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Supporting Web Material: Adaptively-Tuned Particle Swarm Optimization with Application to Spatial Design

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S1. PSO and BBPSO details

In Section 2 we reviewed PSO and BBPSO, and introduced our adaptively tuned variants of both. Here we describe in detail several modifications to both PSO and BBPSO. Most of these are so-called standard modifications from Clerc (2012).

S1.1. PSO

The standard PSO algorithm is given by (1) in the main text. We consider several additions and modifications to this algorithm below. Each of these modifications is combined with adaptively tuned inertia as in (4) to create the AT-PSO algorithms we employ.

S1.1.1. Initialization: In order to initialize the swarm, the number of particles, their initial locations, and their initial velocities have to be chosen. Clerc (2012) suggests making the swarm size a function of the dimension of the search space, but notes that this is known to be hard to do well and suggests a default swarm size of 40. We use this latter suggestion. We assume the search space is a D-dimensional hypercube given by $\bigtimes_{j=1}^{D}[min_j, max_j]$. Then each particle's initial location is randomly generated uniformly on the cube; i.e., $\theta_{ij}(0) \overset{ind}{\sim} U(min_j, max_j)$ for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, D$. Each particle's velocity is initialized based on its location via $v_{ij}(0) \overset{ind}{\sim} U(min_j - x_{ij}(0), max_j - x_{ij}(0))$ for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, D$.

In Section 4.3 each design point is constrained to be in Harris County, TX, which is not a rectangle. We initialize the swarm by placing initial design points on the smallest rectangle containing Harris County. Design points outside of Harris County will quickly move back into the county due to the confinement strategy discussed next.

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S1.1.2. Confinement: Even though the initialization of the swarm is confined to a hypercube, nothing prevents any given particle from leaving the search space. There are a number of things that can be done in order to address this problem and each has advantages and disadvantages depending on the situation. We focus on two approaches. One is to move any particle that leaves the search space to the nearest point still inside the search space and then adjust its velocity, e.g. Clerc (2012) suggests that whenever $x_{ij}(k) > max_j$, it should be set to $x_{ij}(k) = max_j$ and similarly when $x_{ij}(k) < min_j$ it should be set to $x_{ij}(k) = min_j$, and in both cases the velocity along that dimension should be reversed and halved, i.e. $v_{ij}(k) = -0.5v_{ij}(k)$. This causes particles to bounce off the boundary and move back toward the middle of the search space. This is the default strategy we use. This strategy can also be used for more complex search spaces, and in Section 4.3 when a particle leaves Harris County, we move it back to the nearest point in the county and reset the particle's velocity as in the previous paragraph. To move the particle we use the `gNearestPoint` function from the `rgeos` `R` package (Bivand & Rundel, 2016; R Core Team, 2016).

S1.1.3. Redraw Neighborhoods: In Section 2 we briefly described a variant of the stochastic star neighborhood which we use. This neighborhood is stochastic, meaning that each particle's neighbors are randomly drawn when the algorithm is initialized. After an iteration in which the best known value of the objective function is unchanged, each particle's neighbors are randomly redrawn according to the same distribution.

S1.1.4. Asynchronous Updates: The way we defined PSO in (1) each particle can update simultaneously. This means that the algorithm is parallelizable, which is a major advantage for implementation on modern GPUs. However, asynchronously updating the particles typically results in faster converging algorithms when it is computationally feasible. In an asynchronous update from period k to $k+1$, particle i recognizes that particle $i-1$ has already updated its personal best location to $\boldsymbol{p}_{i-1}(k+1)$ by the time it is i's turn to update. So i computes its group best location taking this into account. This causes particle $i = 1$ to behave differently from particle $i = n$ since particle n always has better information in order to perform its update, so every iteration the particles are randomly reordered. More formally, before iteration $k+1$ sample $o_1(k+1), o_2(k+1), \ldots, o_n(k+1)$ from $\{1, 2, \ldots, n\}$ without replacement. Then the particles update starting with $o_1(k+1)$, $o_2(k+1)$, etc., so that $o_j(k+1)$ is the jth particle to be updated in period $k+1$. Then the group best update becomes $\boldsymbol{g}_{o_i}(k+1) = \arg\min_{\{\boldsymbol{p}_{o_j}(k+1)|j\in\mathcal{N}_i\}} Q(\boldsymbol{p}_j(k_j^*))$ where $k_j^* = k+1$ if $j < i$ and k otherwise. We asynchronously update in all of our algorithms.

