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Intern. J. of Research in Marketing 22 (2005) 459–470

International Journal of Research in Marketing

www.elsevier.com/locate/ijresmar

Quick and easy choice sets: Constructing optimal and nearly optimal stated choice experiments

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Abstract

In this paper we compare a number of common strategies for constructing discrete choice experiments. Two of the strategies, including one based on theoretical constructions for optimal discrete choice experiments, produce designs that are better than those that come about from random grouping and from using the L^{MA} construction. A simple account of this theoretical construction is given.

Keywords: Stated choice experiments; Choice modeling; Optimal design theory

1. Introduction and motivation

A *discrete choice experiment* (DCE) consists of several choice sets, each containing two or more options (sometimes called alternatives). Participants are shown the choice sets in turn and are asked which option they prefer. Each option is described by a set of attributes and each attribute can take one of several levels. DCEs are used in marketing to estimate the effect of the attributes on the "attractiveness" of the product under consideration. How well a DCE does this depends in part on which options are used in the choice experiment and how these options are grouped into choice sets. Partly for convenience and partly to try to keep the complexity of choosing between the options in each of the choice sets as equal as possible, we are going to assume that all of the choice sets are of the same size.

We begin by considering an example in which there are five attributes of interest used to describe economy class, long-haul flights of at least 4 h flying time. These attributes, together with the corresponding levels, are as follows:

- A_1: Return airfare ($350, $450, $550, $650)
- A_2: Total travel time, including stops (4, 5, 6, 7 h)
- A_3: Food/beverage (none, beverages only, beverages + cold snack, beverages + hot meal)
- A_4: Audio/Video entertainment (none, audio only, audio + short video clips, audio + movie)
- A_5: Type of airplane (Boeing 737, Boeing 757, Boeing 767, Boeing 777)

The levels of attributes are usually coded and in this paper we use 0, 1, 2 and 3 as the coded levels for the 4 levels for each attribute. If we investigate these five attributes using choice sets of size 2 (a *paired comparison* design) then there are five design strategies that have been routinely adopted in the past. Design strategies typically use an orthogonal main effects plan

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doi:10.1016/j.ijresmar.2005.09.003

(OMEP), which allow the uncorrelated estimation of all main effects under the assumption that all interactions are negligible (see Addelman, 1962). Such designs may be obtained from Hahn and Shapiro (1966), from software packages such as SPEED (Bradley, 1991) or Orthoplan (SPSS, 1989) or from the tables of orthogonal arrays (i.e., OMEPs) at Neil Sloane's website (Sloane, 2003) amongst other options. Below we consider several design strategies commonly found in the published literature on DCEs in marketing, transportation and applied economics. Four of the strategies discussed below involve finding an OMEP for five attributes each with four levels and the fifth requires finding such a design for 10 attributes each with four levels.

The orthogonal main effects design for five 4-level attributes shown in Table 1 has 16 level combinations (or profiles) and was obtained from Sloane's website (oa.16.5.4.2).

2. Strategy 1

The first design strategy is to take one orthogonal main effects design for five 4-level attributes and randomly pair the profiles to give the choice sets. One such design is given in Table 2. Thus the first choice set, using the attributes and uncoded levels, becomes {($350, 4 h, no food or drink, no entertainment, Boeing 737), ($650, 4 h, beverage and hot meal, audio only, Boeing 767)}.

Observe, however, that this pairing has resulted in the second attribute having the same level in all pairs and so no information about the effects of the second attribute is available. Also note the attribute levels are not balanced within each option (e.g., level 0 of the first attribute only appears in the first option). This is only a problem if order of presentation matters.

Table 1
An OMEP for five 4-level attributes

Profile #	A_1	A_2	A_3	A_4	A_5
P1	0	0	0	0	0
P2	0	1	1	1	1
P3	0	2	2	2	2
P4	0	3	3	3	3
P5	1	0	1	2	3
P6	1	1	0	3	2
P7	1	2	3	0	1
P8	1	3	2	1	0
P9	2	0	2	3	1
P10	2	1	3	2	0
P11	2	2	0	1	3
P12	2	3	1	0	2
P13	3	0	3	1	2
P14	3	1	2	0	3
P15	3	2	1	3	0
P16	3	3	0	2	1

Table 2
Strategy 1 choice sets

Pair #	Option 1						Option 2					
	Profile #	A_1	A_2	A_3	A_4	A_5	Profile #	A_1	A_2	A_3	A_4	A_5
1	P1	0	0	0	0	0	P13	3	0	3	1	2
2	P2	0	1	1	1	1	P10	2	1	3	2	0
3	P3	0	2	2	2	2	P11	2	2	0	1	3
4	P4	0	3	3	3	3	P8	1	3	2	1	0
5	P5	1	0	1	2	3	P9	2	0	2	3	1
6	P6	1	1	0	3	2	P14	3	1	2	0	3
7	P7	1	2	3	0	1	P15	3	2	1	3	0
8	P8	2	3	1	0	2	P16	3	3	0	2	1

3. Strategy 2

The second design strategy is similar to the first but uses two different OMEPs, one to represent the profiles that appear as the first option in the choice sets and one to represent the profiles that appear as the second option in the choice sets. One such design is shown in Table 3. Note that each level of each attribute appears equally often in each option but it does not stop the possibility that all pairs may have the same level of one, or more, attributes. This problem is partially overcome in the next two construction techniques.

