Space–Time Dynamic Design of Environmental Monitoring Networks

Christopher K. WIKLE and J. Andrew ROYLE

Methods for constructing optimal spatial sampling designs for environmental monitoring networks, widely applied in a large number of disciplines, generally produce static designs that are optimal under models with no explicit temporal structure. However, environmental processes tend to exhibit both spatial and temporal variability; hence, static networks may not capture the essential spatiotemporal variability of the process. Static designs, often necessary due to geopolitical and economic considerations, could be supplemented with mobile monitoring devices. The design problem is to decide where mobile monitors should be located at time $t + 1$ based on observations through time t. We propose a simple, general, dynamical space–time model that allows estimation of prediction error covariance at time $t + 1$, given information up to time t. We then seek the optimal spatial locations at time $t + 1$ that satisfy some design criterion. Several experiments show the importance of spatial and temporal structure in the selection of optimal designs. Data from the Chicago area ozone monitoring network are used to demonstrate potential dynamical designs under realistic space–time dependence assumptions.

Key Words: Environment, Kalman filter, Network design, Optimal design, Ozone, Sampling, Space–time modeling, Spatial design, Spatial sampling, Spatial statistics.

1. INTRODUCTION

Many environmental processes include variability over both space and time. In practice, it is never possible to completely sample a spatiotemporal environmental process. Traditionally, political, geographic, and/or economic reasons have determined the design of monitoring networks. However, it is intuitive that if the process has significant spatiotemporal interaction, such designs will not be optimal at any particular time. Given the opportunity to move monitors or to sample over different spatial locations through time, it is possible that the design efficiency can be improved by allowing for time-varying designs. An obvious real-world example of maximizing environmental information by time-varying sampling

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occurs with the monitoring of hurricanes via aircraft. In this case, specially equipped aircraft fly into hurricanes at specified intervals to collect observations. Clearly, over time, the location of such monitoring changes according to the evolution of the storm.

The notion of optimal monitoring network design is intuitive. Let Y be a spatiotemporal process, such as ozone, sea surface temperature, or some natural resource variable (e.g., habitat condition), and suppose that the process can be monitored at a set of m spatial locations (the "design") denoted as $\mathbf{D} = (u_1, u_2, \ldots, u_m)$ where $u_i \in \mathcal{R} \subset S$, and S is some geographic region. The design objective is to locate these m points in an optimal fashion, meaning the design minimizes some variance criterion. Common criteria include the average prediction variance, maximum prediction variance, or the variance of regression parameter estimates.

In recent years, substantial effort in the statistical community has been devoted to examining optimal *spatial* designs for environmental processes (e.g., Federov and Mueller 1989; Haas 1992; Guttorp, Le, Sampson, and Zidek 1993; Cox, Cox, and Ensor 1995; Oehlert 1996; Nychka, Yang, and Royle 1997; Bueso, Angulo, and Alonso 1998). Various nonparametric space-filling criteria can also be considered (see Atkinson and Federov [1988] and Federov and Hackl [1997] for general reviews of optimal spatial design). However, given that most environmental processes are temporally dynamic, static designs will not be as efficient as designs that are allowed to evolve over time.

We refer to the problem of constructing time-varying designs as *dynamic* (or adaptive) design, which is not to be confused with adaptive sampling (e.g., Thompson and Seber 1996), although the notions are similar. The primary difference is that the former is model based, whereas the latter occurs in a traditional sample survey framework, but with sample unit selection being modified as observations are made. The term space–time design has been used to describe the design problem for environmental monitoring networks, but generally these approaches have not accounted for the temporally dynamic nature of the process and have been static. For example, Arbia and Lafratta (1997) proposed an approach to space–time design that produces a static spatial design for minimizing the variance of the mean. They used the temporal information for estimating nonstationary spatial variances but otherwise ignored the possibility of temporal evolution of the process under study. Similarly, Le and Zidek (1994) took a Bayesian approach but again ignored the possibility of a dynamic design. Other papers, including Federov and Nachtsheim (1995), considered spatiotemporal models where the optimal design was independent of time. In this paper, we will refer to designs that consider spatiotemporal information but that do not change over time as *static space–time designs*.

The dynamic design problem is difficult. The chief difficulty is that model-based approaches require the specification of the relatively complicated space–time interaction inherent in environmental processes. Many of these models are oversimplified (e.g., they assume space and time are separable). Other models tend to be complicated and are custom developed for specific problems, making generalization to other space–time problems difficult. For example, Haas (1992) and Oehlert (1996) both addressed the problem of design with regard to an acid deposition network using complicated and finely tuned procedures (i.e., Haas 1990; Oehlert 1993). Berliner, Lu, and Snyder (1998) considered adaptive weather observations, allowing the design to change over time according to explicit atmospheric dynamics. Alternatively, Titterington (1980) discussed a general Kalman filter approach to dynamic design, seeking to optimize networks for estimation of regression parameters. His paper introduced much of the optimal dynamic design work from the control theory literature to the general statistics community.

The purpose of this paper is to discuss issues associated with the problem of designing a dynamic network for spatiotemporal processes and to provide a method, based on a relatively simple and generalizable statistical model, for designing such a monitoring network. Specifically, we are interested in the relative importance of spatial and temporal dependence (and their interaction) on potential dynamic design improvements over static space–time designs. Section 2 discusses our space–time dynamic statistical model. Section 3 describes some experiments and results related to the importance of temporal and spatial dependence structures. An example based on Chicago ozone data is presented in Section 4 to illustrate the effect of realistic spatiotemporal covariance structure on dynamic design. Finally, we discuss the results and some related issues in Section 5.

2. SPACE–TIME DYNAMIC MODEL

In order to evaluate potential dynamic designs, we must specify a reasonable space–time model. In principle, assuming that one knows the spatiotemporal covariance structure of a process, it is simple to develop the best linear unbiased predictor and associated prediction variance for some location and time given a sample of observations. Typically, one does *not* know the full joint spatiotemporal covariance structure and, necessarily, must make simplifying assumptions. Traditional approaches to such problems include extending the geostatistical paradigm so that time is treated as another spatial dimension (e.g., Bilonick 1983; Cressie and Majure 1997), multivariate time series methods where the processes are viewed as a set of spatially correlated time series (e.g., Bennett 1979; Rouhani and Wackernagel 1990), and space–time autoregressive moving average methods on a lattice (e.g., Cliff and Ord 1975; Deutsch and Pfeifer 1981). In recent years, more interest has been given to a hybrid space–time dynamic modeling approach, in which explicit temporal structure is prescribed (e.g., Markovian evolution), yet the process is assumed to have descriptive spatial structure (e.g., Guttorp, Meiring, and Sampson 1994; Huang and Cressie 1996; Wikle and Cressie 1997). One advantage of this approach is that the models can be implemented easily via an empirical Bayesian or spatiotemporal Kalman filter procedure. We will focus on such an approach in this paper.

