Computer Graphics

Date

→ Computer Graphics

It deals with the generation, representation, manipulation & display of pictures within the computer.

OR

It usually means images that are created with the help of computer and other specialized graphical hardware and software.

→ Hardware & Software Requirements of Graphics

1. Basic Display Devices
   - Display monitor / video monitor
     - + Raster-Scan Displays
     - + Random-Scan Displays
     - + Colour CRT Monitor
     - + Flat pannel Display
     - + Direct View Storage Tubes

2. Raster-Scan System (Rasterization)

3. Input Devices
   - Keyboard
     Used for non-graphics data. Eg: text strings, screen coordinates, graphic function, etc.
   - Devices for interactive input
     Mouse, trackball, spaceball, Joystick, Digitizer, Data Gloves, Scanner, etc.

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4. Software Packages

The graphic software are divided into 2 categories:

- General Programming Packages

  These provide functions for generating lines, circles, ellipse, polygon. Also apply transformations (scale, rotation, translation, shear), setting intensity values. These packages are for the programmers of high level language C, C++, etc.

- Application Graphic Packages

  These are for non-programmers. Eg: CAD systems, Painting programs, etc.

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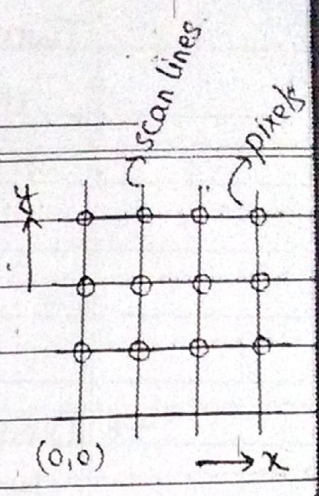
29/07/15.

→ Scan Conversion



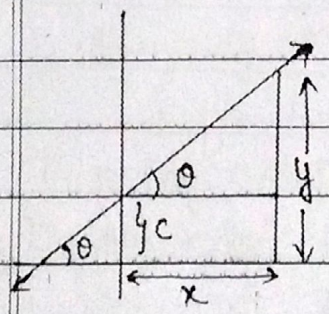
The scan conversion is the process of representing continuous graphic object as a collection of discrete object.

Eg: (2.7, 3) coordinate is not considered by the system because its a floating point values.
(2.7, 3) → (3, 3) This process of conversion of coordinate acc. to the system is called scan conversion.

Scan Conversion

Line	Circle	Ellipse
• Basic Incremental Algo	• Basic Incremental Algo	• Basic Incremental Algo
• Midpoint line Algo	• Midpoint Circle Algo	• Midpoint Ellipse Algo.

→ Scan Conversion of Line.



$y = mx + c$

$m = \tan\theta = \frac{y-c}{x}$

- Slope is +ve : $m = \tan\theta > 0$; θ is acute
- Slope is zero : $m = \tan\theta = 0$
- Slope is −ve : $m = \tan\theta < 0$; θ is obtuse.

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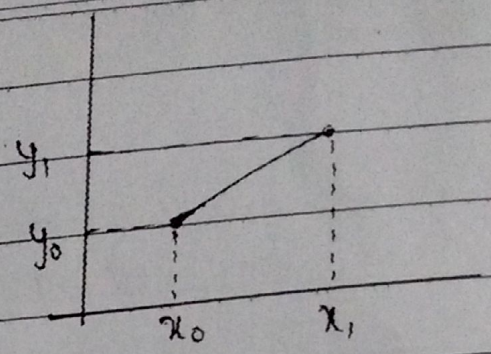
Two point form

$y = mx + B$

$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{dy}{dx}$



→ Basic Incremental Algorithm for line.

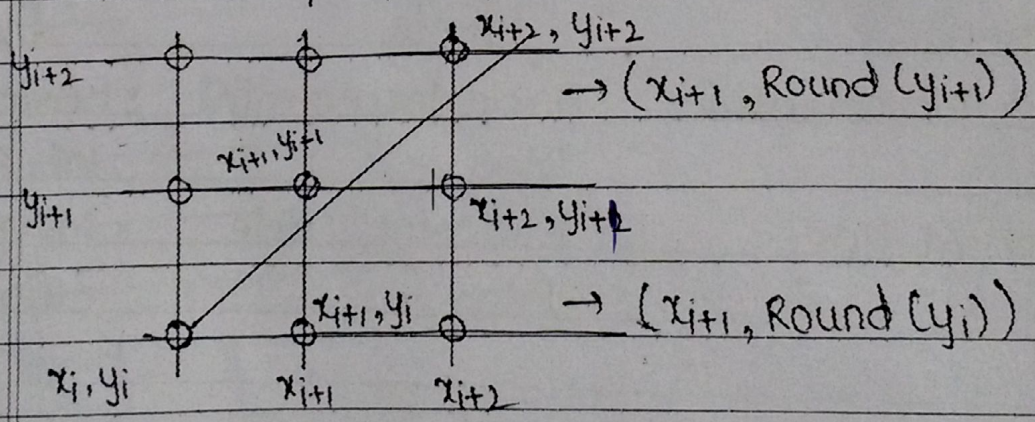
Assumption 1 : slope ≤ 1

$|m| \leq 1$

$-1 \leq m \leq 1$.

Assumption 2 : $x_0 < x_1$; the line is to be processed from left end point (x_0, y_0) to the right end point (x_1, y_1)

The problem is to decide which pixel is turned on between (x_{i+1}, y_i) and (x_{i+1}, y_{i+1}) when we move from x_i to x_{i+1}.



for each x_i we compute

$y_i = mx_i + B$ — (i)

where x_i is incremented by 1

y_i can be a floating point value

therefore we take $\text{Round}(y_i) = \text{floor}(y_i + 0.5)$

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To eliminate multiplication operation

$$y_{i+1} = mx_{i+1} + B$$
$$= m(x_i + 1) + B$$
$$= mx_i + B + m$$

from eq. (i)

$$\boxed{y_{i+1} = y_i + m}$$

- Sumarry

$\lvert m \rvert \leq 1$	$\lvert m \rvert > 1$
+ $y_{i+1} = y_i + m$	+ $x_{i+1} = x_i + 1/m$
+ $x_{i+1} = x_i + 1$	+ $y_{i+1} = y_i + 1$
+ $m = dy/dx$	+ $m = dy/dx$
+ Pixel to be plot is $(x_i, \text{Round}(y_i))$	+ Pixel to be plot is $(\text{Round}(x_i), y_i)$

Note: In both case we assume $x_0 < x_1$; i.e. we move from left end point to right end point.

Note: When we move from right end pt. to left end pt.

$$(x_0, y_0) \leftarrow (x_1, y_1)$$

$$y_{i+1} = y_i - m \qquad x_{i+1} = x_i - 1/m$$

This algorithm is also called DDA (Digital Differential Analyzer)

- Disadvantages
  + Roundoff func. is a complex operation.
  + Roundoff error builds up & pixels get drifted.

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Q. (0,0) to (-8,-4)
Scan convert line using DDA algo.

⇒ Assumption 1 : $|m| \leq 1$

$$m = \frac{-4-0}{-8-0} = \frac{1}{2} < 1.$$

∴ Assumption holds.

Assuptiom 2 : $x_0 < x_1$

$0 \not< -8$

⇒ Assumption doesn't hold, so swap the points.

$(x_0, y_0) = (-8, -4)$

$(x_1, y_1) = (0, 0)$

i	x_i	y_i	Pixel Plot.
0	-8	-4	(-8, -4)
1	-7	-4+0.5	(-7, Round (-3.5)) (-7, -3)
2	-6	-3.5+0.5	(-6, -3)
3	-5	-3+0.5	(-5, Round (-2.5)) (-5, -2)
4	-4	-2.5+0.5	(-4, -2)
5	-3	-2+0.5	(-3, Round (-1.5)) (-3, -1)
6	-2	-1.5+0.5	(-2, -1)
7	-1	-1+0.5	(-1, Round (-0.5)) (-1, 0)
8	0	0	(0, 0)

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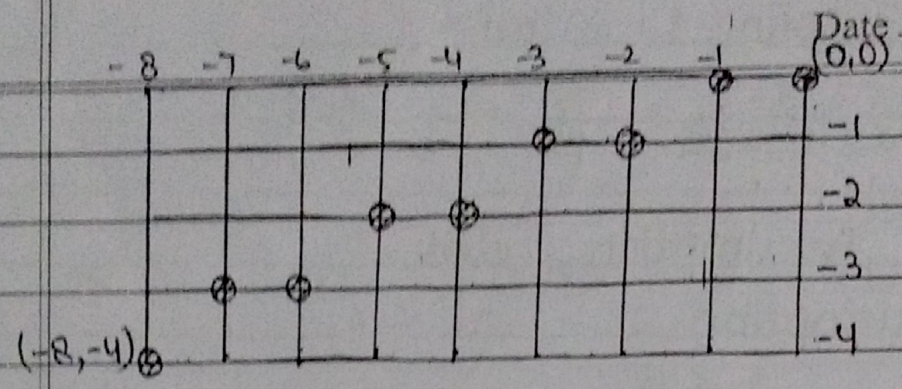

Q. (0,0) to (5,5)

Apply DDA algo to restarize this line.

⇒ Assumption 1 : $|m| \leq 1$.

$$m = \frac{5-0}{5-0} = 1 \leq 1.$$

∴ Assumption holds.

Assumption 2 : $x_0 < x_1$

$0 < 5$

∴ Assumption holds.

i	x_i	y_i	Pixel Plot.
0	0	0	(0,0)
1	1	1	(1,1)
2	2	2	(2,2)
3	3	3	(3,3)
4	4	4	(4,4)
5	5	5	(5,5)

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Q. (2,3) to (12,8)

Apply DDA algo.

⇒ Assumption 1: $|m| \leq 1$

$$m = \frac{8-3}{12-2} = \frac{5}{10} = \frac{1}{2} < 1$$

∴ Assumption holds.

Assumption 2 : $x_0 < x_1$

$2 < 12$

∴ Assumption holds.

i	x_i	y_i	Pixel Plot
0	2	3	(2, 3)
1	3	3.5	(3, Round (3.5)) (3, 4)
2	4	4	(4, 4)
3	5	4.5	(5, Round (4.5)) (5, 5)
4	6	5	(6, 5)
5	7	5.5	(7, Round (5.5)) (7, 6)
6	8	6	(8, 6)
7	9	6.5	(9, Round (6.5)) (9, 7)
8	10	7	(10, 7)

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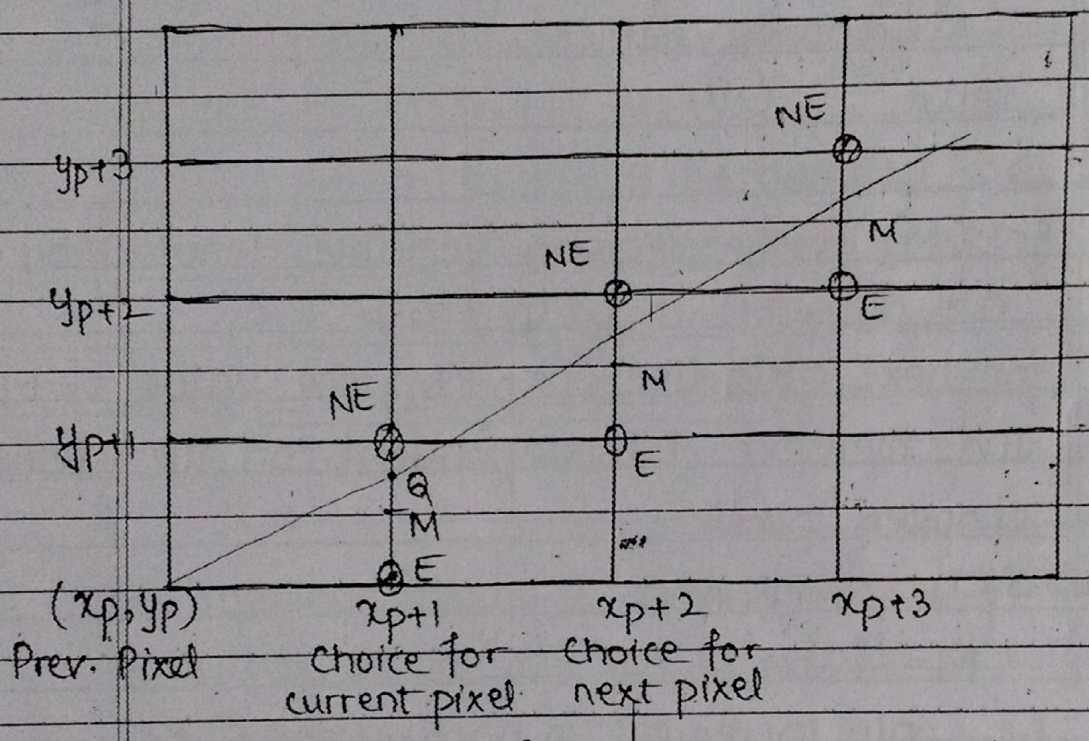
9	11	7.5	(11, Round (7.5)) (11, 8)
10	12	8	(12, 8)

→ Mid-point Line Algorithm.
(also called Bresenham Algo)



Q is the point of intersection b/w line & scan line

$E = (x_p+1, y_p)$; $NE = (x_p+1, y_p+1)$

$\therefore M = (x_p+1, y_p+1/2)$

The basic idea is :

- If M is below the line → choose NE
- If M is above the line → chose E
- If M is on the line → choose E.

Consider the eq. of a line is :

$$F(x, y) = ax + by + c \quad \text{— (i)}$$

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slope intercept form

$$y = x\,dy/dx + B$$

$$y \cdot dx = x \cdot dy + B \cdot dx$$

$$x \cdot dy - y \cdot dx + B \cdot dx = 0 \quad \text{— (ii)}$$

Comparing eq. (i) & (ii)

$$a = dy$$

$$b = -dx$$

$$c = B \cdot dx$$

$$M = (x_p + 1,\ y_p + 1/2)$$

We will compute $F(M)$

```
if F(M) > 0
    choose NE
else
    choose E
```

$\therefore$ $F(M)$ is the decision variable denoted by d.

$$d = a(x_p + 1) + b(y_p + 1/2) + c$$

for next grid line, $x_p + 1$, the value of M and therefore d is dependent on the previously choosen pixel

- If E is choosen

$$M = (x_p + 2,\ y_p + 1/2)$$

i.e. only increment in x direction

$$d_{new} = F(x_p + 2,\ y_p + 1/2)$$

$$= a(x_p + 2) + b(y_p + 1/2) + c$$

$$d_{old} = a(x_p + 1) + b(y_p + 1/2) + c$$

$$d_{new} - d_{old} = a$$

let this increment is called ΔE

$$\boxed{\Delta E = a}$$

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- If NE is choosen
  $M = (x_p+2,\ y_p+1/2+1)$
  i.e. increment in both direction
  $d_{new} = a(x_p+2) + b(y_p+3/2) + c$
  $d_{new} - d_{old} = a+b.$
  $\boxed{\Delta^N E = a+b}$

Suppose: first pixel is (x_0, y_0)
Mid-point will be $(x_0+1,\ y_0+1/2)$
$F(M) = F(x_0+1,\ y_0+1/2)$
$= a(x_0+1) + b(y_0+1/2) + c$
$= (ax_0 + by_0 + c) + (a+b/2)$
$= F(x_0, y_0) + a + b/2$
$d_{start} = F(M) = 0 + a + b/2$
$\boxed{d_{start} = a + \frac{b}{2}}$

Multiplying RHS by 2 to eliminate fraction.
(multiplication will not affect the sign of d, which we are checking)
$d_{start} = 2a + b$
$= 2dy - dx.$

- Advantages
  d_{new} computation requires only simple addition, no time consuming multiplication is required.

note: To eliminate fraction we multiply $f(x,y)$ by 2
this will lead to each constant multiply by 2.