S1.1.5. Coordinate Free Velocity Updates: The standard velocity update in (1) is well known to bias the algorithm toward locations near the coordinate axes and especially the origin (Monson & Seppi, 2005; Spears et al., 2010). In general we may not know if the true optimum is near an axis, so this behavior is undesirable. There are several alternative velocity updates available; e.g., see Monson & Seppi (2005). We use the coordinate free (CF) update suggested by Clerc (2012). First define the center of gravity for particle i to be $\boldsymbol{C}_i(k) = \boldsymbol{\theta}_i(k) + \phi_1\{\boldsymbol{p}_i(k) - \boldsymbol{\theta}_i(k)\}/3 + \phi_2\{\boldsymbol{g}_i(k) - \boldsymbol{\theta}_i(k)\}/3$. Let $\mathcal{H}_i(k)$ denote the hypersphere centered at $\boldsymbol{C}_i(k)$ with radius $||\boldsymbol{C}_i(k) - \boldsymbol{\theta}_i(k)||$ where $||\cdot||$ denotes Euclidean distance. Then a new point $\boldsymbol{\theta}_i'(k)$ is drawn randomly from $\mathcal{H}_i(k)$ by sampling a direction and a radius, each uniformly. This is *not* the same as drawing uniformly over $\mathcal{H}_i(k)$ and in fact favors points near the center. Then the CF velocity update is given by $\boldsymbol{v}_i(k+1) = \omega\boldsymbol{v}_i(k) + \boldsymbol{x}_i'(k)$. We use both the standard and CF velocity updates in our algorithms. The standard PSO algorithm with each feature in this subsection including the CF velocity update is what Clerc (2012) calls SPSO 2011.

S1.1.6. When Personal Best Equals the Group Best: When a particle's personal best and group best locations coincide, it is often advantageous to allow the particle to explore more than usual. In the standard velocity update we do this

by removing the social term so that $\boldsymbol{v}_i(k+1) = \omega \boldsymbol{v}_i(k) + \phi_1 \boldsymbol{r}_{1i}(k) \circ \{\boldsymbol{p}_i(k) - \boldsymbol{\theta}_i(k)\}$. In the CF velocity update we change the center of gravity to ignore the social term so that $\boldsymbol{C}_i(k) = \boldsymbol{\theta}_i(k) + \phi_2\{\boldsymbol{p}_i(k) - \boldsymbol{\theta}_i(k)\}/2$.

S1.2. BBPSO

The standard BBPSO algorithm was introduced by Kennedy (2003) and updates from t to $t+1$ via (2). We use each of the features in Section S1.1 in our BBPSO algorithms, though some of them need to be modified for the BBPSO setting. We list them below along with another modification of BBPSO which we employ. Each of these modifications are combined with adaptively tuning a scale parameter as in (3) to create our AT-BBPSO algorithms.

S1.2.1. BBPSOxp: A commonly used variant of BBPSO, also introduced by Kennedy (2003), is called BBPSOxp. In this variant, each coordinate of each particle has a 50% chance of updating according to (2) and a 50% chance of moving directly to that particle's personal best location on that coordinate. In other words

$$\theta_{ij}(k+1) = \begin{cases} N\left\{\frac{p_{ij}(k)+g_{ij}(k)}{2}, h_{ij}^2(k)\right\} & \text{with probability } 0.5 \\ p_{ij}(k) & \text{otherwise,} \end{cases} \tag{S.1}$$

where $h_{ij}(k) = |p_{ij}(k) - g_{ij}(k)|$. We use both xp and non-xp versions of our BBPSO algorithms.

S1.2.2. CF BBPSO: BBPSO's update also depends on the coordinate system since each coordinate of $\boldsymbol{\theta}$ gets a different standard deviation. We employ BBPSO algorithms using the default standard deviation, but also using a coordinate free standard deviation given by $h_{ij}(k) = ||\boldsymbol{p}_i(k) - \boldsymbol{g}_i(k)||$.

