FAULT DETECTION IN MULTI-INPUT ANALOG CIRCUITS USING POLYNOMIAL REGRESSION MODELLING

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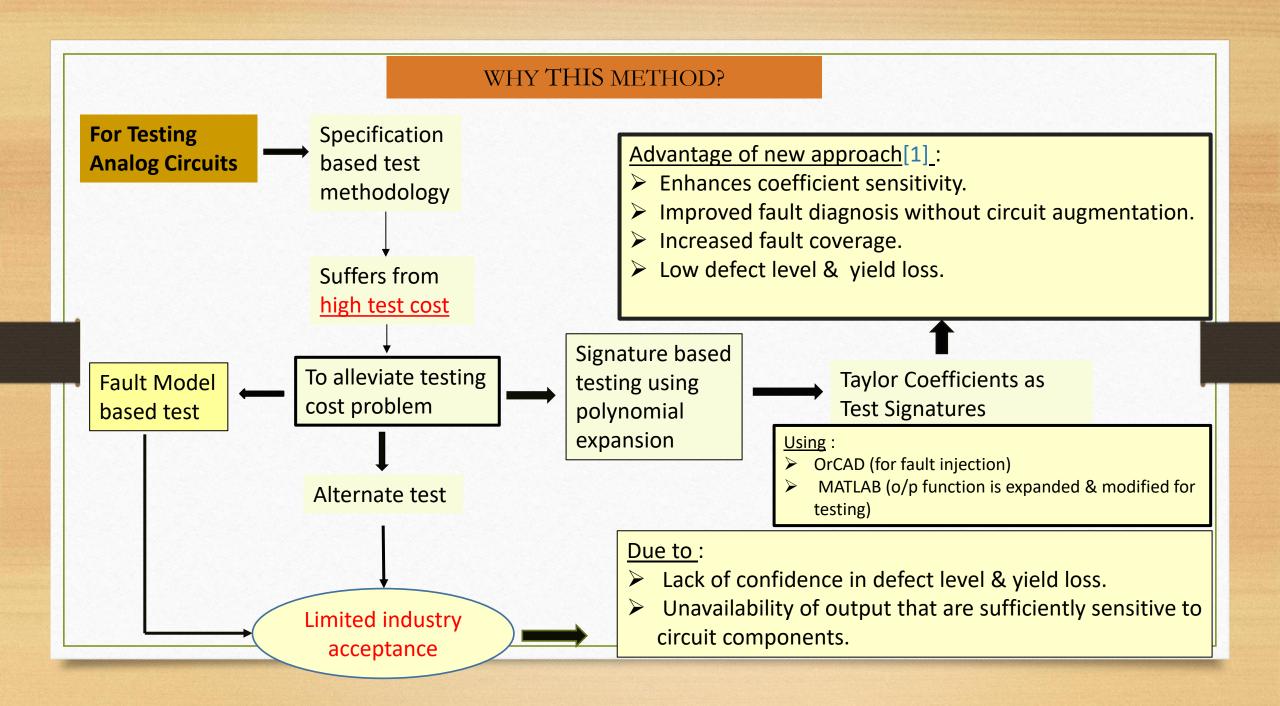
OBJECTIVE

To detect faults in multi-input analog circuits using polynomial regression models.

- Polynomial modelling for both linear and nonlinear circuits.
- Fault detection under DC sweep and sine wave inputs
- Analysis of single and multiple faults, noise robustness.

Literature Survey

| | Citation | Literature | Purpose | Method Used | Research Gap | |
|---|--|--------------------------------|--|---|--|--|
| | S. Sindia, V. D. Agrawal, and V. Singh [1] | 29th VLSI Test Symposium, 2011 | Non-linear analog circuit test using V-transform | Polynomial coefficients made monotonic using V- transform | Fails to handle multiple faults or noise-influenced variation | |
| | S. Sindia and V. D. Agrawal [2] | ITC Conference, 2013 | High-sensitivity test signatures for analog circuits | Needs little circuit augmentation | Considers only one fault at a time | |
| 1 | Piotr Bilski [3] | Measurement, vol. 160, 2020 | Ensemble regression for parametric identification | Uses regression ensembles to analyze faults | Not tested on heterogeneous architectures | |
| | Zhao & Yuzhu He [4] | J. of Electronic Testing, 2015 | Test point selection method | Fault ambiguity minimized using optimization | Ignores circuits with undefined inputs | |
| | Alkis A. Hatzopoulos [5] | IMSTW 2017 | Current spectrum-based analog test | Uses wavelet transforms and supply current | Intermittent faults remain undetected | |
| | H. Kobayashi et al. [6] | ICSICT 2020 | Op-amp and ADC testing techniques | Combines DC-AC, null, and input generators | Not versatile for all circuit types | |
| | Riewruja & Rerkratn [7] | Int. J. of Electronics, 2011 | Design of low-cost analog multiplier using op-amps | Uses quarter-square technique with class AB opamps | No regression-based testing or noise/fault sensitivity analysis provided | |



POLYNOMIAL REGRESSION MODELLING

- Polynomial regression is a form of linear regression where the relationship between input variables and output is modeled as an nth-degree polynomial.
- o It approximates the output Vout of an Analog circuit as a Taylor series expansion around nominal input values Vin1=V1 and Vin2=V2.

$$V_{out} = f(V_{in1}, V_{in2}) = f(V_1, V_2) + (V_{in1} - V_1) \cdot f_1(V_1, V_2) + (V_{in1} - V_2) \cdot f_2(V_1, V_2) + \frac{1}{2!} \{ (V_{in1} - V_1)^2 \cdot f_{11}(V_1, V_2) + 2 \cdot (V_{in1} - V_1) \cdot (V_{in2} - V_2) \cdot f_{12}(V_1, V_2) + (V_{in2} - V_2)^2 \cdot f_{22}(V_1, V_2) \} + \frac{1}{3!} \{ (V_{in1} - V_1)^3 \cdot f_{111}(V_1, V_2) + 3 \cdot (V_{in1} - V_1)^2 \cdot (V_{in2} - V_2) \cdot f_{112}(V_1, V_2) + 3 \cdot (V_{in1} - V_1) \cdot (V_{in2} - V_2)^2 \cdot f_{122}(V_1, V_2) + (V_{in2} - V_2)^3 \cdot f_{222}(V_1, V_2) \} + \dots$$

$$(1)$$

where $f(V_{in1}, V_{in2})$ and all its partial derivatives $f_1 \equiv \frac{\partial f}{\partial V_{in1}}$, $f_2 \equiv \frac{\partial f}{\partial V_{in2}}$, $f_{11} \equiv \frac{\partial^2 f}{\partial V_{in1}^2}$, $f_{22} \equiv \frac{\partial^2 f}{\partial V_{in2}^2}$, $f_{12} \equiv \frac{\partial^2 f}{\partial V_{in1} \partial V_{in2}}$ and so on are continuous and exist at nominal values $V_{in1} = V_1$ and $V_{in2} = V_2$.

O Coefficients ai are estimated using **least squares fitting** (e.g., MATLAB's Polyfitn). Eqn. (1) can be further approximated as

The number of coefficients for an mth degree polynomial regression model which can estimate the output of a two-input CUT can be given as

$$N_{\text{coeff}} = \frac{(m+1)(m+2)}{2}$$

OPTIMAL ORDER

- Selecting the right polynomial degree (order) is crucial to avoid underfitting or overfitting in regression-based fault modeling.
- o AIC/BIC provides an automated, principled, and efficient method for selecting the best polynomial order in regression-based fault modeling without overfitting, thresholds, or guesswork.