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30/July/15

Q. Scan converting a line from the pixel from (1,1) to (8,5) using mid point algo.

$\Rightarrow$ $x_0 = 1$; $y_0 = 1$

$x_1 = 8$; $y_1 = 5$.

$dy = 5-1 = 4$

$dx = 8-1 = 7$.

$d_{start} = 2dy - dx = 8-7 = 1$.

$\Delta E = 2a = \cancel{4}\ \cancel{8}\ \cancel{14}\ 8$

$\Delta NE = 2(a+b) = (4-7)2$
$= \cancel{-8}\ -6$

Both the assumptions are also true.

i	d_{new} d_{old}	x plot	y plot	E/NE
0	$d_{start} = 1 > 0$ –	1	1	E → always start with E
1	$d_1 = d_0 + \Delta NE$ $= -5 < 0$	2	2	NE
2.	$d_2 = d_1 + \Delta E$ $= 3 > 0$	3	2	E
3.	$d_3 = d_2 + \Delta NE$ $= 3-6 = -3 < 0$	4	3	NE
4.	$d_4 = d_3 + \Delta E$ $= -3+8 = 5 > 0$	5	3	E
5	$d_5 = d_4 + \Delta NE$ $= 5-6 = -1 < 0$	6	4	NE
6	$d_6 = d_5 + \Delta E$ $= -1+8 = 7 > 0$	7	4	E
		8	5	NE

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Q. Scan convert a line from pixel (0,0) to (-8,-4)

⇒ $dx = -8-0 = -8$

$dy = -4-0 = -4$.

$|m| = -4/-8 = 1/2 < 1$

Assuption 1 holds.

for assumption 2 : swap the points.

$\Delta E = 2a = 2dy = 8$

$\Delta NE = 2(a+b) = 2(dy-dx) = 2(4-8) = -8$

$d_{start} = 2dy - dx = 8-8 = 0$.

i	d_{new}	d_{old}	E/NE	xplot	yplot
0	$d_{start} = 0 = 0$	–	E	-8	-4
1	$d_1 = d_0 + \Delta E$ $= 8 > 0$	0	E	-7	-3
2	$d_2 = d_1 + \Delta NE$ $= 8-8 = 0 > 0$	8	NE	-6	-3
3	$d_3 = d_1 + \Delta E$ $= 8+8 = 16 > 0$	0	E	-5	-2
4	$d_4 = d_3 + \Delta NE$ $= 16-8 = 8 > 0$	16	NE	-4	-2
5	$d_5 = d_4 + \Delta NE$ $= 8+(-8) = 0 = 0$	8	NE	-3	-2
6	$d_6 = d_5 + \Delta E$ $= 0+8 = 8 > 0$		E	-2	-1
7	$d_7 = d_6 + \Delta E$ $=$		E	-1	0

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Q. (5, 8) to (9, 11)

$\Rightarrow$ $dy = 11 - 8 = 3$

$dx = 9 - 5 = 4$.

$|m| = 3/4 < 1$.

$x_0 < x_1 \Rightarrow 5 < 9$

both assumption holds true.

$\Delta E = 2a = 2dy = 6$.

$\Delta NE = 2(a+b) = 2(dy - dx) = -2$.

$d_{start} = 2dy - dx = 6 - 4 = 2$.

i	d_{new}	d_{old}	E/NE	x plot	y plot
0	$d_{start} = 2 > 0$	–	E	5	8
1	$d_1 = d_0 + \Delta NE$ $= 2 - 2 = 0$	2	NE	6	9
2.	$d_2 = d_1 + \Delta E$ $= 0 + 6 = 6 > 0$	0	E	7	9
3.	$d_3 = d_2 + \Delta NE$ $= 6 - 2 = 4 > 0$	6	NE	8	10
4.		4	NE	9	11

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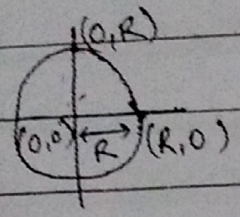
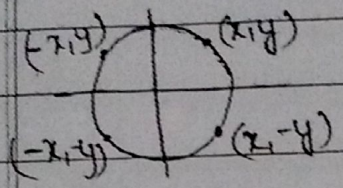
Circle

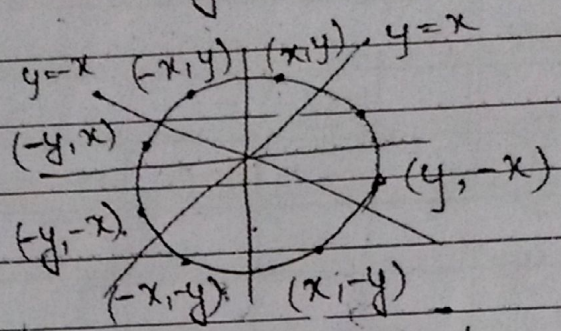


$$x^2+y^2=R^2$$
$$y^2=R^2-x^2$$
$$y=\pm\sqrt{R^2-x^2}$$

Generalize : $y_i = \pm\sqrt{R^2-x_i^2}$



4-way symetry



8-way symmetry.

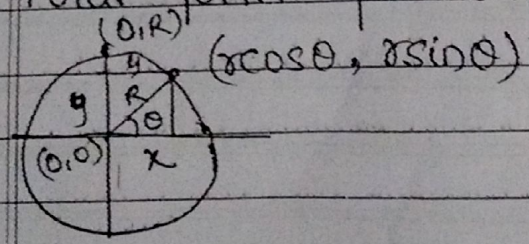
x_i goes from 0 to R in unit steps. At each step, solve for y.

note: Other quadrants are drawn by symmetry.

- Disadvantage

1. Square root function is very slow.
2. Not all pixels filled in as we move from the x close to R, bcoz the slope of the circle becomes infinite there.

→ Polar form of Circle.



$$x = r\cos\theta$$
$$y = r\sin\theta$$

where r is the radius.

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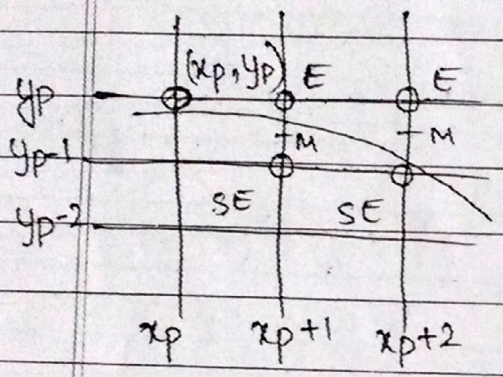
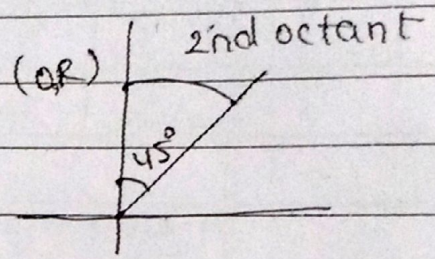
- Algo :

```
θ = 0°
while (θ < 360°)
    x = rcosθ
    y = rsinθ
    putpixel (x,y)
    θ = θ+1°
end while.
```





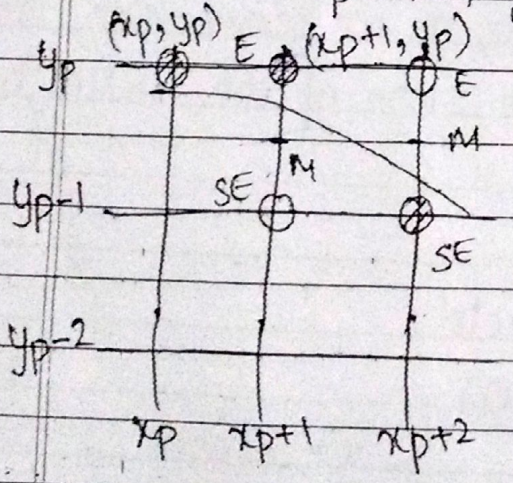
→ <u>Mid-Point Circle Algorithm</u>

$$f(x,y) = x^2 + y^2 - R^2$$

$$d = f(x,y) = \begin{cases} >0 & (x,y) \text{ lie outside circle} \\ =0 & (x,y) \text{ lie on circle} \\ <0 & (x,y) \text{ lie inside circle} \end{cases}$$

(Assuming circle centre (0,0))

$$M = (x_p + 1,\ y_p - 1/2)$$



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$$\boxed{\begin{array}{l} f(M) \le 0 \\ \quad \text{choose } E \\ \text{else} \\ \quad \text{choose } SE \end{array}}$$

Case I: Next mid-point choosen depends on previous choosen pixel E / SE.

If E is choosen, then the mid point is

$$M = (x_p + 2,\ y_p - 1/2)$$

else mid point is

$$M = (x_p + 2,\ y_p - 3/2)$$

Case I: Suppose E is choosen, (when $d_{old} \le 0$),

$$d_{new} = f(x_p + 2,\ y_p - 1/2)$$
$$= (x_p+2)^2 + (y_p - 1/2)^2 - R^2$$
$$= x_p^2 + 4 + 4x_p + y_p^2 + 1/4 - y_p - R^2$$

$$d_{old} = F(x_p + 1,\ y_p - 1/2)$$
$$= (x_p+1)^2 + (y_p - 1/2)^2 - R^2$$
$$= x_p^2 + 1 + 2x_p + y_p^2 + 1/4 - y_p - R^2$$

$$d_{new} - d_{old} = 2x_p + 3$$

$$d_{new} = d_{old} + \underbrace{2x_p + 3}_{\Delta E}$$

$$\boxed{\begin{array}{l} \Delta E = 2x_p + 3 \\ d_{new} = d_{old} + \Delta E \end{array}}$$

Case II: When SE is choosen, i.e. $d_{old} > 0$.

$$d_{new} = f(x_p + 2,\ y_p - 3/2)$$
$$= (x_p+2)^2 + (y_p - 3/2)^2 - R^2$$

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$$= x_p^2 + 4 + 4x_p + y_p^2 + 9/4 - 3y_p - R^2$$

$$d_{new} - d_{old} = 2x_p + 3 - 2y_p + 2$$
$$= 2x_p - 2y_p + 5$$

$$\boxed{\begin{aligned} \Delta SE &= 2x_p - 2y_p + 5 \\ d_{new} &= d_{old} + \Delta SE \end{aligned}}$$

Initial Condition :

$(x_0, y_0) = (0, R)$ = starting pixel

$f(x_0, y_0) = 0 \rightarrow$ means E

midpoint $= (0+1, R - 1/2)$

$$d_{start} = f(1, R - 1/2)$$
$$= 1 + (R - 1/2)^2 - R^2$$
$$= 5/4 - R$$

$$\boxed{d_{start} \simeq 1 - R}$$

Ques : Scan Convert a circle with centre (0,0) & R=10 using mid-point circle algo.

i	d_{new}	d_{old}	E/SE	xplot	yplot
0	1-10 = -9<0	–	–	0	10
1	-9+2x0+3 = -6<0	-9	E	1	10
2	-6+2x1+3 -1<0	-6	E	2	10
3	-1+2x2+3 6>0	-1	E	3	10

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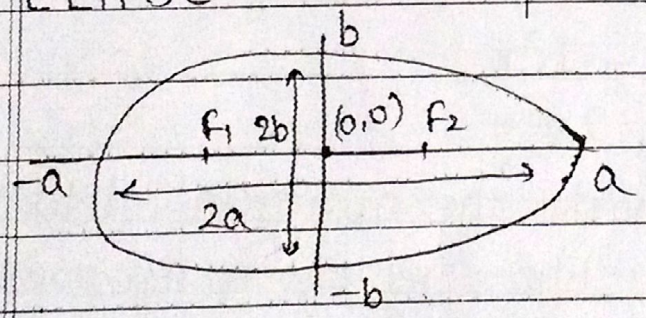
4	$6+2\times3-2\times10+5$ $-3<0$	6	SE	4	9
5	$-3+2\times4+3$ $8>0$	-3	E	5	9
6	$8+10-18+5$ $5>0$	8	SE	6	8
7	$5+12-16+5$ $6>0$	5	SE	7	7
8		6	SE	8	6

note : When E is choosen,
increment only in x, no change in y.
When SE is choosen,
increment both in x, decreement in y.

ELIPSE



Eq. of ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

length of major axis $= 2a$
length of minor axis $= 2b$.

$$F(x,y) = b^2x^2 + a^2y^2 - a^2b^2$$

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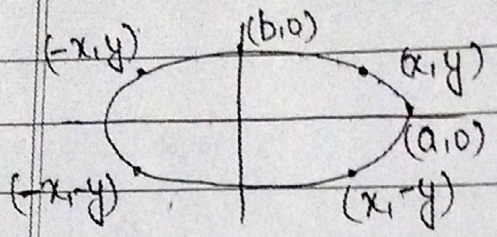
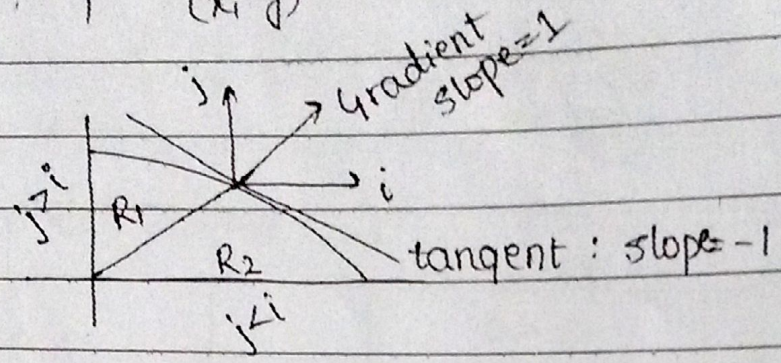


4 way symmetry.



In ellipse there is a symmetry b/w quadrants but two octants of the quadrant is not symmetric.

To simplify the algo., we must calculate the pixel position along elliptical arc through 1 quadrant then we obtain the pixels in the remaining 3 quadrants by symmetry.

Decision parameter $d = F_{ellipse}(x,y)$

$$F_{ellipse}(x,y) = \begin{cases} <0 & \text{if } (x,y) \text{ is inside ellipse} \\ =0 & (x,y) \text{ is on the ellipse} \\ >0 & (x,y) \text{ is outside ellipse} \end{cases}$$

Slope

$$\frac{d}{dx}(b^2x^2 + a^2y^2 - a^2b^2) \Rightarrow -\frac{2b^2x}{2a^2y}$$

$$\frac{dy}{dx} = \frac{-2b^2x}{2a^2y}$$

We first divide the quadrants into two regions. the boundary b/w 2 regions is the point at which the curve has slope of -1.

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$$\text{Gradient } F(x,y) = \frac{\partial}{\partial x} F(x,y)\, i + \frac{\partial}{\partial y} F(x,y)\, j$$

$$= 2b^2x\, i + 2a^2y\, j$$

- In region 1 (R1) j component is greater than i, means $2a^2y > 2b^2x$.
- In region 2 (R) j component is less than i means $2a^2y < 2b^2x$.
- at $j = i$, $dy/dx = -1$.

$$\frac{+2b^2x}{2a^2y} = +1$$

$$2b^2x = 2a^2y.$$

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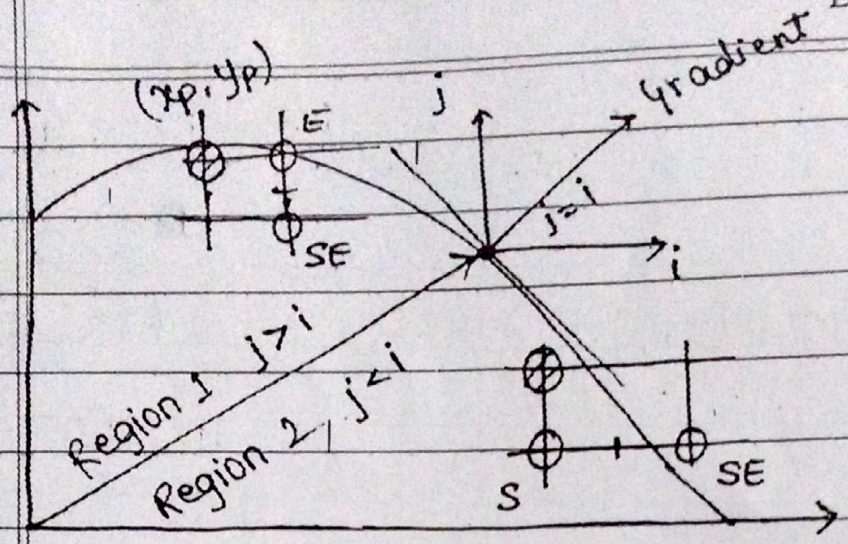
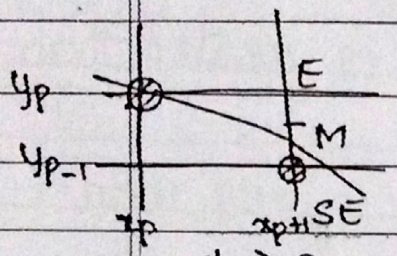
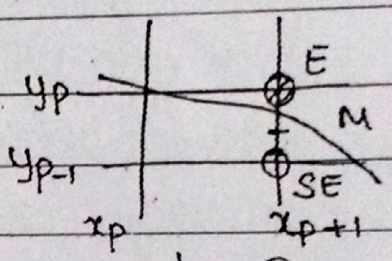




$d_1 > 0$

Midpoint is outside the ellipse



$d_1 < 0$

Midpoint is inside the ellipse.