S1.2.3. When Personal Best Equals The Group Best in BBPSO: A downside of both BBPSO and BBPSOxp is that any particle whose personal best is currently its group best location does not move due to the definition of the standard deviation term. Several methods have been proposed to overcome this; e.g. Hsieh & Lee (2010) and Zhang et al. (2011). Zhang et al. (2011) propose using mutation and crossover operations for the group best particle. To do this, each group best particle randomly selects three other distinct particles from the entire swarm, i_1, i_2, and i_3, and updates according to

$$\theta_{ij}(k+1) = p_{i_1 j}(k) + 0.5\{p_{i_2 j}(k) - p_{i_3 j}(k)\}. \tag{S.2}$$

This combines easily with BBPSOxp to update each coordinate of each particle with $h_{ij}(k) = 0$ according to (S.2) and the rest according to (S.1).

S2. Tables for Comparing PSO and BBPSO algorithms

Tables S2.1–S2.6 contain the simulation results for Objective Functions 1-6 respectively (OF1, OF2, etc.). We use several measures to quantify how well each algorithm finds the global minimum. First, each table includes the mean and standard deviation of the absolute difference between the true global minimum and the algorithm's estimated global minimum across all 40 replications, denoted by Mean and SD. Second, each table includes a convergence criterion — the proportion of the replications that came within 0.01 of the true global minimum, denoted by $\widehat{P}$. Finally, $\widehat{K}$ denotes the median number of iterations until the algorithm reaches the convergence criterion; $\widehat{K} > 1000$ indicates that the algorithm did not converge in the allotted 1,000 iterations in at least of 50% of replications. Mean, $\widehat{P}$, and

$\widehat{K}$ can be thought of as how close on average the algorithm gets to the global minimum, what proportion of the time it converges, and long it takes to converge respectively. Values for the Mean and SD that are greater than 10,000 are omitted for the sake of readable tables. See Section 3 for discussion of these tables.

OF1; Nbhd:	Global				SS3				SS1			
Algorithm	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$
PSO1	0.00	0.00	1.00	205.5	0.00	0.00	1.00	358.5	384.26	891.68	0.32	> 1000
PSO2	0.00	0.00	1.00	113	0.00	0.00	1.00	200.5	724.66	1160.10	0.20	> 1000
PSO1-CF	173.52	201.37	0.00	> 1000	174.60	139.70	0.00	> 1000	1633.30	975.59	0.00	> 1000
PSO2-CF	164.60	117.21	0.00	> 1000	122.56	97.48	0.00	> 1000	1956.30	1074.90	0.00	> 1000
DI-PSO1	0.00	0.00	1.00	214	0.00	0.00	1.00	265.5	1467.50	1543.60	0.00	> 1000
DI-PSO2	0.00	0.00	1.00	187	0.00	0.00	1.00	233.5	1641.30	1566.30	0.00	> 1000
DI-PSO1-CF	1709.90	897.14	0.00	> 1000	694.98	346.65	0.00	> 1000	2418.30	1140.10	0.00	> 1000
DI-PSO2-CF	1535.30	848.11	0.00	> 1000	790.97	429.16	0.00	> 1000	2435.10	1173.80	0.00	> 1000
AT1-PSO1	0.00	0.00	1.00	186	0.00	0.00	1.00	263	1962.50	1777.20	0.00	> 1000
AT1-PSO2	0.00	0.00	1.00	183	0.00	0.00	1.00	247	932.07	1118.20	0.00	> 1000
AT1-PSO1-CF	329.08	260.13	0.00	> 1000	294.80	166.07	0.00	> 1000	2489.30	1040.10	0.00	> 1000
AT1-PSO2-CF	342.09	293.48	0.00	> 1000	268.43	181.01	0.00	> 1000	2096.10	1131.30	0.00	> 1000
AT2-PSO1	0.00	0.00	1.00	112	0.00	0.00	1.00	154	232.14	205.78	0.00	> 1000
AT2-PSO2	0.00	0.00	1.00	117	0.00	0.00	1.00	150.5	127.58	164.99	0.00	> 1000
AT2-PSO1-CF	165.98	133.79	0.00	> 1000	196.39	130.15	0.00	> 1000	1550.60	696.54	0.00	> 1000
AT2-PSO2-CF	118.22	77.93	0.00	> 1000	160.47	118.92	0.00	> 1000	1453.20	922.27	0.00	> 1000
AT1-BBPSO	0.00	0.00	1.00	740	0.00	0.00	1.00	756	0.00	0.00	1.00	679.5
AT1-BBPSOxp	0.00	0.00	1.00	822.5	0.00	0.00	1.00	831	0.00	0.00	1.00	694
AT1-BBPSO-CF	0.00	0.00	1.00	637	0.00	0.00	1.00	642	0.00	0.00	1.00	572.5
AT1-BBPSOxp-CF	0.00	0.00	1.00	724.5	0.00	0.00	1.00	717	0.00	0.00	1.00	607.5
AT2-BBPSO	0.00	0.00	1.00	445.5	0.00	0.00	1.00	465	0.00	0.00	1.00	451
AT2-BBPSOxp	0.00	0.00	1.00	501	0.00	0.00	1.00	511	0.00	0.00	1.00	479
AT2-BBPSO-CF	0.00	0.00	1.00	386.5	0.00	0.00	1.00	404.5	0.00	0.00	1.00	392
AT2-BBPSOxp-CF	0.00	0.00	1.00	436.5	0.00	0.00	1.00	458	0.00	0.00	1.00	420

