞	1246.38	1002.75	0.00	∞	5738.34	3835.37	0.00	∞
BBPSO-MC	9081909.83	1958288.73	0.00	∞	9194595.70	2157840.19	0.00	∞	8830368.63	2234188.42	0.00	∞
BBPSOxp-MC	9138885.89	2265920.11	0.00	∞	8889637.56	2035964.50	0.00	∞	9038770.99	1858659.66	0.00	∞
AT-BBPSO-MC												
$df = 1,\ R^* = 0.1$	950.30	809.12	0.00	∞	690.95	535.97	0.00	∞	743.12	728.28	0.00	∞
$df = 1,\ R^* = 0.3$	300.03	258.36	0.00	∞	685.68	458.39	0.00	∞	1091.45	1375.54	0.00	∞
$df = 1,\ R^* = 0.5$	440.38	436.08	0.00	∞	1419.88	1239.43	0.00	∞	3652.45	4072.28	0.00	∞
$df = 1,\ R^* = 0.7$	1615.81	1539.22	0.00	∞	4330.31	4863.92	0.00	∞	8858.46	4578.27	0.00	∞
$df = 3,\ R^* = 0.1$	196.07	88.33	0.00	∞	226.17	163.16	0.00	∞	252.63	187.08	0.00	∞
$df = 3,\ R^* = 0.3$	23.27	17.11	0.00	∞	54.95	48.43	0.00	∞	189.83	399.03	0.00	∞
$df = 3,\ R^* = 0.5$	30.94	29.21	0.00	∞	185.87	161.77	0.00	∞	1073.22	1233.74	0.00	∞
$df = 3,\ R^* = 0.7$	222.12	524.53	0.00	∞	939.33	789.54	0.00	∞	5897.58	4513.86	0.00	∞
$df = 5,\ R^* = 0.1$	198.27	75.35	0.00	∞	175.31	80.21	0.00	∞	205.86	117.06	0.00	∞
$df = 5,\ R^* = 0.3$	12.25	9.35	0.00	∞	24.76	18.02	0.00	∞	60.79	63.04	0.00	∞
$df = 5,\ R^* = 0.5$	11.51	7.41	0.00	∞	63.91	51.53	0.00	∞	458.47	317.21	0.00	∞
$df = 5,\ R^* = 0.7$	61.67	36.27	0.00	∞	378.27	218.82	0.00	∞	7696.71	5485.99	0.00	∞
$df = \infty,\ R^* = 0.1$	280.79	97.77	0.00	∞	267.44	87.93	0.00	∞	287.37	128.28	0.00	∞
$df = \infty,\ R^* = 0.3$	6.75	3.83	0.00	∞	13.91	9.02	0.00	∞	39.37	32.01	0.00	∞
$df = \infty,\ R^* = 0.5$	4.96	3.73	0.00	∞	30.82	22.83	0.00	∞	269.58	161.10	0.00	∞
$df = \infty,\ R^* = 0.7$	51.31	38.92	0.00	∞	550.48	301.85	0.00	∞	1100097.72	493290.33	0.00	∞
AT-BBPSOxp-MC												
$df = 1,\ R^* = 0.1$	1620.04	724.59	0.00	∞	1434.81	637.94	0.00	∞	1004.77	608.51	0.00	∞
$df = 1,\ R^* = 0.3$	1117.65	614.18	0.00	∞	1541.55	913.10	0.00	∞	1372.47	1108.92	0.00	∞
$df = 1,\ R^* = 0.5$	2424.76	1458.65	0.00	∞	3639.53	2048.74	0.00	∞	5212.21	3221.11	0.00	∞
$df = 1,\ R^* = 0.7$	5685.79	3365.32	0.00	∞	6746.82	4254.76	0.00	∞	13170.58	7291.81	0.00	∞
$df = 3,\ R^* = 0.1$	609.04	289.92	0.00	∞	568.65	326.15	0.00	∞	562.17	301.77	0.00	∞
$df = 3,\ R^* = 0.3$	231.40	167.47	0.00	∞	320.46	308.01	0.00	∞	366.68	274.58	0.00	∞
$df = 3,\ R^* = 0.5$	360.34	224.38	0.00	∞	1026.42	709.48	0.00	∞	5089.66	5857.62	0.00	∞
$df = 3,\ R^* = 0.7$	1426.88	1209.92	0.00	∞	5568.83	7882.29	0.00	∞	38081.89	31747.85	0.00	∞
$df = 5,\ R^* = 0.1$	476.81	215.75	0.00	∞	497.74	194.10	0.00	∞	540.99	263.49	0.00	∞
$df = 5,\ R^* = 0.3$	109.01	45.98	0.00	∞	170.67	120.10	0.00	∞	272.26	192.17	0.00	∞
$df = 5,\ R^* = 0.5$	243.73	154.52	0.00	∞	448.77	241.51	0.00	∞	2147.17	1308.53	0.00	∞
$df = 5,\ R^* = 0.7$	794.68	388.65	0.00	∞	17971.50	16875.10	0.00	∞	507347.01	409181.08	0.00	∞
$df = \infty,\ R^* = 0.1$	609.19	197.59	0.00	∞	585.88	163.05	0.00	∞	536.88	263.19	0.00	∞
$df = \infty,\ R^* = 0.3$	70.82	44.69	0.00	∞	111.39	79.50	0.00	∞	220.30	168.06	0.00	∞
$df = \infty,\ R^* = 0.5$	114.86	88.86	0.00	∞	329.69	169.04	0.00	∞	2889.16	2088.86	0.00	∞