4. Strategy 3

This strategy takes the profiles from an OMEP and pairs them manually in such a way that the pairs satisfy the *minimal overlap* property from Huber and Zwerina

Table 3
Strategy 2 choice sets

Pair #	Option 1					Option 2				
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
1	0	0	0	0	0	1	3	2	0	2
2	0	1	1	1	1	2	1	3	0	3
3	0	2	2	2	2	1	2	0	3	3
4	0	3	3	3	3	0	1	2	3	1
5	1	0	1	2	3	0	0	0	0	0
6	1	1	0	3	2	3	1	0	2	2
7	1	2	3	0	1	3	3	3	3	0
8	1	3	2	1	0	1	1	1	1	0
9	2	0	2	3	1	0	2	3	1	2
10	2	1	3	2	0	3	2	1	0	1
11	2	2	0	1	3	2	3	0	1	1
12	2	3	1	0	2	3	0	2	1	3
13	3	0	3	1	2	0	3	1	2	3
14	3	1	2	0	3	2	2	2	2	0
15	3	2	1	3	0	2	0	1	3	2
16	3	3	0	2	1	1	0	3	2	1

Table 4
Strategy 3 choice sets

Pair #	Option 1					Option 2				
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
1	0	0	0	0	0	3	0	3	1	2
2	0	1	1	1	1	0	2	2	2	2
3	0	2	2	2	2	2	3	1	0	2
4	0	3	3	3	3	1	0	1	2	3
5	1	0	1	2	3	0	1	1	1	1
6	1	1	0	3	2	3	2	1	3	0
7	1	2	3	0	1	1	3	2	1	0
8	1	3	2	1	0	2	0	2	3	1
9	2	0	2	3	1	2	1	3	2	0
10	2	1	3	2	0	0	3	3	3	3
11	2	2	0	1	3	1	1	0	3	2
12	2	3	1	0	2	3	1	2	0	3
13	3	0	3	1	2	2	2	0	1	3
14	3	1	2	0	3	1	2	3	0	1
15	3	2	1	3	0	3	3	0	2	1
16	3	3	0	2	1	0	0	0	0	0

(1996), or as close to it as is possible. In effect, this means that for each attribute there should be the maximum number of different levels in the choice set. Each level appears either 0 or 1 times in each pair and, over the whole choice experiment, each option displays the possible levels of each attribute equally often. One set of pairs that results from this approach is given below. Unfortunately, for this example, it is not possible for any pairs to have no repeated levels. To see this, consider the profile 00000. When this profile is paired with any of the other profiles, one attribute will have a repeated level because every other profile contains one 0. However it is possible to change the attribute that is repeated from choice set to choice set. Table 4 contains a design constructed with this strategy.

5. Strategy 4

The fourth strategy requires an OMEP for ten 4-level attributes. The smallest such design has 64 level combinations. For each level combination in the OMEP the first five attributes are used to represent the profiles of the first option and the final five attributes are used to represent the profiles of the second option. So there are 64 total pairs in the experiment. This strategy is sometimes called an L^{MA} approach (see Louviere, Hensher, & Swait, 2000). As this design is large it is in the Appendix (Table A1).

6. Strategy 5

This strategy uses a software package like SAS (see Kuhfeld, 2004) to generate a starting OMEP and then construct choice sets using a search algorithm. The "goodness" of the design (efficiency) is given, but there is no indication if a design is the best (optimal) design. The user must nominate the number of profiles in the candidate set from which the search algorithm selects profiles for the choice sets. We tried a number of different candidate sets, and used the one with the highest efficiency (1.587), as calculated by SAS (see Table 5).

The first four design construction strategies have the disadvantage that you do not know how good the resulting design will be. Furthermore, it is possible for the effects of interest to be inestimable, and in the case of the L^{MA} approach the number of choice sets required typically is much larger than needed just to estimate the effects of interest. While Strategy 5 does give an efficiency for its designs, one does not know if a better design is available. We discuss the statistical properties of designs constructed from the five strategies in the section entitled "Comparison of strategies".

A recent series of papers (Burgess & Street, 2003, 2005; Street, Bunch, & Moore, 2001; Street & Burgess, 2004) derive proofs for design strategies that allow one to obtain DCEs with good statistical properties for any choice set size and for attributes with any number of levels. The purpose of the present paper is to provide a quick and easy way for academics and practitioners to make use of these recent theoretical results. In particular we describe a construction technique that always gives an optimal or near-optimal design for the estimation of main effects, and gives near-optimal designs for the estimation of main effects plus two-factor interaction effects. In addition the efficiency of any proposed

Table 5
Strategy 5 choice sets

Pair #	Option 1					Option 2				
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
1	3	1	0	2	0	1	3	2	0	3
2	2	0	1	3	3	1	3	2	0	0
3	3	0	2	1	1	1	2	0	3	2
4	2	2	2	2	3	3	3	3	3	2
5	1	0	3	2	1	2	3	0	1	0
6	1	2	0	3	1	0	3	1	2	2
7	0	1	2	3	0	3	2	1	0	1
8	1	1	1	1	3	0	0	0	0	2
9	2	3	0	1	1	3	2	1	0	0
10	0	2	3	1	3	2	0	1	3	0
11	0	3	1	2	1	2	1	3	0	2
12	0	1	2	3	1	1	0	3	2	0
13	3	3	3	3	3	1	1	1	1	2
14	0	0	0	0	3	2	2	2	2	2
15	2	1	3	0	1	3	0	2	1	2
16	3	1	0	2	3	0	2	3	1	0

design can be calculated using the results below, thus allowing any specific designs to be compared.