<u>Region 1</u> : Choice between ~~E & SE~~ E & SE ✓ ~~S & SE~~

current pixel is at (x_p, y_p)

The decision variable for Region 1 is denoted by d_1

$$M = (x_p + 1, \; y_p - 1/2)$$

$$d_1 = f(M)$$

$$= f(x_p + 1, \; y_p - 1/2)$$

$$= b^2 (x_p + 1)^2 + a^2 (y_p - 1/2)^2 - a^2 b^2$$

if $d_1 > 0$

choose ~~SE~~ ~~A~~ SE

else

choose ~~E~~ ~~SE~~ E

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Case 1: If E is choosen, the next mid-point is one increment in x.

$$M = (x_p+2,\ y_p-1/2)$$

$$d_{old} = b^2(x_p+1)^2 + a^2(y_p-1/2)^2 - a^2b^2$$

$$d_{new} = f(x_p+2,\ y_p-1/2)$$

$$= b^2(x_p+2)^2 + a^2(y_p-1/2)^2 - a^2b^2$$

$$d_{new} - d_{old} = b^2(\cancel{x_p^2}+4+4x_p-\cancel{x_p^2}-1-2x_p)$$

$$= b^2(2x_p+3)$$

$$\Rightarrow \boxed{\begin{aligned} \Delta E &= b^2(2x_p+3) \\ d_{new} &= d_{old} + \Delta E \end{aligned}}$$

Case 2: If SE is choosen, the next midpoint is one increment in x & one decrement in y.

$$M = (x_p+2,\ y_p-3/2)$$

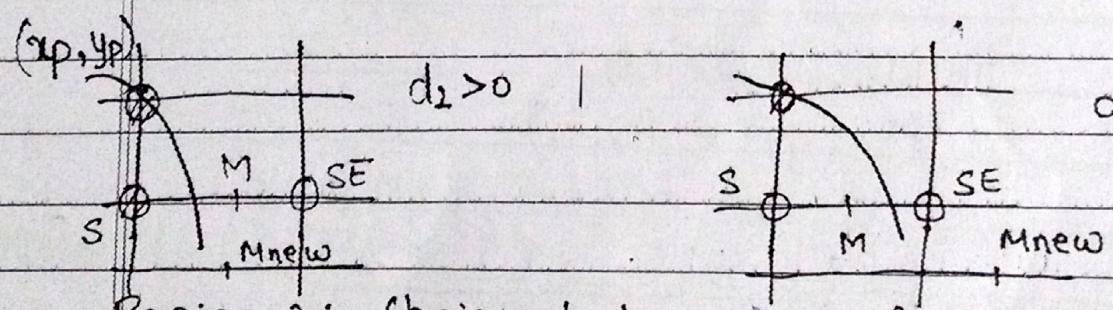
$$d_{new} = f(x_p+2,\ y_p-3/2)$$

$$= b^2(x_p+2)^2 + a^2(y_p-3/2)^2 - a^2b^2$$

$$d_{new} - d_{old} = b^2(2x_p+3) + a^2(\cancel{y_p^2}+9/4-3y_p-\cancel{y_p^2}-1/4+y_p)$$

$$= b^2(2x_p+3) + a^2(-2y_p+2)$$

$$\Rightarrow \boxed{\begin{aligned} \Delta SE &= b^2(2x_p+3) + a^2(-2y_p+2) \\ d_{new} &= d_{old} + \Delta SE \end{aligned}}$$



Region 2: Choice between S & SE

$$S = (x_p,\ y_p-1)$$

$$SE = (x_p+1,\ y_p-1)$$

$$M = (x_p+1/2,\ y_p-1)$$

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The decision variable is denoted by d_2.

if $d_2 > 0$

choose S

else

choose SE.

Case 1: If S is choosen, the next mid point is

$$M = (x_p + 1/2,\ y_p - 2)$$

$$d_{old} = b^2(x_p + 1/2)^2 + a^2(y_p - 1)^2 - a^2b^2$$

$$d_{new} = b^2(x_p + 1/2)^2 + a^2(y_p - 2)^2 - a^2b^2$$

$$d_{new} - d_{old} = a^2(\cancel{y_p^2} + 4 - 4y_p - \cancel{y_p^2} - 1 + 2y_p)$$

$$= a^2(-2y_p + 3)$$

$$\boxed{\begin{aligned} \Delta S &= a^2(-2y_p + 3) \\ d_{new} &= d_{old} + \Delta S \end{aligned}}$$

Case 2: If SE is choosen, the next mid-point is

$$M = (x_p + 3/2,\ y_p - 2)$$

$$d_{new} = b^2(x_p + 3/2)^2 + a^2(y_p - 2)^2 - a^2b^2$$

$$d_{new} - d_{old} = b^2(x_p^2 + 9/4 + 3x_p - x_p^2 - 1/4 - x_p) + a^2(-2y_p + 3)$$

$$= b^2(2 + 2x_p) + a^2(-2y_p + 3)$$

$$\boxed{\begin{aligned} \Delta SE &= b^2(2x_p + 2) + a^2(-2y_p + 3) \\ d_{new} &= d_{old} + \Delta SE \end{aligned}}$$

Initial Condition :

$$(x_0, y_0) = (0, b)$$

Mid-point $= (0+1,\ b - 1/2)$

$$f(1, b - 1/2) = b^2(1)^2 + a^2(b - 1/2)^2 - a^2b^2$$

$$\boxed{d_0 = b^2 + a^2(-b + 1/4)}$$

$\cancel{b^2 + a^2(-b + 1/4)}$

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$36(13) + 64(-8)$ $\quad 360 - 108 = 252$ $\quad 252 - 152 = 100$ $\quad 152$

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Ques : $a = 8$, $b = 6$. Scan convert ellipse in first quadrant.

$\Rightarrow$ $d_0 = b^2 + a^2(-b + 1/4)$

$= 36 + 64(-6 + 1/4) = 36 + (-23/4)\,64$

i	d_{new}	d_{old}	E/SE	Plotx	Ploty	$2a^2y > 2b^2x$
0	$d_0 = -332 < 0$	–	–	0	6	$128 \times 6 > 0$
1.	$d_1 = -332 + (36(5))$ $= -152 < 0$	-332	E	1	6	
2.	$d_2 = -152 + 36 \times 5$ $= 108 - 44 < 0$	-152	E	2	6	
3.	$d_3 = 208 > 0$	-44	E	3	6	
4.	$d_4 = 208 - 36 \times 9 + 64 \times 10$ $= -108 < 0$	208	SE	4	5	
5	$d_5 = -108 + 36 \times 10$ $= 288 > 0$	-108	E	5	5	
6	$d_6 = 288 + 36 \times 13 - 64 \times 8$ $244 > 0$	288	SE	6	4	
		244	SE	7	3	$384 < 504$

note : (7, 3) is the last point which will be the initial point in the region 2.

Region 2 :

$(x_0, y_0) = (7, 3)$

$M = (7 + 1/2,\ 3 - 1) = (15/2, 2)$

$d_2 = F(15/2, 2) = -151$

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i	dnew	dold	S/SE	Plot x	Plot y	$2a^2y < 2b^2x$
0	$-151 < 0$	–	–	7	3	$384 < 504$
1	$+233 > 0$	-151	SE	8	2	$256 < 576$
2	$745 > 0$	233	S	8	1	$128 < 576$
3			SE	8	0	

2D - TRANSFORMATIONS

→ Transformation

These are used to change the coordinates of the object within the graphical c. Changing th of the coordinate means, change the object position, its orientation & its size.

→ Possible Transformation

1. Translation

Move all the points of an object in fixed distance in a specified direction.

2. Scaling

Process of resizing an object either enlarge the object or compress the object.

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3. Rotation
View the object at different orientation or angle.

4. Shearing.
It is a transformation that distort the shape of an object along either or both of the axis.

5. Reflection.
Like a mirror image of the object.

note: These basic transformations can be combined to obtain more complex transformation.

- Point $P = (x, y)$
- 2D object = $\{P_1, P_2, \ldots P_n\}$
  & set of edges $\{e_1, e_2, \ldots e_n\}$
  where $e = \{P_i P_j\}$

- In Matrix form
  $P = [x \;\; y]$ or $\begin{bmatrix} x \\ y \end{bmatrix}$
  Object $A = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix}$ if A is a Δ.

Baseline of Transformation

$$[A][T] = [A^*]$$

Position vector of obj

Transformation Matrix

Position vector of Transformed object.

$[T] = 2 \times 2$ matrix.

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→ Transformation of Point

$P = [x \quad y]$

$$[x \quad y]\begin{bmatrix} a & b \\ c & d \end{bmatrix} = [ax+cy \quad bx+dy]$$
$$= [x^* \quad y^*]$$

Translation

$T = [tx \quad ty]$

$P = [x \quad y]$

$[P] + [T] = [P^*]$

$[x \quad y] + [tx \quad ty] = [x+tx \quad y+ty] = [x' \quad y']$

t_x = change in x coordinate

t_y = change in y coordinate

Eg: $P = [\cancel{5}, \cancel{2}]$ $[2 \quad 1]$

where will be the point if u move it 3 units to right & 1 unit up.

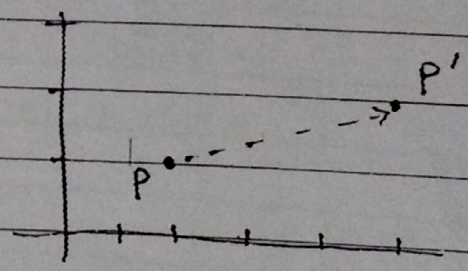
$t_x = 3 \qquad t_y = 1.$

$T = [3 \quad 1]$

$[P^*] = [P] + [T]$

$= [2 \quad 1] + [3 \quad 1]$

$= [5 \quad 2]$



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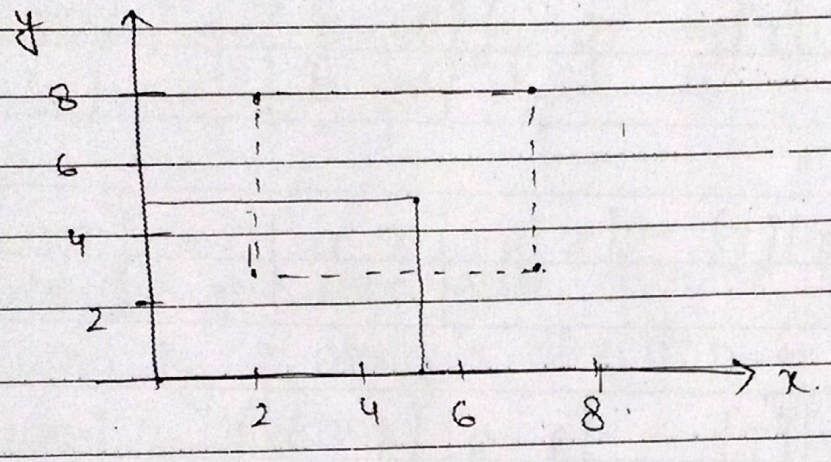
Ques: Translate a square ABCD, A(0,0), B(5,0), C(5,5), D(0,5) by 2 units in x-direction & 3 units in y-direction.

$$\Rightarrow \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 0 \\ 5 & 0 \\ 5 & 5 \\ 0 & 5 \end{bmatrix} \xrightarrow[+3(y)]{+2(x)} \begin{bmatrix} 2 & 3 \\ 7 & 3 \\ 7 & 8 \\ 2 & 8 \end{bmatrix} \begin{array}{c} A' \\ B' \\ C' \\ D' \end{array}$$



Scaling

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} ; \quad P = [x \ y]$$

→ scaling along both x & y direction

$$[T] = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

→ scaling along x-direction

$$[T] = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$$

→ scaling along y-direction

$$[T] = \begin{bmatrix} 1 & 0 \\ 0 & d \end{bmatrix}$$

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note : 1) $a = d > 1$

It means enlargement of the object's coordinates occur.

2) $a = d < 1$

It means compression of the object's coordinates occur.

3) $a \neq d$.

Scaling is unequal. The resulting object is distorted.

Case a) $[P][T] = [x \;\; y]\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} = [ax \;\; dy]$
$= [x' \;\; y']$

Case b) $[P][T] = [x \;\; y]\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} = [ax \;\; y]$
$= [x' \;\; y']$

Case c) $[P][T] = [x \;\; y]\begin{bmatrix} 1 & 0 \\ 0 & d \end{bmatrix} = [x \;\; dy]$
$= [x' \;\; y']$

Ques : Find the new coordinates of the ΔABC, $A(0,0)$ $B(1,1)$ & $C(5,2)$ after it has been

a) magnified to twice its size.

b) reduced to half its size.

$\Rightarrow$ a) $T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$[P][T] = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 10 & 4 \end{bmatrix}$$

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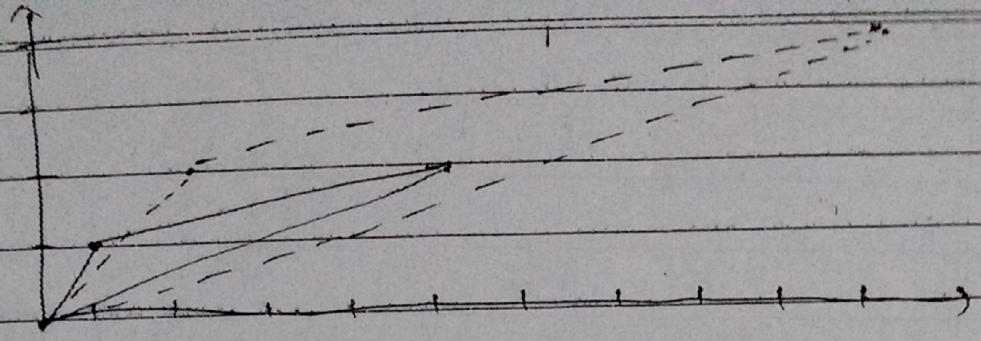

b) $T = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$

$$[P][T] = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1/2 & 1/2 \\ 5/2 & 1 \end{bmatrix}$$

Ques: find the new coordinate of a pt. (1.5, 2) scaled by 2 along x direction & by 0.5 along y direction

$\Rightarrow T = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$

$$[P][T] = \begin{bmatrix} 1.5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix}$$

note: scaling always cause the objects move away from the origin, if scaling is more than 1 &

Rotation

$$[T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

[T] is a transformation for a general rotation by an arbitary angle θ in counter clockwise direction.

