

# Backtracking

Date.....

Ques N-Queen Problem.

N-Queen problem is to place  $n$  Queens in such a manner on  $n \times n$  chessboard -d that no Queen attack each other by being in the same row, same column or same diagonal.

We've given  $4 \times 4$  chessboard and 4 Queens:-

	1	2	3	4
1				
2				
3				
4				

4x4 chess  
Board

Since we have to place 4 Queens such as  $Q_1, Q_2, Q_3, Q_4$  on the chess board such that no two Queens attack each other.

In such condition each Queen must be placed on a different row, i.e., we put ' $Q_i$ ' on row ' $i$ '.



Let suppose we place first Queen at  $(2, 3)$ , i.e.,

	1	2	3	4
1				
2			Q <sub>2</sub>	
3				
4				

The position of Q<sub>2</sub> is  $(2, 3)$ . It means, we can't place the next Queen in the same row & same column.

\* The working algorithm to solve the 4-Queen problem :-

- 1.) create a  $4 \times 4$  chessboard.
- 2.) Place the Queen Q<sub>1</sub> at the left most position which means row 1 and column 1.

	1	2	3	4
1	Q <sub>1</sub>			
2				
3				
4				

Mark the cells of the chessboard with cross marks that are under attack from Q<sub>1</sub>.



	1	2	3	4
1	Q <sub>1</sub>	X	X	X
2	X	X		
3	X		X	
4	X			X

3.) The possible safe cells for Queen Q<sub>2</sub> at row 2 are of column 3 & column 4, because these cells do not come under the attack from Queen Q<sub>1</sub>. So we place Q<sub>2</sub> at first possible safe cells which is row 2 & column 3.

And again make the cell of the board with cross mark that are under attack.

	1	2	3	4
1	Q <sub>1</sub>	X	X	X
2	X	X	Q <sub>2</sub>	X
3	X	X	X	X
4	X		X	X

4.) Now we see that no safe place is left for Q<sub>3</sub> if we place Q<sub>2</sub> at (2, 3)

So, make position (2, 3) false & backtrack.



5.) Now, we place  $Q_2$  at second possible safe cell which is row 2 & column 4.

	1	2	3	4
1	$Q_1$	X	X	X
2	X	X	X	$Q_2$
3	X		X	X
4	X	X		X

6.) The only remaining cell for  $Q_3$  is row 3 & column 2.  
So we place  $Q_3$  at row 3 & column 2.

	1	2	3	4
1	$Q_1$	X	X	X
2	X	X	X	$Q_2$
3	X	$Q_3$	X	X
4	X	X	X	X

7.) Now, we don't have safe place for Queen  $Q_4$ , if we place  $Q_3$  at (3, 2).  
Therefore, make position (3, 2) false & backtrack.

We can't move  $Q_2$  at other position, also we can't move  $Q_3$  also at some other position.



8.) This time we backtrack to the first Queen  $Q_1$ .  
change the position of  $Q_1$ .

	1	2	3	4
1	X	$Q_1$	X	X
2	X	X	X	
3		X		X
4		X		

9.) The safe place for  $Q_2$  is (2, 4)

	1	2	3	4
1	X	$Q_1$	X	X
2	X	X	X	$Q_2$
3		X		X
4		X		

10.) Place  $Q_3$  at (3, 1)

	1	2	3	4
1	X	$Q_1$	X	X
2	X	X	X	$Q_2$
3	$Q_3$	X	X	X
4	X	X		

11.) Place  $Q_4$  at (4, 3)



	1	2	3	4
1	x	Q <sub>1</sub>	x	x
2	x	x	x	Q <sub>2</sub>
3	Q <sub>3</sub>	x	x	x
4	x	x	Q <sub>4</sub>	x

12.) Now, here we got the solution for 4 Queen problem because all 4 Queens are placed exactly in each row / column individual.

### \* Backtracking Algorithm for N - Queen problem:-

The idea is to place Queens one by one in different columns, starting from the leftmost column. When we place a Queen in a column, we check for clashes with already placed Queens. In the current position, if we find a row which there is no clash, we mark the current row & column as a part of the solution. If we do not find such a row due to clashes, then we mark backtrack & return false.



code:-

```
#include <bits/stdc++.h>
using namespace std;

void printSolution(vector<vector<char>> &board, int n)
{
    cout << endl;
    for(int i = 0; i < n; i++)
    {
        for(int j = 0; j < n; j++)
        {
            cout << board[i][j] << " ";
        }
        cout << endl;
    }
    cout << endl;
}
```

// this function let us know that  
placing the queen at current cell  
is safe or not.

// by checking the previous left row,  
upper left diagonal & lower left  
diagonal

```
bool isSafe(int row, int column,  
            vector<vector<char>> &board, int n)  
{
```

// checking the left row

int i = row;

int j = column;

*Spiral*

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while (j >= 0)

{

if (board[i][j] == 'Q')

{

return false;

}

j--;

}

// checking the upper left diagonal

i = row;

j = column;

while (i >= 0 and j >= 0)

{

if (board[i][j] == 'Q')

{

return false;

}

i--;

j--;

}

// checking the lower left diagonal

i = row;

j = column;

while (i < n and j >= 0)

{

if (board[i][j] == 'Q')

{

return false;

}

i++;

Teacher's Sign .....



```
    } j--;
```

// if false is not returned till now means the cell is safe so we return true

```
    return true;
```

```
}
```

```
void nQueenSolve (vector<vector  
    <char>> &board, int column,  
    int n)
```

```
{
```

// base case --> we stop when we find 1 solution, i.e., when we place our Queen in all columns

```
    if (column >= n)
```

```
    {
```

```
        printSolution (board, n);
```

```
        return;
```

```
    }
```

// now we try 1 Queen in all rows of current column & rest recursion will handle.

```
    for (int row = 0; row < n; row++)
```

```
    {
```

// at every iteration, first we check that placing Queen on current cell is safe or not

```
        if (isSafe (row, column, board,  
                    n))
```



{

board[row][column] = 'Q';

// we place 1 Queen above, now  
recursion will place rest

nQueensolve(board, column  
+ 1, n);

// backtracking step

board[row][column] = '-';

}

}

}

int main()

{

int n;

cout << "enter no. of queen, (n)  
you want to place in n x n  
matrix : ";

cin >> n;

if (n == 0 || n == 1 || n == 2 || n == 3)

{

cout << "solution not exist";

return 0;

}

vector<vector<char>> board(n,  
vector<char>(n, '-'));

int column = 0;

cout << "Possible solutions are: ";

nQueensolve(board, column, n);

return 0;

}



Algorithm!:-

- i) Start in the leftmost column.
- ii) If all Queens are placed return true.
- iii) Try all rows in the current column. Do the following for every tried row :-
  - a) If the Queen can be placed safely in this row, then mark current [row][column] as part of the solution & recursively check if placing Queen here leads to a solution.
  - b) If placing the Queen in [row][column] leads to a solution then return true.
  - c) If placing the Queen doesn't lead to a solution then unmark the current [row][column] (Backtracking) & go to step a to try other rows.
- iv) If all rows have been tried & nothing worked, return false to trigger Backtracking.