2.1 MODEL FORMULATION

Let the spatial-temporal process of interest be denoted by $Y(s;t)$, where $s \in S$, with S some continuous spatial domain in two-dimensional Euclidean space, and $t \in \{1, 2, \ldots, \}$, a discrete index of times. We further assume that at each time t the process is observed at some finite subset of S and that the data are given by $\mathbf{Z}_t = (Z(s_1;t), \ldots, Z(s_{m_t};t))'$. Now, assuming that observations occur with error, we can write a measurement equation as

$$\mathbf{Z}_t = \mathbf{K}_t \mathbf{Y}_t + \boldsymbol{\epsilon}_t, \tag{2.1}$$

where $\mathbf{Y}_t$ is an $n \times 1$ vector of n prediction locations at time t, $\boldsymbol{\epsilon}_t$ is the zero mean measurement error process with covariance matrix $\boldsymbol{\Sigma}_\epsilon$, and $\mathbf{K}_t$ is an $m_t \times n$ matrix that maps the true process (Y) at prediction locations to the data (Z) at observation locations. $\mathbf{K}_t$ simply determines which of the spatial locations in $\mathbf{Y}_t$ are observed and so is a sparse matrix of 0's and 1's (Wikle, Berliner, and Cressie 1998). The form of this matrix can, in general, be much more complicated. It is critical that we let $\mathbf{K}_t$ vary in time to accommodate different potential observation networks (i.e., designs) at each time.

A model is specified for the spatiotemporal process (Y) at the n prediction locations according to a first-order vector Markov process:

$$\mathbf{Y}_t = \mathbf{H}_t \mathbf{Y}_{t-1} + \boldsymbol{\eta}_t, \tag{2.2}$$

where $\mathbf{H}_t$ is the first-order Markov parameter matrix, and $\boldsymbol{\eta}_t$ is the conditional spatio-temporal noise process with covariance matrix $\boldsymbol{\Sigma}_\eta$. Note that $\mathbf{H}_t$ represents the dynamics of the process and can be timevarying. This is particularly relevant for linear approximations to nonlinear systems (e.g., Berliner et al. 1998). Such a formulation typically requires substantial knowledge of the underlying process (e.g., the governing partial differential equations). Similarly, one could allow the conditional covariance to change with time, with similar complications. Furthermore, we could allow additional autoregressive lags in (2.2) as well as moving average structures. In the current case, the problem is well defined if we know or can estimate $\mathbf{K}_t$, $\boldsymbol{\Sigma}_\epsilon$, $\mathbf{H}_t$, and $\boldsymbol{\Sigma}_\eta$.

Given the measurement equation (2.1) and state equation (2.2), a space–time Kalman filter can be derived either by Bayesian arguments (e.g., Meinhold and Singpurwalla 1983; West and Harrison 1997) or projection arguments (e.g., Hamilton 1994). In either case, one gets the following recursive equations for the prediction error covariance:

$$\begin{aligned} \mathbf{A}_t &\equiv \text{var}(\mathbf{Y}_t \mid \mathbf{Z}_t, \ldots, \mathbf{Z}_1) && (2.3) \\ &= [\mathbf{K}_t \boldsymbol{\Sigma}_\epsilon^{-1} \mathbf{K}_t + \mathbf{B}_t^{-1}]^{-1} \\ &= \mathbf{B}_t - \mathbf{B}_t \mathbf{K}_t' [\mathbf{K}_t \mathbf{B}_t \mathbf{K}_t' + \boldsymbol{\Sigma}_\epsilon]^{-1} \mathbf{K}_t \mathbf{B}_t && (2.4) \end{aligned}$$

where

$$\begin{aligned} \mathbf{B}_t &\equiv \text{var}(\mathbf{Y}_t \mid \mathbf{Z}_{t-1}, \ldots, \mathbf{Z}_1) && (2.5) \\ &= \mathbf{H}_t \mathbf{A}_{t-1} \mathbf{H}_t' + \boldsymbol{\Sigma}_\eta. && (2.6) \end{aligned}$$

To start the recursion, $\mathbf{A}_0$ must be specified and is typically chosen to be the unconditional (i.e., marginal) variance–covariance matrix of the Y process at the prediction locations (e.g., Harvey 1993, p. 88). Furthermore, to obtain optimal predictions in a Kalman filter, the parameter matrices $\mathbf{H}_t$, $\mathbf{K}_t$, $\boldsymbol{\Sigma}_\epsilon$, and $\boldsymbol{\Sigma}_\eta$ must be known. In practice, we seldom know these and must either specify or estimate them. In this case, we no longer obtain exactly the conditional variance. However, our approach is analogous to Kalman filtering in time and kriging in space, where the covariance or variogram parameters must also be estimated. For a discussion of the difficulties in treating estimated parameters as known when forming predictive distributions, see Cressie and Zimmerman (1992). Although one could employ a fully Bayesian model (e.g., Wikle et al. 1998) to obtain more realistic estimates of precision, one loses the computational efficiency and simplicity realized by the Kalman filter approach. Such trade-offs must always be considered when building stochastic models.

2.2 MODEL-BASED DYNAMIC DESIGN

The recursive formulation of equations (2.3)–(2.6) provides the mechanism for dynamic design of monitoring networks. The two features of the model that allow the prediction variance to change over time are the time-varying parameter matrices $\mathbf{H}_t$ and $\mathbf{K}_t$. Of these, the Markov parameter matrix ($\mathbf{H}_t$) is most clearly related to changes in the dynamical nature of the process with time. As mentioned previously, without fundamental knowledge of the underlying process, it is not always beneficial to allow this parameter matrix (or the conditional covariance matrix) to vary with time. For the remainder of this paper we assume that $\mathbf{H}_t = \mathbf{H}$ is fixed in time. Thus, the prediction variance $\mathbf{A}_t$ changes through time only due to the effect of $\mathbf{K}_t$; that is, it changes only as a function of the observation locations at each time.

Algorithmically, one obtains the optimal design for time t given information up to time $t-1$ by (a) calculating $\mathbf{B}_t$ based on $\mathbf{A}_{t-1}$ as in Equation (2.6) and (b) minimizing some function of $\mathbf{A}_t$, the *design criterion*, over all potential designs $\mathbf{D}$ (e.g., one common criterion is the average spatial prediction error variance).

This procedure is clearly adaptive if $\mathbf{H}$, $\boldsymbol{\Sigma}_\epsilon$, $\boldsymbol{\Sigma}_\eta$, and $\mathbf{A}_0$ have been specified. Note that if we are interested in the one-step-ahead prediction variance, then the appropriate minimization in Step b would be some function of $\mathbf{B}_t$, rather than $\mathbf{A}_t$. The minimization in Step b can be carried out using a simple exchange algorithm. The basic idea is that the criterion is evaluated successively for different designs, and the design is updated by exchanging bad points for better points. Such algorithms are widely used in practice and many variations on the basic theme exist (see Cook and Nachtsheim 1980; Atkinson and Federov 1988; Nychka et al. 1997). Although these algorithms are somewhat greedy and tend to find local optima, for relatively small problems experience indicates that they do find the global optimum. For larger problems, the solutions tend to be arbitrarily close to the global optimum depending on how long the algorithm is allowed to run.