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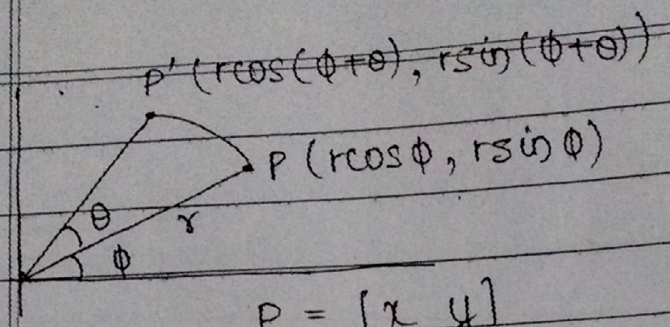

$$P = [x \quad y]$$
$$= [r\cos\phi \quad r\sin\phi]$$

$$P' = [x' \quad y']$$
$$= [r\cos(\phi+\theta) \quad r\sin(\phi+\theta)]$$
$$= [r(\cos\theta\cos\phi - \sin\theta\sin\phi) \quad r(\sin\theta\cos\phi + \cos\theta\sin\phi)]$$
$$= [r\cos\theta\cos\phi - r\sin\theta\sin\phi \quad r\sin\theta\cos\phi + r\cos\theta\sin\phi]$$
$$= [x\cos\theta - y\sin\theta \quad x\sin\theta + y\cos\theta]$$
$$= [x \quad y]\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$
$$[P'] = [P][T]$$

for $\theta = 90°$

$$[T] = \begin{bmatrix}\cos 90 & \sin 90 \\ -\sin 90 & \cos 90\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -1 & 0\end{bmatrix}$$

for $\theta = 180°$

$$[T] = \begin{bmatrix}\cos 180 & \sin 180 \\ -\sin 180 & \cos 180\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

for $\theta = 270°$

$$[T] = \begin{bmatrix}\cos 270 & \sin 270 \\ -\sin 270 & \cos 270\end{bmatrix} = \begin{bmatrix}0 & -1 \\ 1 & 0\end{bmatrix}$$

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for $\theta = 360°$

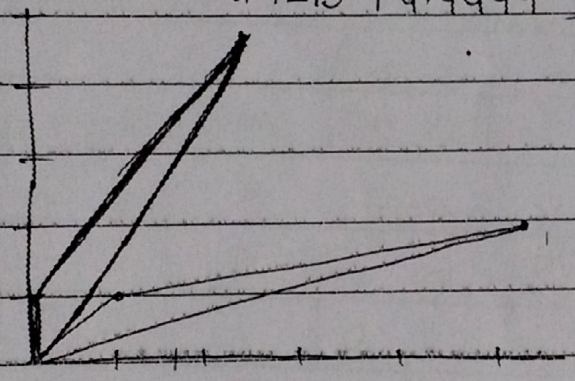
$$[T] = \begin{bmatrix} \cos 360 & \sin 360 \\ -\sin 360 & \cos 360 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Ques. Perform a 45° rotation of ΔABC, $A(0,0)$, $B(1,1)$ & $C(5,2)$ about origin.

$$[P'] = [P][T]$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{2} \\ 3/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1.414 \\ 2.1213 & 4.9497 \end{bmatrix}$$



Q. A point (3,3) makes a rotation of 45° about the origin and then translating in the direc. of vector $V = 5I + 6J$. Find the new location of P.

$$\Rightarrow [P'] = [P][T] = [3 \;\; 3] \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 & 6/\sqrt{2} \\ 0 & 6/\sqrt{2} \end{bmatrix}$$

P after rotation $= (0, 4.2)$

$$P^* = [P'] + [T] = [0 \;\; 4.2] + [5 \;\; 6]$$

$$= [5 \;\; 10.2]$$

$\therefore$ new location $= (5, 10.2)$

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note: for rotating clockwise by an angle θ about the origin, we just put $\theta = -\theta$ in the transformation matrix.

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

→ Pure Rotation.

The pure rotation is which the determinant of the transformation matrix is 1.

$$\det(T) = 1.$$

Reflection.

a) Along x-axis

$$[T] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

b) Along y-axis

$$[T] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

c) Along $x=y$ axis

$$[T] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

d) Along $y=-x$ axis

$$[T] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

e) Along origin

$$[T] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

with $a = d < 1$.

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→ Pure Reflection.

Its a reflection in which determinant of transformation matrix is -1.

$$\det(T) = -1.$$

Ques: Prove that two pure reflection about the line passing through origin is same as a pure rotation. ~~reflection~~. A triangle with coordinates $(4,1)$, $(5,2)$ & $(4,3)$.

⇒ let the line be $y = -x$, after reflection coordinates $= (-1,-4), (-2,-5), (-3,-4)$.

now reflecting about the x axis

coordinates $= (-1,4), (-2,5), (-3,4)$. — A.

Now rotating the coordinates with $\theta = 90$.

$$\begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -2 & 5 \\ -3 & 4 \end{bmatrix} \quad \text{— B}$$

now $A = B$

hence two pure reflection = pure rotation.

Shearing.

It is used to modify the shape of 2D or 3D objects.

The effect of this transformation looks like pushing a geometric obj. in a direction that is parallel to coordinate plane (3D) or coordinate axis (2D).

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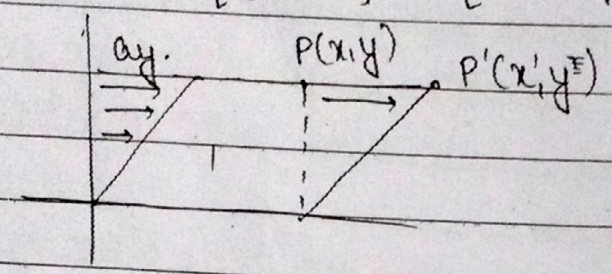
How far a direction is pushed is determined by shearing factor.

→ x-shearing :

One can push in x-direction, +ve or -ve, and keep the y direction unchanged.

$$[T] = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$$

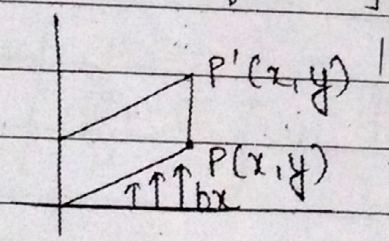
$$[x \quad y]\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} = [x+ay \quad y]$$



→ y-shearing :

$$[T] = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$

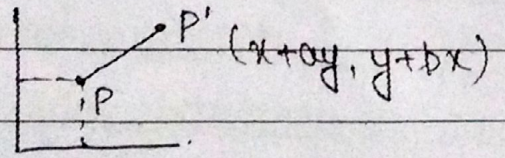
$$[x \quad y]\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = [x \quad y+bx]$$



→ x-y shearing :

$$[T] = \begin{bmatrix} 1 & b \\ a & 1 \end{bmatrix}$$

$$[x \quad y]\begin{bmatrix} 1 & b \\ a & 1 \end{bmatrix} = [x+ay \quad y+bx]$$



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Ques: A square ABCD, A (0,0), B (1,0), C (1,1) D (0,1). a = 2 ; b = 3. Illustrate the effect of x-shear, y-shear & x-y shear.

⇒ x shear.

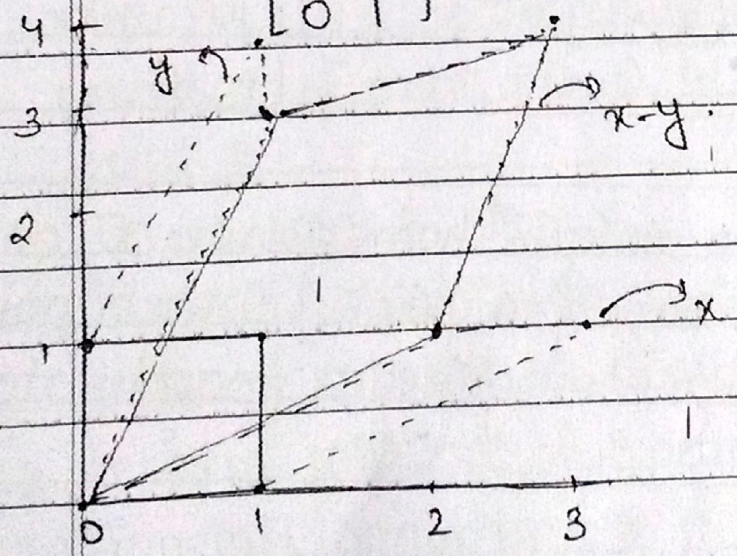
$$P' = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{matrix} A \\ C \\ B \\ D \end{matrix}$$

y-shear

$$P' = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 1 & 4 \\ 0 & 1 \end{bmatrix}$$

x-y shear

$$P' = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$$



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note: The Origin ~~of~~ is invariant under general 2x2 transformation.
This limitation is solved by use of Homogeneous Coordinate System (HCS)

$$\begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Homogeneous Coordinate System

$(x, y) \rightarrow (x, y, H)$, where $H \neq 0$.

$(x/H, y/H, 1) \leftarrow (x, y, H)$; $H = 1$.

For 2-dimensional system, $H=1$. If $H=0$, this represents a point at infinity that is the generation of horizons.

(x, y)

one way

$(x, y, 1)$ $\qquad$ (hx, hy, h)

eg:

$(3, 2)$

$(3, 2, 1)$ $\qquad$ $(6, 4, 2)$ or $(12, 8, 4)$

note: There is no unique homogeneous coordinate representation of a point, i.e. each point may have many homogeneous coordinates.

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→ General Transformation Matrix in HCS

$$\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ Translation Matrix in HCS

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & n & 1 \end{bmatrix}$$, where m = translation in x & n = translation in y.

$$[x^* \; y^* \; 1] = [x \; y \; 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & n & 1 \end{bmatrix}$$

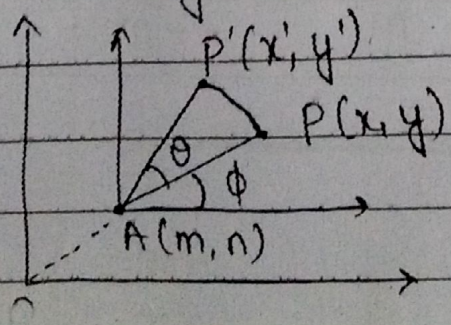
$$= [x+m \quad y+n \quad 1]$$

imp # Rotation About a Point A(m,n)

Given a 2D point P(x,y) which we want to rotate with respect to an arbitary point A(m,n) let P'(x',y') be the result of the anticlockwise rotation of point P(x,y) by an angle θ about a point A.

It requires 3 steps:

1. Translate a point (m,n) to the origin O, so that centre of rotation A is at origin.
2. Perform rotation of θ degree about origin.
3. Translate origin back to the original position A(m,n)



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$$R_{\theta,A} = T_{-V}\, R_\theta\, T_V$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & n & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ m(1-\cos\theta)+n\sin\theta & m(1-\cos\theta)-n\sin\theta & 1 \end{bmatrix}$$

Ques: Perform a 45° rotation of A(0,0), B(1,1), C(5,2) about an arbitary point (-1,-1)

$\Rightarrow$ Points : $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}$ $\begin{matrix} A \\ B \\ C \end{matrix}$

$$R_{(45,A)} = \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ -1(1-\cos 45)-\sin 45 & -1(1-\cos 45)+\sin 45 & 1 \end{bmatrix}$$

$$R_{(45,A)} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -1 & (\sqrt{2}-1) & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -1 & (\sqrt{2}-1) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & \sqrt{2}-1 & 1 \\ -1 & 2\sqrt{2}-1 & 1 \\ 3\sqrt{2}/2-1 & 7\sqrt{2}/2+\sqrt{2}-1 & 1 \end{bmatrix}$$

$\therefore$ points = (-1, 0.4), (-1, 1.8), (1.1, 5.3)

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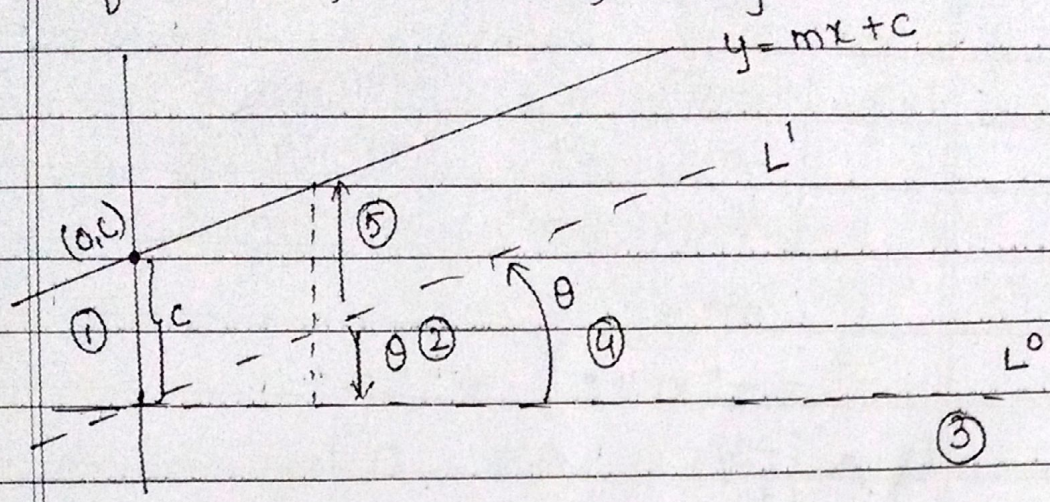
Reflection About an Arbitary line.



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -c & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 0 \\ -\sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$$

$$[M_L] = [T_{-V}][R_{-\theta}][M_N][R_\theta][T_V]$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\cos\theta\sin\theta & 0 \\ 2\sin\theta\cos\theta & \sin^2\theta - \cos^2\theta & 0 \\ -2c\sin\theta\cos\theta & -c(\sin^2\theta - \cos^2\theta) + c & 1 \end{bmatrix}$$

Putting, $\tan\theta = m$

$$\sin\theta = \frac{m}{\sqrt{m^2+1}} \qquad \cos\theta = \frac{1}{\sqrt{m^2+1}}$$

Putting these values in $[M_L]$

$$[M_L] = \begin{bmatrix} \frac{1-m^2}{m^2+1} & \frac{2m}{m^2+1} & 0 \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} & 0 \\ \frac{-2cm}{m^2+1} & \frac{-c(m^2-1)+c}{m^2+1} & 1 \end{bmatrix}$$

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Ques: Obtain a mirror transformation of a Δ formed by the vertices A(0,3), B(2,0), C(3,2), About the line passing through (1,3) & (-1,-1)

$\Rightarrow \quad m = \frac{-1-3}{-1-1} = 2$

$$[T] = \begin{bmatrix} (1-m^2)/(1+m^2) & 2m/(1+m^2) & 0 \\ 2m/(1+m^2) & (-m^2-1)/(1+m^2) & 0 \\ -2mc/(1+m^2) & 2c/m^2+1 & 1 \end{bmatrix}$$

$y = mx + c$

$-1 = -2 \times 1 + c \quad \Rightarrow c = 1$

$$[T] = \begin{bmatrix} -3/5 & 4/5 & 0 \\ 4/5 & 3/5 & 0 \\ -4/5 & 2/5 & 1 \end{bmatrix}$$

$[A \ B \ C][T] = [A' \ B' \ C']$

$$\begin{bmatrix} 0 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3/5 & 4/5 & 0 \\ 4/5 & 3/5 & 0 \\ -4/5 & 2/5 & 1 \end{bmatrix} = \begin{bmatrix} 8/5 & 11/5 & 1 \\ -2 & 2 & 1 \\ -1 & 4 & 1 \end{bmatrix}$$

Ques: Reflect a diamond shaped polygon whose vertex are A(-1,0) B(0,-2) C(1,0) D(0,2) about a line:

a) $y=2$ b) $x=2$ c) $y=x+2$

$\Rightarrow$

scaling/ rotation/ reflection/ shearing

$$\left[\begin{array}{cc|c} a & b & p \\ c & d & q \\ m & n & s \end{array}\right]$$

$p \rightarrow$ projection

$m, n \rightarrow$ translation

$s \rightarrow$ overall scaling

→ Overall scaling

$$[x \ y \ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix} = [x \ y \ s]$$

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$x^* = x/s = [x/s \quad y/s \quad 1]$

$y^* = y/s = \begin{cases} s = 1/2 & [2x \quad 2y \quad 1] \quad (ee) \\ s = 2 & [x/2 \quad y/2 \quad 1] \quad (ec) \end{cases}$

$\Downarrow$

$s > 1$ equal compression (ec)

$s < 1$ equal expansion (ee)

PROJECTION

$(p \neq 0, q \neq 0)$

$$[x \quad y \quad 1] \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix} = [x \quad y \quad xp + yq + 1]$$

$x^* = x / px + qy + 1$
$y^* = y / px + py + 1$ } Transforming the position vector from $h \neq 1$ plane to $h = 1$.