Table S2.1. Simulation results for OF1. Each algorithm was run for 1,000 iterations over 40 replications. Mean and SD denote the mean and standard deviation of minimum objective function values found over all replications, while $\widehat{P}$ denotes the proportion of all replications that came within 0.01 of the true global minimum (equal to zero), and $\widehat{K}$ denotes the median number of iterations until the algorithm came within 0.01 of the global minimum.

OF2; Nbhd:	Global				SS3				SS1			
Algorithm	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$
PSO1	0.00	0.00	0.92	882.5	42.08	41.70	0.00	> 1000	3121.10	1900.60	0.00	> 1000
PSO2	0.00	0.00	1.00	455	0.05	0.09	0.28	> 1000	2128.20	2264.80	0.00	> 1000
PSO1-CF	1198.70	727.12	0.00	> 1000	718.76	477.04	0.00	> 1000	3467.90	1973.20	0.00	> 1000
PSO2-CF	892.45	593.77	0.00	> 1000	639.33	342.00	0.00	> 1000	2966.50	1426.90	0.00	> 1000
DI-PSO1	52.59	108.12	0.00	> 1000	225.76	210.47	0.00	> 1000	5494.70	2604.70	0.00	> 1000
DI-PSO2	151.31	169.29	0.00	> 1000	516.50	282.07	0.00	> 1000	4323.90	1872.50	0.00	> 1000
DI-PSO1-CF	3793.40	1717.40	0.00	> 1000	1401.90	678.72	0.00	> 1000	4510.60	2058.90	0.00	> 1000
DI-PSO2-CF	3873.30	1935.70	0.00	> 1000	1566.30	733.85	0.00	> 1000	3637.10	1528.20	0.00	> 1000
AT1-PSO1	0.00	0.00	1.00	478.5	0.08	0.18	0.25	> 1000	7808.60	2929.70	0.00	> 1000
AT1-PSO2	0.00	0.00	1.00	481.5	0.01	0.01	0.85	924	4071.70	1647.50	0.00	> 1000
AT1-PSO1-CF	1424.90	620.02	0.00	> 1000	1021.10	509.66	0.00	> 1000	2910.80	1528.70	0.00	> 1000
AT1-PSO2-CF	1517.60	1070.00	0.00	> 1000	883.77	458.02	0.00	> 1000	2480.30	1419.50	0.00	> 1000
AT2-PSO1	91.83	356.78	0.10	> 1000	0.40	0.85	0.12	> 1000	4323.20	1925.80	0.00	> 1000
AT2-PSO2	5.98	20.86	0.52	979	0.23	1.24	0.60	965.5	2122.30	1402.00	0.00	> 1000
AT2-PSO1-CF	1358.10	766.94	0.00	> 1000	969.11	476.07	0.00	> 1000	2737.70	1082.90	0.00	> 1000
AT2-PSO2-CF	1078.80	640.14	0.00	> 1000	878.00	410.46	0.00	> 1000	2342.40	1427.20	0.00	> 1000
AT1-BBPSO	0.06	0.03	0.00	> 1000	0.01	0.01	0.62	989	0.00	0.00	0.90	916.5
AT1-BBPSOxp	0.67	0.41	0.00	> 1000	0.02	0.01	0.15	> 1000	0.01	0.00	0.85	944
AT1-BBPSO-CF	0.03	0.01	0.05	> 1000	0.00	0.00	0.97	916	0.00	0.00	1.00	819.5
AT1-BBPSOxp-CF	0.35	0.22	0.00	> 1000	0.01	0.01	0.72	972	0.00	0.00	1.00	873.5
AT2-BBPSO	0.00	0.00	1.00	852	0.00	0.00	0.92	846.5	0.00	0.00	1.00	660
AT2-BBPSOxp	0.45	0.63	0.00	> 1000	0.05	0.13	0.50	972	0.00	0.00	1.00	718
AT2-BBPSO-CF	0.00	0.00	1.00	821	0.01	0.02	0.92	825	0.00	0.00	1.00	636.5
AT2-BBPSOxp-CF	0.24	0.20	0.00	> 1000	0.04	0.08	0.52	962	0.00	0.00	1.00	682