$df = \infty,\ R^* = 0.7$	2469.49	2799.80	0.00	∞	1483405.94	407996.51	0.00	∞	2535125.64	778910.46	0.00	∞
DI-PSO												
$\alpha = 50,\ \beta = 1$	40714.56	24208.86	0.00	∞	16548.67	9312.34	0.00	∞	7839.43	5031.49	0.00	∞
$\alpha = 50,\ \beta = 2$	47763.21	31361.92	0.00	∞	24627.59	11610.10	0.00	∞	11660.25	6362.56	0.00	∞
$\alpha = 50,\ \beta = 4$	62225.74	28459.35	0.00	∞	31511.13	15145.07	0.00	∞	16416.52	7831.71	0.00	∞
$\alpha = 100,\ \beta = 1$	26613.11	14611.63	0.00	∞	10168.37	6029.10	0.00	∞	5378.87	3302.21	0.00	∞
$\alpha = 100,\ \beta = 2$	46177.71	37974.29	0.00	∞	17795.63	8897.56	0.00	∞	9818.51	4708.72	0.00	∞
$\alpha = 100,\ \beta = 4$	64833.73	36534.52	0.00	∞	28627.71	14257.04	0.00	∞	14536.17	7921.94	0.00	∞
$\alpha = 200,\ \beta = 1$	21611.55	12914.90	0.00	∞	5699.97	3916.44	0.00	∞	3499.49	1865.32	0.00	∞
$\alpha = 200,\ \beta = 2$	36405.44	27997.69	0.00	∞	12492.90	6829.93	0.00	∞	6944.87	4559.84	0.00	∞
$\alpha = 200,\ \beta = 4$	57723.16	30860.32	0.00	∞	21807.81	11567.18	0.00	∞	12201.41	5365.69	0.00	∞
AT-PSO												
$R^* = 0.1$	8823.66	9727.43	0.00	∞	8852.96	6912.25	0.00	∞	58369.56	38288.17	0.00	∞
$R^* = 0.3$	6658.84	6659.87	0.00	∞	769.74	850.01	0.00	∞	5183.03	4046.47	0.00	∞
$R^* = 0.5$	20994.61	16398.66	0.00	∞	1713.89	2589.20	0.00	∞	1877.08	1465.51	0.00	∞
$R^* = 0.7$	35128.87	17886.90	0.00	∞	7126.76	5756.50	0.00	∞	5771.37	3581.95	0.00	∞

Table 3: Simulation results for OF2. See text for description.

OF3	Global nbhd				Ring-3 nbhd				Ring-1 nbhd			
Algorithm	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$
PSO	1242764.67	2075592.00	0.00	∞	121.33	104.18	0.00	∞	241.45	255.69	0.00	∞
BBPSO-MC	232830400.95	37473167.09	0.00	∞	247662755.14	43820300.43	0.00	∞	247321508.63	39096259.33	0.00	∞
BBPSOxp-MC	235491326.54	42124202.61	0.00	∞	242679907.63	42872974.55	0.00	∞	253599464.23	43615037.04	0.00	∞
AT-BBPSO-MC												
$df=1,\ R^*=0.1$	382.54	325.35	0.00	∞	486.70	533.46	0.00	∞	669.09	932.66	0.00	∞
$df=1,\ R^*=0.3$	233.17	429.80	0.00	∞	182.89	272.06	0.00	∞	352.64	465.63	0.00	∞
$df=1,\ R^*=0.5$	315.43	497.55	0.00	∞	159.55	216.03	0.00	∞	254.78	669.72	0.00	∞
$df=1,\ R^*=0.7$	235.08	455.38	0.00	∞	96.32	106.25	0.00	∞	189.20	288.45	0.00	∞
$df=3,\ R^*=0.1$	545.05	481.33	0.00	∞	629.79	657.37	0.00	∞	567.75	585.07	0.00	∞
$df=3,\ R^*=0.3$	176.81	186.22	0.00	∞	172.84	204.18	0.00	∞	256.32	426.46	0.00	∞
$df=3,\ R^*=0.5$	159.09	170.77	0.00	∞	128.99	127.25	0.00	∞	137.56	202.68	0.00	∞
$df=3,\ R^*=0.7$	155.74	399.33	0.00	∞	132.16	339.64	0.00	∞	82.12	170.21	0.00	∞
$df=5,\ R^*=0.1$	833.71	627.25	0.00	∞	928.42	982.63	0.00	∞	450.93	444.86	0.00	∞
$df=5,\ R^*=0.3$	283.78	431.37	0.00	∞	135.95	98.01	0.00	∞	229.39	314.25	0.00	∞
$df=5,\ R^*=0.5$	198.53	185.97	0.00	∞	173.66	278.73	0.00	∞	118.79	114.49	0.00	∞
$df=5,\ R^*=0.7$	142.06	190.31	0.00	∞	64.57	98.26	0.00	∞	108.71	538.33	0.00	∞
$df=\infty,\ R^*=0.1$	1343.31	757.87	0.00	∞	1056.96	522.89	0.00	∞	958.18	773.33	0.00	∞
