Ancillarity–Sufficiency or not

Interweaving to improve MCMC estimation of the local level model

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Model

For $t = 1, 2, \cdots, T$:

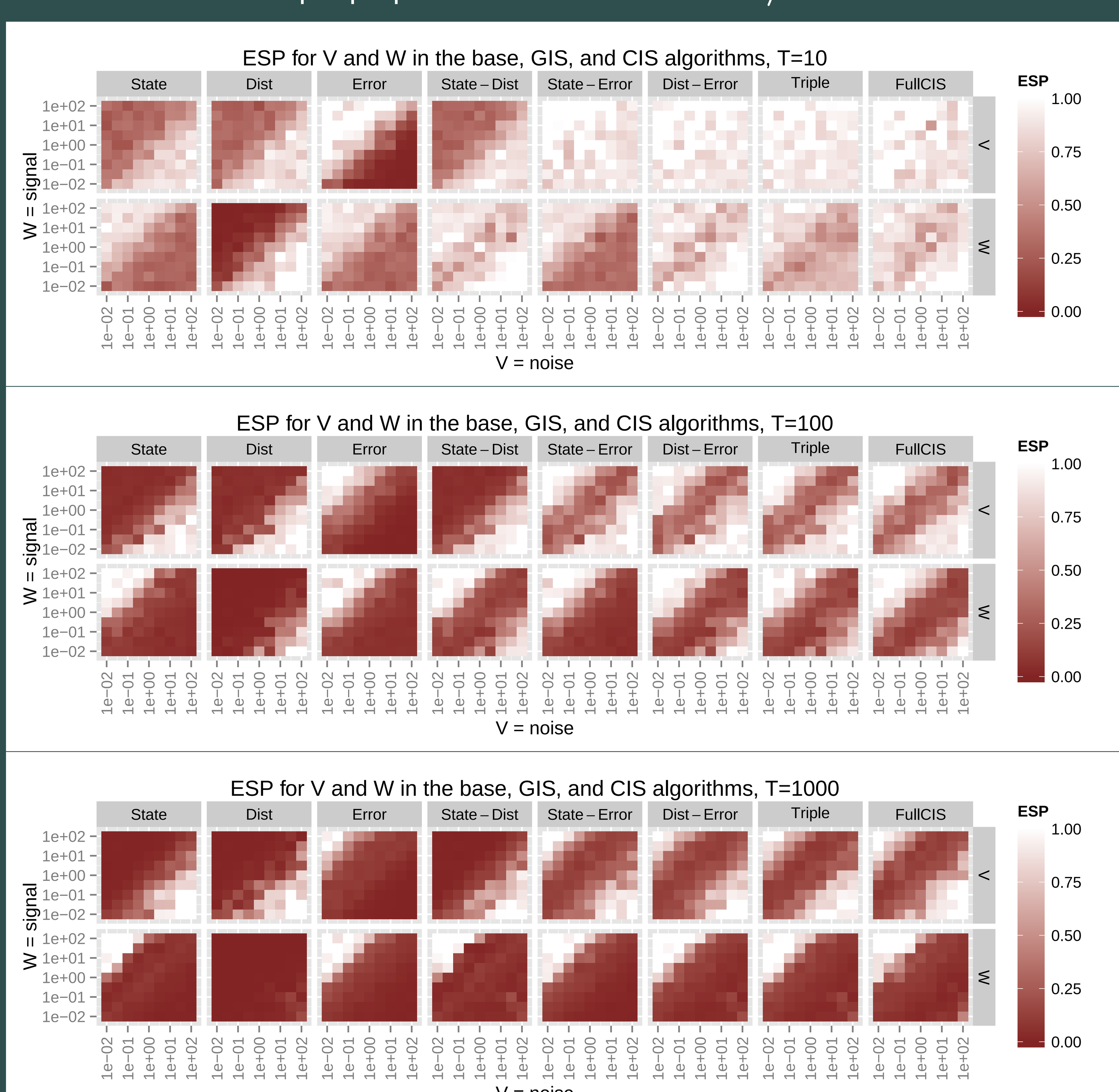
$$y_t|\theta_{0:T} \overset{ind}{\sim} N(\theta_t, V)$$
$$\theta_t|\theta_{0:t-1} \sim N(\theta_{t-1}, W)$$

Data Augmentations (DAs)

- States (standard): $\theta_{0:T}$
  - Sufficient augmentation (SA) for W given V
  - Ancillary augmentation (AA) for V given W
- Scaled disturbances: $\gamma_0 = \theta_0$ and for $t = 1, 2, \cdots, T$
  $$\gamma_t = (\theta_t - \theta_{t-1})/\sqrt{W}$$
  - Ancillary augmentation for (V, W)
- Scaled errors: $\psi_0 = \theta_0$ and for $t = 1, 2, \cdots, T$
  $$\psi_t = (y_t - \theta_t)/\sqrt{V}$$
  - Ancillary augmentation for (V, W)