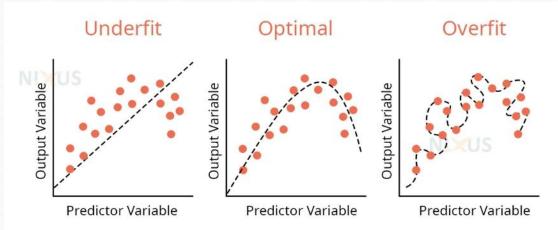
AIC = $n \cdot ln(MSE) + 2 \cdot k$ BIC = $n \cdot ln(MSE) + k \cdot ln(n)$

Where:

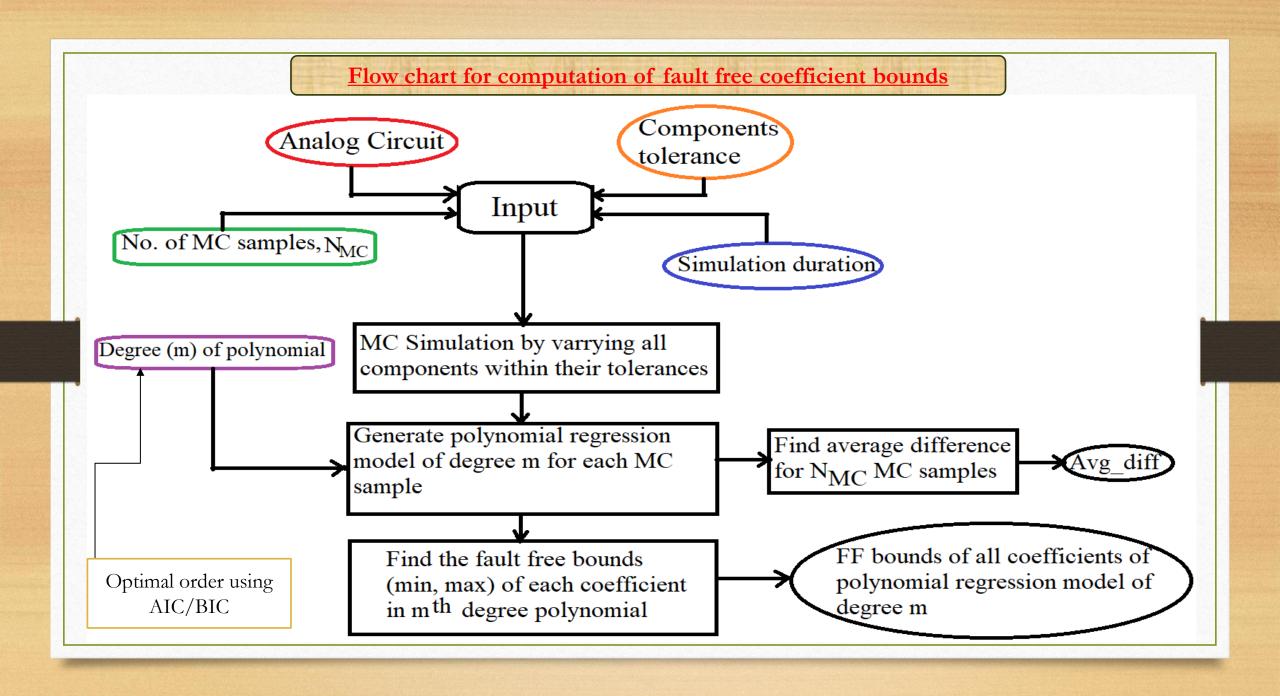
n = Number of data points.

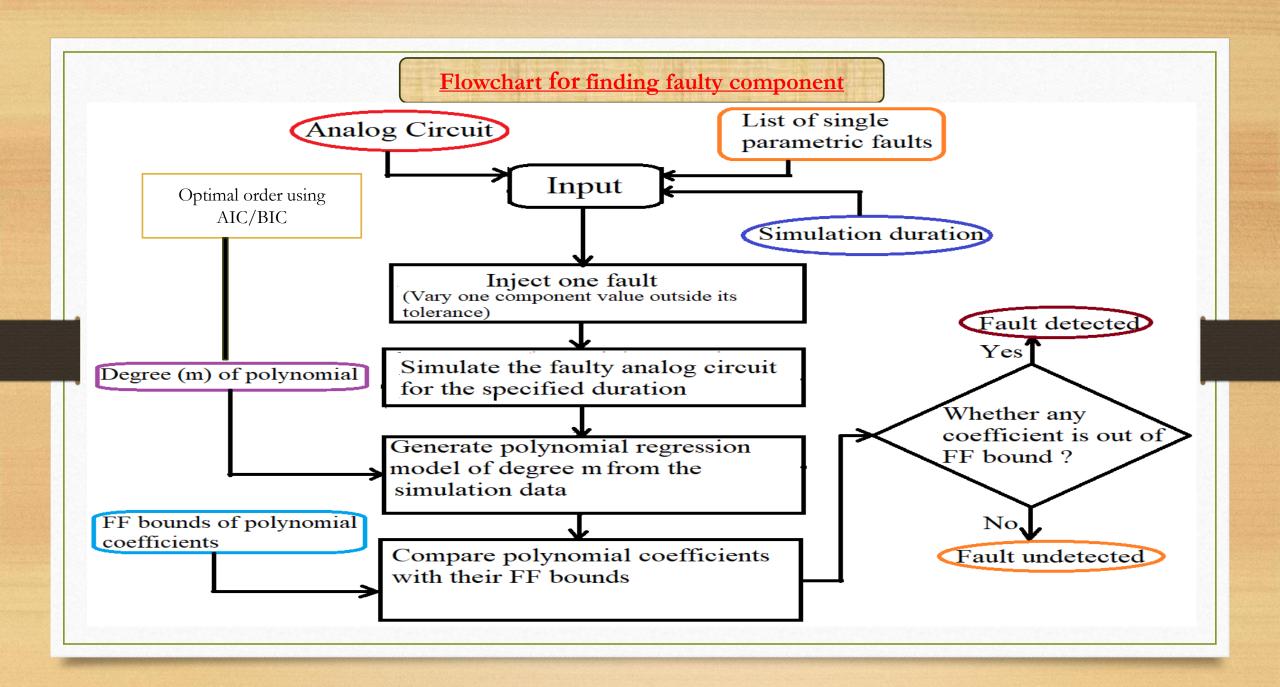
k = Number of parameters (Polynomial order).

MSE = Mean Squared Error.



- AIC/BIC minimizes overfitting by balancing model complexity and error.
- This ensures optimal polynomial order for reliable fault detection in analog circuits.





CIRCUITS UNDER STUDY

LINEAR CIRCUIT

- Lead-lag Circuit with sine input
- Lead-lag circuit with DC sweep input varied at rate of 50 Hz & 100Hz frequency
- PI Compensator of a Buck Converter

NON-LINEAR CIRCUIT

- Analog multiplier with sine input
- Analog Multiplier with DC sweep input varied at the rate of 50 Hz and 100Hz frequency

Case Study-1: Lead-lag Circuit

A 100 Hz sinusoidal signal at 200 mV amplitude is applied at the inverting input, while a 10 Hz sinusoidal signal at 400 mV amplitude is applied at the non-inverting input.

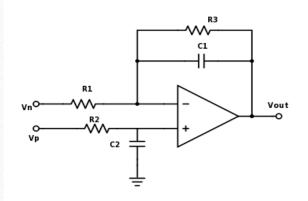
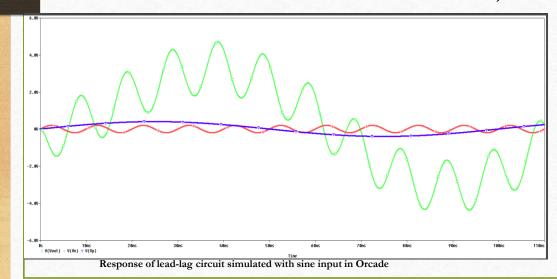


Fig. - Lead-lag circuit with two low-pass filters

• A DC sweep input at the rate of 100 Hz from 0V to 0.4V is applied at the inverting input, while a DC sweep input at the rate of 50 Hz from 0V to 0.5V is applied at the non-inverting input.