$df=\infty,\ R^*=0.3$	230.32	320.08	0.00	∞	138.02	120.68	0.00	∞	212.78	331.90	0.00	∞
$df=\infty,\ R^*=0.5$	225.28	219.75	0.00	∞	156.78	132.24	0.00	∞	120.23	106.23	0.00	∞
$df=\infty,\ R^*=0.7$	131.34	91.53	0.00	∞	56.10	76.80	0.00	∞	179.69	683.04	0.00	∞
AT-BBPSOxp-MC												
$df=1,\ R^*=0.1$	616.80	278.83	0.00	∞	633.00	276.25	0.00	∞	490.75	350.79	0.00	∞
$df=1,\ R^*=0.3$	152.80	179.75	0.00	∞	163.64	131.42	0.00	∞	140.13	158.06	0.00	∞
$df=1,\ R^*=0.5$	139.49	269.65	0.00	∞	99.39	225.41	0.00	∞	202.26	429.55	0.00	∞
$df=1,\ R^*=0.7$	149.01	459.29	0.00	∞	73.54	126.07	0.00	∞	188.28	370.67	0.00	∞
$df=3,\ R^*=0.1$	961.94	370.97	0.00	∞	931.38	294.49	0.00	∞	709.41	452.45	0.00	∞
$df=3,\ R^*=0.3$	168.44	279.24	0.00	∞	144.15	200.78	0.00	∞	120.33	48.68	0.00	∞
$df=3,\ R^*=0.5$	123.39	102.98	0.00	∞	97.56	50.88	0.00	∞	77.47	81.83	0.00	∞
$df=3,\ R^*=0.7$	49.75	70.72	0.00	∞	33.96	52.12	0.00	∞	3586.02	9973.24	0.00	∞
$df=5,\ R^*=0.1$	1229.87	526.97	0.00	∞	1171.93	406.81	0.00	∞	727.95	436.07	0.00	∞
$df=5,\ R^*=0.3$	267.86	435.67	0.00	∞	124.80	75.86	0.00	∞	157.72	217.77	0.00	∞
$df=5,\ R^*=0.5$	137.08	104.13	0.00	∞	91.44	86.38	0.00	∞	75.74	95.04	0.00	∞
$df=5,\ R^*=0.7$	66.87	126.23	0.00	∞	19.28	17.08	0.00	∞	82498.29	102684.12	0.00	∞
$df=\infty,\ R^*=0.1$	2002.70	822.46	0.00	∞	1718.67	698.34	0.00	∞	1158.47	903.16	0.00	∞
$df=\infty,\ R^*=0.3$	137.09	84.51	0.00	∞	116.46	66.60	0.00	∞	114.47	57.68	0.00	∞
$df=\infty,\ R^*=0.5$	121.71	103.55	0.00	∞	104.78	67.31	0.00	∞	41.15	33.00	0.00	∞
$df=\infty,\ R^*=0.7$	24.31	24.58	0.00	∞	24.31	20.57	0.00	∞	2088154.00	1258187.91	0.00	∞
DI-PSO												
$\alpha=50,\ \beta=1$	4180196.82	4677639.48	0.00	∞	354071.40	579847.34	0.00	∞	70128.62	165037.08	0.00	∞
$\alpha=50,\ \beta=2$	6964209.15	8322398.93	0.00	∞	2341768.78	4277548.98	0.00	∞	1232022.80	1464906.82	0.00	∞
$\alpha=50,\ \beta=4$	20143670.82	15026478.57	0.00	∞	11740754.91	8374232.25	0.00	∞	6766696.51	4472751.26	0.00	∞
$\alpha=100,\ \beta=1$	1610827.18	3342137.70	0.00	∞	55104.82	171942.01	0.00	∞	1691.16	3160.57	0.00	∞
$\alpha=100,\ \beta=2$	6945649.64	6844592.19	0.00	∞	1001725.17	1774982.73	0.00	∞	150478.59	213009.16	0.00	∞
$\alpha=100,\ \beta=4$	21536033.79	15337048.80	0.00	∞	6691579.83	5932202.88	0.00	∞	4328277.03	4060546.38	0.00	∞
$\alpha=200,\ \beta=1$	1543367.07	2077471.63	0.00	∞	4350.55	16257.10	0.00	∞	266.44	453.75	0.00	∞
$\alpha=200,\ \beta=2$	6626917.67	9093854.31	0.00	∞	299834.83	1040979.02	0.00	∞	5848.88	14329.18	0.00	∞
$\alpha=200,\ \beta=4$	17830796.63	14245007.26	0.00	∞	3367710.63	3194612.95	0.00	∞	776424.69	865443.75	0.00	∞
AT-PSO												
$R^*=0.1$	5779.95	14928.86	0.00	∞	297272.41	1261108.56	0.00	∞	33785179.04	23020834.78	0.00	∞
$R^*=0.3$	869.69	2100.98	0.00	∞	131.15	175.33	0.00	∞	184.15	186.17	0.00	∞
$R^*=0.5$	71363.37	218211.42	0.00	∞	190.13	271.20	0.00	∞	134.32	200.92	0.00	∞
$R^*=0.7$	1572669.78	3884482.27	0.00	∞	391.10	857.02	0.00	∞	535.09	1277.83	0.00	∞

Table 4: Simulation results for OF3. See text for description.

