CS1571
Fall 2019
10/7 In-Class Worksheet

Name: Simran Gidwani

Where were you sitting in class today: Back left

A. Pre-Reflection

On a scale of 1-5, with 5 being most confident, how well do you think you could execute these learning objectives:

10.1 Compare model checking, theorem proving, and resolution as ways of inferring new knowledge from a knowledge base. ___2__

10.2 Prove that a KB entails a sentence *alpha* through logical inference rules. __2___

10.3 Prove that a KB entails a sentence *alpha* through resolution. __2___

B. Model Checking

1. Is Model Checking sound? T

2. Is Model Checking complete? T

3. What is the time complexity of Model Checking for determining entailment.
   a. **Exponential - $O(c^n)$**
   b. Quadratic - $O(n^2)$
   c. Linear - $O(n)$

C. Theorem Proving

The following are a list of some logical equivalencies (from the text) and inference rules.

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Modus Ponens

(if (P⇒ Q) is true, and P is true, then we can infer that Q is true)

And Elimination

(if (P^Q) is true, then we can infer that P is true)

And Introduction

(if P is true, and Q is true, then we can infer that P^Q is true)

Or Introduction

(if P is true, then we can infer that P v Q is true)

KB is: P^Q, P=>R, Q^R=>S

Does KB ⊨ S? Write your proof in the table below. For each line of the proof, indicate what rule you're using to introduce a new sentence into the KB.

Sentence #	Sentence	Rule & Sentences Used
1	P^Q (if P ^ Q is true, P & Q is true)	N/A
2	P=>R (P is true can infer R is true)	N/A
3	Q^R=>S (Q is true and R is true so S is true)	N/A
4	P is true	1, And Elimination
5	Q is true	1, And Elimination
6	R is true	4, 2 Knowledge base
7	Q ^ R is true	5, 6 And introduction
8	S is true	7, Modus Ponens

4. Is Theorem Proving sound?

Yes if the logical inference rules will give you correct truth values

5. Is Theorem Proving complete?

N but only if your set of inference rules are complete

6. What is the time complexity of Theorem Proving for determining entailment.
   a. Exponential
   b. Quadratic
   c. Linear
   d. Other

D. Theorem Proving with Resolution

Let's go back to the Wumpus world. Suppose our KB is currently:
$(B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1}$

We want to prove that KB |= α, where α says "there's no pit in Room 1,2" ($\neg P_{1,2}$)

First, figure out the CNF for:

$(B_{1,1} \Leftrightarrow (P_{1,2} \text{ v } P_{2,1})) \land \neg B_{1,1}$

Step 1: Remove the biconditional using biconditional elimination.

Step 2: Remove the implication using implication elimination.

Step 3: Move ¬ inwards using de Morgan's rules and double-negation ($\neg(\neg P) \Leftrightarrow P$):

Step 4: Apply distributivity law to get conjunctions of disjunctions.

Step 5: To prove by contradiction with resolution, we need to negate the sentence we want to prove and add it to the KB. So KB ^ $\neg\alpha$ is:

Step 6: Apply the resolution rule to create new clauses until you get a contradiction. You can use the table to track your work.

Clause #	Clause	Clauses Used
1	$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1})$	
2	$(\neg P_{1,2} \vee B_{1,1})$	
3	$(\neg P_{2,1} \vee B_{1,1})$	
4	$\neg B_{1,1}$	
5	$P_{1,2}$	
6		
7		
8		
9		
10		
11		

E. Post-Reflection

On a scale of 1-5, with 5 being most confident, how well do you think you could execute these learning objectives:

10.1 Compare model checking, theorem proving, and resolution as ways of inferring new knowledge from a knowledge base. ______

10.2 Prove that a KB entails a sentence alpha through logical inference rules. ______

10.3 Prove that a KB entails a sentence alpha through resolution. ______