The nominal values of the circuit components of the lead-lag circuit shown in Fig. 3 are- R1=1k Ω , R2= 10k Ω , R3=10k Ω , C1=0.1595 μ F, C2=1.595 μ F



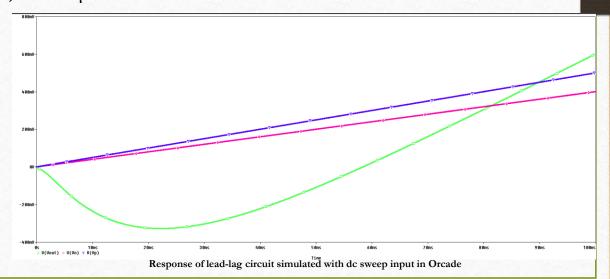
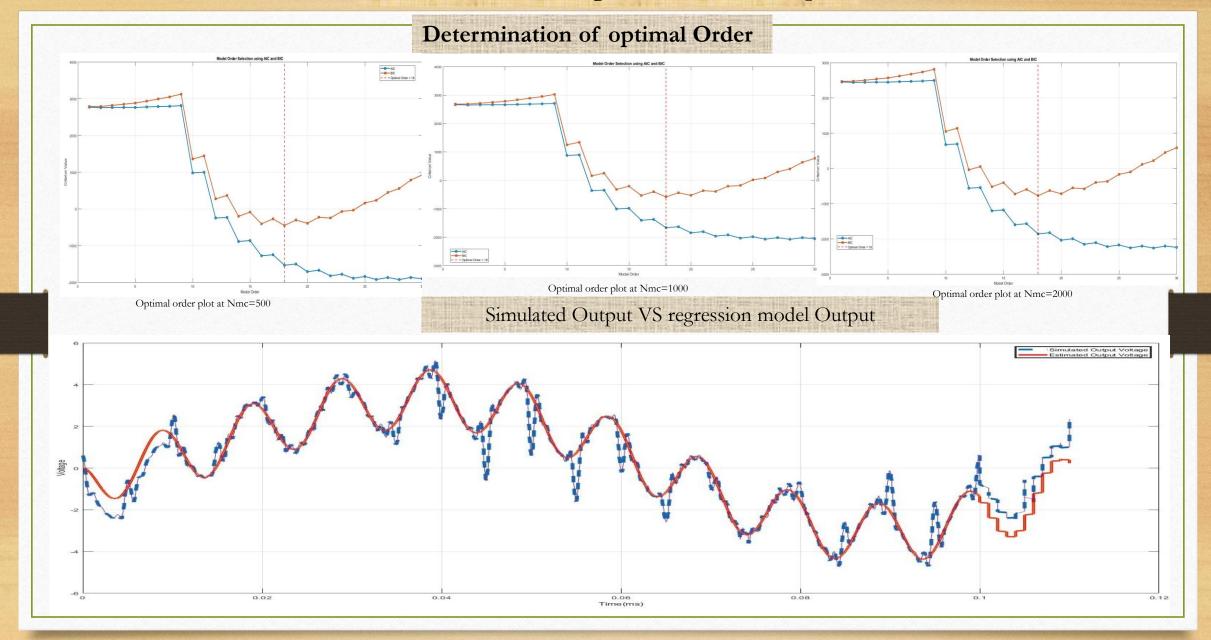
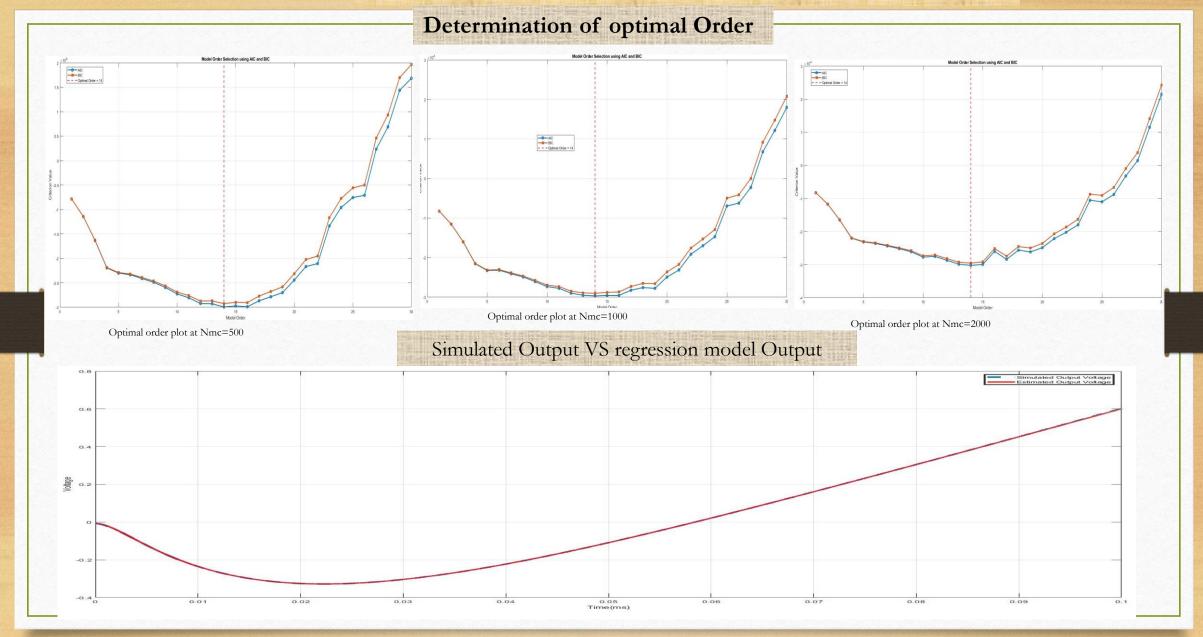


Illustration-1: Lead-lag Circuit with sine input



| | IEADI | AG - in case of | AC INDUTY SI | VE WAVE | LEADLAG - in case of AC INPUT(SINE WAVE) | | | | | | | | | | | |
|---|---|--------------------------|-------------------------|------------------------|--|-----------|--|--|--|------------------------------|--------------------------------|------------------------------|---------------------|--|--|--|
| | | | ` | , | 4=0/\ = /=00/ | | V | | | AG in the presence | | viations (>5%) | | | | |
| Various Detected | Faults in LEADLA | G in the presence | ce of faults with | deviations (5% – | · 15%) & (20%-47) | 7%) | Injected | N_{MC}=500 , Avg_dif = 0.0420 Optimal Order – 18th | | N _{MC} =1000, A | $vg_dif = 0.0420$ | N _{MC} =2000, Av | | | | |
| Injected Faults | <i>N</i> _{MC} =500, Avg | dif = 0.0420 | N _{MC} =1000 A | vg_dif = 0.0420 | N _{MC} =2000, Av | g dif = | Faults | | | Optimal Order – 18th | | 0.0420 | | | | |
| injected i date | Optimal Or | | | Order – 18th | 0.0420 | 5_w.r | | NY 6 | E 1 E : | N. C. E. I.D. | | Optimal Order – 18th | | | | |
| | Орина Ок | 1011 | Орина | 71dc1 — 10tii | Optimal Order | 1 0+b | | No. of Coefficients | Fault Detection Status | No. of Coefficients | Fault Detection Status | No. of Coefficients Out | Fault Detecti | | | |
| | | | | | | | | Out of Bound | Status | Out of Bound | Status | of Bound | on | | | |
| | No. of | Fault Detection | No. of | Fault Detection | No. of Fault | | | out of Bound | | out of Bound | | or Bound | Status | | | |
| | Coefficients Out | Status | Coefficients | Status | Coefficients | Detecti | R1 3%↑ | 0 | X | 0 | X | 0 | X | | | |
| | of Bound | | Out of Bound | | Out of Bound | on | R2 2% ↓ | 0 | X | 0 | X | 0 | X | | | |
| | | | | | | Status | R3 4% ↑ | 0 | X | 0 | X | 0 | X | | | |
| R1 12%↑ | 44.0 | 2/ | 72 | V | 64 | | C1 2% ↓ | 0 | X | 0 | X | 0 | X | | | |
| · | 113 | .1 | 73 | | 64 | | C2 4% ↑ | 0 | X | 0 | X | 0 | X | | | |
| R1 8% ↓ | 1 | V | 0 | X | 0 | X | R1 1%↓ | 0 | X | 0 | X | 0 | X | | | |
| R2 9% ↑ | 0 | X | 0 | X | 0 | X | R2 1%↑ | 0 | X | 0 | X | 0 | X | | | |
| R2 6%↓ | 0 | X | 0 | X | 0 | X | R3 4% ↓ C1 3% ↑ | 0 | X X | 0 | X X | 0 | X X | | | |
| R3 10% ↑ | 71 | $\sqrt{}$ | 57 | $\sqrt{}$ | 38 | $\sqrt{}$ | | | | , and the second | | Ť | | | | |
| R3 7%↓ | 0 | X | 0 | X | 0 | X | C2 4% ↓ | 0 | X | 0 | X | 0 | X | | | |
| C1 13%↑ | 0 | X | 0 | X | 0 | X | LEADLAG - in case of AC INPUT(SINE WAVE) | | | | | | | | | |
| C1 15%↓ | 3 | | 3 | $\sqrt{}$ | 3 | $\sqrt{}$ | Injected Faults | Various Detected Faults in LEADLAG in the presence of MULTIPLE faults with devia llts N_{MC} =500, $Avg_dif = 0.0420$ N_{MC} =1000, $Avg_dif = 0.0420$ N_{MC} =20 | | | | | | | | |
| C2 11% ↑ | 0 | X | | X | | X | Injected Faults | | v g_a II = 0.0420 Order – 18th | | vg_an = 0.0420 Order – 18th | N _{MC} =2000, Avg_o | | | | |
| C2 1170 C2 14%↓ | · · · · · · · · · · · · · · · · · · · | Λ .l | 0 | 1 | 0 | Λ √ | | | | | | Spuniai Older – Ioth | | | | |
| · | 10 | V | 7 | V | 6 | | | No. of | Fault Detection | No. of | Fault Detection | No. of | Fault | | | |
| R1 20%↑ | 163 | V | 156 | V | 153 | √ , | | Coefficients Out of Bound | Status | Coefficients Out of Bound | Status | Coefficients Out of Bound | Detection Status | | | |
| R1 30% ↓ R2 32% ↑ | 183 | V | 183 | V | 183 | √ √ | R1 15%↑ R2 20%↓ | 162 | \checkmark | 155 | $\sqrt{}$ | 151 | √ | | | |
| R2 40% ↓ | 34 | 2/ | 33 | N 2 | 32 | 1 | R2 8%↓ | | √ | | V | | √ | | | |
| R3 25% ↑ | 104 | N al | 93 | N al | 89 | √ √ | R3 12% ↑ R3 20% ↑ | 100 | · | 96 | · | 90 | | | | |
| R3 45%↓ | 174 | 2/ | 173 | N N | 172 | 1 | C1 10% ↑ | 450 | \checkmark | 450 | $\sqrt{}$ | 454 | √ | | | |
| C1 42%↑ | 185 | N al | 184 | N N | 183 | √ √ | C2 10% ↓ R1 18% ↓ | 158 | | 152 | | 151 | | | | |
| C1 42% C1 25% \ | 17 | 2/ | 14 | N N | 13 | √ √ | C1 10% ↑ C2 10% L | 166 | √ | 162 | √ | 160 | √ | | | |
| C2 35% ↑ | 15 | 2/ | 14 | 2/ | 12 | √ √ | R2 9% ↑ | 166 | | 102 | | 100 | | | | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | 38 | V | 33 | V | 31 | | R3 15% ↓ C1 12% ↑ | | \checkmark | | $\sqrt{}$ | | \checkmark | | | |
| C2 4770 ↓ | 140 | ٧ | 135 | V | 132 | √ | C1 1276 C2 8% \ | 153 | | 142 | | 137 | | | | |