The basic idea of the construction technique is simple: start with an OMEP to represent the profiles in the first option in the choice sets. Choose some systematic set of level changes to get from the profiles in the first option in the choice sets to the profiles in the second option in the choice sets, and then choose another systematic set of changes to get from the profiles in the first option to the profiles in the third option, and so on. The benefit of this approach is that the nature of the systematic changes required to make the resulting choice sets optimal has been determined for the estimation of main effects. Bunch, Louviere, and Anderson (1996) introduced cyclic or shifted designs, but their designs only work well for estimating main effects, and the numbers of levels for all the attributes must be at least equal to the size of the choice sets. Systematic changes to get near-optimal sets for estimating main effects plus two-factor interaction effects also have been determined. These systematic changes are discussed below.

7. The information matrix and statistical efficiency

In general, each profile in each option in a choice set is described by k attributes and each choice set contains m options. We assume that the qth attribute has l_q levels, represented by 0, 1, ..., l_q-1 and that attributes may have different numbers of levels (i.e., a design can be asymmetric).

We discuss experiments that are consistent with the multinomial logit model (MNL), where the results from a DCE are to be used to estimate the main effects or the main effects plus two-factor interactions. A common way to compare designs is by using the generalized variance of the parameter estimates; designs for which the generalized variance is as small as possible are required. Since the variance–covariance matrix of the parameter estimates is the inverse of the Fisher information matrix, optimal designs will, when using the D-optimality criterion, have the maximum determinant of the information matrix. In El Helbawy and Bradley (1978) the information matrix is defined to be $C=B\Lambda B'$, where B is the matrix of contrasts for the effects to be estimated (i.e. main effects or main effects plus two-factor interactions), and Λ is the matrix of second derivatives of the likelihood function. Under the null hypothesis of no differences between the effects of the levels of each attribute, it turns out that Λ contains the proportions of choice sets in which pairs of profiles appear together (for details see, for instance, Burgess & Street, 2005). The next example illustrates these calculations.

Example 1. In order to illustrate the calculations, suppose that there are only two attributes from the example in the previous section, one with two levels and the other with three levels.

A_1: Return airfare (\$350, \$650, coded as 0,1)
A_2: Total travel time, including stops (4, 5, 6 h, coded as 0,1,2)

Therefore $l_1=2$ and $l_2=3$ and the possible profiles are 00, 01, 02, 10, 11, 12. The entries in Λ can be evaluated by counting the occurrences of pairs of profiles (see El Helbawy & Bradley, 1978) and dividing by m^2N where N is the number of choice sets. The diagonal entries are chosen so that the row and column sums of Λ are 0. To evaluate Λ label the rows and columns of Λ by the profiles. Thus, Λ is a 6×6 matrix with rows and columns labeled by 00, 01, 02, 10, 11, 12. Let each choice set have three options, so $m=3$; and let the choice sets in the experiment be (00, 11, 02) and (10, 02, 12). Then using the profiles in the order given, the first row of Λ is 1/18 (2, 0, −1, 0, −1, 0), second row is 1/18 (0, 0, 0, 0, 0, 0) (since 01 occurs in neither of the choice sets) and so on. Thus, we get the matrix

$$\Lambda=\frac{1}{18}\begin{bmatrix} 2 & 0 & -1 & 0 & -1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -1 & 0 & 4 & -1 & -1 & -1\\ 0 & 0 & -1 & 2 & 0 & -1\\ -1 & 0 & -1 & 0 & 1 & 0\\ 0 & 0 & -1 & -1 & 0 & 2 \end{bmatrix}.$$

The matrix B is a matrix of contrasts for the effects of interest. If the main effect of attribute q, which has l_q levels, is of interest then B will contain l_q-1 rows that correspond to l_q-1 independent contrasts (one for each degree of freedom) associated with the attribute. Any set of l_q-1 independent contrasts will result in the same covariance matrix of the parameter estimates, and hence in the same generalized variance (determinant of the covariance matrix of the parameter estimates). For each attribute we find an appropriate set of contrasts. We then use these contrasts as the rows of the matrix B and calculate the information matrix $C=B\Lambda B'$.

Example 1 (continued). Suppose that the main effects of each attribute are of interest. Then, since $l_1=2$, there will be one row in B for the one contrast for the main effect of the first attribute and, since $l_2=3$, there will be two rows in B for the two contrasts for the main effect of the second attribute. The entries in B for the main effect for the first attribute are −1, corresponding to the level 0; and 1, which corresponds to level 1. There will be two rows for the second attribute—in the first row, which is the linear contrast, attribute level 0 corresponds to -1 in the contrast, attribute level 1 corresponds to 0 in the contrast and attribute level 2 corresponds to 1 in the contrast. In the second row, which is the quadratic contrast, attribute level 0 corresponds to -1, attribute level 1 to 2 and attribute level 2 to -1. The only thing left to do is to normalize the contrasts, which means dividing the entries in each row by the square root of the sum of the squares in each row. The entries in the first row are divided by $\sqrt{6}$, those in the second row by $\sqrt{4}$ and those in the third row by $\sqrt{12}$, so that $BB'=I$. Thus we get

$$B = \begin{bmatrix} \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{2} & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & \frac{1}{2} \\ \frac{-1}{\sqrt{12}} & \frac{2}{\sqrt{12}} & \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{12}} & \frac{2}{\sqrt{12}} & \frac{-1}{\sqrt{12}} \end{bmatrix}$$

and

$$C = B\Lambda B' = \begin{pmatrix} \frac{4}{27} & \frac{-1}{9\sqrt{6}} & \frac{-1}{9\sqrt{2}} \\ \frac{-1}{9\sqrt{6}} & \frac{7}{36} & 0 \\ \frac{-1}{9\sqrt{2}} & 0 & \frac{1}{12} \end{pmatrix}.$$

Note that since the C matrix is not diagonal, the estimates of the main effects will be correlated although the two components of the attribute with three levels are independent.