Ques: For a line AB, A(1,3) B(4,1) $p=1, q=1$. Project this line.

$$\Rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 4 & 1 & 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 4 & 1 & 6 \end{bmatrix}$$

But, it is in the form of $h = x + y + 1$

So, $C^* = \dfrac{C}{x+y+1} = \dfrac{1}{1+3+1} = \dfrac{1}{5}$

$= \dfrac{3}{1+3+1} = \dfrac{3}{5}$

$D^* = \dfrac{D}{x+y+1} = \dfrac{4}{4+1+1} = \dfrac{4}{6} = \dfrac{2}{3}$

$= 1$

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$$\therefore \begin{bmatrix} C^* \\ D^* \end{bmatrix} = \begin{bmatrix} 1/5 & 3/5 & 1 \\ 4/6 & 1/6 & 1 \end{bmatrix}$$

3D - TRANSFORMATION

HCS 3D $\longrightarrow$ (x, y, z, H)

(x, y, z) when $H \neq 0$

$(x/H, y/H, z/H) \longleftarrow (x, y, z, H)$

$$[T] = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & h & i & r \\ l & m & n & s \end{bmatrix}$$

$$= \left[\begin{array}{c|c} 3\times3 & 3\times1 \\ \hline 1\times3 & 1\times1 \end{array}\right]$$

scaling / Rotation / Reflection / Sheasing $\leftarrow$ 3×3

$3\times1 \rightarrow$ prespective transformation

$1\times3 \rightarrow$ translation

$1\times1 \rightarrow$ overall scaling.

Transformation for 3D translation.

$$[x \;\; y \;\; z \;\; 1] = [x+l \;\; y+m \;\; z+n \;\; 1]$$

$$[x^* \;\; y^* \;\; z^* \;\; 1] = [x \;\; y \;\; z \;\; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix}$$

Scaling

$$[x^* \;\; y^* \;\; z^* \;\; 1] = [x \;\; y \;\; z \;\; 1] \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [ax \;\; ey \;\; iz \;\; 1]$$

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This is called as local scaling transformation.

- $a, e, i > 1 \rightarrow$ expand
- $a, e, i < 1 \rightarrow$ compress

→ Overall Scaling

$$[x^* \; y^* \; z^* \; 1] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$

$$= [x \; y \; z \; s]$$

$$\underbrace{[x \; y \; z \; 1] = \left[\frac{x^*}{s} \; \frac{y^*}{s} \; \frac{z^*}{s} \; 1\right]}_{\text{after normalizing}}$$

- $s > 1 \rightarrow$ compression
- $s < 1 \rightarrow$ expansion

Shearing

$$[x^* \; y^* \; z^* \; 1] = [x \; y \; z \; 1] \begin{bmatrix} 1 & b & c & 0 \\ d & 1 & j & 0 \\ g & h & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [x+dy+gz \quad bx+y+hz \quad cx+jy+z \quad 1]$$

This is shearing along x, y & z direction.

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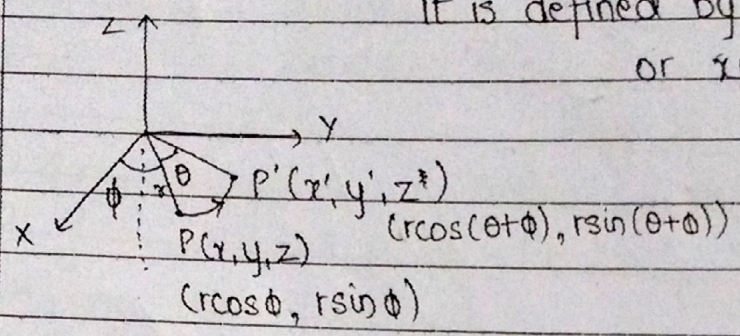
Transformation for 3D rotation

requirements : i) θ

ii) axis of rotation (x, y, z axis)

→ Rotation about z-axis.

It is defined by $z=0$ plane or xy plane.



$P(x, y, 0) = (r\cos\phi, r\sin\phi, 0)$

$P^*(x', y', 0) = (r\cos(\theta+\phi), r\sin(\theta+\phi), 0)$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

where $x = r\cos\phi$, $y = r\sin\phi$.

$$[T_z] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

In HCS, $$[T_z] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}$$

note: Since both P & P^* lies on x-y plane as shown in fig. means their z component remains the same.

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$x \to y \to z \to x$
cyclic transformation.

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→ Rotation about x-axis.

It is defined by $x=0$ or zy plane.

$$x' = x$$
$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta.$$

$$[T_x] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ Rotation about y-axis

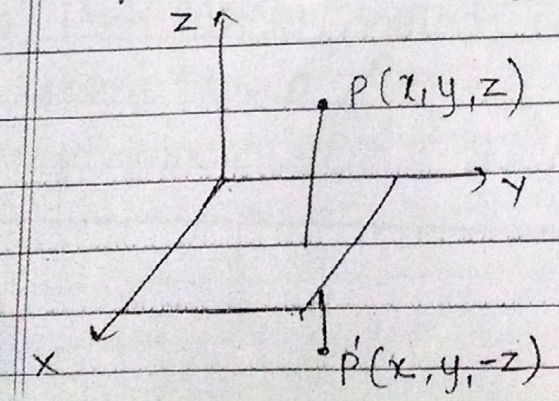
It is defined by $y=0$ plane or xz plane.

$$[T_y] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation for 3D reflection.

In 3D, reflections are performed about arbitary plane. Such planes are also called reference plane. For each reference plane, the point lying on the plane will remain same after reflection.

→ Reflection about xy plane.



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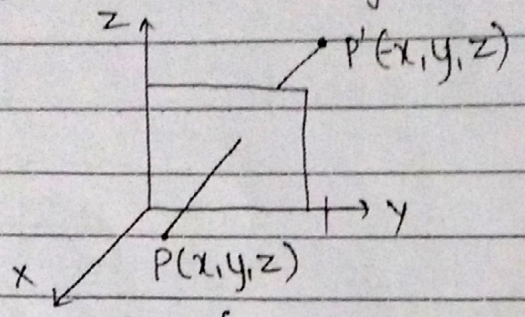
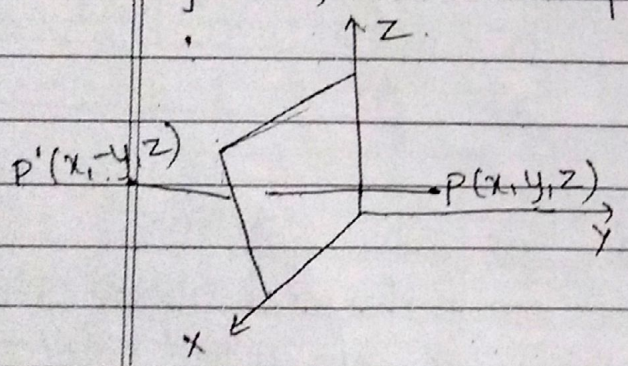
$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ Reflection about yz plane.



$$[T] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ Reflection about xz plane.



$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ques: Consider an object that has a position vector given by $[x]$.
Find the new position vector $[x']$ after the rotation of $\theta = -90°$ about x axis followed by rotation of $\phi = 90°$ about y axis.

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$$[X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

$$[T] = [T_{-90,x}]\,[T_{90,y}]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & \sin(-90) & 0 \\ 0 & -\sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & 0 & -\sin 90 & 0 \\ 0 & 1 & 0 & 0 \\ \sin 90 & 0 & \cos 90 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 \\ -3 & 1 & -2 & 1 \\ -2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 1 \\ -3 & 0 & -2 & 1 \\ -2 & 0 & 0 & 1 \end{bmatrix}$$

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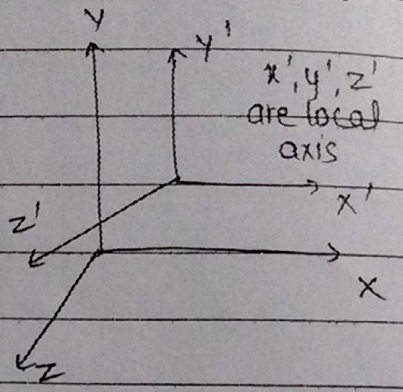
Rotation about an axis parallel to the coordinate axis

We can rotate an obj about an axis other than x, y or z axis.

Steps :

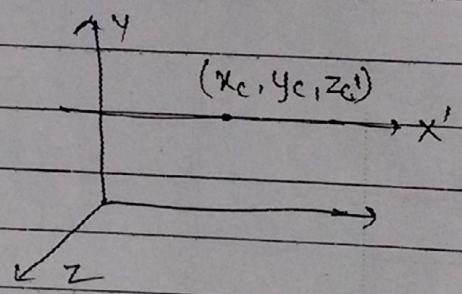
1. Translate the local axis to the coordinate axis.
2. Perform rotation about specified axis.
3. Translate the transformed local axis back to the original position.



Pg-118

Ques: Consider a block with position vectors A to H. Rotate a block 30° about local axis x' passing through the centroid of the block. Centroid = [3/2 3/2 3/2 1]

$$\Rightarrow [T_r] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -y_c & -z_c & 1 \end{bmatrix}$$



$$[R] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_r]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & y_c & z_c & 1 \end{bmatrix}$$

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$$[X^A] = [X][T_r][R_x][T_r]^{-1}$$

→ Suppose if we do multiple rotation in local axis system parallel to the global axis system then we have to make the origin of the local axis system coincident with the origin of global axis system.

Steps :

1. Translate the origin of LAS to the origin of GAS.
2. Perform rotation about specified axises.
3. Translate the local axis back to original position.

Ques Rotate a block -45° about y' axis followed by the rotation of +30° about x' axis. Suppose the origin of the LAS (x_c, y_c, z_c)

$$\Rightarrow [X^A] = [X][T_r][R_{y,-45}][R_{x,30}][T_r]^{-1}$$

$$[T_r] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x_c & -y_c & -z_c & 1 \end{bmatrix}$$

$$[R_{y,-45}] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\theta=-45} = \begin{bmatrix} 1/\sqrt{2} & 0 & +1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[R_{x,30}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\theta=30} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_r]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x_c & y_c & z_c & 1 \end{bmatrix}$$

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Projection

1. It is a mapping of 3D object onto 2D screen. The easiest way to think about converting 3D world into 2D image, is with the camera. The surface projection is formed on which the projection is formed is called view plane / display surface / plane of projection.

2. Projectors are the lines that generates from an arbitary point called center of projection, passing through each point of the object. These are also called projection lines or rays.

3. Projection of object is formed when projection lines intersect with the view plane.

Types :

Parallel	Perspective
• Projection lines from the object converge at ∞	• Centre of Projection is located at a finite ~~pos~~ distance resulting in perspective projection
• The camera is at infinite location from the view plane.	• The camera is placed at finite location from view plane.
• The projection lines are parallel to each other.	
imp. • The projection line will have a direction, which is called direction of projection.	

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Plane Geom. Projection

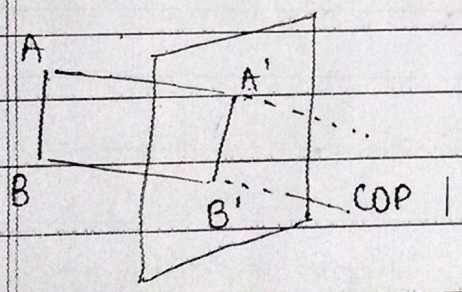
COP = ∞ | COP is at finite location fixed point.

- **Parallel**
  - (Projector ⊥ VP) → Orthographic (Multiview)
    - → Left side
    - → Right side
    - → rear
    - → top
    - → front
    - → bottom
  - (VP is not parallel to princ axis) (Projector ⊥ VP)
    - → Axonometric
      - → Trimetric
      - → Dimetric
      - → Isometric
    - → Oblique
      - → Cavalier
      - → Cabinet
- **Perspective**
  - → Single Point
  - → Two Point
  - → Three Point

VP is ∥ to princ axis

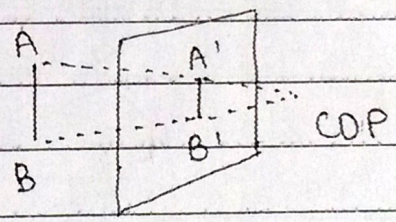
13/Aug/15

Parallel



1. This projection preserve the shape of the object. OR preserve the relative proportion of the object.
2. The accurate view of the various sites of an obj is obtained by parallel projection.

Perspective



1. It does not preserve the relative proportion of the object.
3. Produce realistic view of 3D object.
2. Projection of a distant object is smaller than the projection of the

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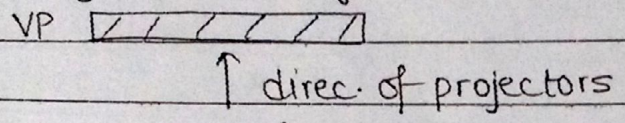
3 → Does not give the realistic view. To get the 3D feeling of the obj. we usually need multiple views.

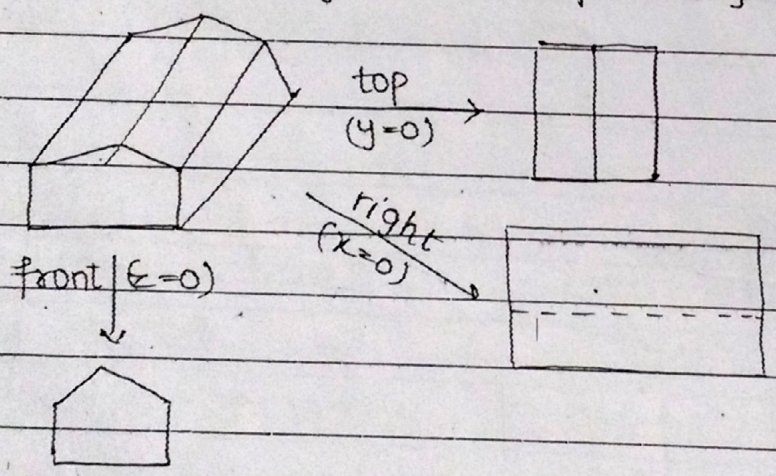
same size that are closer to projection plane.

Orthographic Projection.



- The direction of projectors is ⊥ to the view plane
- The rays coming from the objects are parallel.

note: This projection accurately preserve the true shape and size of single phase (face) of an obj.



- Transformation for single orthographic plan projection onto one of the plane $x=0, y=0, z=0$

1. Matrix for projection onto $z=0$ plane.

$$[P_z] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(we get front view)

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2. Matrix for projection onto $x=0$ plane.