Illustration-2: Lead-lag Circuit with DC sweep input



| | LE | ADLAG- in cas | e of DC INPU | T(slow dc wit | h 100Hz & 50H | Iz frequency | LEADLAG- in case of DC INPUT(slow dc with 100Hz & 50Hz frequency) | | | | | | | | | | | |
|-------|-----------------|----------------------------------|-------------------------|---------------|------------------------|------------------------------------|---|----------------------|------------------------------|---|------------------------------|--|--------------------------------------|----------------------|--|--|--|--|
| | | ed Faults in LEA | | | | | %) & (20%- | Injected Faults | | Faults in LEADL. | | of faults with devia | ntions (>5%) N _{MC} =20 | 000 | | | | |
| | | N 500 A | 116 5 4004 | 47%) | | | 116 | injected rauns | Avg_dif = | –500, =5.1931e-04 Order – 14th | Avg_dif | = 1000, = 5.1978e-04 Order = 14th | Avg_dif = 5 Optimal Ord | .2020e-04 | | | | |
| 8 - | Injected Faults | <i>N</i> _{MC} =500, Avg | _ aif = 5.1931e- | | | | | | No. of | Fault Detection | No. of | Fault Detection | No. of Coefficien | | | | | |
| | | Optimal Or | + :der – 14th | | /8e-04 Order – 14th | 5.2020e-04 Optimal Order – 14th | | | Coefficients Out of Bound | Status | Coefficients Out of Bound | Status | Out of Bound | Detectio n Status | | | | |
| | | No. of | Fault | No. of | Fault | No. of | Fault | R1 3%↑ | 1 | $\sqrt{}$ | 1 | V | 1 | 1 | | | | |
| | | Coefficients | Detection | Coefficients | Detection | Coefficients | Detection | R2 2% ↓ | 1 | | 1 | $\sqrt{}$ | 0 | X | | | | |
| | | Out of Bound | Status | Out of | Status | Out of | Status | R3 4%↑ | 0 | X X | 0 | X | 0 | X | | | | |
| | | Out of Bound | Status | Bound | Status | Bound | Status | C1 2% ↓ | | | | X | · · | X | | | | |
| | | | | Dound | | Dound | | C2 4% ↑ R1 1%↓ | 0 | X X | 0 | X | 0 | X X | | | | |
| - | R1 12%↑ | 0 | X | 0 | X | 0 | X | R1 176↓ R2 1%↑ | 0 | X | 0 | X | 0 | X | | | | |
| | R1 8% ↓ | 0 | X | 0 | X | 0 | X | | 0 | A | 0 | X X | 0 | | | | | |
| 1 | R2 9% ↑ | 0 | X | 0 | X | 0 | X | R3 4% ↓ C1 3% ↑ | 0 | X | 0 | | 0 | X X | | | | |
| 1 | R2 6%↓ | 0 | X | 0 | X | 0 | X | | v | -1- | | X | Ů | | | | | |
| Ī | R3 10% ↑ | 0 | X | 0 | X | 0 | X | C2 4% ↓ | 0 | X | 0 | X | 0 | X | | | | |
| 1 | R3 7%↓ | 0 | X | 0 | X | 0 | X | _ | | | , | T(slow dc with 100Hz & 50Hz frequency) the presence of MULTIPLE faults with deviations | | | | | | |
| | | U | | U | | 0 | | Injected Faults | | aults in LEADLAG _ dif = 5.1931e-04 | | MULTIPLE faults g_dif = 5.1978e-04 | va dif - | | | | | |
| | C1 13%↑ | 0 | X | 0 | X | 0 | X | injected raults | | order – 14th | 1.10 | 3_uii = 3.1978e-04 Order = 14th | N_{MC} =2000, Avg_dif = 5.2020e-04 | | | | | |
| | C1 15%↓ | 3 | $\sqrt{}$ | 2 | $\sqrt{}$ | 2 | $\sqrt{}$ | | Optimia Order 1 till | | | | Optimal Orde | | | | | |
| | C2 11%↑ | 0 | X | 0 | X | 0 | X | | No. of | Fault Detection | No. of | Fault Detection | No. of | Fault | | | | |
| | C2 14%↓ | 1 | $\sqrt{}$ | 1 | V | 1 | $\sqrt{}$ | | Coefficients Out of Bound | Status | Coefficients Out of Bound | Status | Coefficients Out of Bound | Detection Status | | | | |
| - 1 = | R1 20%↑ | 1 | V | 0 | X | 0 | X | R1 15%↑ | | √ | | √ | | | | | | |
| - | R1 30% ↓ | 3 | V | 3 | √ | 3 | $\sqrt{}$ | R2 20% ↓ R2 8% ↓ | 32 | | 32 | | 29 | | | | | |
| | R2 32% ↑ | 1 | V | 1 | V | 1 | V | R2 870 ↓ R3 12% ↑ | 30 | \checkmark | 30 | √ | 29 | $\sqrt{}$ | | | | |
| 1 | R2 40% ↓ | 1 | V | 1 | √ | 1 | √ | R3 20% ↑ | | | | | | | | | | |
| - | R3 25% ↑ | 10 | $\sqrt{}$ | 8 | $\sqrt{}$ | 8 | V | C1 10% ↑ C2 10% ↓ | -0 | $\sqrt{}$ | _ | $\sqrt{}$ | | V | | | | |
| 1 | R3 45%↓ | 4 | $\sqrt{}$ | 4 | $\sqrt{}$ | 4 | $\sqrt{}$ | R1 18% ↓ | 28 | | 27 | | 25 | | | | | |
| - | C1 42%↑ | 4 | V | 4 | V | 3 | $\sqrt{}$ | K1 18% ↓ C1 10% ↑ | | √ | | √ | | \checkmark | | | | |
| | C1 25% \ | 2 | | 2 | $\sqrt{}$ | 2 | $\sqrt{}$ | C2 10% ↓ R2 9% ↑ | 34 | | 34 | | 32 | | | | | |
| | □2 35% ↑ | 0 | X | 0 | X | 0 | X | R3 15% ↓ | | | | | | | | | | |
| | C2 47% | 35 | V | 30 | V | 26 | V | C1 12% ↑ C2 8% ↓ | 32 | V | 32 | V | 29 | V | | | | |

Case Study-2: Analog Multiplier Circuit



Key Features of the Proposed Analog Multiplier:

- Utilizes translinear characteristic of the class-AB output stage of op-amps.
- Employs op-amp supply current sensing technique to obtain:
 - Square of sum and Square of difference of two input signals.
 - $vo = (1/4) \times [(v1 + v2)^2 (v1 v2)^2] = v1 \times v2$
- Multiplication performed using quarter-square algebraic identity.
- Achieves low-voltage operation with general-purpose op-amps.
- Implements a four-quadrant analog multiplier using only op-amps (no extra active devices).