OF4	Global nbhd				Ring-3 nbhd				Ring-1 nbhd			
Algorithm	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$
PSO	29.94	15.21	0.00	∞	5.37	3.47	0.02	∞	3.24	1.44	0.00	∞
BBPSO-MC	237.31	18.67	0.00	∞	232.86	24.40	0.00	∞	232.69	21.86	0.00	∞
BBPSOxp-MC	229.80	22.24	0.00	∞	236.60	25.20	0.00	∞	230.84	21.05	0.00	∞
AT-BBPSO-MC												
$df = 1,\ R^* = 0.1$	4.96	1.58	0.00	∞	3.93	1.29	0.00	∞	4.07	1.88	0.00	∞
$df = 1,\ R^* = 0.3$	5.14	1.89	0.00	∞	2.76	1.45	0.06	∞	2.48	1.57	0.06	∞
$df = 1,\ R^* = 0.5$	6.32	2.16	0.00	∞	4.68	1.94	0.00	∞	4.24	2.36	0.02	∞
$df = 1,\ R^* = 0.7$	8.56	2.84	0.00	∞	5.78	1.67	0.00	∞	6.03	2.01	0.00	∞
$df = 3,\ R^* = 0.1$	5.44	1.30	0.00	∞	5.40	1.17	0.00	∞	5.39	1.54	0.00	∞
$df = 3,\ R^* = 0.3$	3.56	1.76	0.00	∞	1.59	1.24	0.20	∞	1.43	1.00	0.18	∞
$df = 3,\ R^* = 0.5$	4.07	1.85	0.00	∞	2.70	1.57	0.08	∞	2.57	1.44	0.02	∞
$df = 3,\ R^* = 0.7$	4.70	1.73	0.00	∞	3.83	1.72	0.00	∞	3.43	1.75	0.06	∞
$df = 5,\ R^* = 0.1$	6.63	1.34	0.00	∞	6.45	1.92	0.00	∞	6.72	1.86	0.00	∞
$df = 5,\ R^* = 0.3$	2.80	1.51	0.02	∞	1.24	1.12	0.24	∞	1.13	1.21	0.30	∞
$df = 5,\ R^* = 0.5$	4.11	2.16	0.02	∞	2.26	1.60	0.12	∞	1.90	1.43	0.16	∞
$df = 5,\ R^* = 0.7$	4.40	2.00	0.00	∞	2.91	1.48	0.00	∞	3.56	1.84	0.00	∞
$df = \infty,\ R^* = 0.1$	9.12	1.88	0.00	∞	9.17	1.58	0.00	∞	9.42	1.70	0.00	∞
$df = \infty,\ R^* = 0.3$	2.15	1.31	0.00	∞	0.72	0.92	0.00	∞	0.61	0.74	0.04	∞
$df = \infty,\ R^* = 0.5$	2.74	1.55	0.04	∞	1.62	1.38	0.18	∞	1.01	0.91	0.26	∞
$df = \infty,\ R^* = 0.7$	3.22	1.78	0.06	∞	2.11	1.56	0.18	∞	2.13	1.30	0.10	∞
AT-BBPSOxp-MC												
$df = 1,\ R^* = 0.1$	4.25	1.24	0.00	∞	4.70	1.25	0.00	∞	6.34	1.56	0.00	∞
$df = 1,\ R^* = 0.3$	0.74	0.87	0.04	∞	0.23	0.44	0.02	∞	0.51	0.74	0.00	∞
$df = 1,\ R^* = 0.5$	1.75	1.24	0.12	∞	0.95	1.00	0.38	∞	1.41	1.16	0.18	∞
$df = 1,\ R^* = 0.7$	3.07	1.65	0.02	∞	2.14	1.20	0.06	∞	3.10	1.62	0.02	∞
$df = 3,\ R^* = 0.1$	5.34	1.01	0.00	∞	6.29	1.31	0.00	∞	7.39	1.59	0.00	∞
