General dynamic linear model (DLM):

$$
\begin{aligned}
y_t &= F_t\theta_t + v_t & v_t &\stackrel{ind}{\sim} N(0, V_t) \\
\theta_t &= G_t\theta_{t-1} + w_t & w_t &\stackrel{ind}{\sim} N(0, W_t) \\
\theta_0 &\sim N(m_0, C_0)
\end{aligned}
$$

where y_t has dimension d and θ_t has dimension p.

Full conditional distributions are

$$
\begin{aligned}
p(\theta_t|\theta_{-t}, y_{1:t}) &= p(\theta_t|\theta_{t-1}, \theta_{t+1}, y_t) \\
&\propto p(\theta_t|\theta_{t-1})p(y_t|\theta_t)p(\theta_{t+1}|\theta_t) \\
&= N(\theta_t; G_t\theta_{t-1}, W_t)N(y_t; F_t'\theta_t, V_t)N(\theta_{t+1}; G_{t+1}\theta_t, W_{t+1}) \\
&= N(u_t, U_t) \\
U_t &= \left(W_t^{-1} + F_t^\top V_t^{-1} F_t + G_t^\top W_{t+1}^{-1} G_t\right)^{-1} \\
u_t &= U_t^{-1}\left(W_t^{-1} G_t \theta_{t-1} + F_t^\top V_t^{-1} y_t + G_t^\top W_{t+1}^{-1}\theta_{t+1}\right)
\end{aligned}
$$

for with $W_0^{-1} \stackrel{d}{=} 0$ and $V_0^{-1} \stackrel{d}{=} 0$.

One-at-a-time sampler

One-at-a-time state sampler proceeds sequentially through t to sample from the full conditionals using the most recent values of the state. More specifically, given a sample $\theta_{0:T}^{(i)}$ at iteration i, perform the following steps:

- Step 0: $\theta_0^{(i+1)} \sim p\left(\theta_0 \middle| \theta_1^{(i)}\right)$.
- Step t: $\theta_t^{(i+1)} \sim p\left(\theta_t \middle| \theta_{t-1}^{(i+1)}, \theta_{t+1}^{(i)}, y_t\right)$ for $t = 1, \ldots, T-1$.
- Step T: $\theta_T^{(i+1)} \sim p\left(\theta_T \middle| \theta_{T-1}^{(i+1)}, y_T\right)$.

to obtain a a sample $\theta_{0:T}^{(i+1)}$ at iteration $i+1$.

Odd/even sampler

The odd/even sampler jointly samples all odd indices and then jointly samples all even indices. The key is that conditional on the states with even indices, the states with odd indices are independent and vice versa. So, given a sample $\theta_{0:T}^{(i)}$ at iteration i, perform the following steps:

- Step 1: For t odd, sample independently $\theta_t^{(i+1)} \sim p\left(\theta_t \middle| \theta_{t-1}^{(i)}, \theta_{t+1}^{(i)}, y_t\right)$.
- Step 2: For t even, sample independently $\theta_t^{(i+1)} \sim p\left(\theta_t \middle| \theta_{t-1}^{(i+1)}, \theta_{t+1}^{(i+1)}, y_t\right)$.