The nominal component values used in the simulation are: R11=R12 = $10 \text{ k}\Omega$,R21=R22= $1 \text{ k}\Omega$,R31 =R32= $250 \text{ k}\Omega$,R41 =R42= $500 \text{ k}\Omega$, R51 = R52= $250 \text{ k}\Omega$

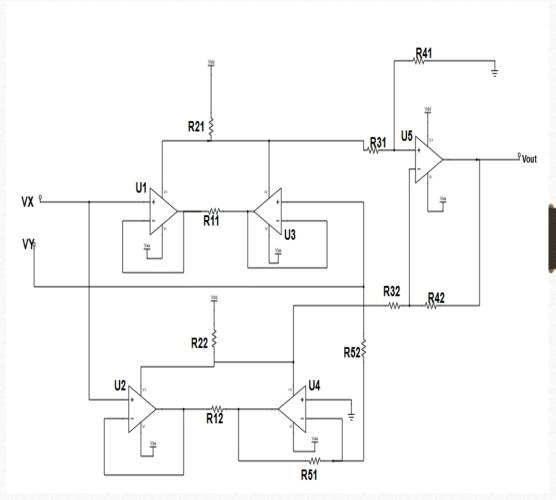


Fig:- Four Quadrant Analog Multiplier using op-amp

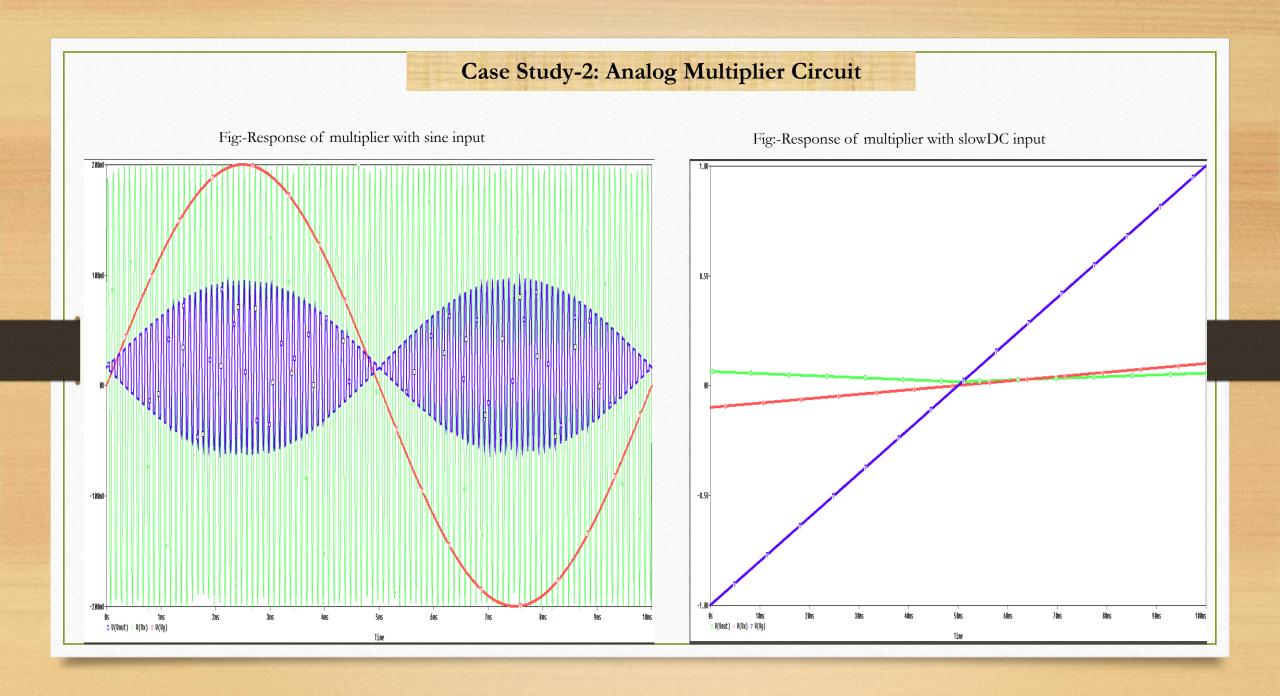
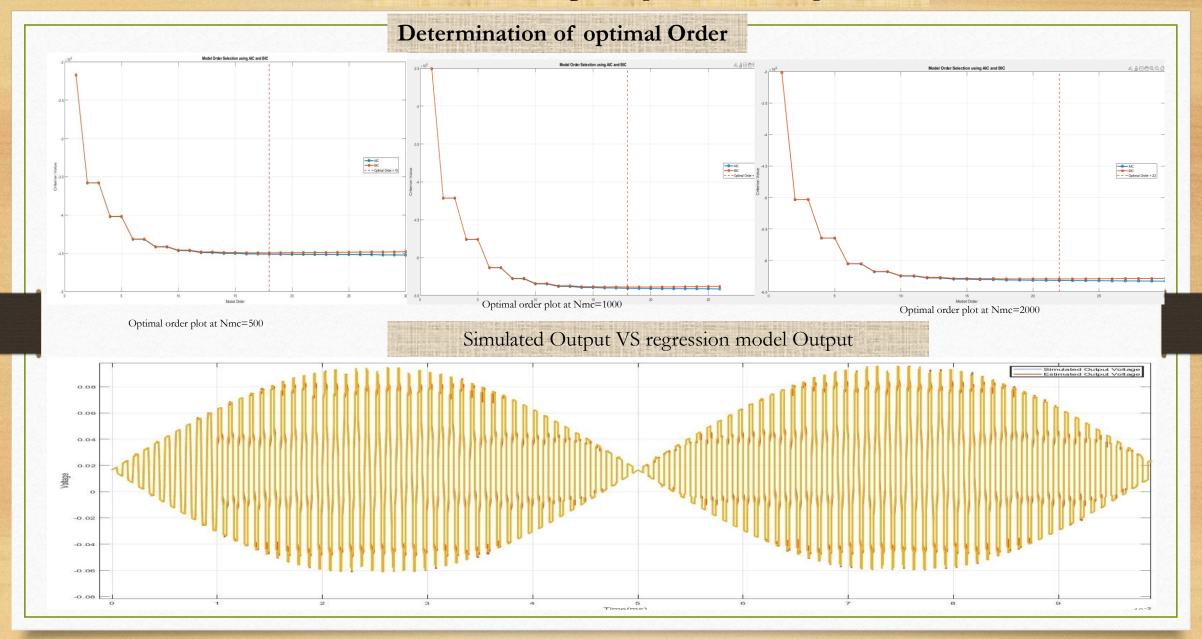
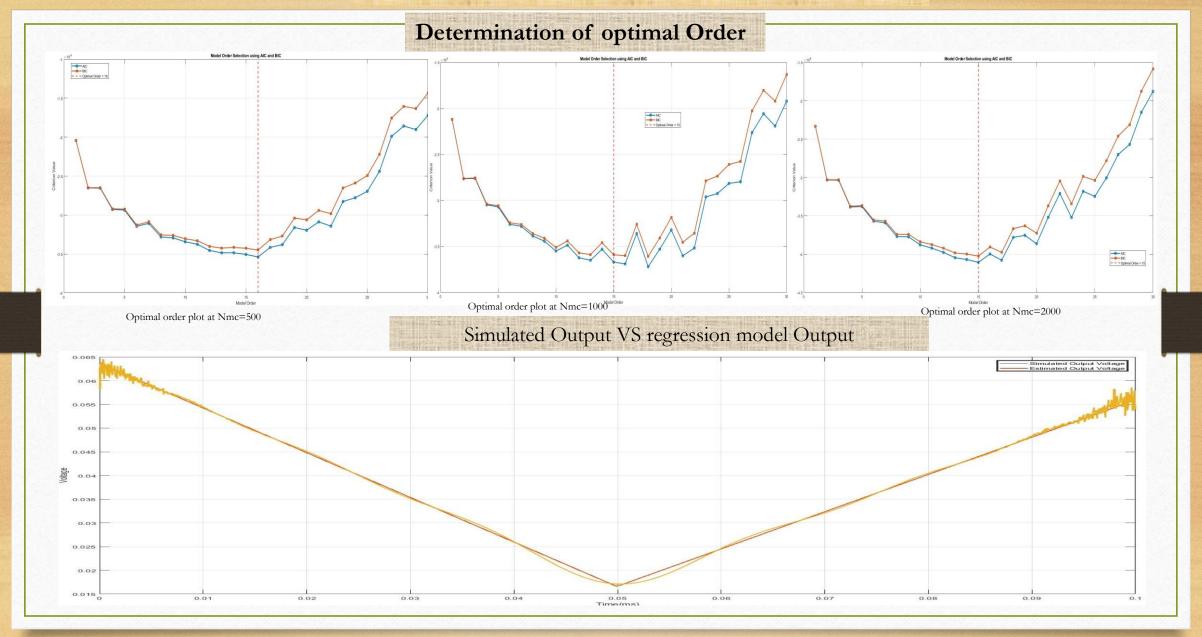