$df = 3,\ R^* = 0.3$	0.25	0.45	0.00	∞	0.08	0.23	0.00	∞	0.19	0.41	0.02	∞
$df = 3,\ R^* = 0.5$	0.65	0.78	0.52	345.50	0.30	0.52	0.72	342.00	0.56	0.66	0.46	∞
$df = 3,\ R^* = 0.7$	1.43	1.21	0.22	∞	1.24	1.02	0.24	∞	1.85	1.12	0.08	∞
$df = 5,\ R^* = 0.1$	6.02	1.08	0.00	∞	6.50	1.08	0.00	∞	7.94	1.31	0.00	∞
$df = 5,\ R^* = 0.3$	0.10	0.26	0.00	∞	0.08	0.23	0.00	∞	0.10	0.33	0.02	∞
$df = 5,\ R^* = 0.5$	0.40	0.61	0.66	338.00	0.25	0.42	0.74	338.50	0.46	0.70	0.64	367.00
$df = 5,\ R^* = 0.7$	1.13	1.01	0.28	∞	0.75	0.85	0.44	∞	1.74	1.10	0.04	∞
$df = \infty,\ R^* = 0.1$	7.81	1.19	0.00	∞	8.10	1.36	0.00	∞	9.02	1.36	0.00	∞
$df = \infty,\ R^* = 0.3$	0.09	0.23	0.00	∞	0.03	0.01	0.00	∞	0.05	0.13	0.00	∞
$df = \infty,\ R^* = 0.5$	0.25	0.57	0.80	346.00	0.13	0.33	0.86	345.00	0.17	0.42	0.84	352.50
$df = \infty,\ R^* = 0.7$	0.90	0.89	0.36	∞	0.77	0.92	0.46	∞	2.66	1.39	0.00	∞
DI-PSO												
$\alpha = 50,\ \beta = 1$	32.08	11.51	0.00	∞	13.04	5.65	0.00	∞	6.33	3.37	0.00	∞
$\alpha = 50,\ \beta = 2$	38.18	17.28	0.00	∞	15.90	5.67	0.00	∞	10.13	4.19	0.00	∞
$\alpha = 50,\ \beta = 4$	56.49	22.89	0.00	∞	28.28	11.13	0.00	∞	15.83	7.08	0.00	∞
$\alpha = 100,\ \beta = 1$	28.51	12.71	0.00	∞	11.32	5.72	0.00	∞	4.66	2.48	0.00	∞
$\alpha = 100,\ \beta = 2$	32.85	13.96	0.00	∞	14.35	7.57	0.00	∞	6.86	2.78	0.00	∞
$\alpha = 100,\ \beta = 4$	49.67	16.91	0.00	∞	24.47	9.82	0.00	∞	13.74	6.18	0.00	∞
$\alpha = 200,\ \beta = 1$	22.39	12.14	0.00	∞	7.53	4.31	0.00	∞	3.92	2.11	0.02	∞
$\alpha = 200,\ \beta = 2$	36.56	14.29	0.00	∞	11.21	5.42	0.00	∞	5.28	2.77	0.00	∞
$\alpha = 200,\ \beta = 4$	55.67	19.91	0.00	∞	19.67	8.61	0.00	∞	11.18	5.41	0.00	∞
AT-PSO												
$R^* = 0.1$	7.73	5.50	0.00	∞	5.72	1.90	0.00	∞	23.02	10.87	0.00	∞
$R^* = 0.3$	14.10	6.74	0.00	∞	6.67	3.74	0.00	∞	3.77	2.04	0.02	∞
$R^* = 0.5$	23.89	8.43	0.00	∞	11.39	5.91	0.00	∞	6.06	2.92	0.00	∞
$R^* = 0.7$	34.91	12.20	0.00	∞	14.91	5.84	0.00	∞	9.80	3.78	0.00	∞

Table 5: Simulation results for OF4. See text for description.