Table S2.2. Simulation results for OF2. Each algorithm was run for 1,000 iterations over 40 replications. Mean and SD denote the mean and standard deviation of minimum objective function values found over all replications, while $\widehat{P}$ denotes the proportion of all replications that came within 0.01 of the true global minimum (equal to zero), and $\widehat{K}$ denotes the median number of iterations until the algorithm came within 0.01 of the global minimum.

OF3; Nbhd:	Global				SS3				SS1			
Algorithm	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$
PSO1	51.84	81.96	0.00	> 1000	32.87	35.51	0.00	> 1000			0.00	> 1000
PSO2	24.09	40.64	0.00	> 1000	34.97	56.51	0.00	> 1000			0.00	> 1000
PSO1-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
PSO2-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
DI-PSO1	73.91	118.06	0.00	> 1000	138.79	237.03	0.00	> 1000			0.00	> 1000
DI-PSO2	73.15	101.70	0.00	> 1000	285.84	515.43	0.00	> 1000			0.00	> 1000
DI-PSO1-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
DI-PSO2-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
AT1-PSO1	42.13	62.63	0.00	> 1000	27.96	32.20	0.00	> 1000			0.00	> 1000
AT1-PSO2	29.68	49.48	0.02	> 1000	31.32	39.42	0.00	> 1000			0.00	> 1000
AT1-PSO1-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
AT1-PSO2-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
AT2-PSO1	63.40	150.86	0.02	> 1000	27.31	45.22	0.00	> 1000			0.00	> 1000
AT2-PSO2	45.50	127.02	0.00	> 1000	21.57	33.17	0.00	> 1000			0.00	> 1000
AT2-PSO1-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
AT2-PSO2-CF			0.00	> 1000			0.00	> 1000			0.00	> 1000
AT1-BBPSO	286.56	733.87	0.00	> 1000	163.98	568.42	0.00	> 1000	55.96	68.75	0.00	> 1000
AT1-BBPSOxp	152.06	495.54	0.00	> 1000	53.66	40.22	0.00	> 1000	38.12	36.67	0.00	> 1000
AT1-BBPSO-CF	121.97	358.24	0.00	> 1000	199.26	573.65	0.00	> 1000	36.63	52.87	0.00	> 1000
AT1-BBPSOxp-CF	390.89	857.50	0.00	> 1000	39.49	42.56	0.00	> 1000	31.37	35.33	0.00	> 1000
AT2-BBPSO	192.88	490.38	0.00	> 1000	63.77	83.78	0.00	> 1000	84.38	132.31	0.00	> 1000
AT2-BBPSOxp	124.92	235.06	0.00	> 1000	41.88	46.31	0.00	> 1000	34.63	36.02	0.00	> 1000
AT2-BBPSO-CF	325.19	767.79	0.00	> 1000	115.76	398.59	0.00	> 1000	71.89	121.28	0.00	> 1000
AT2-BBPSOxp-CF	229.62	646.40	0.00	> 1000	45.22	55.65	0.00	> 1000	37.75	59.87	0.00	> 1000

Table S2.3. Simulation results for OF3. Each algorithm was run for 1,000 iterations over 40 replications. Mean and SD denote the mean and standard deviation of minimum objective function values found over all replications, while $\widehat{P}$ denotes the proportion of all replications that came within 0.01 of the true global minimum (equal to zero), and $\widehat{K}$ denotes the median number of iterations until the algorithm came within 0.01 of the global minimum.