Illustration-3: Analog Multiplier with sine input



| | ANAL | OG MULTIPLIER- | in case of AC INPU | T(SINE WAVE) | | | | ANALO | G MULTIPLIER- | in case of AC IN | PUT (SINE WAVE | Ξ) | |
|----------------------|-------------------------------------|--|-------------------------------------|---|---|--------------------|--|----------------------------|--|----------------------------|--|---|------------|
| Variou | s Detected Faults in A | nalog Multiplier in th | ne presence of faults v | vith deviations (5% – | 15%) & (20%-46%) | | Various Detected Faults in Analog Multiplier in the presence of faults with deviations (>5%) | | | | | | |
| Injected Faults | N _{MC} =500, A | vg_dif = 0.0098 Order – 18th | N _{MC} =1000, A | Avg_dif = 0.0097 Order – 18th | N _{MC} =2000, Avg_d Optimal Order | | Injected Faults | | N _{MC} =500, Avg_dif = 0.0098 Optimal Order – 18th | | N_{MC} =1000, Avg_dif = 0.0097 Optimal Order – 18th | | Avg_dif = |
| | No. of Coefficients Out of Bound | Fault Detection Status | No. of Coefficients Out of Bound | Fault Detection Status | No. of Coefficients Out of Bound | Fault Detection | | No. of Coefficients Out | Fault Detection Status | No. of Coefficients Out | Fault Detection Status | No. of Coefficients Out | Fault |
| | | | | | | Status | | of Bound | Status | of Bound | Status | of Bound | on |
| R11 6%↑ | 19 | $\sqrt{}$ | 16 | $\sqrt{}$ | 23 | $\sqrt{}$ | | or bound | | or bound | | or bound | Status |
| R11 6% ↓ | 12 | $\sqrt{}$ | 7 | | 12 | | R11 1%↓ | 0 | X | 0 | X | 1 | V |
| R12 7% ↑ | 10 | $\sqrt{}$ | 4 | $\sqrt{}$ | 23 | | R11 3% ↑ | 5 | √ V | 5 | √ V | 0 | X |
| R12 7% ↓ | 27 | $\sqrt{}$ | 20 | | 64 | | R12 2% ↑ | 1 | V | 0 | X | 0 | X |
| R21 8% ↑ | 25 | \checkmark | 16 | V | 12 | $\sqrt{}$ | R12 1%↓ | 5 | √ | 5 | √ | 5 | √ |
| R21 8%↓ | 15 | $\sqrt{}$ | 8 | V | 12 | $\sqrt{}$ | R21 3% ↑ | 0 | X | 1 | √ | 0 | X |
| R22 9%↑ | 20 | V | 15 | V | 32 | | R21 2.5%↓ | 0 | X | 0 | X | 0 | X |
| R22 9% ↓ | 21 | $\sqrt{}$ | 17 | V | 30 | V | R22 2%↑ | 1 | V | 0 | X | 0 | X |
| R31 10% ↑ | 15 | V | 12 | V | 11 | V | R22 3% ↓ | 3 | V | 1 | V | 0 | X |
| R31 10% ↓ | 30 | √ | 30 | V | 27 | V | R31 4% ↑ | 0 | X | 0 | X | 0 | X |
| R32 11% ↑ | 22 | V | 15 | V | 30 | V | R31 3.5% ↓ | 3 | V | 0 | X | 0 | X |
| R32 11% \ | 36 | V | 30 | V | 32 | V | R32 1% ↑ | 1 | V | 0 | X | 0 | X |
| R41 12% ↑ | 38 | V | 35 | V | 13 | V | R32 3% ↓ | 7 | √ | 5 | V | 3 | √ |
| R41 12% J | 17 | V | 15 | , v | 17 | , V | R41 2%↑ | 0 | X | 0 | X | 0 | X |
| R42 13% ↑ | 29 | √ √ | 18 | V | 32 | V | R41 1% | 0 | X | 0 | X | 0 | X |
| R42 13% ↓ | 21 | 1 | 0 | V | 18 | V | R41 176↓ R42 3%↑ | 3 | Λ √ | 1 | Λ √ | 0 | X |
| R51 14% ↑ | 3 | 2 | 5 | N N | 12 | 2 | R42 1%↓ | 1 | 1 | 1 | V | 0 | X |
| R51 14% ↓ | 16 | 2 | 16 | 7 | 35 | 2 | R51 1.5% ↑ | 0 | X | 2 | , v | 1 | <i>X</i> √ |
| R52 15% ↑ | 41 | 2/ | 38 | 2 | 18 | 2 | R51 1.8% ↓ | 2 | √ V | 0 | X | 0 | X |
| R52 15% \ | 26 | 2 | 24 | 2 | 13 | 2 | R52 2.3% ↑ | 0 | X | 0 | X | 0 | X |
| R11 20% ↑ | 17 | V | 9 | 2 | 25 | 2 | R52 2.9%↓ | 0 | X | 0 | X | 0 | X |
| R11 20% | | V | | V al | | 2 | | ANALO | C MIII TIDI IED | in and of AC IN | PUT(SINE WAVE | 7.\ | |
| R12 25% ↑ | 58 | N al | 37 | N al | 39 | N al | Vario | us Detected Faults | | | | | |
| R12 25% R12 25% | 17 | V | 14 | V | 27 | V | Injected Faults | N_{MC} =500, Avg | | N_{MC} =1000, Av | | | |
| 08 | 40 | N | 20 | N | 15 | N | Injected Faults | Optimal O | | Optimal Orde | - | N_{MC} =2000, $Avg_dif = 0.009$ ° Optimal Order – 22nd | |
| R21 30% ↑ | 33 | V | 28 | V | 39 | V | | | Fault Detection | | | | Fault |
| R21 30% ↓ | 4 | V | 3 | V | 9 | V | | Coefficients Out | | Coefficients Out | | | Detection |
| R22 35% ↑ | 40 | V | 27 | V | 22 | V | | of Bound | Status | of Bound | | | Status |
| R22 35% ↓ | 20 | V | 20 | V | 14 | V | R11 8%↑ | Or Dound | | or bound | | or Bound | Status |
| R31 36% ↑ | 28 | V | 27 | V | 8 | ٧ | R11 8% R12 10% ↑ | 15 | \checkmark | 12 | $\sqrt{}$ | 17 | $\sqrt{}$ |
| R31 36% ↓ | 38 | V | 26 | V | 18 | V | R12 10% R21 20% ↑ | 15 | | 13 | | 17 | |
| R32 38% ↑ | 7 | V | 7 | V | 2 | V | R21 20% R22 15% ↓ | | 2/ | | V | | $\sqrt{}$ |
| R32 38% ↓ | 75 | V | 57 | V | 96 | V | K 22 1576 ↓ | 25 | V | 21 | V | 38 | V |
| R41 40% ↑ | 32 | V | 23 | V | 26 | V | R31 20% ↓ | 23 | | ۷1 | | 50 | |
| R41 40% ↓ | 18 | | 10 | | 30 | $\sqrt{}$ | R31 20% ↓ R32 20% ↑ | | V | | V | | V |
| R42 42% ↑ | 46 | $\sqrt{}$ | 44 | $\sqrt{}$ | 34 | $\sqrt{}$ | R32 20% R41 10% ↑ | 18 | ٧ | 17 | V | 16 | ٧ |
| R42 42% ↓ | 3 | $\sqrt{}$ | 4 | $\sqrt{}$ | 4 | $\sqrt{}$ | R52 30% | 10 | | 1 / | | 10 | |
| R51 44% ↑ | 30 | \checkmark | 26 | $\sqrt{}$ | 19 | $\sqrt{}$ | R51 20% ↑ | 21 | | 16 | \checkmark | 23 | $\sqrt{}$ |
| R51 44% ↓ | 68 | $\sqrt{}$ | 58 | $\sqrt{}$ | 53 | $\sqrt{}$ | R11 15% ↑ | 21 | | 10 | | 43 | |
| R52 46% ↑ | 66 | √ | 65 | V | 71 | V | R12 20% ↓ | | $\sqrt{}$ | | V | | V |
| R52 46% \ | 28 | √ | 24 | √ | 36 | V | R42 18 % ↑ | 34 | • | 31 | , | 55 | , |
| 1070 1 | 20 | ٧ | 24 | V | 30 | ٧ | 10 70 | 54 | | JI | | JJ | |

