Particle Swarm Optimization for Spatio-Temporal Design

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Abstract

KEY WORDS:

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1 Introduction

2 Model

Let $Y(\boldsymbol{s}, t)$ denote the true (latent) log ozone (in log PPM) at location $\boldsymbol{s} \in \mathcal{D}$ at time point $t \in \mathcal{T}$. Then we assume the following process model

$$Y(\boldsymbol{s}, t) = \mu(\boldsymbol{s}; t) + \psi(\boldsymbol{s}) + \tau(t) + \kappa(\boldsymbol{s}; t) + \delta(\boldsymbol{s}, t)$$

where $\mu(.;.)$ is a deterministic mean term, $\psi(.)$ is a spatial random effect, $\tau(t)$ is a temporal random effect, $\kappa(.;.)$ is a spatio-temporal interaction random effect, and $\delta(.;.)$ is a white noise fine scale variation term. To complete the model, we need to specify the mean structure, and covariance structures for $\psi(.)$, $\tau(.)$, and $\kappa(.,.)$. Let $\gamma(\boldsymbol{s}; t) = \psi(\boldsymbol{s}) + \tau(t) + \kappa(\boldsymbol{s}; t)$. We are unlikely to have access to many relevant covariates, so a nonstationary covariance structure for $\gamma(.;.)$ seems appropriate. Similarly, we expect spatio-temporal interaction so a nonseparable covariance structure seems appropriate, and additional an assymetric structure seems appropriate because, e.g., the wind tends to blow in certain directions. This makes the modeling problem somewhat challenging, but we can simplify it by taking advantage of the fact that we do not need to model the dynamics in continuous time. We need continuous space in order to apply PSO to the design problem, but since measurements are typically made daily, we can work in the discrete time setting.

In discrete time, the process model can be conceived of as a time series of geostatistical spatial processes. Then the model can be written as

$$Y_t(\boldsymbol{s}) = \mu_t(\boldsymbol{s}) + \mathcal{M}_t(\boldsymbol{s}) + \psi(\boldsymbol{s}) + \delta_t(\boldsymbol{s})$$

where $\mathcal{M}_t(\boldsymbol{s}) = \int_{\mathcal{D}} m(\boldsymbol{s}; \boldsymbol{u})[Y_{t-1}(\boldsymbol{u}) - \mu_{t-1}(\boldsymbol{s})]d\boldsymbol{u}$, $\mu_t(\boldsymbol{s})$ is a deterministic mean term, $\psi(.)$ is a spatially correlated process, and $\delta_t(.)$ is white noise. Spatio-temporal interaction, i.e. how locations in period $t-1$ impact a location in period t, is controlled by $\mathcal{M}_t$ through a first order autoregressive structure. The weight function $m(\boldsymbol{s};\boldsymbol{u})$ controls this interaction and must be specified. Similarly, a spatial covariance structure for the $\psi(.)$s must be specified.

Supposing both of pieces of the model are specified, let $\boldsymbol{d}_i^{(t)} \in \mathcal{D}$ for $i = 1, 2, \ldots, I_t$ denote the locations of the I_t monitoring stations in period t. Then the data model is

$$Z_t(\boldsymbol{d}_i^{(t)}) = Y_t(\boldsymbol{d}_i^{(t)}) + \varepsilon_t(\boldsymbol{d}_i^{(t)}) \text{ for } i = 1, 2, \ldots, I_t$$

where $\varepsilon_t(.)$ is white noise.