3. THE ROLE OF SPATIAL AND TEMPORAL DEPENDENCE

In order to gain some understanding of the relative role of spatial and temporal correlation in space–time dynamic design, we conducted some simple experiments. Specifically, we considered a spatial domain with 49 potential sampling locations on a 7×7 unit spacing regular grid. We assume that the conditional covariance between $Y(\mathbf{s}_i; t)$

and $Y(\mathbf{s}_j;t)$ given Y at all prediction sites at the previous time [i.e., the (i,j) element of $\mathbf{\Sigma}_\eta$] has a stationary isotropic exponential structure $c_\eta(\|\mathbf{s}_i-\mathbf{s}_j\|)=\sigma^2\exp[\|\mathbf{s}_i-\mathbf{s}_j\|\log(\rho)]$, where ρ is the spatial dependence between locations $\mathbf{s}_i$ and $\mathbf{s}_j$ when $\|\mathbf{s}_i-\mathbf{s}_j\|=1$. For the experiments presented here, we assume that $\sigma^2=19$. Furthermore, we let $\mathbf{K}_t$ be a simple incidence matrix (i.e., potential observation locations coincide with prediction locations). We further assume that the measurement error covariance structure is spatial white noise, $\mathbf{\Sigma}_\epsilon=\sigma_\epsilon^2\mathbf{I}$, where $\sigma_\epsilon^2=1$. Five monitors are assumed to be available for our 7×7 grid. In one set of experiments, we fix four monitors and allow one of them to move freely about the domain. In a second set of experiments, we allow all five monitor locations to move freely. For these experiments, our design criterion is the average prediction variance (APV), which is the mean of the diagonal elements of $\mathbf{A}_t$.

3.1 Simple Dynamical Structure

We first consider a simple structure on the Markov parameter matrix, $\mathbf{H}=h\mathbf{I}$, in which h is an autoregressive temporal dependence parameter. This model is essentially the separable spatiotemporal model described in Huang and Cressie (1996). Three values are considered for the temporal dependence parameter $(h=\{.9,.75,.5\})$ and three values for the spatial dependence parameter $(\rho=\{.95,.90,.80\})$. Note in the case of the exponential spatial model described in the previous section that these values of ρ correspond to high, medium, and low spatial dependence, respectively. This characterization is based on our subjective assessment of the strength of spatial correlation in air quality monitoring networks. For example, in the Chicago ozone data discussed in Section 4, we find $\hat{h}$ to vary from 0.4 to 0.7 across the network, while $\hat{\rho}$ is approximately 0.9. Although we feel that the values of h and ρ used in this experiment are representative of many atmospheric processes on local scales, these parameters only have meaning relative to the units of measurement. For example, in the urban ozone data case, $\rho=0.9$ may be realistic when distance is measured in kilometers, but may not be realistic when distance is measured in hundreds of kilometers. If one were studying soil nitrogen levels in an agricultural field, distance might be measured in meters and one might expect large values of ρ on this scale, but small values of ρ if distance were measured in kilometers. Similarly, the temporal correlation parameter h is scale dependent. Larger values are appropriate for high frequency measurements (minutes, hours, days), depending on the temporal smoothness of the process under consideration. Two ozone measurements separated by one day might be highly correlated, but two precipitation measurements separated by one day are likely to be much less correlated.

The effect of high temporal correlation $(h=0.9)$ on dynamic design is well illustrated in Figure 1. The first plot in this figure shows the optimal static space–time design under the case of moderate spatial dependence $(\rho=0.9)$, which is the starting design. In this case, the monitor in the middle of the domain is allowed to move, and the other four are held fixed. Other plots in this figure show how the optimal design changes with time. Clearly, under this strong temporal dependence, the information from a monitor at time $t-1$ is still very much available to reduce the prediction variance in that region during the next few periods,

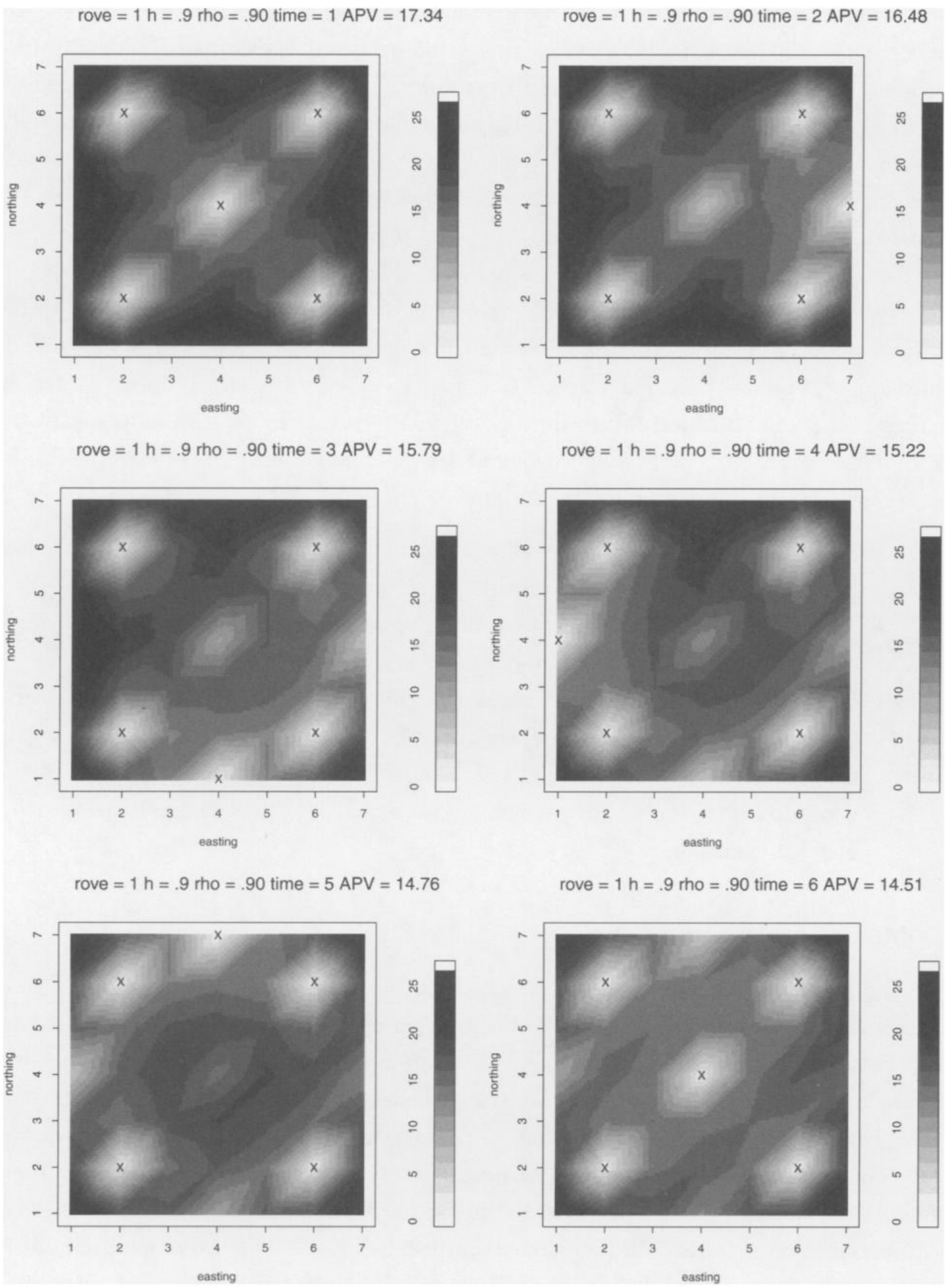

Figure 1. Change in Optimal Design Over Time. Optimal design locations (×) and prediction variances for the first six times under the model with moderate spatial correlation ($\rho = 0.9$), high temporal correlation ($h = 0.9$), and one roving monitor. The APV is averaged over space.

so we want to move sites at the next time to reduce the redundancy in the observations. For the most part, the roving monitor cycles around the outside of the domain. Figure 2 shows the identical experiment except that all five monitoring locations can move. As expected, there is a substantial reduction in APV when all monitors are allowed to move. By contrast, if we have relatively low temporal dependence (e.g., $h = 0.5$), then the dynamic design does not change from the optimal static space–time design if only one monitor can rove. However, if all five monitors are free to rove, then there is a difference between the static and the dynamic designs, and the APV is reduced, although only slightly.