Illustration-4: Analog Multiplier with DC sweep input



| | ANALOG MULT | IPLIER - in case of I | OC INPUT(slow dc wi | th 100Hz & 50Hz freq | uency) | | AN | ALOG MULTIPL | IER - in case of 1 | DC INPUT(slow of | lc with 100Hz & 50 | 0Hz frequency) | |
|------------------------|---|---------------------------------------|--|------------------------|---|------------------------------|-------------------------|----------------------------|---|----------------------------|--|------------------------------------|-----------------|
| Var | Various Detected Faults in Analog Multiplier in the presence of faults with deviations (5% – 15%) & (20%-46%) | | | | | | | | | plier in the presen | | | |
| Injected Faults | N _{MC} =500, Av | g_dif = 0.0103 Order - 16th | N_{MC} =1000, Avg_dif = 0.0108 Optimal Order - 15th | | N _{MC} =2000, Avg_d Optimal Order | | Injected Faults | N _{MC} =500, Av | N_{MC} =500, Avg_dif = 0.0103 Optimal Order - 16th | | N_{MC} =1000, Avg_dif = 0.0108 Optimal Order - 15th | | vg_dif = |
| | No. of Coefficients Out of Bound | Fault Detection Status | No. of Coefficients Out of Bound | Fault Detection Status | No. of Coefficients Out of Bound | Fault Detection Status | | No. of Coefficients Out | Fault Detection Status | No. of Coefficients Out | Fault Detection Status | Optimal Ord No. of Coefficients Ou | Fault |
| R11 6%↑ | 10 | V | 12 | | 3 | V | | of Bound | | of Bound | | of Bound | on |
| R11 6% ↓ | 2 | $\sqrt{}$ | 8 | $\sqrt{}$ | 2 | $\sqrt{}$ | | | | | | | Status |
| R12 7% ↑ | 4 | $\sqrt{}$ | 3 | $\sqrt{}$ | 2 | | R11 1%↓ | 0 | X | 0 | X | 1 | V |
| R12 7% ↓ | 6 | V | 9 | v | 5 | √ | R11 3% ↑ | 8 | ٧ | 8 | \checkmark | 2 | V |
| R21 8% ↑ | 2 | V | 4 | V | 2 | V | R12 2% ↑ | 1 | √ | 0 | X | 1 | √ |
| R21 8%↓ | 12 | V | 13 | V | 2 | V | R12 1% ↓ | 7 | √ | 5 | V | 2 | √ |
| R22 9%↑ | 12 | V | 8 | V | 2 | √ | R21 3% ↑ | 0 | X | 1 | V | 3 | V |
| R22 9% ↓ | 1 | V | 3 | V | 0 | X | R21 2.5%↓ | 0 | X | 1 | V | 1 | V |
| R31 10% ↑ | 8 | V | 7 | V | 2 | √ | R22 2%↑ | 1 | V | 2 | V | 3 | V |
| R31 10% ↓ | 6 | V | 8 | V | 2 | ٧ | R22 3% ↓ | 3 | V | 1 | V | 1 | N |
| R32 11% ↑ | 2 | V | 7 | V | 3 | V | R31 4% ↑ | 0 | X | 0 | X | 3 | ٧, |
| R32 11% ↓ | 10 | V | 6 | V | 3 | √ | R31 3.5% ↓ | 3 | V | 6 | √ | 1 | √ |
| R41 12% ↑ | 5 | V | 7 | V | 2 | V | R32 1% ↑ | 1 | N | 0 | X | 0 | X |
| R41 12% ↓ | 5 | V | 13 | V | 2 | V | R32 3% ↓ | 8 | V | 10 | ٧ | 3 | ٧ |
| R42 13% ↑ | 10 | N | 8 | V | 4 | V | R41 2%↑ | 0 | X | 0 | X | 0 | X |
| R42 13% ↓ R51 14% ↑ | 1 | N | 4 | N . l | 3 | ν | R41 1%↓ | 0 | X | 0 | X | 0 | V |
| R51 14% ↓ | 12 | V | 2 | N al | 0 | X √ | R42 3%↑ | 3 | X | 0 | X √ | 1 | \ \ |
| R52 15% ↑ | 13 12 | V | 12 14 | √ √ | 7 | V | R42 1%1 | 1 | V | 1 | V | 1 | Ž |
| R52 15% ↓ | 12 | V | 3 | √ √ | 0 | X | R51 1.5% ↑ | 0 | X | 2 | Ž | 1 | į |
| R11 20% ↑ | 13 | V | 16 | V | 9 | √ √ | R51 1.8% J | 3 | √ × | 0 | X | 2 | V |
| R11 20% ↓ | 1 | , V | 2 | V | 0 | X | R52 2.3% ↑ | 0 | X | 0 | X | 1 | √ |
| R12 25% ↑ | 1 | V | 2 | V | 0 | X | R52 2.9%↓ | 0 | X | 0 | X | 0 | X |
| R12 25% ↓ | 28 | V | 43 | $\sqrt{}$ | 28 | $\sqrt{}$ | | Optimal O | rder - 16th | Optimal O | rder - 15th | Optimal Order - 15th | |
| R21 30% ↑ | 3 | V | 6 | | 2 | V | | No. of | Fault Detection | No. of | Fault Detection | No. of | Fault |
| R21 30% ↓ | 15 | $\sqrt{}$ | 25 | | 10 | $\sqrt{}$ | | Coefficients Out | Status | Coefficients Out | Status | Coefficients Out | Detection |
| R22 35% ↑ | 16 | $\sqrt{}$ | 22 | $\sqrt{}$ | 10 | $\sqrt{}$ | | of Bound | | of Bound | | of Bound | Status |
| R22 35% ↓ | 5 | V | 4 | $\sqrt{}$ | 5 | $\sqrt{}$ | R11 8%↑ | | V | | √ | | ا |
| R31 36% ↑ | 9 | V | 11 | V | 7 | √ | R12 10% ↑ | 5 | V | 6 | V | 3 | V |
| R31 36% ↓ | 3 | V | 2 | $\sqrt{}$ | 1 | V | R21 20% ↑ | | | | | | |
| R32 38% ↑ | 2 | V | 4 | V | 1 | V | R22 15% ↓ | | \checkmark | | \checkmark | | |
| R32 38% ↓ | 22 | V | 46 | V | 35 | V | | 6 | | 5 | | 2 | |
| R41 40% ↑ | 3 | V | 5 | V | 1 | √ , | R31 20% ↓ | | | | | | |
| R41 40% \ | 13 | V | 12 | V | 9 | V | R32 20% ↑ | | \checkmark | | \checkmark | | $\sqrt{}$ |
| R42 42% ↑ | 19 | V | 23 | V | 13 | √ ./ | R41 10% ↑ | 23 | | 13 | | 5 | |
| R42 42% ↓ | 2 | N .1 | 3 | . l | 1 | N | R52 30%↓ | | V | | \checkmark | | $\sqrt{}$ |
| R51 44% ↑ R51 44% ↓ | 3 | V | 4 | V | 3 | ٧ ما | R51 20% ↑ | 5 | Y | 5 | • | 7 | , |
| R51 44% ↓ R52 46% ↑ | 60 | V | 68 | N N | 57 | √ √ | R11 15% ↑ | | | | , | | ı |
| R52 46% ↓ | 42 | V | 30 | N N | 30 | V | R12 20% ↓ R42 18 % ↑ | =0 | $\sqrt{}$ | | \checkmark | 25 | V |
| | 5 | V | 9 | V | 3 | V | K42 18 % | 78 | | 57 | | 37 | |