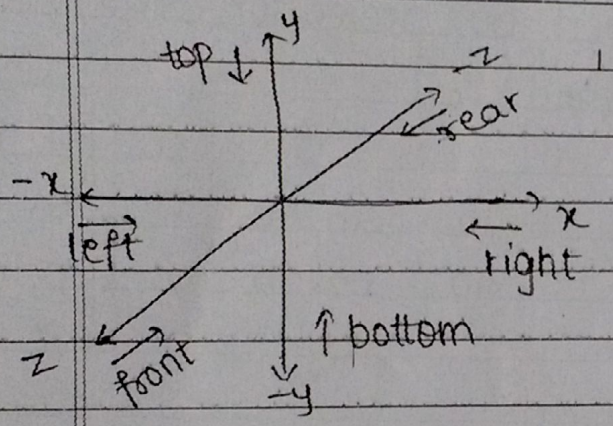
$$[P_x] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(right view)

3. Matrix for projection onto $y=0$ plane.

$$[P_y] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(top view)



- The projection matrix for rear view is obtained by reflection through $z=0$ plane followed by the projection onto $z=0$ plane.

$$[P_{rear}] = [M_{xy}][P_z]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

in HCS

- The proj. matrix for left view is obtained by rotation about y axis by 90° followed by the projection onto $z=0$ plane.

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$$[P_{left}] = [P_{y,90^\circ}][P_z]$$

$$= \begin{bmatrix} \cos 90 & 0 & -\sin 90 & 0 \\ 0 & 1 & 0 & 0 \\ \sin 90 & 0 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

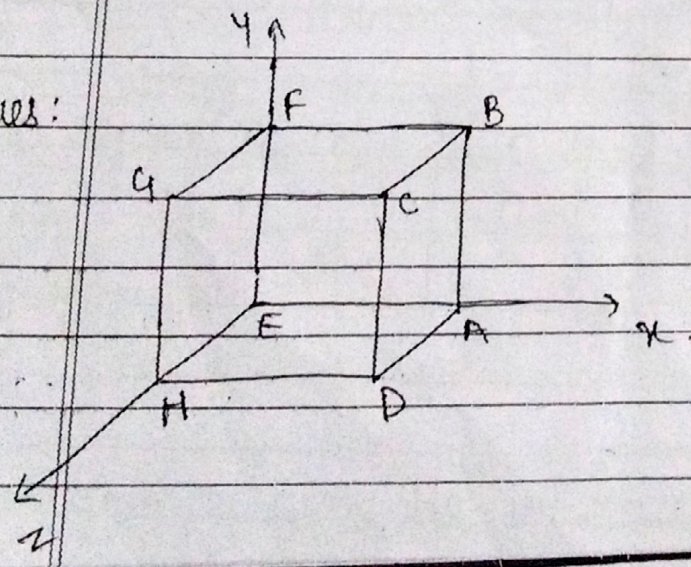
The proj. matrix for bottom view is obtained by ~~projection onto y=0 pla~~ rotation about ~~y~~ x axis by -90° followed by projection onto $z=0$ plane.

$$[P_{bottom}] = [P_{x,-90}][P_z]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos -90 & \sin -90 & 0 \\ 0 & -\sin -90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ques:



Show all the six views of A(4,0,0), B(4,4,0), C(4,4,8), D(4,0,4), E(0,0,0), F(0,4,0), G(0,4,8), H(0,0,4)

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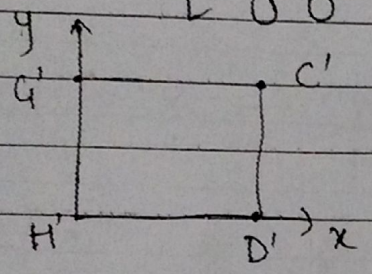
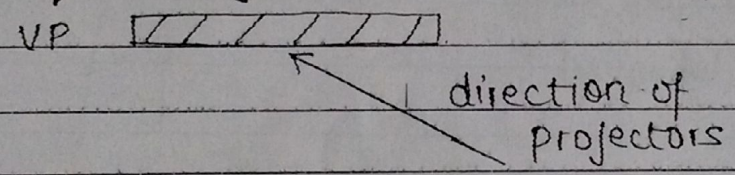
for front view

$$[M_F] = \begin{bmatrix} 4 & 0 & 0 & 1 \\ 4 & 4 & 0 & 1 \\ 4 & 4 & 8 & 1 \\ 4 & 0 & 4 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 4 & 0 & 1 \\ 0 & 4 & 8 & 1 \\ 0 & 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 1 \\ 4 & 4 & 0 & 1 \\ 4 & 4 & 0 & 1 \\ 4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 4 & 0 & 1 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

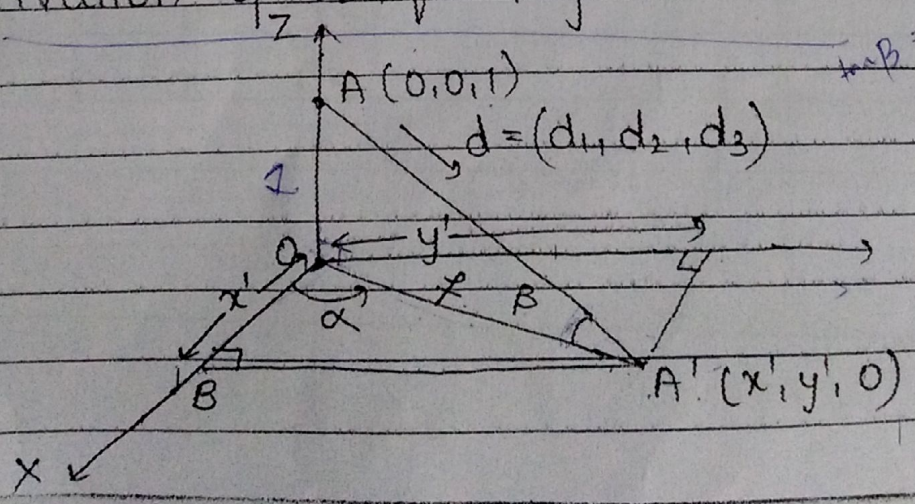


Oblique Projection



The direction of projectors in not ⊥ to view plane. This projection shows all 3 dimensions of an obj in a single view.

→ Derivation of Oblique Projection Matrix.



$\tan\beta = 1/f$

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$f = \frac{OA'}{OA}$

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The oblique projection of a point A (0,0,1) to the position A' (x', y', 0) on the view plane z = 0 plane.
α is the angle between projected line OA' and +x axis. β is the angle b/w oblique projector & plane of projection (angle b/w OA' & AA').
f is the foreshortening factor which is defined as length of the projected line in the view plane.
β is the angle which decides whether the projection is cavalier or cabinet.

In Δ OA'B

$OB = x'$

$BA' = y'$

$\cos\alpha = \frac{x'}{f} \Rightarrow x' = f\cos\alpha$

$\sin\alpha = \frac{y'}{f} \Rightarrow y' = f\sin\alpha$

$\boxed{f = \frac{(OA)'}{OA}} = \frac{OA}{\|}$ $\boxed{f = OA'}$

The transformation matrix for the oblique projection is given by:

$$[T_{oblique}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x' & -y' & 1 \end{bmatrix}$$

~~The projector OA'~~

Oblique Projection onto z = 0 plane

$$[T_{oblique}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -x' & -y' & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\tan\beta = \frac{1}{f}}$

$f = 1$

$f = \frac{1}{2}$

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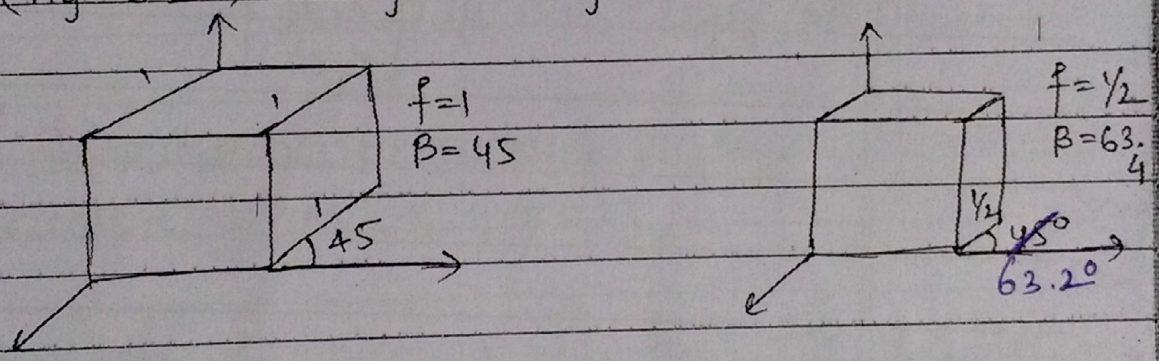
Cavalier	Cabinet
→ All the lines ⊥ to the projection plane are projected with no change in length. $f = 1$	→ The lines which are ⊥ to the view plane are projected at half the length of original.
→ The angle b/w projectors and view plane is 45°. $\tan\beta = \frac{1}{f}$	→ The angle b/w projector and view plane is 63.4°
→ The foreshortning factor = 1 means the edges ⊥ to the projection plane are not foreshortnend	→ The forshortning = 1/2

note: The common value of α is either 30° or 45°. The values of $(180° - \alpha)$ is also acceptable.

(fig: 3.24) → diagram. Pg-156.



Transformation of Straight line & Mid Point (Self).

Transformation of Intersecting lines.

consider two lines:

$$y = m_1 x + b_1 \rightarrow -m_1 x + y = b_1$$
$$y = m_2 x + b_2 \rightarrow -m_2 x + y = b_2$$

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$[X][M] = [B]$

$$[x \quad y]\begin{bmatrix} -m_1 & -m_2 \\ 1 & 1 \end{bmatrix} = [b_1 \quad b_2]$$

$$[X][M][M]^{-1} = [B][M]^{-1}$$

$$[X_i] = [x_i \quad y_i] = [b_1 \quad b_2][M^{-1}]$$

$$= [b_1 \quad b_2]\begin{bmatrix} 1 & m_2 \\ -1 & -m_1 \end{bmatrix}\frac{1}{-m_1+m_2}$$

$$= \begin{bmatrix} \frac{b_1 - b_2 m_2}{-m_1+m_2} & \frac{b_1 m_2 - b_2 m_1}{-m_1+m_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{b_1 - b_2}{m_2 - m_1} & \frac{m_2 b_1 - m_1 b_2}{m_2 - m_1} \end{bmatrix}$$

Assume the transformation matrix

$$[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$y' = m_1' x_i' + b_1'$$

$$y' = m_2' x'' + b_2'$$

$$[x_i'] = [x_i' \quad y_i'] = \begin{bmatrix} \frac{b_1' - b_2'}{m_2' - m_1'} & \frac{b_1' m_2' - b_2' m_1'}{m_2' - m_1'} \end{bmatrix}$$

After transformation

$$[x_i' \quad y_i'] = [x_i \quad y_i]\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= [x_i a + y_i c \quad x_i b + y_i d]$$

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Axonometric Projection

→ Projection lines are ⊥ to the view plane.
→ View plane is not parallel to principle axis.
With this projection we can display more than one face of an object or we can say that in a single view all three dimensions of an object is shown.

Trimetric	Dimetric	Isometric
• $f_x \neq f_y \neq f_z$	• Any 2 of f_x, f_y, f_z are same	• $f_x = f_y = f_z$.
• The direction of projection makes unequal angles with all of the three principle axis.	• The direction of projection makes equal angles with exactly two of the three principle axises.	• The direction of projection makes equal angle with all of the three principle axises.

f is a foreshortening ratio for each projected principle axises (x, y or z)

note: This type of projection was first formed by rotation about y-axis, then about x-axis followed by the orthographic projection onto z=0 plane

→ How to calculate the foreshortning factors along the projected principle axises??
[T] is a transformation matrix or concatenated matrix for rotation, translation, etc. & projection about z=0 plane.

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[u] is the matrix of unit vectors along the principle axises.

$$[u] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$[u][T] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} [T]$$

$$= \begin{bmatrix} x_x^* & y_x^* & 0 & 1 \\ x_y^* & y_y^* & 0 & 1 \\ x_z^* & y_z^* & 0 & 1 \end{bmatrix}$$

$$\cancel{0} \quad \cancel{1}$$

$$f_x = \sqrt{x_x^{*2} + y_x^{*2}}$$

$$f_y = \sqrt{x_y^{*2} + y_y^{*2}}$$

$$f_z = \sqrt{x_z^{*2} + y_z^{*2}}$$

Pg-144

Ques: Find the axonometric projection of a cube whose position vectors are represented by matrix [x]. The projection is obtained by first rotating the cube about y-axis by an angle 30° followed by rotation about x-axis by ~~30°~~ 45°.

Step 1: $[T] = [R_{y,30}][R_{x,45°}][P_{z=0}]$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$$

Step 2: $[x^*] = [x][T]$

Step 3: then $[u][T]$

then f_x, f_y, f_z.

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→ Dimetric

Perspective Projection Transformation

Its a transformation from one 3D space to another 3D space.

Transformation matrix for perspective transformation is given by $[T] = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$

This is called the 3-point perspective transformation.

$[x \ y \ z \ 1][T] = [x \ y \ z \ \ xp+yq+rz+1]$

$[x^* \ y^* \ z^* \ 1] = \left[\frac{x}{px+qy+rz+1} \ \frac{y}{px+\dots+1} \ \frac{z}{px+\dots 1} \ 1\right]$

- Two-point perspective transformation.

$[T] = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- One-point perspective transformation

$[T] = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

single point perspective transformation wrt x axis

$[T_x] = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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wrt y axis

$$[T_y] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

wrt z axis

$$[T_z] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ About perspective transformation

1. Any set of parallel lines that are not parallel to the view planes are projected ~~lin~~ into converging line & point of convergence is called centre of projection.

imp 2. Any set of parallel lines which are parallel to the view planes will be projected as parallel lines. The point at which these lines appears to be converging is called <u>vanishing point</u>. The no. of vanishing point depends upon the set of parallel line.

3. Principle Vanishing Point are the vanishing points whose set of parallel lines are parallel to one of the principle axises.

4. No. of principle vanishing point can be 1, 2 or 3.

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→ Single Point Perspective Projection about z-axis.

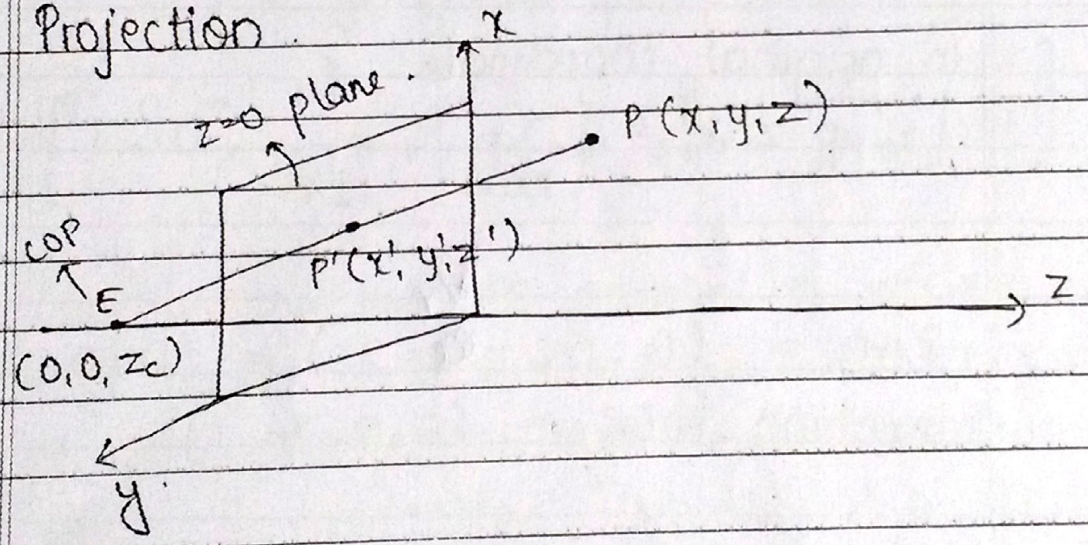
$[T] = [P_1][P_2]$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[x \ y \ z \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} = [x \ y \ 0 \ zr+1]$$

$$[x' \ y' \ z' \ 1] = \left[\frac{x}{zr+1} \ \frac{y}{zr+1} \ 0 \ 1 \right]$$

→ Mathematical Description of Perspective Projection



find $P'(x', y', z')$ on $z = 0$ plane.