Once the C matrix for a DCE has been calculated, the statistical efficiency of the design can also be calculated if the C matrix of the optimal design is known. In general the D-efficiency of any design is given by $[\det(C)/\det(C_{\text{optimal}})]^{1/p}$, where p is the number of parameters that have to be estimated in the model. For designs that estimate main effects, $p=\Sigma_i(l_i-1)$. For designs that estimate both main effects and two factor interactions, $p=\Sigma_i(l_i-1)+\Sigma_i\Sigma_{j,i<j}(l_i-1)(l_j-1)$.

An optimal design has an efficiency of 100%. A design is nearly optimal if the efficiency is high but there is no formal definition of this phrase. To construct optimal or nearly optimal choice sets it is desirable to start with an orthogonal design so that the estimates of the main effects or the main effects plus two-factor interactions from the choice experiment are most likely to be uncorrelated.

8. Designs for estimating main effects

Burgess and Street (2005) provide an upper bound for $\det(C)$ for estimating main effects for any choice set size with any number of attributes each having any number of levels. The maximum value of the determinant of C is

$$\det(C_{\text{optimal}}) = \prod_{q=1}^{k}\left(\frac{2S_q}{m^2(l_q-1)\prod_{i=1,i\neq q}^{k} l_i}\right)^{l_q-1} \quad (1)$$

where

$$S_q = \begin{cases} (m^2-1)/4 & l_q=2, m \text{ odd}, \\ m^2/4 & l_q=2, m \text{ even}, \\ \left(m^2-\left(l_q x^2+2xy+y\right)\right)/2 & 2<l_q\leq m, \\ m(m-1)/2 & l_q\geq m \end{cases} \quad (2)$$

and positive integers x and y satisfy the equation $m=l_q x+y$ for $0\leq y<l_q$. The value S_q is the largest number of pairs of profiles that can have different levels for attribute q in a choice set. In other words S_q is the maximum number of differences in the levels of attribute q in each choice set.

Example 1 (continued). For the two choice sets given before, $\det(C)=1/972$. Now in this example $m=3$ and since $l_1=2$, $S_1=(m^2-1)/4=2$, and since $l_2=3$, $S_2=m(m-1)/2=3$. So the maximum value of $\det(C)$ is $\det(C_{\text{optimal}})=(2\times2)/(9\times1\times3)\times((2\times3)/(9\times2\times2))^2=1/243$ and $p=3$. Hence the efficiency of the two choice sets is $((1/972)/(1/243))^{1/3}\times100=63\%$. These choice sets are not optimal because the second choice set (10, 02, 12) does not have the maximum possible number of differences in the levels of the second attribute. Now consider the choice sets (00, 11, 02) and (10, 01, 12). In both of the choice sets the number of level differences for the first attribute is 2 and the number of level differences for the second attribute is 3. The C matrix is diagonal so the estimates of the main effects will be uncorrelated and this design is 100% efficient.

So how do we go about finding choice sets that are optimal? One way that works is to use an OMEP to make the profiles in the first option of the choice sets, and then make systematic level changes so that as many pairs of profiles as possible have different levels for each attribute.

For binary attributes to be presented in choice sets of size two (pairs), we need to choose an OMEP to make the profiles in the first option, and interchange the 0's and 1's to make the profiles for the second option. This interchange process is known as using the foldover of the profiles in the first option to make the profiles in the second option. The resulting pairs are optimal and are shown in Table 6 for an example with 5 attributes.

Table 6
Optimal pairs for estimating main effects for 5 binary attributes

Set #	Option 1					Option 2				
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
1	0	0	0	0	0	1	1	1	1	1
2	1	0	0	1	1	0	1	1	0	0
3	0	1	0	1	0	1	0	1	0	1
4	0	0	1	0	1	1	1	0	1	0
5	1	1	0	0	1	0	0	1	1	0
6	1	0	1	1	0	0	1	0	0	1
7	0	1	1	1	1	1	0	0	0	0
8	1	1	1	0	0	0	0	0	1	1

For binary attributes to be presented in choice sets of size 3 (triples), we need to have $S_q=2$ to get an optimal design; hence we need to systematically change the levels so that either 0 or 1 appears twice in each choice set, with the other level appearing once in each set. For example, for a DCE with 5 attributes the design in Table 7 is optimal and it is clear that $S_q=2$ for all of the attributes. For instance in the first choice set (00000, 11100, 00011) it can be seen that for each attribute the levels differ twice when comparing each pair of options. For the first attribute the level is different when comparing options 1 and 2; it is the same when comparing options 1 and 3; and the levels differ when comparing options 2 and 3. In Table 7 the profiles in the first option form an OMEP for 5 binary attributes. The profiles in the second option have been obtained by interchanging 1's and 0's in the first three attributes and the profiles in the third option have been obtained by interchanging 0's and 1's in the fourth and fifth attributes.