OF5	Global nbhd				Ring-3 nbhd				Ring-1 nbhd			
Algorithm	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$
PSO	27.55	20.12	0.00	∞	0.03	0.03	0.28	∞	0.08	0.07	0.04	∞
BBPSO-MC	762.36	82.36	0.00	∞	785.33	69.84	0.00	∞	751.88	80.37	0.00	∞
BBPSOxp-MC	766.30	80.16	0.00	∞	785.43	64.92	0.00	∞	771.17	76.11	0.00	∞
AT-BBPSO-MC												
$df = 1,\ R^* = 0.1$	0.83	0.10	0.00	∞	0.88	0.09	0.00	∞	0.92	0.06	0.00	∞
$df = 1,\ R^* = 0.3$	0.02	0.01	0.24	∞	0.02	0.01	0.34	∞	0.05	0.03	0.02	∞
$df = 1,\ R^* = 0.5$	0.01	0.01	0.52	455.50	0.01	0.01	0.56	455.00	0.01	0.01	0.48	∞
$df = 1,\ R^* = 0.7$	0.02	0.01	0.36	∞	0.02	0.02	0.46	∞	0.04	0.04	0.16	∞
$df = 3,\ R^* = 0.1$	1.06	0.02	0.00	∞	1.05	0.02	0.00	∞	1.04	0.02	0.00	∞
$df = 3,\ R^* = 0.3$	0.08	0.03	0.00	∞	0.12	0.04	0.00	∞	0.32	0.15	0.00	∞
$df = 3,\ R^* = 0.5$	0.01	0.01	0.54	459.00	0.01	0.01	0.62	481.00	0.09	0.07	0.02	∞
$df = 3,\ R^* = 0.7$	0.01	0.01	0.46	∞	0.12	0.15	0.04	∞	205.76	57.74	0.00	∞
$df = 5,\ R^* = 0.1$	1.11	0.03	0.00	∞	1.11	0.03	0.00	∞	1.09	0.03	0.00	∞
$df = 5,\ R^* = 0.3$	0.18	0.05	0.00	∞	0.29	0.09	0.00	∞	0.66	0.15	0.00	∞
$df = 5,\ R^* = 0.5$	0.02	0.01	0.32	∞	0.02	0.01	0.26	∞	0.69	0.21	0.00	∞
$df = 5,\ R^* = 0.7$	0.01	0.01	0.32	∞	39.11	26.34	0.00	∞	480.85	48.89	0.00	∞
$df = \infty,\ R^* = 0.1$	1.47	0.11	0.00	∞	1.45	0.09	0.00	∞	1.25	0.08	0.00	∞
$df = \infty,\ R^* = 0.3$	0.69	0.11	0.00	∞	0.85	0.07	0.00	∞	0.97	0.05	0.00	∞
$df = \infty,\ R^* = 0.5$	0.04	0.02	0.00	∞	0.26	0.09	0.00	∞	9.61	10.88	0.00	∞
$df = \infty,\ R^* = 0.7$	0.82	0.19	0.00	∞	395.75	42.10	0.00	∞	638.33	59.56	0.00	∞
AT-BBPSOxp-MC												
$df = 1,\ R^* = 0.1$	1.01	0.02	0.00	∞	1.01	0.02	0.00	∞	1.00	0.03	0.00	∞
$df = 1,\ R^* = 0.3$	0.07	0.04	0.00	∞	0.13	0.06	0.00	∞	0.25	0.13	0.00	∞
$df = 1,\ R^* = 0.5$	0.01	0.01	0.54	465.50	0.01	0.01	0.44	∞	0.06	0.08	0.06	∞
$df = 1,\ R^* = 0.7$	0.02	0.02	0.46	∞	0.02	0.03	0.46	∞	1.20	0.54	0.00	∞
$df = 3,\ R^* = 0.1$	1.17	0.04	0.00	∞	1.16	0.04	0.00	∞	1.09	0.03	0.00	∞
$df = 3,\ R^* = 0.3$	0.56	0.12	0.00	∞	0.73	0.08	0.00	∞	0.87	0.09	0.00	∞
$df = 3,\ R^* = 0.5$	0.04	0.02	0.00	∞	0.33	0.14	0.00	∞	5.84	7.01	0.00	∞
$df = 3,\ R^* = 0.7$	7.20	9.68	0.00	∞	295.56	44.31	0.00	∞	439.15	53.40	0.00	∞
$df = 5,\ R^* = 0.1$	1.26	0.06	0.00	∞	1.30	0.06	0.00	∞	1.16	0.06	0.00	∞
$df = 5,\ R^* = 0.3$	0.80	0.08	0.00	∞	0.92	0.05	0.00	∞	1.00	0.02	0.00	∞
$df = 5,\ R^* = 0.5$	0.22	0.10	0.00	∞	0.93	0.09	0.00	∞	72.53	27.81	0.00	∞
$df = 5,\ R^* = 0.7$	172.86	53.14	0.00	∞	491.44	39.13	0.00	∞	527.93	56.57	0.00	∞
$df = \infty,\ R^* = 0.1$	1.71	0.13	0.00	∞	1.70	0.17	0.00	∞	1.34	0.14	0.00	∞
$df = \infty,\ R^* = 0.3$	0.99	0.04	0.00	∞	1.03	0.01	0.00	∞	1.06	0.03	0.00	∞