Write the parametric eq. of line start with E & passing through P.

$E + t(P - E) \qquad 0 < t < \infty$

$[(0,0,z_c) + t[(x,y,z) - (0,0,z_c)]]$

$= [(0,0,z_c) + [tx, ty, t(z - z_c)]]$

$= (tx, ty, z_c + t(z - z_c))$

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Point P' is obtained when $t = t'$

$= [xt', yt', z_c + t'(z - z_c)]$

Since P' lie on $z=0$ plane

$z_c + t'(z - z_c) = 0$ must be true.

$$\Rightarrow t' = \frac{z_c}{z_c - z}$$

$$= \frac{1}{1 - z/z_c}$$

put $\boxed{r = -1/z_c}$

$$t' = \frac{1}{1 + rz}$$

Thus, $x' = t'x = \dfrac{x}{rz+1}$

$$y' = t'y = \frac{y}{rz+1}$$

$$z' = 0$$

In HCS,

$(x, y, 0, zr+1)$

In original coordinate

$$[x' \; y' \; z' \; 1] = \left[\frac{x}{rz+1}, \frac{y}{rz+1}, 0, 1\right]$$

$$COP = (0, 0, -\cancel{r}\ 1/r\ 1) = (0 \; 0 \; z_c \; 1)$$

Vanishing Point $= (0 \; 0 \; 1/r \; 1)$

→ Ques Find out the single point perspective projection about x-axis.

$$COP = [-Y_p \;\; 0 \;\; 0 \;\; 1]$$

Vanishing Point $= [Y_p \;\; 0 \;\; 0 \;\; 1]$

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$$[T] = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[x' \; y' \; z' \; 1] = \left[\frac{x}{px+1} \quad \frac{y}{px+1} \quad \frac{z}{px+1} \quad 1 \right]$$

→ Single Point perspective projection about y-axis

COP = $[0 \; -Y_p \; 0 \; 1]$

Vanishing Point = $[0 \; Y_p \; 0 \; 1]$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[x' \; y' \; z' \; 1] = \left[\frac{x}{qy+1} \quad \frac{y}{qy+1} \quad \frac{z}{qy+1} \quad 1 \right]$$

Q. Consider the line AB parallel to z-axis with end points A (3, 2, 4) B (3, 2, 8). Perform a perspective projection onto z = 0 plane from centre of projection (0, 0, −2) also find the vanishing point.

⇒ $r = -1/z_c = -1/-2 = 1/2$.

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$z_c = -2 = \frac{-1}{r}$

$r = +1/2$

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 3 & 2 & 4 & 1 \\ 3 & 2 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 0 & 3 \\ 3 & 2 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2/3 & 0 & 1 \\ 3/5 & 2/5 & 0 & 1 \end{bmatrix}$$

Vanishing Point $(0, 0, 2, 1)$

2-Point Perspective Transformation:

$$[x \ y \ z \ 1] \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [x \ y \ z \ px+qy+1]$$

Ordinary Coordinate

$$[x' \ y' \ z' \ 1] = \left[\frac{x}{px+yq+1} \ \frac{y}{px+yq+1} \ \frac{z}{px+yq+1} \ 1 \right].$$

2 COP : on x axis $(-1/p \ 0 \ 0 \ 1)$
on y axis $(0 \ -1/q \ 0 \ 1)$

2 vanishing pt : on x axis $(+1/p \ 0 \ 0 \ 1)$
on y axis $(0 \ 1/q \ 0 \ 1)$

$$[P_{pq}] = [P_p][P_q]$$

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→ Three-Point Perspective Transformation

$$[x \ y \ z \ 1]\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} = [x \ y \ z \ px+yq+rz+1]$$

$$[x' \ y' \ z' \ 1] = \left[\frac{x}{px+qy+rz+1} \quad \frac{y}{px+\dots 1} \quad \frac{z}{px+\dots 1} \quad 1\right]$$

3 COP: on x axis $(-1/p \ 0 \ 0 \ 1)$
on y axis $(0 \ -1/q \ 0 \ 1)$
on z axis $(0 \ 0 \ -1/r \ 1)$

3 Vanishing pt: on x axis $(1/p \ 0 \ 0 \ 1)$
on y axis $(0 \ 1/q \ 0 \ 1)$
on z axis $(0 \ 0 \ 1/r \ 1)$

Ques: Consider a view whose pos. vectors are given by the ve matrix x. The cube can be centred on the z-axis by translating it -0.5 unit in x & y direction, perform a single point perspective transformation onto z=0 plane with COP at $z_e = 10$ on the z-axis.

⇒ [T] =

Step 1: Translate the cube by -0.5 unit in x & y direction to get the centred cube on z-axis.

Step 2: The transformation matrix for translation is

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.5 & -0.5 & 0 & 1 \end{bmatrix}$$

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$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.5 & -0.5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.1 \\ -0.5 & -0.5 & 0 & 1 \end{bmatrix}$$

then :

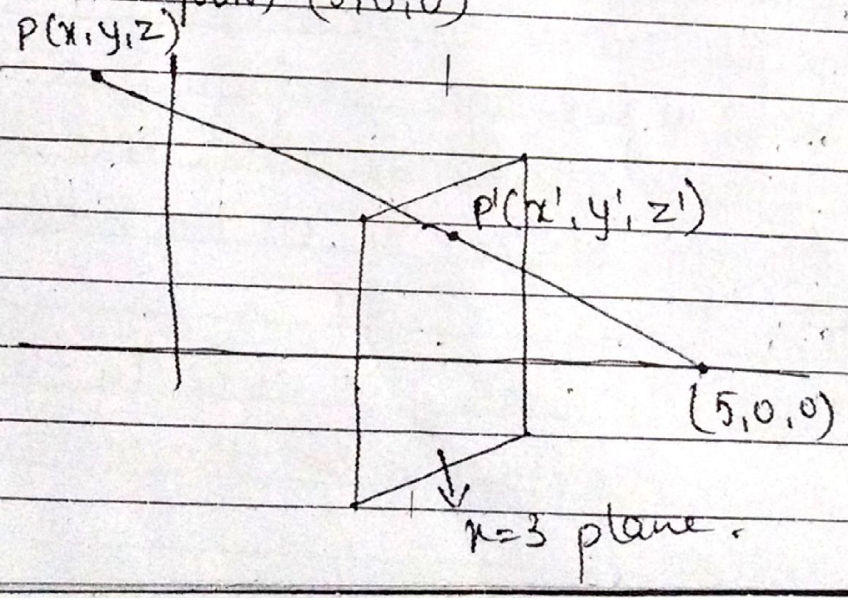
$$[X][T] = [X']$$

Ques : Find the 3-pt. perspective transformation with the vanishing point $V_x = 5$ $V_y = 5$ $V_z = -5$.

$\Rightarrow$ COP $C_x = -5$ $C_y = -5$ $C_z = 5$

$$\begin{bmatrix} 1 & 0 & 0 & 1/5 \\ 0 & 1 & 0 & 1/5 \\ 0 & 0 & 1 & -1/5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ques : Obtain a transformation matrix for perspective projection of given obj. projected on $x=3$ plane as viewed from $(5,0,0)$



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$E + (t(P-E)) \qquad 0 < t < \infty$

$(5,0,0) + t((x,y,z) - (5,0,0))$

$(5,0,0) + (tx-5t, ty, tz)$

$(5+t(x-5), ty, tz)$

P' is obtained when $t = t'$

$[5+t(x-5), ty, tz]$

Since P' lie on $x=3$ plane.

$5 + t'(x-5) = 3$

$t'(x-5) = -2$

$t' = \frac{-2}{x-5}$

$P' = (x', y', z')$

$= \left(3, \frac{-2y}{x-5}, \frac{-2z}{x-5}\right)$

$= \left(\frac{3(x-5)}{(x-5)}, \frac{-2y}{x-5}, \frac{-2z}{x-5}\right)$

$P' = (x', y', z', 1)$

$= \left(\frac{3x-15}{x-5}, \frac{-2y}{x-5}, \frac{-2z}{x-5}, 1\right)$

$= (3x-15, -2y, -2z, x-5)$

In matrix form

$$[x' \; y' \; z' \; 1] = (x \; y \; z \; 1)\begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ -15 & 0 & 0 & -5 \end{bmatrix}$$

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Ques: let the two COP on x axis (10,0,0) & z axis (0, 0, -10). find the 2-point perspective projection matrix when the plane of projection is $x+5=0$.

$\Rightarrow$ On x axis with view plane $x=-5$

$$E + t(P-E)$$

$$(10,0,0) + t\,((x,y,z)-(10,0,0))$$

$$(10,0,0) + (t(x-10), ty, tz)$$

$$(10+t(x-10), ty, tz)$$

Since P' lie on $x=-5$ plane.

$$10 + t(x-10) = -5$$

$$t(x-10) = -15$$

$$t = \frac{-15}{x-10}$$

$$P' = (x' \quad y' \quad z')$$

$$= \left(-5 \quad \frac{-15y}{x-10} \quad \frac{-15z}{x-10}\right)$$

$$= \left(\frac{-5(x-10)}{x-10} \quad \frac{-15y}{x-10} \quad \frac{-15z}{x-10}\right)$$

$$P' = (x' \quad y' \quad z' \quad 1)$$

$$= \left(\frac{-5x+50}{x-10} \quad \frac{-15y}{x-10} \quad \frac{-15z}{x-10} \quad 1\right)$$

$$= (-5x+50 \quad -15y \quad -15z \quad x-10)$$

matrix form

$$[x' \; y' \; z' \; 1] = (x \; y \; z \; 1)\begin{bmatrix} -5 & 0 & 0 & 1 \\ 0 & -15 & 0 & 0 \\ 0 & 0 & -15 & 0 \\ 50 & 0 & 0 & -10 \end{bmatrix}$$

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On z-axis with view plane $x = -5$

$E + t(P - E)$

$(0,0,-10) + t((x,y,z) - (0,0,-10))$

$(0,0,-10) + [t(x), ty, t(z+10))$

$(tx, ty, t(z+10) - 10)$

Since P' lie on $x = -5$ plane.

$tx = -5$

$t = -5/x$.

$P' = (x' \quad y' \quad z' \quad 1)$

$= \left[-5 \quad \frac{-5y}{x} \quad \frac{-5}{x}(z+10) - 10 \quad 1\right]$

$= [-5x \quad -5y \quad -5z - 50 - 10x \quad x]$

$$= (x \; y \; z \; 1) \begin{bmatrix} -5 & 0 & -10 & 1 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -50 & 0 \end{bmatrix}$$

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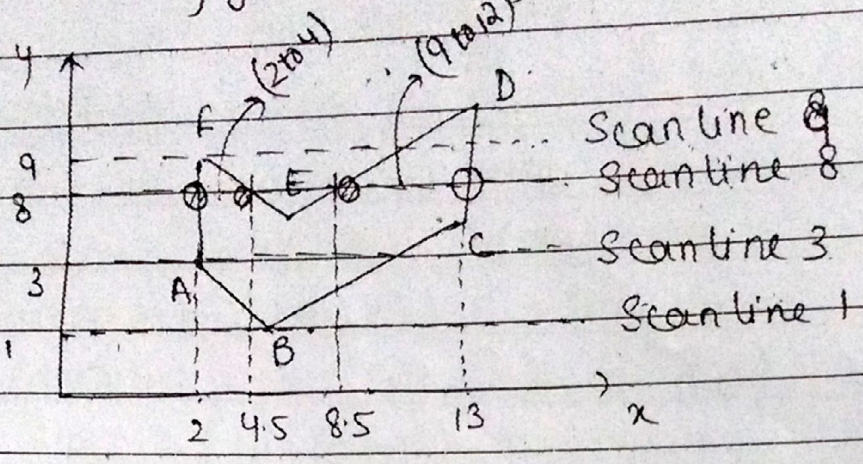
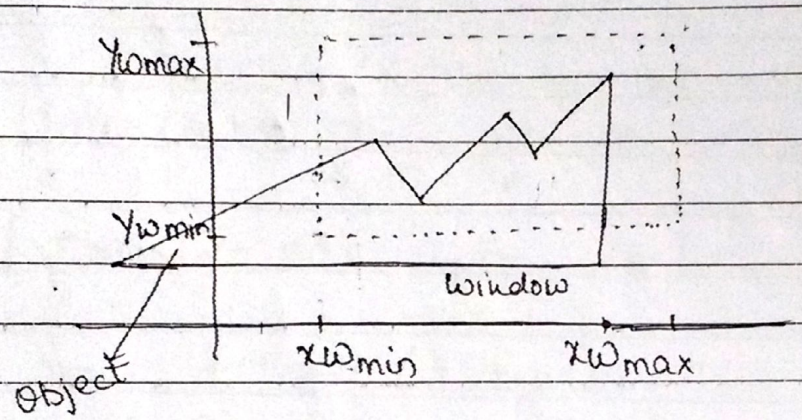
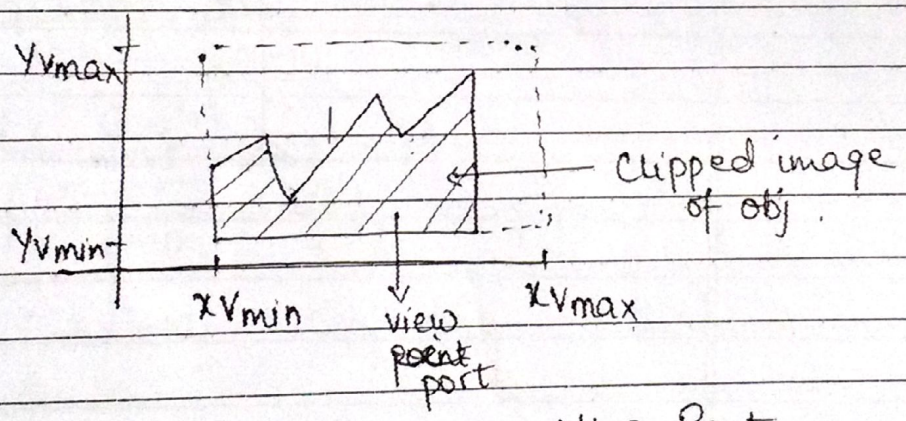
Polygon Filling



Cohen Sutherland Line Clipping Algorithm.





Window

- world coordinate area selected for the display is called window.

View Port

- A area on a display device to which a window is mapped is called view port.

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- It defines "what is to be viewed". It defines "where it is to be displayed".

Assuming the size of window & view port are in rectangular shape but they can be of any shape like concave, convex or both.

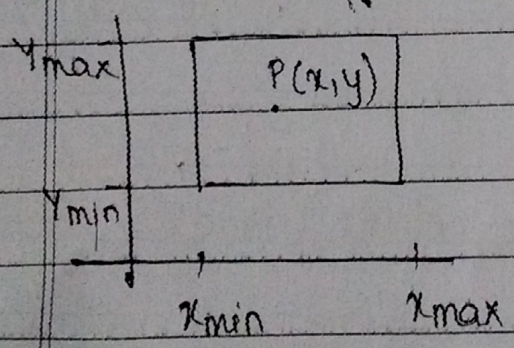
→ Clipping

The procedure that identify the portion of a picture lie inside the window, and therefore should be drawn, picture lie outside the window will not be drawn.

→ Clipping Algo

1. Point Clipping
2. line Clipping → Cohen Sutherland, Cyrus-Beck line, Liang Barskey
3. Polygon Clipping → Sutherland Hodgman Algo.
4. Text
5. Curve

→ Point Clipping



for P(x,y) to be inside cliping rectangle, It should satisfy the 4 inequalities :

$x_{min} \le x \le x_{max}$

$y_{min} \le y \le y_{max}$.

If any one of the inequality fails, point will be clipped.

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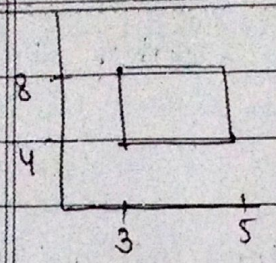
Eg:



a) P(4,6)

$3 \le 4 \le 5$

$4 \le 6 \le 8$

It will be drawn.