These systematic level changes are equivalent to adding a generator to the profiles in Option 1 to obtain the profiles in Option 2, and adding another generator to the profiles in Option 1 to obtain the profiles in Option 3. The addition is performed in modulo arithmetic according to the number of levels for a particular attribute. The generator 11100 is added (modulo 2) to the profiles in Option 1: $0000+11100 \equiv 11100$, $10011+11100 \equiv 01111$, $01010+11100 \equiv 10110$, $00101+11100 \equiv 11001$, and so on, to obtain the profiles in Option 2. Recall that $0+0 \equiv 0$, $0+1 \equiv 1$, $1+0 \equiv 1$ and $1+1 \equiv 0$ in modulo 2 arithmetic. Similarly the generator 00011 is added (modulo 2) to each of the profiles in Option 1 to obtain the profiles in Option 3.

If an attribute has more than two levels, the process is similar but more complicated. Suppose all attributes have three levels. Then $l_q=3$ and hence $S_q=m(m-1)/2=1$ if $m=2$, $S_q=m(m-1)/2=3$ if $m=3$, and $S_q=(m^2-(3x^2+2xy+y))/2$ for all other values of m. So if there are 4 attributes each with 3 levels, and pairs are to be used to estimate the main effects, then the systematic level changes that result from adding 1 or 2 modulo 3 to all the levels in each attribute give an optimal design. (Recall that $1+1 \equiv 2$ (mod 3), $1+2 \equiv 2+1 \equiv 0$ (mod 3) and $2+2 \equiv 1$ (mod 3).) The design in Table 8 was obtained by using an OMEP to make the profiles in the first option, and then adding 1 (mod 3) to the levels in the first and third attributes and adding 2 (mod 3) to the levels in the second and fourth attributes to get the profiles in the second option. All attribute levels could have been changed by adding 1 (mod 3) or all by adding 2 (mod 3) or any group by adding 1 (mod 3) and the remaining attributes could be changed by adding 2 (mod 3) and the resulting set of pairs would be optimal for estimating main effects. Similarly optimal triples can be obtained by changing levels in an attribute by adding 1 (mod 3) for the second option and adding 2 (mod 3) for the profiles in the third option or vice versa. These systematic level changes are equivalent to adding the generator 1212, using modulo 3 arithmetic, to the OMEP to obtain the profiles in Option 2.

Now consider 4 attributes, each with 3 levels, for choice sets of size 4 (quadruples). We need to calcu-

Table 7
Optimal triples for estimating main effects for 5 binary attributes

Set #	Option 1					Option 2					Option 3				
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
1	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1
2	1	0	0	1	1	0	1	1	1	1	1	0	0	0	0
3	0	1	0	1	0	1	0	1	1	0	0	1	0	0	1
4	0	0	1	0	1	1	1	0	0	1	0	0	1	1	0
5	1	1	0	0	1	0	0	1	0	1	1	1	0	1	0
6	1	0	1	1	0	0	1	0	1	0	1	0	1	0	1
7	0	1	1	1	1	1	0	0	1	1	0	1	1	0	0
8	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1

Table 8
Optimal pairs for estimating main effects for 4 ternary attributes

Set #	Option 1				Option 2			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
1	0	0	0	0	1	2	1	2
2	0	1	1	2	1	0	2	1
3	0	2	2	1	1	1	0	0
4	1	0	1	1	2	2	2	0
5	1	1	2	0	2	0	0	2
6	1	2	0	2	2	1	1	1
7	2	0	2	2	0	2	0	1
8	2	1	0	1	0	0	1	0
9	2	2	1	0	0	1	2	2

late $S_q = (m^2 - (3x^2 + 2xy + y))/2$ where $m = 3x + y$ and $0 \leq y < 3$. Thus $4 = 3 \times 1 + 1$, so $x = y = 1$. Hence $S_q = (4^2 - (3 \times 1^2 + 2 \times 1 \times 1 + 1))/2 = 5$. This means that when considering the six possible pairs of profiles in a choice set, the maximum number of level changes for one attribute is 5. One such choice set is (0000, 1212, 1021, 2101). Note the levels of attribute 1 in the six possible pairs of profiles in the choice set: 0 and 1, 0 and 1, 0 and 2, 1 and 1, 1 and 2, 1 and 2. In five of the pairs the levels of attribute 1 differ. One can construct the choice sets by adding generators 1212, 1021 and 2101, using modulo 3 arithmetic, to the profiles in Option 1 in Table 8 to obtain the profiles in Options 2, 3 and 4, respectively. The C matrix for this design is diagonal and the design is 100% efficient.

The choice of which systematic level changes to make, and the need to calculate the efficiency of the resulting design, becomes more critical as the number of levels increases relative to the choice set size. For example, consider finding optimal choice sets of size 3 to estimate the main effects of four asymmetric attributes, two with 2 levels and two with 4 levels. We know that $S_1 = S_2 = 2$ and $S_3 = S_4 = 6$. For each attribute in each choice set there are three possible pairs of profiles, so we need to systematically change the levels of the attributes so that there are 2 differences in the levels of the first two attributes and 6 differences in the levels of the last two attributes. One way to obtain an optimal design is to change the levels of the first and second attributes by adding 1 (mod 2), and changing the levels of the third and fourth attributes by adding 1 (mod 4) to get the profiles in the second option in each choice set. To obtain the profiles in the third option in the choice sets we could add 0 to the first attribute, 1 (mod 2) to the second attribute and 2 (mod 4) to the third and fourth attributes. The design is shown in Table 9 and is 100% efficient.