$df = \infty,\ R^* = 0.5$	0.82	0.12	0.00	∞	2.65	1.75	0.00	∞	255.87	50.83	0.00	∞
$df = \infty,\ R^* = 0.7$	451.87	38.28	0.00	∞	622.22	35.60	0.00	∞	586.09	47.80	0.00	∞
DI-PSO												
$\alpha = 50,\ \beta = 1$	55.37	25.87	0.00	∞	7.99	9.84	0.00	∞	1.07	0.35	0.00	∞
$\alpha = 50,\ \beta = 2$	73.04	43.14	0.00	∞	21.12	16.40	0.00	∞	6.20	4.72	0.00	∞
$\alpha = 50,\ \beta = 4$	142.00	66.28	0.00	∞	61.48	36.04	0.00	∞	27.71	18.29	0.00	∞
$\alpha = 100,\ \beta = 1$	32.00	20.76	0.00	∞	2.52	2.93	0.00	∞	0.22	0.19	0.00	∞
$\alpha = 100,\ \beta = 2$	78.45	51.91	0.00	∞	12.10	18.31	0.00	∞	1.76	1.50	0.00	∞
$\alpha = 100,\ \beta = 4$	139.13	62.71	0.00	∞	42.32	21.23	0.00	∞	15.40	12.26	0.00	∞
$\alpha = 200,\ \beta = 1$	28.74	20.48	0.00	∞	0.56	0.70	0.00	∞	0.06	0.07	0.18	∞
$\alpha = 200,\ \beta = 2$	76.70	56.65	0.00	∞	4.61	6.06	0.00	∞	0.52	0.34	0.00	∞
$\alpha = 200,\ \beta = 4$	130.62	67.27	0.00	∞	21.48	17.97	0.00	∞	5.24	5.82	0.00	∞
AT-PSO												
$R^* = 0.1$	3.28	6.03	0.00	∞	2.22	2.69	0.00	∞	111.55	65.99	0.00	∞
$R^* = 0.3$	0.74	1.00	0.02	∞	0.02	0.02	0.34	∞	0.06	0.06	0.16	∞
$R^* = 0.5$	4.06	8.44	0.00	∞	0.07	0.13	0.22	∞	0.01	0.02	0.56	414.50
$R^* = 0.7$	28.82	33.05	0.00	∞	0.43	1.25	0.04	∞	0.07	0.16	0.22	∞

Table 6: Simulation results for OF5. See text for description.

OF6	Global nbhd				Ring-3 nbhd				Ring-1 nbhd			
Algorithm	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$	Mean	SD	$\widehat{p}$	$\widehat{t}$
PSO	19.66	0.56	0.00	∞	14.77	8.35	0.02	∞	19.48	2.63	0.00	∞
BBPSO-MC	20.72	0.23	0.00	∞	20.74	0.22	0.00	∞	20.74	0.21	0.00	∞
BBPSOxp-MC	20.75	0.18	0.00	∞	20.73	0.18	0.00	∞	20.73	0.18	0.00	∞
AT-BBPSO-MC												
$df = 1,\ R^* = 0.1$	18.35	5.39	0.00	∞	9.73	9.19	0.00	∞	6.24	7.88	0.00	∞
$df = 1,\ R^* = 0.3$	18.65	4.66	0.00	∞	14.66	8.70	0.00	∞	11.40	9.71	0.00	∞
$df = 1,\ R^* = 0.5$	19.93	0.37	0.00	∞	18.90	4.71	0.04	∞	16.91	6.85	0.06	∞
$df = 1,\ R^* = 0.7$	19.69	2.00	0.00	∞	19.52	2.28	0.00	∞	18.05	5.13	0.00	∞
$df = 3,\ R^* = 0.1$	19.98	0.09	0.00	∞	19.48	3.55	0.00	∞	14.60	8.47	0.00	∞
$df = 3,\ R^* = 0.3$	19.82	0.05	0.00	∞	19.48	2.82	0.00	∞	19.94	0.54	0.00	∞
$df = 3,\ R^* = 0.5$	19.83	0.06	0.00	∞	19.95	0.30	0.00	∞	20.24	0.38	0.00	∞
$df = 3,\ R^* = 0.7$	19.85	0.10	0.00	∞	20.05	0.37	0.00	∞	20.34	0.39	0.00	∞
$df = 5,\ R^* = 0.1$	20.00	0.07	0.00	∞	20.12	0.25	0.00	∞	17.83	6.21	0.00	∞
$df = 5,\ R^* = 0.3$	19.83	0.04	0.00	∞	19.87	0.21	0.00	∞	20.12	0.45	0.00	∞
$df = 5,\ R^* = 0.5$	19.83	0.04	0.00	∞	19.90	0.26	0.00	∞	20.32	0.45	0.00	∞
$df = 5,\ R^* = 0.7$	19.85	0.12	0.00	∞	20.06	0.35	0.00	∞	20.42	0.39	0.00	∞
$df = \infty,\ R^* = 0.1$	20.09	0.08	0.00	∞	20.16	0.14	0.00	∞	20.32	0.27	0.00	∞