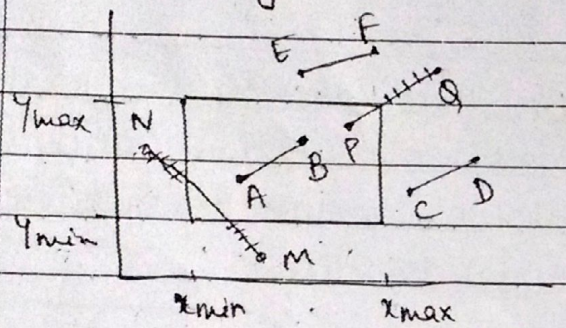
b) P(4,9)

$3 \le 4 \le 5$

$4 \le 9 \not\le 8$

It will be clipped.

→ Line Clipping



① Trivially Accepted Case

- If both the end points of the line lie inside the CR (Cliping Rectangle) then the line is trivially accepted. Hence need no clipping.

Eg line AB

② Trivially Rejected Case

- If both the end points of the line lie entirely to one side of the CR

Eg: line CD's end points lie to the right of x_{max} Hence it is trivially rejected & needs neither to be cliped nor displayed

③ Partially R Visible

lines PQ & MN are neither trivially accepted nor trivially rejected.

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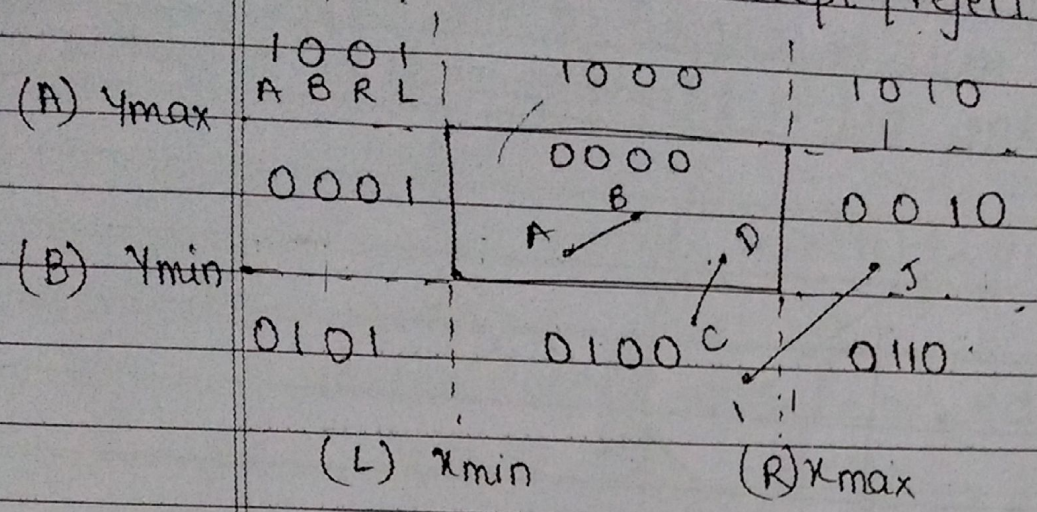
→ To perform trivial accept / reject test.



ABRL code (4 bit)

$y > y_{max}$ A : 1 if region lies above of above edge.

$y < y_{min}$ B

$x > x_{max}$ R

$x < x_{min}$ L

Step 1: Given a line with end points $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$, first compute the 4 bit code for each of the end points.

Step 2: Test for trivial acceptance. If 4-bit code of the end-points P_1 & P_2 of of the line is 0000, then the line is completely inside the CR.

note: This is checked by taking the logical OR of the end points.

Step 3: Test for trivial rejection. If the logical AND of the end points is non-zero then the line is trivially rejected.

Step 4: When the line is not trivially accepted nor trivially rejected.

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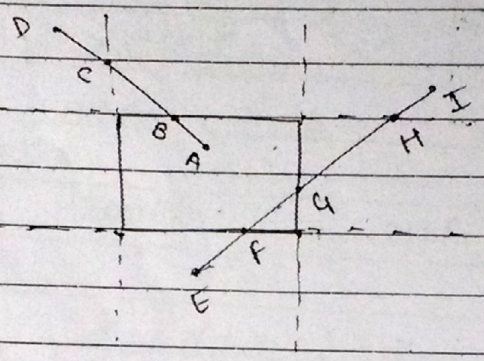
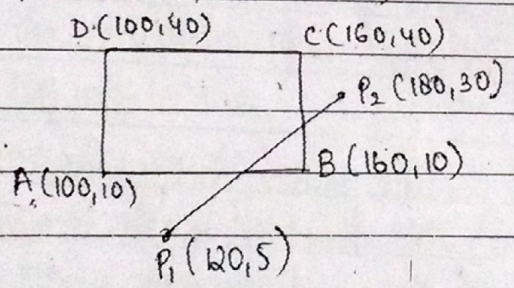
I 0100	I 0100
J 0010	J 0010
OR 0110	AND 0000
≠ 0000	≠ non zero

Date _____

Step 5: If a line can not be trivially accepted / rejected, atleast one of the 2-end points must lie outside the window & the line segment process a window edge.
This line must be clipped at the clipping rectangle edge before beigng passed to drawing routine.



Ques Consider a CR ABCD using the Cohen Sutherland find the visible portion of line seg with end points $P_1(120,5)$, $P_2(180,30)$



⇒ P_1 = 0100

P_2 = 0010

1. Test for trivially acceptance.

P_1	0100
P_2	0010
OR	0110 ≠ 0000

not trivially accepted.

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2. Test for trivially rejection.

$P_1 = 0100$

$P_2 = 0010$

AND $= 0000 \neq$ non zero.

not trivially rejected.

3. Choosing P_1 (0100)

Intersection with below edge.

$y = 10$ & $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\Rightarrow \frac{y-5}{x-120} = \frac{30-5}{180-120}$

$5y-25 = x-120.$

$-x+5y = -95$

putting $y=10$

$-x = -95-50$

$x = +\cancel{145}\ 132$

Intersection pt. $P_1' = (+\cancel{145}\ 132, 10)$.

code of $P_1' = 0000$

line $P_1P_2 \rightarrow P_1'P_2$.

4. Trivially acceptance of $P_1'P_2$

$P_1' = 0000$

$P_2 = 0010$

$0010 \neq 0000$

not trivially accepted.

5. Trivally rejection of $P_1'P_2$

$P_1'B_1'$ AND $P_2 = 0000 \neq$ non zero

not trivially rejected.

6. Outside Point $= P_2$. (0010)

intersection with right edge $x=160$.

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$-160 + 5y = -95$

$35y = 65$

$y = 13$ 201.66

intersection pt. $P_2' = (160, 13)$ 201.66

bit code of $P_2' = 0000$.

The line $P_1'P_2$ is clipped to $P_1'P_2'$

6. Trivially acceptance test for $P_1'P_2'$

P_1' 0000

P_2' 0000

OR 0000 = 0000

Hence $P_1'P_2'$ is trivially accepted.

Q. Consider the CR (0,0) (1,0), (1,1) & (0,1) Clip the line segment with end points P(0,5) & Q(1,5)

$\Rightarrow$ P (0,5) =

Date

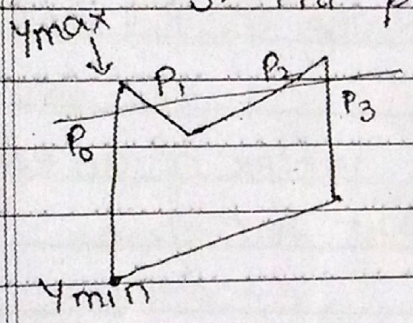
Filling Polygon

The basic idea is intersect the scan line with polygon edges & fill all the pixel b/w pair of intersection.

→ Basic Algo Steps.

for $y = y_{min}$ to y_{max}

1. Intersect scan line y with each edge.
2. Sort intersection pair by increasing x.
   $[P_0, P_1, P_2, P_3]$
3. Fill pairwise ($P_0 \to P_1$, $P_2 \to P_3$ ....)



→ Special Cases

Fill only interior pixels

for a given pair of intersection points

$(x_i, y), (x_j, y)$

fill : from ceil(x_i) to floor (x_j)

note : If intersection has an integer x coordinate

- If x_i is integer – we define it interior.
- If x_j is integer – we define it exterior.

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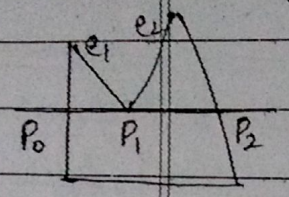
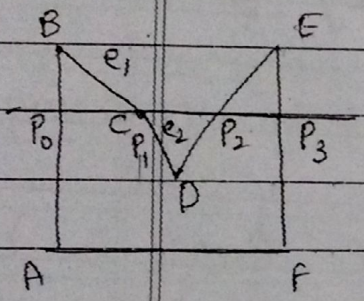
→ Case 2 :

If intersection is a edge end point (vertices of a polygon)



In this case the intersection point would be $[P_0, P_1, P_1, P_2]$. Infact if we compute the intersection of the scan line e_1 & e_2 seperately, we will get the intersection point P_1 twice. So we keep both of the P_1

→ Case 2.1 :



Intersection is a edge end point.

In this case the intersection point would be $[P_0, P_1, P_2, P_3]$ and the pair would be (P_0, P_1) & (P_2, P_3)

note: We only count y_{min} vertex of an edge for parity calculation but not y_{max}.

The horizontal edges doesn't contribute to the parity calculation. Hence they always be ignored.

The local extrema / maxima also do not contribute in the parity & never be filled.

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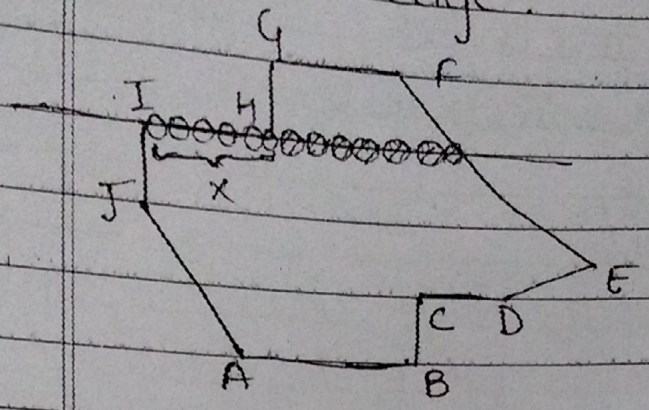
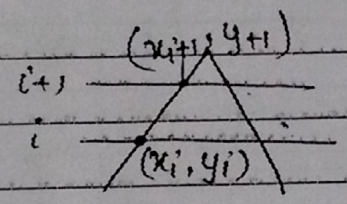
Horizontal Edge



note : The bottom horizontal edges are drawn but top horizontal edges are not drawn.

Edge Coherence

It says that the edges intersect by the scan line i will also be intersected by the scan line $i+1$.



$$y = mx + b$$

$$y = i$$

$$x_i = \frac{i-b}{m}$$

for scan line $y = i+1$

$$x_{i+1} = (i+1-b)/m$$

$$= x_i + \frac{1}{m}$$

Data structures Used for Polygon Fill Algo.

1. Global Edge Table
2. Active Edge Table

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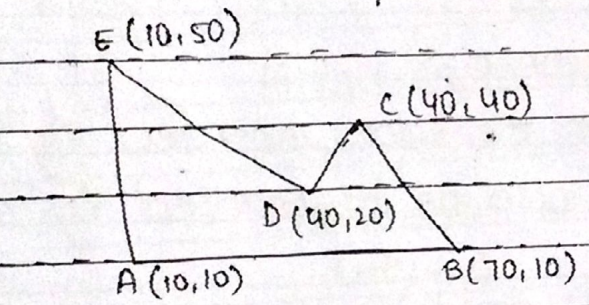
Global Edge Table

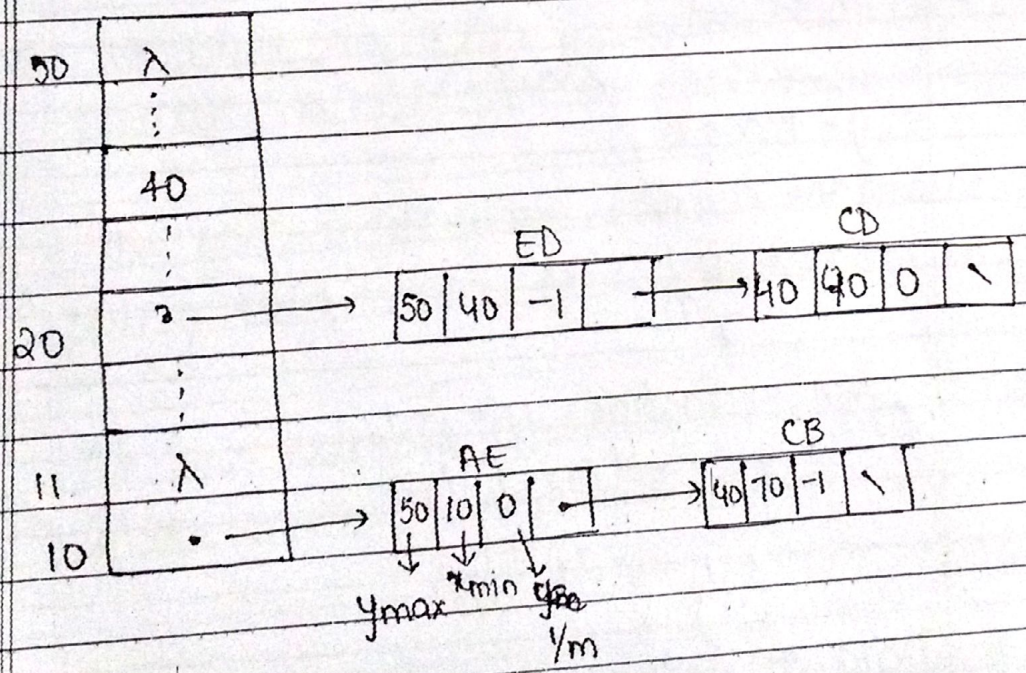
It contain all the edges sorted by their smaller y coordinate. It is built by using a bucket sort with as many buckets as there are scan lines.

Each bucket has a list of entries stored in the increasing x coordinate order.

Active Edge Table

It contain all the edges crossed by the scan line at the correct stage of iteration or we can say that it is a list of edges that are active for the scan line, sorted by increasing x coordinate of the intersection pair.





Global Edge Table

Date

AET:

y=10

EA CB

•	→	50	10	0	→	40	70	-1	\

for y = 10 ;

the parity turns on at x=10 and turns off at x=70. Therefore all the pixels from x=10 to 69 will be drawn.

y=11

	→	50	10	0	→	40	69	-1	\

$x_{11} = x_{10} + 1/m$
$= 10 + 0$
$= 10$

$x_{11} = x_{10} + 1/m$
$= 70 - 1$
$= 69$

the pixels from x = 10 to 68 will be drawn

⋮

y=20

AE ED

•	→	50	10	0	→	50	40	-1	→

CD

40	40	0	↓

CB

40	60	-1	/

$x_{20} = x_{19} + 1/m$
$= 61 - 1$
$= 60$

intersection pair :

(10, 40) (40, 60)

pixels from 10 to 39 & 40 to 59 will be drawn.

y=21

→	50	10	0	→	50	39	-1	→	40	40	0	→	40	59	-1	/

intersection pair:

(10, 39) (40, 59)

∴ pixels from 10 to 38 & 40 to 58 will be drawn.

⋮

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y = 40

	EA				ED			
→	50	10	0	→	50	20	-1	1

This process continues till y = 40, at which, the right most two edges dropout, i.e. CD & CB will be dropout

The algo proceeds ~~till~~ untill y reaches 50. the remaining two edges EA & ED will also be removed from the AET.

note: We always remove those entries from AET for which $y = y_{max}$.

Thick Primitives

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