Caution should be exercised when constructing pairs when the number levels of a particular attribute is not a prime number. One can make systematic level changes so that the number of level changes is equal to S_q, but one may be unable to estimate the main effect of that attribute. Consider the previous example in which there were 4 attributes, two with 2 levels and two with 4 levels. For the 4 level attributes $S_3 = S_4 = m(m-1)/2 = 1$, so we only know that there must be a systematic level change to get the levels of the two attributes in the second option from those in the first option. Ignoring the 2 level attributes, suppose that we choose to change the levels of the 4 level attributes by adding 2 (mod 4).

Table 9
Optimal design for two 2-level attributes and two 4-level attributes

Set #	Option 1				Option 2				Option 3			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
1	0	0	0	0	1	1	1	1	0	1	2	2
2	0	1	0	2	1	0	1	3	0	0	2	0
3	1	0	2	0	0	1	3	1	1	1	0	2
4	1	1	2	2	0	0	3	3	1	0	0	0
5	1	1	0	3	0	0	1	0	1	0	2	1
6	1	0	0	1	0	1	1	2	1	1	2	3
7	0	1	2	3	1	0	3	0	0	0	0	1
8	0	0	2	1	1	1	3	2	0	1	0	3
9	1	1	3	0	0	0	0	1	1	0	1	2
10	1	0	3	2	0	1	0	3	1	1	1	0
11	0	1	1	0	1	0	2	1	0	0	3	2
12	0	0	1	2	1	1	2	3	0	1	3	0
13	0	0	3	3	1	1	0	0	0	1	1	1
14	0	1	3	1	1	0	0	2	0	0	1	3
15	1	0	1	3	0	1	2	0	1	1	3	1
16	1	1	1	1	0	0	2	2	1	0	3	3

Then we would get pairs of 0 with 2, 1 with 3, 2 with 0 and 3 with 1. Thus, only two (02, 13) of the six possible ordered pairs (01, 02, 03, 12, 13, 23) result compared to four of the six (01, 12, 23, 30) if we add either 1 or 3 (mod 4). This means that it is not possible to estimate the main effects of the 4 level attributes. This situation arises because $2 + 2 \equiv 0 \pmod 4$ and it is always an issue when constructing pairs when the number of levels of an attribute is not prime. In this case a design which is 95.84% efficient can be constructed by adding 1 (mod 2) to the levels of the 2 level attributes and adding 1 or 3 (mod 4) to the levels of the 4 level attributes to obtain the profiles for the second option. Furthermore, one can construct an optimal design by making three sets of systematic changes, or equivalently, adding three different generators, resulting in 48 pairs. Three such generators are 1112, 1121 and 1133.

9. Designs for estimating main effects and two-factor interactions

The situation is more complicated if one wants to estimate both main effects and two factor interactions. If all attributes have two levels and choice sets are of size 2, the optimal designs consist of all pairs with $(k+1)/2$ attributes different (if k is odd), or all pairs with either $k/2$ or $k/2+1$ attributes different (k even) as established in Street et al. (2001).

If all attributes have two levels, and choice sets have more than two options, the optimal design consists of all choice sets in which the number of attributes that differ between any pair of profiles in the choice set is

Table 10
Optimal pairs design for 3 binary attributes

Set #	Option 1			Option 2		
	A_1	A_2	A_3	A_1	A_2	A_3
1	0	0	0	0	1	1
2	0	0	1	0	1	0
3	1	0	0	1	1	1
4	1	0	1	1	1	0
5	0	0	0	1	0	1
6	0	0	1	1	0	0
7	1	1	1	0	1	0
8	1	1	0	0	1	1
9	0	0	0	1	1	0
10	0	0	1	1	1	1
11	1	0	0	0	1	0
12	1	0	1	0	1	1

$(k+1)/2$, if k is odd, or $k/2$ or $k/2+1$ if k is even (see Burgess & Street, 2003). Furthermore, the maximum possible determinant of C for any choice set size has been determined and is given by

$$\det(C) = \begin{cases} \left(\frac{(m-1)(k+2)}{m(k+1)2^k}\right)^{k+k(k-1)/2} & k \text{ even} \\ \left(\frac{(m-1)(k+1)}{mk2^k}\right)^{k+k(k-1)/2} & k \text{ odd.} \end{cases} \tag{3}$$

Burgess and Street (2003) show that it is possible to construct choice sets from which all main effects and two-factor interactions can be estimated orthogonally for binary attributes with choice sets of size m. They also discuss some isolated cases where it is known that no design can exist that will realize this bound.

For the general case, where the attributes can have any number of levels and the choice set can be of any size, an explicit expression for $\det(C)$ is provided by Burgess and Street (2005) in terms of the differences between the levels of each attribute in the choice sets. No general constructions are known, although Burgess and Street (2005) give optimal designs for some specific values of k and m. We now illustrate the case when all attributes have two levels.

Consider a DCE for pairs with profiles described by 3 attributes each with 2 levels for estimating the main effects and all two-factor interactions. First we require either a fractional or complete factorial design of resolution 5 (i.e. all main effects and all two-factor interactions can be estimated). In this case no fraction is resolution 5, so we use the complete factorial designs to make the profiles in the first option in the choice sets. Because the number of attributes (k) is odd, the optimal design consists of all pairs with $(k+1)/2=2$ attributes different, so we need to systematically change the levels of two of the attributes in each choice set and leave the level of the third attribute unchanged. However, this does not allow the main effects to be estimated so the process is repeated with the level of a different attribute remaining unchanged, while the levels of the other two attributes are systematically changed.