A summary of the decrease in APV (averaged over time steps 10–20) relative to the optimal static design (i.e., the time 1 design) for each simulation is shown in Table 1. Furthermore, Figures 3 and 4 show plots of the APV decrease from the optimal static (i.e., starting) design as a function of time for the one and five roving monitor cases, respectively. From these figures and Table 1, the dynamic design experiments relative to APV can be summarized by the following intuitive results.

- All other factors being constant, the improvement of a dynamic design over a static design is largest when temporal structure is strong.
- All other factors being constant, the improvement of a dynamic design over a static design is largest when more monitors are allowed to rove.
- When only one monitor can rove, the improvement of a dynamic design over a static design is generally not dependent on the strength of the spatial correlation in the process. However, when all five are allowed to rove, processes with low spatial dependence typically show dynamic designs with relatively large improvements over static designs.

3.2 More Complicated Dynamics

The parameter matrix $\mathbf{H}$ in the previous section was assumed to be diagonal and constant across all spatial locations. Although such a model can be useful (e.g., Huang and Cressie 1996), it is inherently separable and does not capture complicated dynamics. A simple extension is to let neighboring spatial locations at the previous time contribute to the process at the current time. In that case, $\mathbf{H}$ has a lagged nearest neighbor structure, with diagonals of $\mathbf{H}$ corresponding to each neighbor (e.g., Wikle et al. 1998). As a simple example, we consider the case where the elements of $\mathbf{H}$ corresponding to the east and west neighbors are both equal to 0.25, the elements corresponding to the north and south neighbors are assumed to be zero, and the same-site (i.e., main-diagonal) parameters of $\mathbf{H}$ are defined to be 0.5. Given a moderate spatial dependence of 0.9 and the conditional covariance model described in Section 3, dynamically varying designs were examined. Figure 5 shows the first six of these designs where five monitors are allowed to rove. Thus, even with relatively low same-site temporal correlation, the simple off-site (lagged neighbor) structure in $\mathbf{H}$ still allows nonintuitive dynamically varying designs of some complexity.

Finally, consider a very simple nonseparable space–time model in which $\mathbf{H} = diag(\mathbf{h})$, where the elements of $\mathbf{h}$ vary with location. Although not shown here, such a model produces

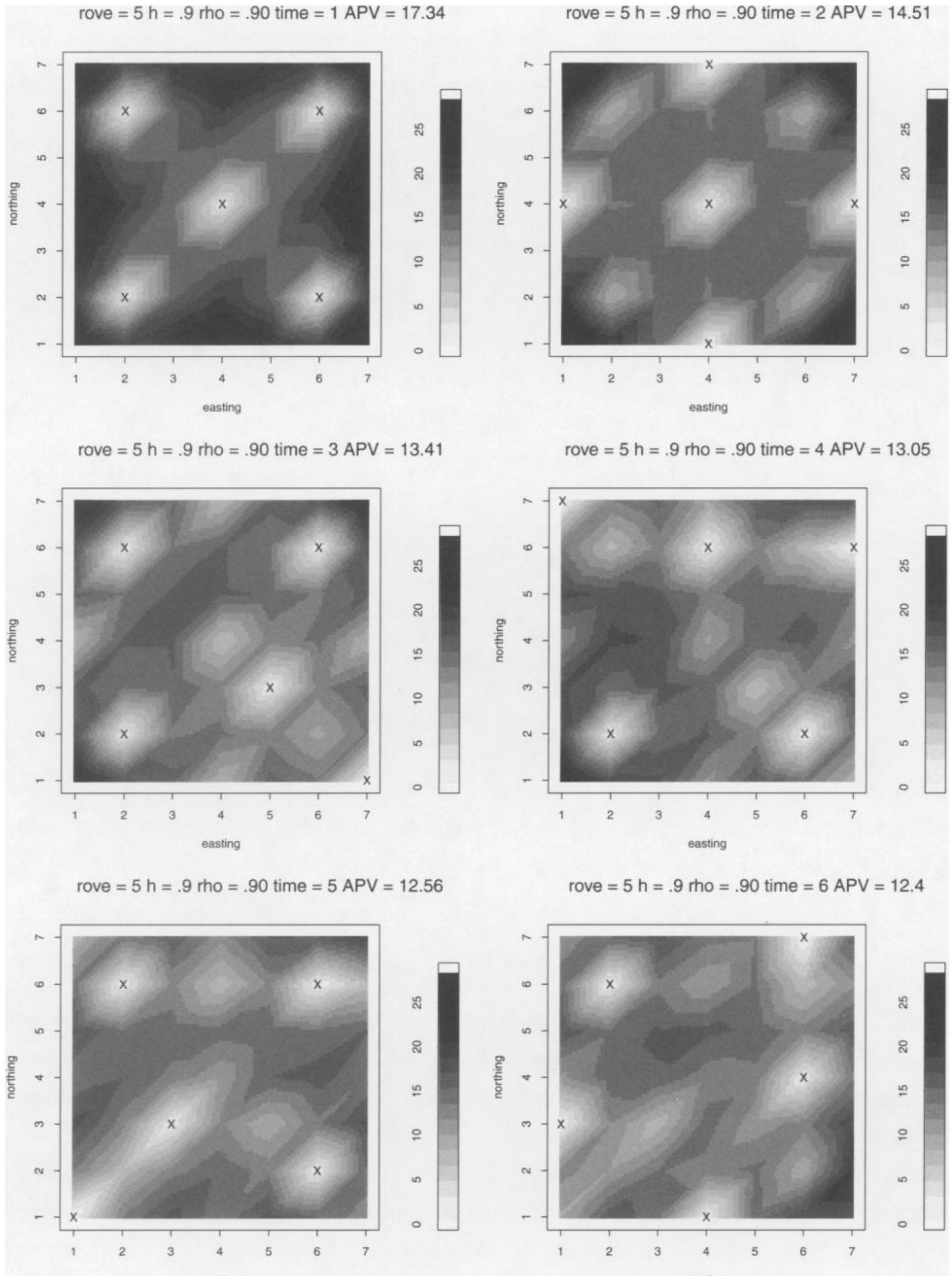

Figure 2. Change in Optimal Design Over Time. Same as Figure 1 except with five roving monitors.

complicated asymmetrical designs. Most environmental processes exhibit substantially more complicated nonseparable structures for **H** than the simple structures presented here. Efficient designs are likely to be more complicated and asymmetrical than spatial designs obtained with separable and/or stationary covariance models. Wikle and Cressie (1997)