OF4; Nbhd:	Global				SS3				SS1			
Algorithm	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$
PSO1	3.78	1.53	0.00	> 1000	0.90	1.12	0.47	> 1000	411.24	913.72	0.00	> 1000
PSO2	8.82	4.25	0.00	> 1000	2.62	1.62	0.07	> 1000	814.28	1218.90	0.00	> 1000
PSO1-CF	504.88	318.43	0.00	> 1000	337.63	182.82	0.00	> 1000	1951.90	1010.30	0.00	> 1000
PSO2-CF	337.14	160.49	0.00	> 1000	253.11	127.14	0.00	> 1000	1973.70	971.52	0.00	> 1000
DI-PSO1	7.68	2.34	0.00	> 1000	3.02	1.76	0.07	> 1000	1499.60	1649.80	0.00	> 1000
DI-PSO2	11.15	4.86	0.00	> 1000	4.71	2.28	0.00	> 1000	1497.30	1511.10	0.00	> 1000
DI-PSO1-CF	1796.40	910.83	0.00	> 1000	780.47	623.55	0.00	> 1000	2276.80	1045.00	0.00	> 1000
DI-PSO2-CF	1652.70	826.63	0.00	> 1000	717.85	335.94	0.00	> 1000	2295.10	941.03	0.00	> 1000
AT1-PSO1	4.76	2.15	0.00	> 1000	2.16	1.76	0.17	> 1000	1584.60	1101.90	0.00	> 1000
AT1-PSO2	6.85	2.12	0.00	> 1000	2.43	1.49	0.05	> 1000	772.81	909.62	0.00	> 1000
AT1-PSO1-CF	963.98	737.17	0.00	> 1000	470.14	242.39	0.00	> 1000	2032.40	937.66	0.00	> 1000
AT1-PSO2-CF	721.45	403.80	0.00	> 1000	358.45	172.38	0.00	> 1000	1780.40	650.79	0.00	> 1000
AT2-PSO1	7.44	3.39	0.00	> 1000	3.50	1.66	0.00	> 1000	108.91	112.21	0.00	> 1000
AT2-PSO2	10.01	4.71	0.00	> 1000	5.02	2.70	0.00	> 1000	122.32	191.30	0.00	> 1000
AT2-PSO1-CF	495.75	240.51	0.00	> 1000	454.87	259.16	0.00	> 1000	1355.10	663.30	0.00	> 1000
AT2-PSO2-CF	425.66	293.16	0.00	> 1000	363.88	192.89	0.00	> 1000	1209.60	810.10	0.00	> 1000
AT1-BBPSO	3.22	1.39	0.05	> 1000	0.46	0.53	0.30	> 1000	1.15	1.23	0.35	> 1000
AT1-BBPSOxp	0.05	0.15	0.00	> 1000	0.03	0.01	0.00	> 1000	0.14	0.35	0.65	974.5
AT1-BBPSO-CF	3.59	1.63	0.00	> 1000	0.24	0.52	0.80	898.5	1.17	1.15	0.35	> 1000
AT1-BBPSOxp-CF	0.15	0.34	0.75	985	0.01	0.00	0.85	973.5	0.12	0.32	0.88	865
AT2-BBPSO	4.59	1.93	0.00	> 1000	0.62	0.79	0.52	633.5	1.14	1.18	0.40	> 1000
AT2-BBPSOxp	0.17	0.43	0.85	656	0.00	0.00	1.00	672	0.12	0.32	0.88	633.5
AT2-BBPSO-CF	3.54	1.85	0.00	> 1000	0.59	0.74	0.52	583.5	1.57	1.30	0.22	> 1000
AT2-BBPSOxp-CF	0.09	0.36	0.92	591.5	0.00	0.00	1.00	614	0.10	0.29	0.90	598.5

Table S2.4. Simulation results for OF4. Each algorithm was run for 1,000 iterations over 40 replications. Mean and SD denote the mean and standard deviation of minimum objective function values found over all replications, while $\widehat{P}$ denotes the proportion of all replications that came within 0.01 of the true global minimum (equal to zero), and $\widehat{K}$ denotes the median number of iterations until the algorithm came within 0.01 of the global minimum.