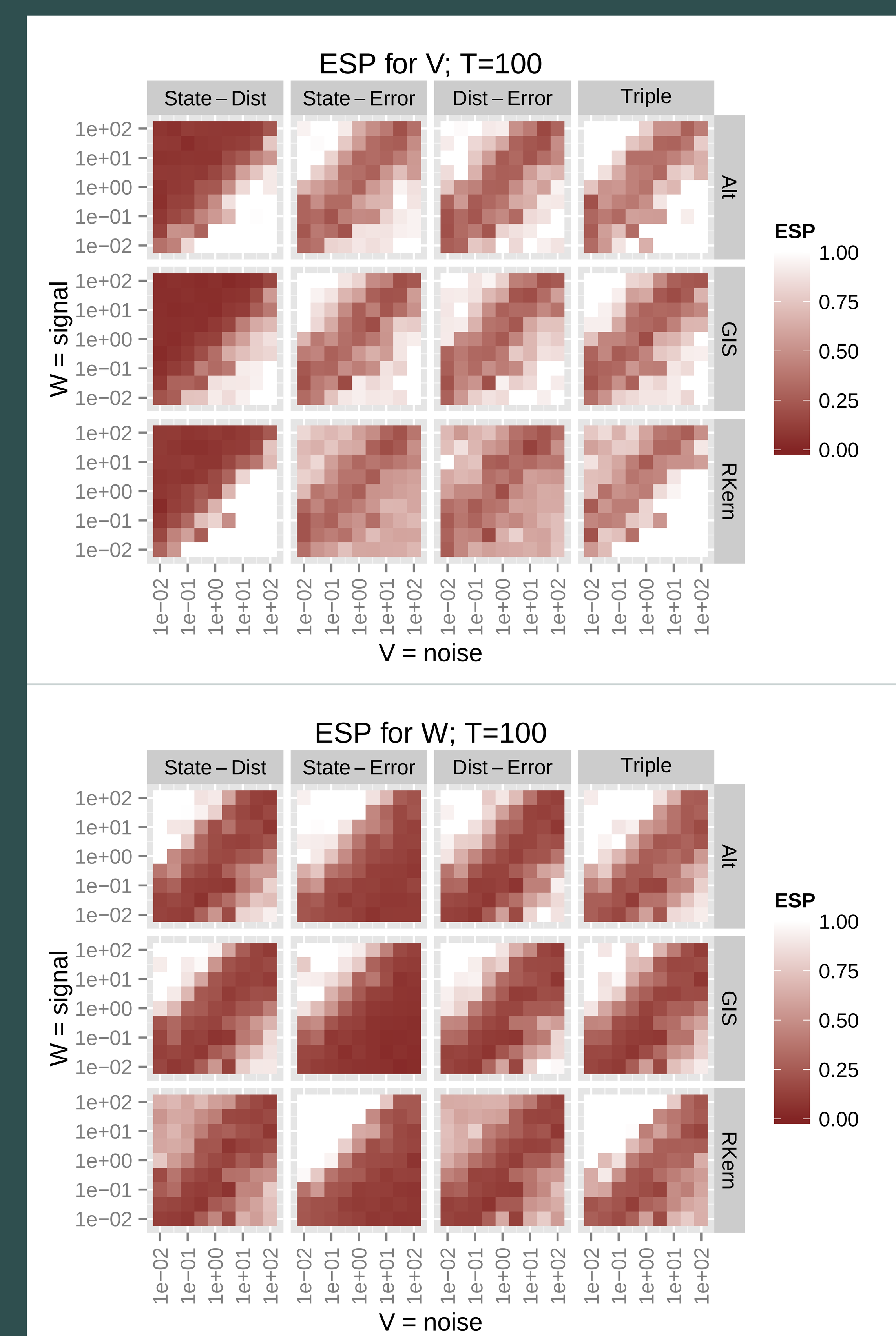
Interweaving vs Base

- GIS: let $\phi = (V, W)$
  $$[\theta|\phi^{(k)}, y] \to [\phi^{(k+0.5)}|\theta, y] \to [\gamma|\theta, \phi^{(k+0.5)}, y] \to [\phi^{(k+1)}|\gamma, y]$$
- ASIS: GIS, but require θ to be a SA and γ an AA for (V, W), or vice versa
- CIS: GIS for $V^{(k+1)}|W^{(k)}$, then for $W^{(k+1)}|V^{(k+1)}$ (similar to Gibbs steps)
- Let effective sample proportion $\equiv ESP \equiv ESS/n$



GIS vs Hybrid

- Alternating: let $\phi = (V, W)$
  $$[\theta|\phi^{(k)}, y] \to [\phi^{(k+0.5)}|\theta, y] \to [\gamma|\phi^{(k+0.5)}, y] \to [\phi^{(k+1)}|\gamma, y]$$
- Random Kernel: randomly select a DA at every iteration.



Conclusions

- Hard to find a SA for (V, W), so ASIS is elusive
- GIS gives no better mixing than alternating algorithms
- Full CIS gives no better mixing than non-ASIS GIS
- GIS is computationally cheaper than full CIS and hybrid algorithms
- GIS improves mixing over base DA algorithms, but trade-off with computation
- New "scaled" DAs generalize easily to most DLMs

References

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