One way of doing this is to leave the level of the first attribute unchanged and add 1 (mod 2) to the second and third attributes, then to leave the level of the second attribute unchanged while adding 1 (mod 2) to the first and third attributes. This will result in the choice sets (000,011), (001,010), (100,111), (101,110) and (000,101), (001,100), (110,011), (111,010) from the two steps after removing repeated choice sets. This design is 94.5% efficient. By repeating the process a third time, this time leaving the levels of the third attribute unchanged while adding 1 (mod 2) to the levels of the first and second attributes, we obtain the design in Table 10. This design is 100% efficient.

These choice sets also can also constructed by adding (modulo 2) generators 011, then 101 and finally 110 to the profiles in option 1 to obtain the profiles in option 2, after removing repeated choice sets.

For choice sets with three options, the profiles in option 2 can be obtained by leaving the levels of the second attribute unchanged and systematically changing the levels of the first and third attributes by adding 1 (mod 2) to the respective attributes of the profiles in option 1. Similarly the profiles in option 3 are obtained by leaving the levels of the third attribute unchanged and systematically changing the levels of the first and second attributes by adding 1 (mod 2) to the respective attributes of the profiles in option 1. This design is 100% efficient and is shown in Table 11.

Alternatively, by adding (modulo 2) the generator 101 to the profiles in the first option we obtain the profiles in the second option. Similarly by adding

Table 11
Optimal triples design for 3 binary attributes

Set #	Option 1			Option 2			Option 3		
	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
1	0	0	0	1	0	1	1	1	0
2	0	0	1	1	0	0	1	1	1
3	0	1	0	1	1	1	1	0	0
4	0	1	1	1	1	0	1	0	1
5	1	0	0	0	0	1	0	1	0
6	1	0	1	0	0	0	0	1	1
7	1	1	0	0	1	1	0	0	0
8	1	1	1	0	1	0	0	0	1

Table 12
Comparison of construction methods for main effects only

Construction method	$m=2$		$m=3$		$m=4$	
	# Choice sets	Eff (%)	# Choice sets	Eff (%)	# Choice sets	Eff (%)
Strategy 1	8	0	N/A	N/A	4	0
	16	44.4*	N/A	N/A	16	68.1*
Strategy 2	16	36.1*	16	65.6*	16	71.3*
Strategy 3	16	0	16	72.0*	16	76.2*
Strategy 4	64	75.0	64	75.0	64	75.0
Strategy 5	16	94.5*	16	100	16	100
Strategy 6	16	94.5	16	100	16	100
	48	100				

(modulo 2) the generator 110 to the profiles in the first option we obtain the profiles in the third option.

The main effects and all two-factor interactions can be estimated independently in all designs discussed in this section.

10. Comparison of strategies

In this section we compare the information matrices of DCEs constructed using Strategies 1, 2, 3, 4 and 5 with a design constructed using the method of Burgess and Street (Strategy 6). This comparison depends only on the design used and it is not dependent on the data collected. Each design is to be used to estimate only main effects. The results are shown in Table 12. An asterisk denotes a choice experiment in which the main effects cannot be estimated independently.

Table 12 shows that the Strategy 5 and 6 designs are the most efficient, but only Strategy 6 designs always provide uncorrelated estimates of the main effects. The designs constructed using the two random methods, Strategies 1 and 2, have low statistical efficiency and the estimates of the main effects are correlated. Strategy 3 designs do not perform well in this particular example, but it is possible to construct optimal designs using this method. The designs constructed using Strategy 4 have uncorrelated estimates of the main effects, but the number of choice sets is larger than necessary, and the efficiencies are less than those of Strategy 5 and 6 designs. However, strategy 4 (L^{MA}) designs may be useful for other purposes, such as testing violations of IIA. Finally, the Strategy 5 designs are just as efficient, in this example, as the Strategy 6 designs but the design for pairs does not give uncorrelated estimates of the main effects, which often is the case with Strategy 5 designs.

Now consider designs for main effects and two-factor interactions. Strategies 4, 5 and 6 are the only strategies that routinely generate designs that permit these effects to be estimated, and we compare the designs from the three strategies for a small example. Suppose there are four 2-level attributes, and we wish to estimate all main effects and two-factor interactions. The results are in Table 13; an asterisk denotes a choice experiment in which the main effects and two-factor interactions are correlated.

Strategy 5 designs are the most efficient, but none of these designs allow main effects and two-factor interactions to be estimated independently. Strategy 4 designs have a large number of choice sets, with efficiencies less than designs based on Strategies 5 and 6. However, Strategy 6 designs are very efficient and can always estimate the effects of interest independently.

11. Discussion and conclusions

The Burgess and Street (2005) method of design construction for DCEs outlined in this paper will lead to “good” designs but not necessarily to designs that are the smallest and/or best possible (nor is it necessarily

Table 13
Comparison of construction methods for main effects and all two-factor interactions

Construction method	$m=2$		$m=3$		$m=4$	
	# Choice sets	Eff (%)	# Choice sets	Eff (%)	# Choice sets	Eff (%)
Strategy 4	128	88.4*	256	93.8	256	93.8
Strategy 5	24	98.5*	32	99.8*	16	99.8*
Strategy 6	24	94.0	32	96.7	16	99.0

the case the smallest possible designs should be used, as noted by Louviere et al., 2000). However, these designs allow independent estimation of all effects, and they generally are superior to most designs in the published literature. Indeed, our review of that literature suggests that the efficiency of many designs is less than 50%. By way of contrast, using the approach described in this paper, we have encountered only one instance in which a design was less than 90% efficient, and that design was 87% efficient.