Table 1. Results from Simple–Structure Dynamic Design Experiments

Experiment	Number of roving monitors	Temporal structure (h)	Spatial structure (ρ)	Average predicted variance decrease (%)[a]
1	1	0.90	0.95	−16.1
2	1	0.90	0.90	−16.7
3	1	0.90	0.80	−16.7
4	1	0.75	0.95	−5.7
5	1	0.75	0.90	−5.5
6	1	0.75	0.80	−5.4
7	1	0.50	0.95	−0.6
8	1	0.50	0.90	−0.2
9	1	0.50	0.80	−0.1
10	5	0.90	0.95	−29.6
11	5	0.90	0.90	−31.5
12	5	0.90	0.80	−35.0
13	5	0.75	0.95	−11.0
14	5	0.75	0.90	−12.5
15	5	0.75	0.80	−13.3
16	5	0.50	0.95	−3.1
17	5	0.50	0.90	−3.3
18	5	0.50	0.80	−2.3

[a] Prediction variance decrease relative to the optimal static space–time design, averaged over time steps 10–20.

provide a dimension reduction space–time Kalman filter mechanism whereby nonseparable and spatially nonstationary processes can be modeled. Such approaches would be a natural choice for examining dynamical designs for complicated processes.

4. CHICAGO AREA OZONE MONITORING NETWORK

To examine the potential for dynamic design of monitoring networks with realistic and complicated covariance and parameter matrices, our methodology was applied to a collection of 21 stations used to monitor ambient ozone in the Chicago area (see Figure 6). The data consist of eight-hour average ozone (from 9 A.M.–5 P.M.) measurements taken each day from the period June 3 to August 21, 1987. These data are available in AIRS, the EPA air quality database, and have been examined in numerous studies (e.g., Bloomfield, Royle, Steinberg, and Yang 1996; Nychka et al. 1997). We consider the existing monitoring network to be our universe of potential site locations. Under the condition that there are only five monitors available and that these monitors can be moved on a daily basis, our goal is to examine how the optimal spatial design might change with time. That is, we are interested in the optimal "thinning" of an existing network that finds the most efficient designs for a five-point subset of that network. For this example, we focus on the minimization of the maximum prediction variance (MPV) as our design criterion. This is an appropriate design criterion for ozone, as we are interested in preventing an exceedance of the ambient standards at the locations in the existing network. Thus, we do not want to predict at any

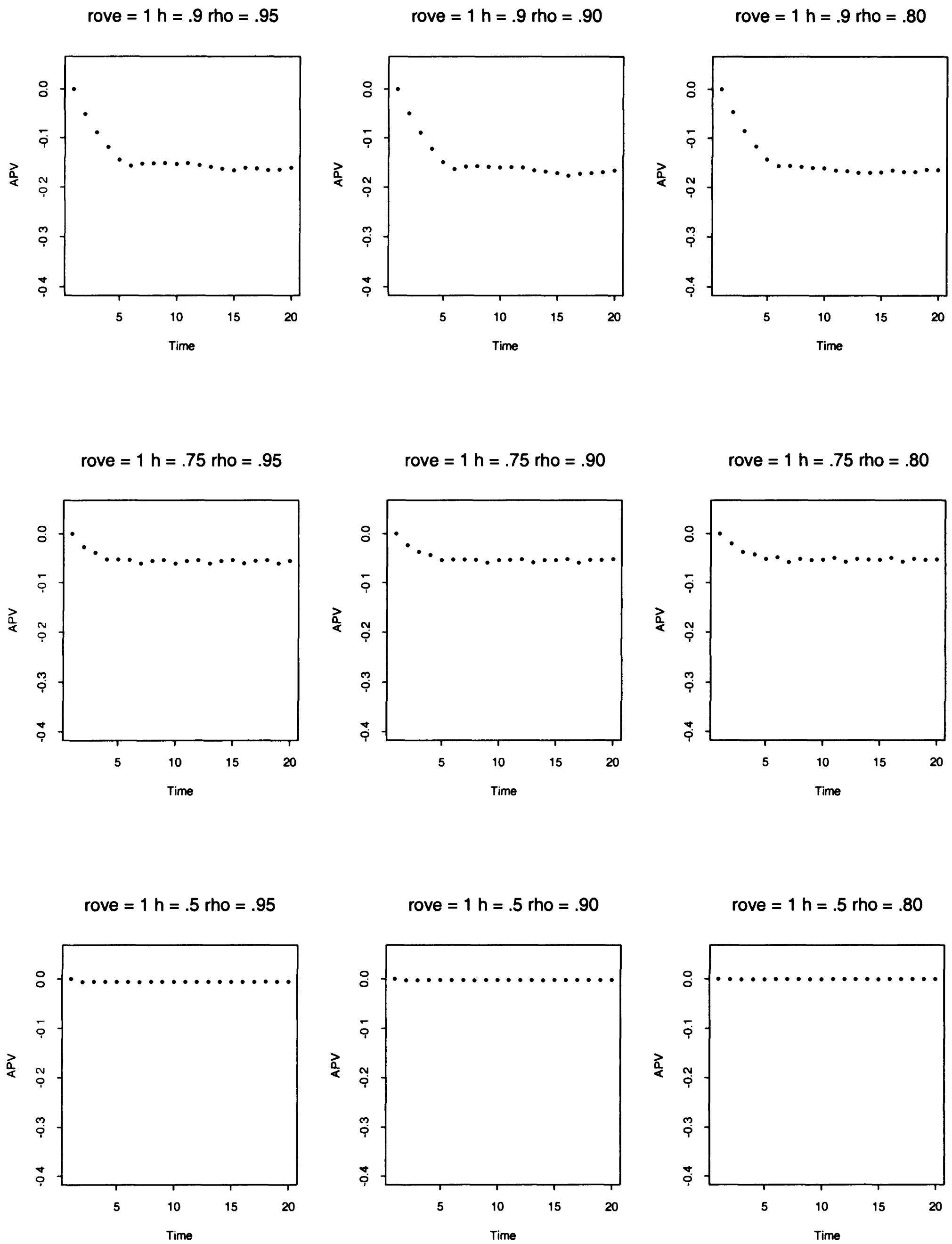

Figure 3. Decrease in APV as a Function of Time. Percent decrease in spatially averaged APV relative to the optimal static space–time design (i.e., design at time 1) as a function of time for one roving monitor. Note that the spatial dependence changes from left to right (high to low) such that the three plots on the left all have high spatial dependence ($\rho = 0.95$), the three plots in the center all have moderate spatial dependence ($\rho = 0.90$), and the three plots on the right all have low spatial dependence ($\rho = 0.80$). Similarly, temporal correlation changes from top to bottom, with high temporal correlation ($h = 0.9$) for the uppermost three figures and low temporal correlation ($h = 0.5$) for the lowest three figures.