$df = \infty,\ R^* = 0.3$	19.83	0.03	0.00	∞	19.85	0.17	0.00	∞	19.98	0.33	0.00	∞
$df = \infty,\ R^* = 0.5$	19.83	0.03	0.00	∞	19.89	0.20	0.00	∞	20.14	0.38	0.00	∞
$df = \infty,\ R^* = 0.7$	19.83	0.06	0.00	∞	20.06	0.36	0.00	∞	20.36	0.37	0.00	∞
AT-BBPSOxp-MC												
$df = 1,\ R^* = 0.1$	15.26	7.90	0.00	∞	11.33	8.97	0.00	∞	9.05	8.54	0.00	∞
$df = 1,\ R^* = 0.3$	18.42	5.50	0.00	∞	17.06	6.87	0.00	∞	14.76	8.83	0.00	∞
$df = 1,\ R^* = 0.5$	20.19	0.20	0.00	∞	19.48	3.44	0.02	∞	17.05	6.20	0.00	∞
$df = 1,\ R^* = 0.7$	20.22	0.26	0.00	∞	20.08	0.67	0.00	∞	19.74	2.08	0.00	∞
$df = 3,\ R^* = 0.1$	20.19	0.16	0.00	∞	20.26	0.13	0.00	∞	19.58	3.36	0.00	∞
$df = 3,\ R^* = 0.3$	20.08	0.17	0.00	∞	20.23	0.18	0.00	∞	20.27	0.18	0.00	∞
$df = 3,\ R^* = 0.5$	20.19	0.13	0.00	∞	20.27	0.13	0.00	∞	20.33	0.25	0.00	∞
$df = 3,\ R^* = 0.7$	20.21	0.13	0.00	∞	20.32	0.11	0.00	∞	20.39	0.11	0.00	∞
$df = 5,\ R^* = 0.1$	20.25	0.08	0.00	∞	20.27	0.11	0.00	∞	20.32	0.12	0.00	∞
$df = 5,\ R^* = 0.3$	20.07	0.16	0.00	∞	20.20	0.17	0.00	∞	20.21	0.18	0.00	∞
$df = 5,\ R^* = 0.5$	20.14	0.15	0.00	∞	20.23	0.12	0.00	∞	20.33	0.14	0.00	∞
$df = 5,\ R^* = 0.7$	20.20	0.11	0.00	∞	20.31	0.13	0.00	∞	20.37	0.11	0.00	∞
$df = \infty,\ R^* = 0.1$	20.18	0.08	0.00	∞	20.27	0.08	0.00	∞	20.29	0.12	0.00	∞
$df = \infty,\ R^* = 0.3$	20.01	0.17	0.00	∞	20.14	0.18	0.00	∞	20.20	0.19	0.00	∞
$df = \infty,\ R^* = 0.5$	20.06	0.15	0.00	∞	20.22	0.14	0.00	∞	20.27	0.15	0.00	∞
$df = \infty,\ R^* = 0.7$	20.14	0.15	0.00	∞	20.30	0.13	0.00	∞	20.36	0.14	0.00	∞
DI-PSO												
$\alpha = 50,\ \beta = 1$	19.42	1.48	0.00	∞	18.52	3.08	0.00	∞	19.02	3.09	0.00	∞
$\alpha = 50,\ \beta = 2$	19.50	1.04	0.00	∞	17.94	3.01	0.00	∞	19.33	1.89	0.00	∞
$\alpha = 50,\ \beta = 4$	19.83	0.64	0.00	∞	19.66	1.00	0.00	∞	19.76	1.07	0.00	∞
$\alpha = 100,\ \beta = 1$	18.71	2.25	0.00	∞	16.04	5.48	0.00	∞	17.63	5.10	0.00	∞
$\alpha = 100,\ \beta = 2$	19.75	1.02	0.00	∞	19.27	1.95	0.00	∞	18.96	3.30	0.00	∞
$\alpha = 100,\ \beta = 4$	19.99	1.04	0.00	∞	19.80	1.11	0.00	∞	20.13	0.75	0.00	∞
$\alpha = 200,\ \beta = 1$	19.13	2.10	0.00	∞	14.83	6.47	0.00	∞	16.77	5.92	0.00	∞
$\alpha = 200,\ \beta = 2$	20.00	1.29	0.00	∞	19.63	2.37	0.00	∞	20.27	0.32	0.00	∞
$\alpha = 200,\ \beta = 4$	20.38	0.65	0.00	∞	20.19	0.65	0.00	∞	20.47	0.31	0.00	∞
AT-PSO												
$R^* = 0.1$	18.67	4.43	0.00	∞	18.18	5.06	0.00	∞	20.18	0.66	0.00	∞
$R^* = 0.3$	19.44	1.34	0.00	∞	16.84	6.30	0.04	∞	15.62	7.72	0.00	∞
$R^* = 0.5$	19.69	0.92	0.00	∞	18.88	3.16	0.00	∞	19.66	2.29	0.00	∞
$R^* = 0.7$	19.97	0.53	0.00	∞	19.76	0.69	0.00	∞	20.12	0.32	0.00	∞

Table 7: Simulation results for OF6. See text for description.

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