OF5; Nbhd:	Global				SS3				SS1			
Algorithm	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$
PSO1	0.03	0.03	0.30	> 1000	0.01	0.01	0.57	441	0.52	0.65	0.07	> 1000
PSO2	0.02	0.02	0.38	> 1000	0.01	0.01	0.60	205	0.56	0.60	0.07	> 1000
PSO1-CF	0.89	0.20	0.00	> 1000	0.96	0.14	0.00	> 1000	1.45	0.28	0.00	> 1000
PSO2-CF	0.94	0.15	0.00	> 1000	0.99	0.09	0.00	> 1000	1.48	0.31	0.00	> 1000
DI-PSO1	0.02	0.02	0.28	> 1000	0.01	0.01	0.65	258.5	1.27	0.45	0.00	> 1000
DI-PSO2	0.02	0.03	0.38	> 1000	0.01	0.02	0.65	239	1.36	0.42	0.00	> 1000
DI-PSO1-CF	1.42	0.26	0.00	> 1000	1.16	0.08	0.00	> 1000	1.60	0.30	0.00	> 1000
DI-PSO2-CF	1.33	0.18	0.00	> 1000	1.15	0.09	0.00	> 1000	1.58	0.28	0.00	> 1000
AT1-PSO1	0.02	0.02	0.42	> 1000	0.01	0.01	0.78	253	1.37	0.28	0.00	> 1000
AT1-PSO2	0.02	0.02	0.32	> 1000	0.01	0.01	0.82	239.5	1.44	0.56	0.00	> 1000
AT1-PSO1-CF	1.09	0.22	0.00	> 1000	1.04	0.10	0.00	> 1000	1.64	0.26	0.00	> 1000
AT1-PSO2-CF	1.03	0.21	0.00	> 1000	1.04	0.11	0.00	> 1000	1.61	0.26	0.00	> 1000
AT2-PSO1	0.03	0.03	0.38	> 1000	0.01	0.01	0.52	167	0.93	0.28	0.00	> 1000
AT2-PSO2	0.03	0.03	0.35	> 1000	0.01	0.01	0.65	132.5	0.85	0.27	0.00	> 1000
AT2-PSO1-CF	0.97	0.25	0.00	> 1000	1.02	0.10	0.00	> 1000	1.42	0.21	0.00	> 1000
AT2-PSO2-CF	0.91	0.19	0.00	> 1000	0.97	0.10	0.00	> 1000	1.38	0.21	0.00	> 1000
AT1-BBPSO	0.01	0.01	0.55	875.5	0.00	0.01	0.85	555.5	0.00	0.01	0.82	535.5
AT1-BBPSOxp	0.01	0.01	0.72	631	0.00	0.00	1.00	623.5	0.00	0.00	0.95	566.5
AT1-BBPSO-CF	0.02	0.02	0.35	> 1000	0.00	0.01	0.85	475	0.01	0.01	0.68	472
AT1-BBPSOxp-CF	0.01	0.01	0.78	518.5	0.00	0.00	1.00	549	0.00	0.00	0.97	469.5
AT2-BBPSO	0.02	0.02	0.40	> 1000	0.00	0.01	0.92	366.5	0.01	0.01	0.72	406.5
AT2-BBPSOxp	0.01	0.01	0.78	387	0.00	0.00	0.97	400.5	0.00	0.01	0.90	380.5
AT2-BBPSO-CF	0.01	0.02	0.52	478.5	0.00	0.01	0.82	308.5	0.00	0.01	0.78	307
AT2-BBPSOxp-CF	0.01	0.01	0.72	370.5	0.00	0.00	1.00	342.5	0.00	0.01	0.80	349

Table S2.5. Simulation results for OF5. Each algorithm was run for 1,000 iterations over 40 replications. Mean and SD denote the mean and standard deviation of minimum objective function values found over all replications, while $\widehat{P}$ denotes the proportion of all replications that came within 0.01 of the true global minimum (equal to zero), and $\widehat{K}$ denotes the median number of iterations until the algorithm came within 0.01 of the global minimum.