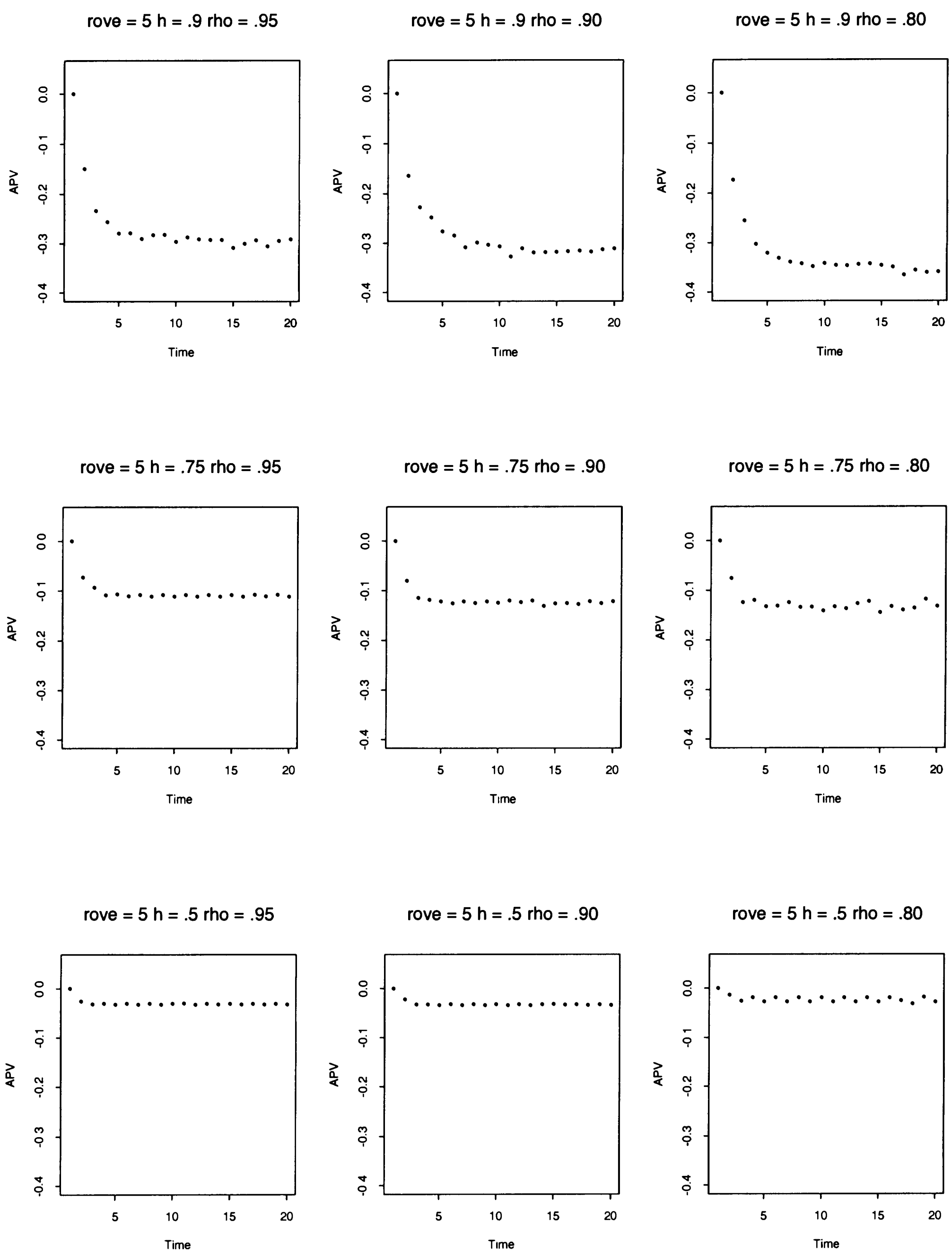

Figure 4. Decrease in APV as a Function of Time. Same as Figure 3 except with five roving monitors.

site too poorly, and minimizing the maximum prediction variance will ensure that the worst prediction at existing sites is not too poor. The ability to use a variety of design criteria with the approach considered here is advantageous. For example, as a comparison, these data were also examined based on the APV criterion, and the network changes with time were not substantial.

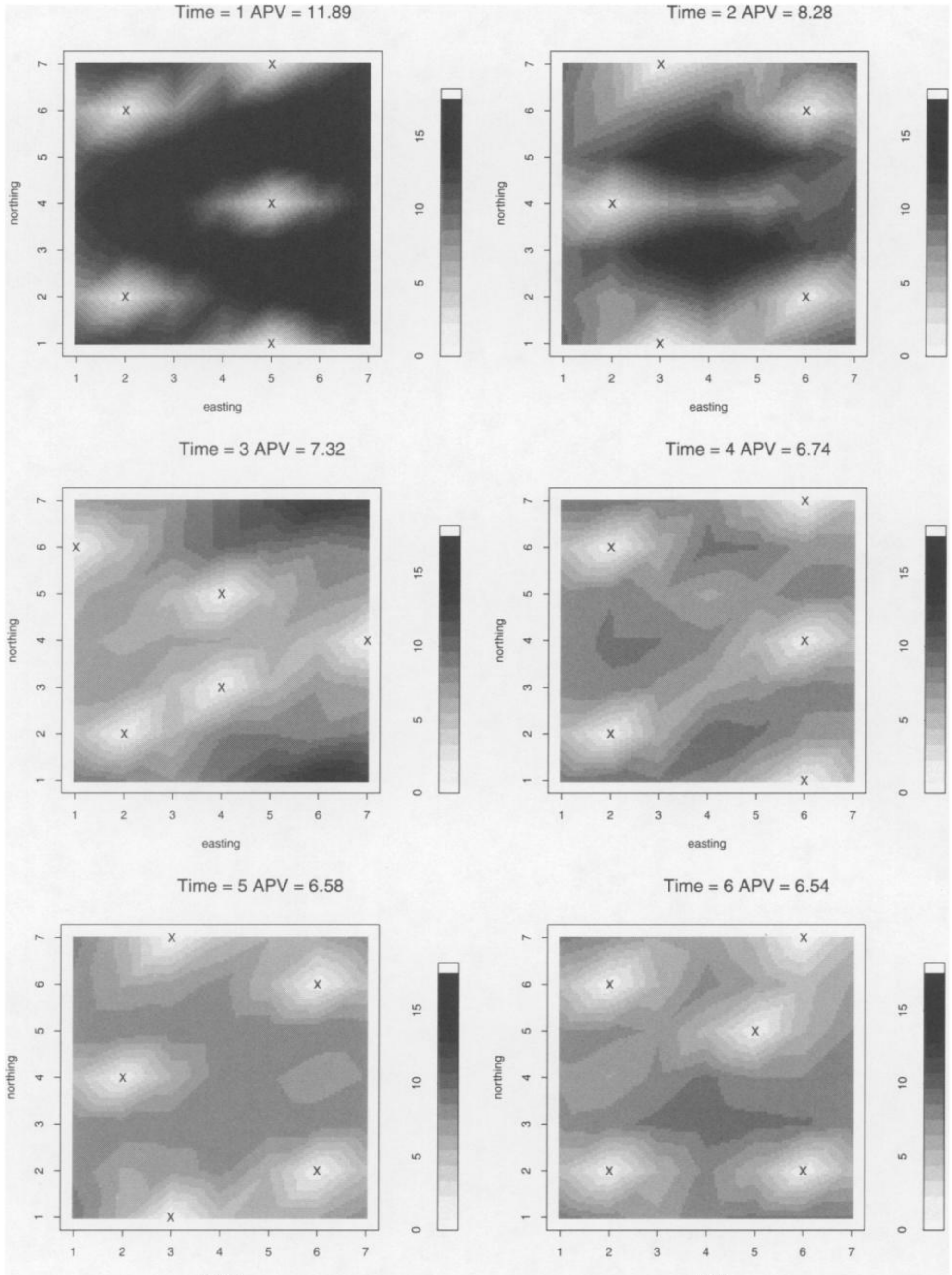

Figure 5. Dynamically Varying Designs. Optimal design locations (×) and prediction variances for the first six times under the lagged nearest-neighbor structure for $\mathbf{H}$ *(described in Section 3.2) with five roving monitors.*