The construction method and examples assume that one wants to minimize the number of choice sets to estimate the effects of interest. To maximize the number of observations relative to the number of parameters to be estimated, one can repeat a construction one or more times and combine the resulting choice sets.

As part of our strategy to minimize the number of choice sets we have removed duplicate choice sets. On occasion this will result in unequal replication of levels within each option (and indeed will mean that the set of first options, the set of second options and so on are not main effects plans). One can avoid this by leaving in the duplicate options.

The relative efficiencies quoted in the examples assume no prior knowledge about the values of the coefficients in the utility function or the choice probabilities. If the values of the coefficients are known it may be possible to improve on the designs described in this paper. However, it is worth noting that the design approach that we described and discussed will be optimal unless there are one or two options that are extremely popular (or unpopular). In these cases the designs easily identify the extreme options.

Similar techniques to the ones given here can be used to obtain results about the best designs if at most s attributes can vary between the pairs in a choice set (see Burgess & Street, 2005; Grasshoff, Grossmann, Holling, & Schwabe, 2002).

As noted, the methods for constructing designs for DCEs discussed in this paper generally lead to optimal or nearly optimal designs for estimating the parameters of MNL models. These designs have now been used in field applications in marketing, transportation and applied economics for the past 3–4 years. The empirical experience with these designs suggests that they indeed deliver superior efficiency in practice as well as in theory. However, research in this area could benefit from additional comparisons of this construction method with others so that the research community can better understand what works well in what circumstances.

Acknowledgements

The authors gratefully acknowledge the support of the Australian Research Council, Grant Number DP0343632, entitled "Modelling the Choices of Individuals".

Appendix A. Table A1

Strategy 4 choice sets

Pair	Option 1					Option 2					Pair	Option 1					Option 2				
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
1	0	1	2	3	0	1	2	3	0	1	33	0	1	0	0	2	0	2	2	0	3
2	1	0	3	1	0	3	2	0	2	3	34	1	0	1	2	2	2	2	1	2	1
3	2	2	0	2	0	0	2	1	3	0	35	2	2	2	1	2	1	2	0	3	2
4	3	3	1	0	0	2	2	2	1	2	36	3	3	3	3	2	3	2	3	1	0
5	0	2	1	0	2	1	0	3	1	3	37	0	2	3	3	0	0	0	2	1	1
6	1	3	0	2	2	3	0	0	3	1	38	1	3	2	1	0	2	0	1	3	3
7	2	1	3	1	2	0	0	1	2	2	39	2	1	1	2	0	1	0	0	2	0
8	3	0	2	3	2	2	0	2	0	0	40	3	0	0	0	0	3	0	3	0	2
9	0	3	3	1	3	1	1	3	2	0	41	0	3	1	2	1	0	1	2	2	2
10	1	2	2	3	3	3	1	0	0	2	42	1	2	0	0	1	2	1	1	0	0
11	2	0	1	0	3	0	1	1	1	1	43	2	0	3	3	1	1	1	0	1	3
12	3	1	0	2	3	2	1	2	3	3	44	3	1	2	1	1	3	1	3	3	1
13	0	0	0	2	1	1	3	3	3	2	45	0	0	2	1	3	0	3	2	3	0
14	1	1	1	0	1	3	3	0	1	0	46	1	1	3	3	3	2	3	1	1	2
15	2	3	2	3	1	0	3	1	0	3	47	2	3	0	0	3	1	3	0	0	1
16	3	2	3	1	1	2	3	2	2	1	48	3	2	1	2	3	3	3	3	2	3
17	0	1	3	2	1	3	2	1	0	2	49	0	1	1	1	3	2	2	0	0	0
18	1	0	2	0	1	1	2	2	2	0	50	1	0	0	3	3	0	2	3	2	2
19	2	2	1	3	1	2	2	3	3	3	51	2	2	3	0	3	3	2	2	3	1
20	3	3	0	1	1	0	2	0	1	1	52	3	3	2	2	3	1	2	1	1	3
21	0	2	0	1	3	3	0	1	1	0	53	0	2	2	2	1	2	0	0	1	2
22	1	3	1	3	3	1	0	2	3	2	54	1	3	3	0	1	0	0	3	3	0
23	2	1	2	0	3	2	0	3	2	1	55	2	1	0	3	1	3	0	2	2	3
24	3	0	3	2	3	0	0	0	0	3	56	3	0	1	1	1	1	0	1	0	1
25	0	3	2	0	2	3	1	1	2	3	57	0	3	0	3	0	2	1	0	2	1
26	1	2	3	2	2	1	1	2	0	1	58	1	2	1	1	0	0	1	3	0	3
27	2	0	0	1	2	2	1	3	1	2	59	2	0	2	2	0	3	1	2	1	0
28	3	1	1	3	2	0	1	0	3	0	60	3	1	3	0	0	1	1	1	3	2
29	0	0	1	3	0	3	3	1	3	1	61	0	0	3	0	2	2	3	0	3	3
30	1	1	0	1	0	1	3	2	1	3	62	1	1	2	2	2	0	3	3	1	1
31	2	3	3	2	0	2	3	3	0	0	63	2	3	1	1	2	3	3	2	0	2
32	3	2	2	0	0	0	3	0	2	2	64	3	2	0	3	2	1	3	1	2	0

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