OF6; Nbhd:	Global				SS3				SS1			
Algorithm	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$	Mean	SD	$\widehat{P}$	$\widehat{K}$
PSO1	20.01	0.02	0.00	> 1000	20.13	0.07	0.00	> 1000	20.26	0.10	0.00	> 1000
PSO2	20.00	0.01	0.00	> 1000	20.21	0.11	0.00	> 1000	20.37	0.13	0.00	> 1000
PSO1-CF	20.67	0.14	0.00	> 1000	20.59	0.11	0.00	> 1000	20.56	0.13	0.00	> 1000
PSO2-CF	20.68	0.19	0.00	> 1000	20.62	0.11	0.00	> 1000	20.62	0.13	0.00	> 1000
DI-PSO1	20.02	0.03	0.00	> 1000	20.06	0.04	0.00	> 1000	20.11	0.05	0.00	> 1000
DI-PSO2	20.13	0.14	0.00	> 1000	20.18	0.11	0.00	> 1000	20.23	0.09	0.00	> 1000
DI-PSO1-CF	20.43	0.11	0.00	> 1000	20.33	0.10	0.00	> 1000	20.29	0.11	0.00	> 1000
DI-PSO2-CF	20.48	0.13	0.00	> 1000	20.38	0.11	0.00	> 1000	20.32	0.11	0.00	> 1000
AT1-PSO1	20.00	0.00	0.00	> 1000	20.01	0.01	0.00	> 1000	20.00	0.00	0.00	> 1000
AT1-PSO2	20.02	0.04	0.00	> 1000	20.08	0.06	0.00	> 1000	20.00	0.00	0.00	> 1000
AT1-PSO1-CF	20.06	0.06	0.00	> 1000	20.14	0.07	0.00	> 1000	20.00	0.00	0.00	> 1000
AT1-PSO2-CF	20.23	0.09	0.00	> 1000	20.16	0.09	0.00	> 1000	19.68	2.06	0.00	> 1000
AT2-PSO1	20.01	0.02	0.00	> 1000	20.07	0.05	0.00	> 1000	20.22	0.09	0.00	> 1000
AT2-PSO2	20.12	0.10	0.00	> 1000	20.27	0.08	0.00	> 1000	20.33	0.11	0.00	> 1000
AT2-PSO1-CF	20.33	0.10	0.00	> 1000	20.33	0.10	0.00	> 1000	20.32	0.11	0.00	> 1000
AT2-PSO2-CF	20.38	0.12	0.00	> 1000	20.32	0.13	0.00	> 1000	20.32	0.11	0.00	> 1000
AT1-BBPSO	17.22	8.02	0.00	> 1000	5.23	9.13	0.00	> 1000	18.71	6.28	0.00	> 1000
AT1-BBPSOxp	18.34	6.18	0.00	> 1000	17.58	6.79	0.00	> 1000	20.42	0.21	0.00	> 1000
AT1-BBPSO-CF	15.63	9.14	0.07	> 1000	0.53	3.30	0.42	> 1000	13.68	9.62	0.22	> 1000
AT1-BBPSOxp-CF	14.75	9.19	0.00	> 1000	10.35	9.46	0.00	> 1000	20.40	0.33	0.00	> 1000
AT2-BBPSO	17.76	7.56	0.15	> 1000	5.95	8.70	0.65	684.5	19.04	5.54	0.07	> 1000
AT2-BBPSOxp	18.22	6.18	0.10	> 1000	17.65	5.79	0.05	> 1000	20.17	1.84	0.00	> 1000
AT2-BBPSO-CF	18.33	6.76	0.07	> 1000	2.06	6.25	0.90	628	14.47	9.01	0.22	> 1000
AT2-BBPSOxp-CF	14.49	9.15	0.28	> 1000	10.03	8.80	0.32	> 1000	18.77	4.41	0.00	> 1000

Table S2.6. Simulation results for OF6. Each algorithm was run for 1,000 iterations over 40 replications. Mean and SD denote the mean and standard deviation of minimum objective function values found over all replications, while $\widehat{P}$ denotes the proportion of all replications that came within 0.01 of the true global minimum (equal to zero), and $\widehat{K}$ denotes the median number of iterations until the algorithm came within 0.01 of the global minimum.