We estimated the $\mathbf{H}$ and $\boldsymbol{\Sigma}_\eta$ matrices and σ_ϵ^2 directly from the data. In particular, we used a two-stage procedure where, in the first stage, σ_ϵ^2 was estimated ($\hat{\sigma}_\epsilon^2 = 10$) by assuming the measurement error process is a nugget effect as is commonly done in spatial analysis (e.g., Cressie 1993). After obtaining the measurement error variance estimate, we obtained

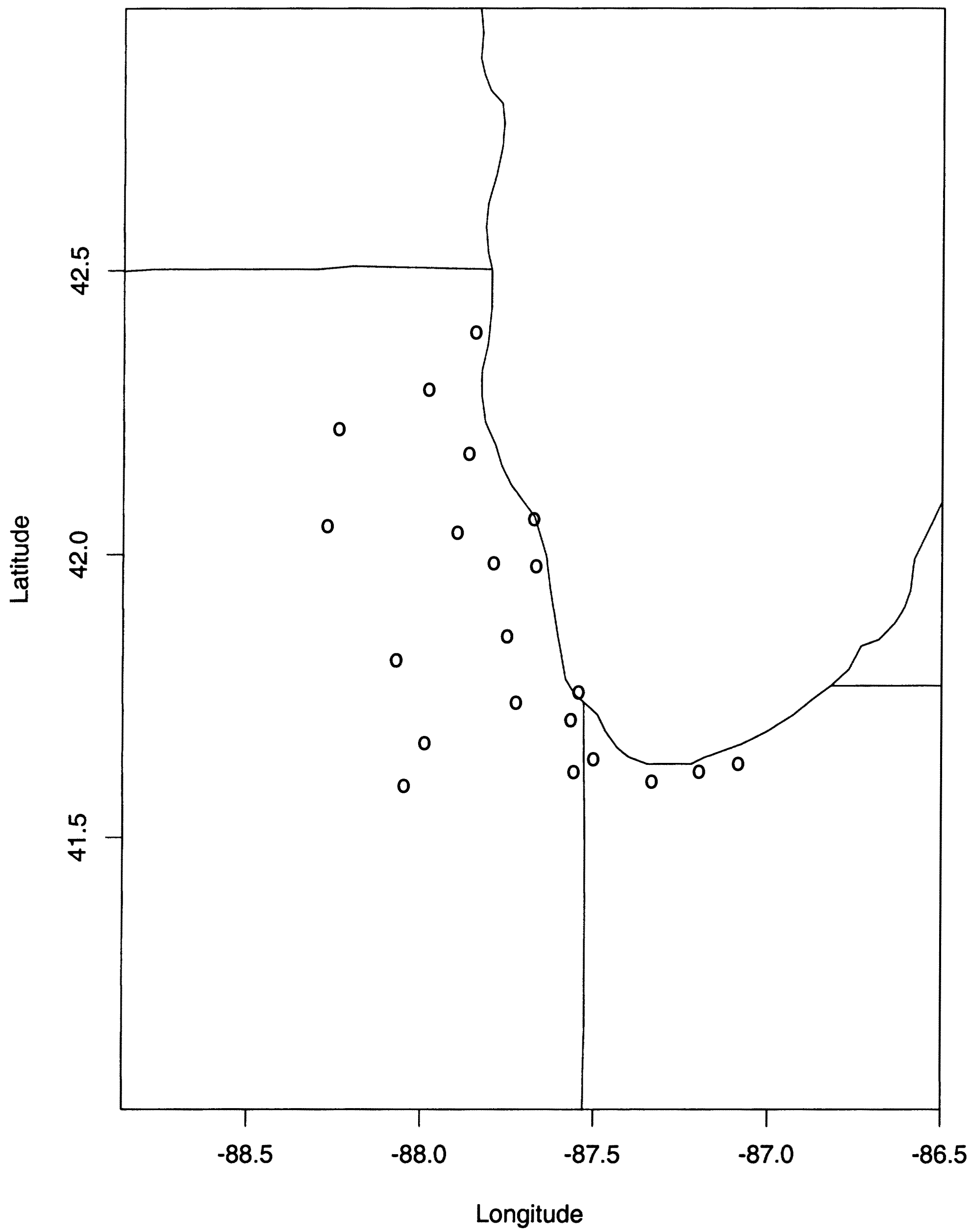

Figure 6. Locations of 21 Ozone Monitoring Stations in the Chicago Urban Area.

the lag-0 and lag-1 covariance estimates of the Y process by method of moments, and the parameter matrices follow by ordinary least squares or conditional maximum likelihood estimation (e.g., Wikle and Cressie 1997). Although somewhat inefficient, such estimation procedures are sufficient to illustrate the methodology presented here. For greater efficiency, one could use an EM algorithm approach for parameter estimation (e.g., Shumway and Stoffer 1982).

Figure 7 shows the first nine designs based on this data set. Note that due to the complicated structure of the estimated parameter matrices, we see that these designs tend

