

**CS1571**  
**Fall 2019**  
**10/9 In-Class Worksheet**

Name: \_\_\_\_\_

Where were you sitting in class today: \_\_\_\_\_

**A. Pre-Reflection**

On a scale of 1-5, with 5 being most confident, how well do you think you could execute these learning objectives:

10.1 Compare model checking, theorem proving, and resolution as ways of inferring new knowledge from a knowledge base. \_\_\_\_\_

10.3 Prove that a KB entails a sentence  $\alpha$  through resolution. \_\_\_\_\_

11.3 Compare first-order logic to propositional logic. \_\_\_\_\_

11.4 Translate English sentences into first order logic. \_\_\_\_\_

**B. Theorem Proving with Resolution**

1. Let's go back to the Wumpus world. Suppose our KB is currently:

$$(B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$$

We want to prove that  $KB \models \alpha$ , where  $\alpha$  says "there's no pit in Room 1,2" ( $\neg P_{1,2}$ )

First, figure out the CNF for:

$$(B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$$

Step 1: Remove the biconditional using biconditional elimination.

$$((B_{1,1} \rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \rightarrow B_{1,1})) \wedge \neg B_{1,1}$$

Step 2: Remove the implication using implication elimination.

$$(\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1}) \wedge \neg B_{1,1}$$

Step 3: Move  $\neg$  inwards using de Morgan's rules and double-negation ( $\neg(\neg P) \Leftrightarrow P$ ):

$$(\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1}) \wedge \neg B_{1,1}$$

Step 4: Apply distributivity law to get conjunctions of disjunctions.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1}) \wedge \neg B_{1,1}$$

Step 5: To prove by contradiction with resolution, we need to negate the sentence we want to prove and add it to the KB. So  $KB \wedge \neg \alpha$  is

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1}) \wedge \neg B_{1,1} \wedge P_{1,2}$$

Step 6: Apply the resolution rule to create new clauses until you get a contradiction. You can use the table to track your work.

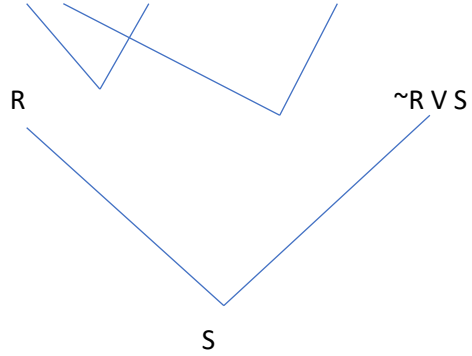
Clause #	Clause	Clauses Used
1	$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1})$	
2	$(\neg P_{1,2} \vee B_{1,1})$	
3	$(\neg P_{2,1} \vee B_{1,1})$	
4	$\neg B_{1,1}$	
5	$P_{1,2}$	
6	$\neg P_{1,2}$	2, 4
7		

2.  $(P \wedge Q) \wedge (P \Rightarrow R) \wedge [(Q \wedge R) \Rightarrow S]$  , Prove S

$$(P \wedge Q) \wedge (\neg P \vee R) \wedge [\neg(Q \wedge R) \vee S]$$

$$(P \wedge Q) \wedge (\neg P \vee R) \wedge [\neg Q \vee \neg R \vee S]$$

$$(P \wedge Q) \wedge (\neg P \vee R) \wedge [\neg Q \vee \neg R \vee S] \wedge \neg S$$



End at:  $S \wedge \neg S == \text{contradiction}$

### C. First-Order Logic

You are all experts in something. Express something about what you know in First-Order Logic. I will be using these answers (non-anonymized) as discussion items next class.

X = plays the piano

For all x, if x is a piano, then there exists a person Y that can play it

$\forall x \text{ Piano}(X) \wedge \exists y (\text{Person}(Y), X)$

### D. Post-Reflection

On a scale of 1-5, with 5 being most confident, how well do you think you could execute these learning objectives:

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