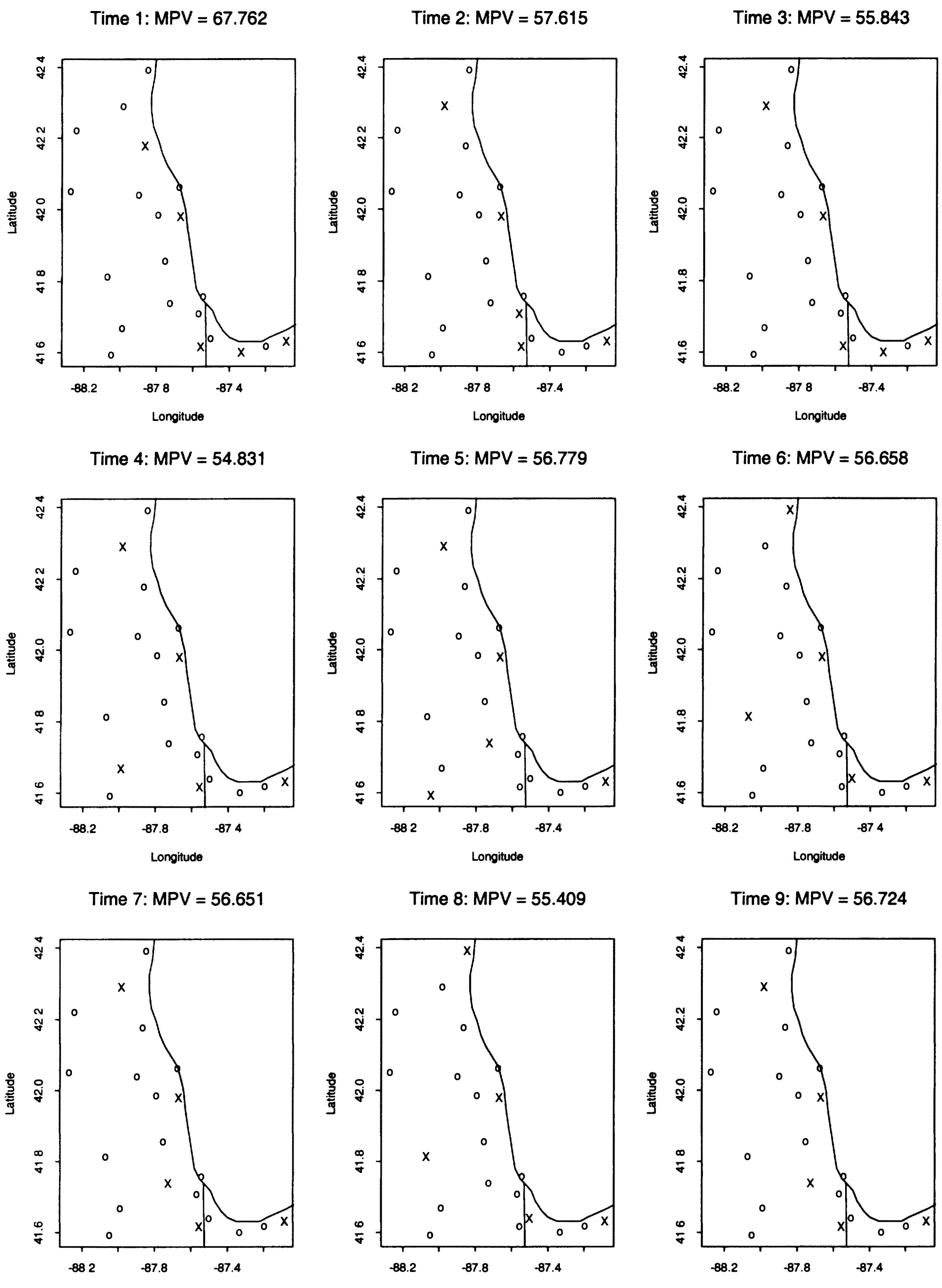

Figure 7. Optimal Design Locations (×) for the Chicago Monitoring Network Over the First Nine Times, Allowing Five Roving Monitors.

not to provide a uniform coverage of the region, but rather, the designs tend to favor locations that have high variance and low correlation with other sites. For comparison, Table 2 shows the minimum MPV over time for the dynamic designs from Figure 7, as well as the optimal static space–time design for this model and summary results from 500 random initial designs

Table 2. Minimized MPV Results from Chicago Area Ozone Network

Design	*Time 1*	*Time 3*	*Time 5*	*Time 7*	*Time 9*
Dynamic	67.76	55.84	56.78	56.65	56.72
Optimal fixed	67.76	64.14	63.10	63.03	63.01
Random[a] (min.)[b]	73.59	62.29	61.61	61.40	61.31
Random (mean)[c]	157.90	149.90	148.60	148.60	148.50

[a] Based on 500 initial random designs fixed over all times.
[b] The minimum MPV of all 500 random designs at each time.
[c] The mean MPV of the 500 random designs at each time.

fixed over time. The dynamic designs are clearly superior through time. It is illustrative to note that the best of the random designs are superior to the optimal static space–time design at all but the first time. This is further evidence of the inefficiency of static designs in the presence of spatiotemporal variability.

Note that one might be interested in evaluating potential monitoring sites that are not part of the current Chicago area network. In that case, one needs to extrapolate the spatiotemporal structure to any potential location in the continuous domain of interest. To do this, one needs to consider complicated nonseparable space–time models (e.g., Guttorp et al. 1994; Wikle and Cressie 1997). Indeed, this is the fundamental challenge in space–time modeling and is beyond the scope of the present paper.

5. DISCUSSION

Sampling plans for environmental processes that have spatiotemporal structure can be improved if allowed to change with time. We have demonstrated that this is true, even with relatively simple spatiotemporal structure, if the temporal association is strong. However, for certain situations, for example, if the temporal dependence is weak to moderate and there is not space–time interaction, purely spatial designs are efficient, justifying their widespread use even for space–time processes. This statement must be qualified by the number of monitors that are allowed to move. Even with weak to moderate temporal dependence, dynamic designs are considerably more efficient (relative to APV) than static designs when several monitors can move (although, perhaps not efficient enough to justify the potential cost). The role of spatial dependence appears to be less crucial. When only one monitor in a spatially efficient network is allowed to move, the efficiency of dynamic designs is not highly dependent on the degree of spatial correlation. However, when all monitors are allowed to move, designs are more efficient under models with weaker spatial dependence. As the dynamical structure $\mathbf{H}$ becomes more complicated, the dynamic designs are less intuitive, but the designs are typically much more efficient than the optimal static space–time design.

The methodology was applied to a set of ozone monitoring locations in the Chicago area. This analysis used the data to estimate the complicated dynamical and conditional covariance matrices needed for the space–time dynamic model. The results showed that if a subset of monitoring stations is desired, significant improvements in efficiency (approximately 20%) can be attained by allowing monitoring sites to change with time.

We note that the time step of the spatiotemporal model is critical. This time increment is generally determined by the sampling requirements for the process of interest. However, if one has the freedom to alter monitor location within the required time step, then more efficiency could be obtained from the dynamic design. For instance, when monitoring daily ozone concentrations, if a monitor is moved at noon, then one essentially gains another monitor since the temporal correlation is expected to be much higher within a day than among days.

It is clear, based on Equations (2.4) and (2.5), that the prediction error variance will be sensitive over time to changes in the Markovian parameter matrix $\mathbf{H}_t$. As mentioned previously, allowing this matrix to vary in time typically requires some fundamental knowledge about the underlying dynamics of the process. Alternatively, one can imagine different regimes for a process in which the structure of $\mathbf{H}_j$ would change according to regime j. In modeling such a process, it is plausible to treat $\mathbf{H}_j$ as random and to model it in another stage of the model hierarchy. Such an approach requires a fully Bayesian treatment. There have been some recent examples of hierarchical spatial (e.g., Royle, Berliner, Wikle, and Milliff 1999) and spatiotemporal models (e.g., Waller, Carlin, Xia, and Gelfand 1997; Wikle et al. 1998). Such models could be used to allow $\mathbf{H}_t$ to vary in time and appropriate dynamic designs could be examined.

As was demonstrated by the experiments and the ozone data, the methodology used here can consider different design criteria, depending on the issues at hand. For example, in the case of ambient air quality monitoring, we may be interested in preventing an exceedance. In such cases, we could consider minimizing the maximum prediction variance. Alternatively, for temperature monitoring in global warming analyses, we might be interested in a time trend. Although not discussed here, it can be shown that if the spatiotemporal covariance is separable, which is a special case of our model when $\mathbf{H} = h\mathbf{I}$, then optimal designs for estimating temporal regression parameters will be static and, thus, will be the optimal design for any given time. These issues will be addressed elsewhere.

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