Case Study-3: PI Compensator of a Buck Converter

- ☐ It is the peak current-mode controlled DC-DC Buck converter using PWM control IC UC3843.
- A key functional block of this controller is the PI compensator (shown in Fig.), which is built using an error amplifier.
- ☐ The output voltage (Vout) is divided by the resistor dividetr network (Rf1 and Rf2) and then fed to the error amplifier

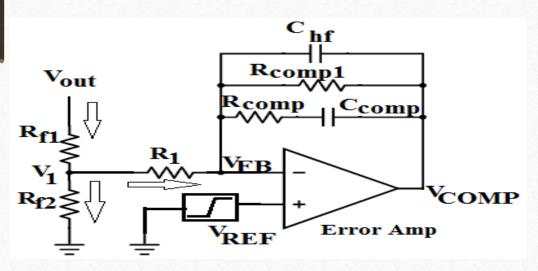


Fig.. PWM controller circuit of buck converter

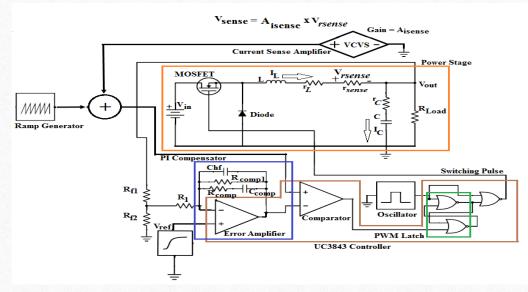
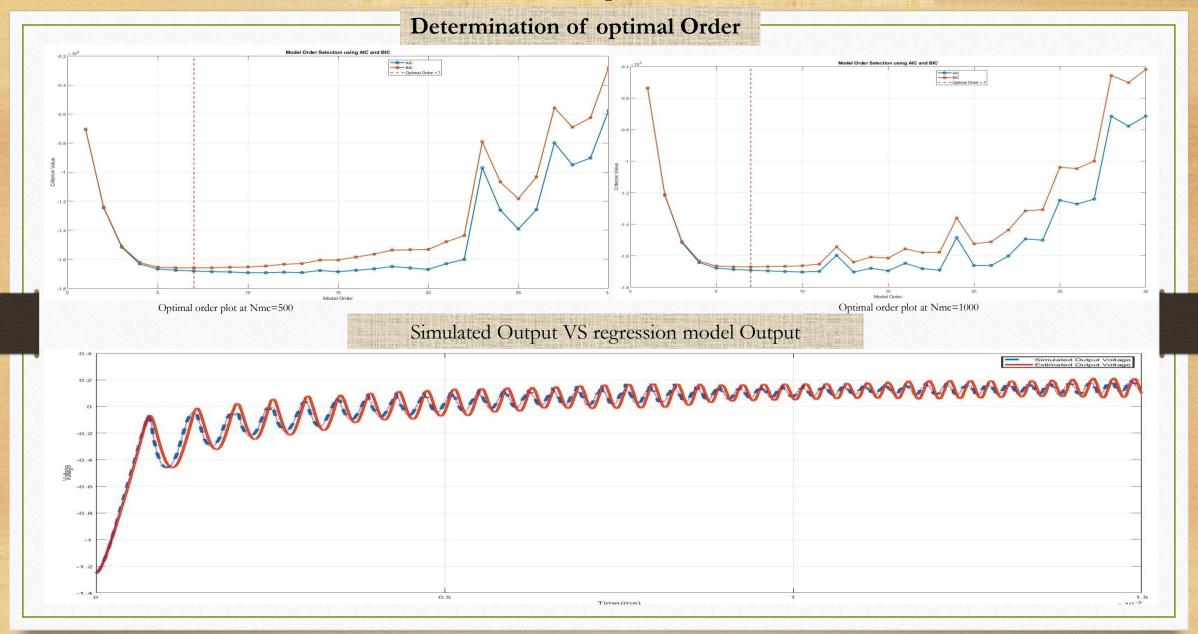


Fig.. Schematic of a current programmed control buck converter

- ☐ The controller is crucial to maintain a constant output voltage.
- ☐ Therefore, the controller circuit must be properly designed and tested.
- ☐ In this study, we focus on parametric fault detection in the controller part of the Buck converter circuit.

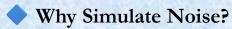
The nominal values are:- R_{f1} =9.76 k Ω , R_{f2} =3.25 k Ω , R_{1} =1k Ω , C_{hf} =30pF, C_{comp} =2nF, R_{comp1} =10 M Ω , R_{comp} =80 k Ω

Illustration-5: PI Compensator of a Buck Converter



| | | PI CO | MPENSATOR | OF A BU | CK CONVERT | ER | | | 1 | | PI C | OMPENSATO | R OF A BUC | CK CONVERT | ER | | 1 | |
|-----------------------------|--|----------------------------------|--|----------------------------------|--|-------------------------------|---|----------------------------------|-------------------------------------|--|--|---|--|--|--|---|-------------------------------------|--|
| Various Detec | ted Faults PI c | | in the presence | | | |) & (20%-50% |) | | Various Det | | | in the prese | nce of faults wi | | | | |
| Injected Faults | | N _{MC} | =500 | | | $N_{\rm MC}$ = | 1000 | | Injected Faults | | N_{MC} | =500 | | | N _{MC} | =1000 | | |
| | Optimal Order – 6 th , Avg_dif =0.0162 | | Optimal Order – 7 th , Avg_dif =0.0161 | | Optimal Order – 6 th , Avg_dif =0.0164 | | Optimal Ore Avg_dif = | |] | | Optimal Order – 6 th , Avg_dif =0.0162 | | Optimal Order – 7 th , Avg_dif =0.0161 | | Optimal Order – 6 th , Avg_dif =0.0164 | | rder – 7 th , =0.0162 | |
| | No. of Coefficients Out of Bound | Fault Detecti on Status | No. of Coefficients Out of Bound | Fault Detecti on Status | No. of Coefficients Out of Bound | Fault Detectio n Status | No. of Coefficients Out of Bound | Fault Detecti on Status | | No. of Coefficients Out of Bound | Fault Detectio n Status | No. of Coefficients Out of Bound | Fault Detectio n Status | No. of Coefficients Out of Bound | Fault Detectio n Status | No. of Coefficients Out of Bound | Fault Detectio n Status | |
| R1 6% ↑ | 0 | Х | 1 | √ | 0 | х | 6 | √ | Rf1 2%↓ | 0 | х | 0 | х | 0 | х | 0 | x | |
| R1 5.5% ↓ | 0 | х | 1 | √ | 0 | х | 26 | √ | Rf2 1%↑ | 0 | х | 14 | √ | 0 | х | 14 | √ | |
| R _{£2} 5.8% ↑ | 0 | х | 28 | √ | 0 | х | 27 | √ | R1 3%↑ | 0 | х | 0 | х | 0 | х | 1 | √ | |
| R _{£2} 6% ↓ | 5 | √ | 35 | √ | 5 | √ | 1 | √ | Chf 3%↓ | 0 | х | 12 | √ | 0 | х | 14 | √ | |
| R _{fl} 7% ↑ | 0 | х | 3 | √ | 0 | х | 1 | √ | Rcomp1 | | Х | | √ | | Х | | √ | |
| R _{fl} 8.5% ↓ | 0 | х | 4 | √ | 0 | х | 32 | √ | 2.5% ↑ | 0 | x | 34 | x | 0 | x | 12 | x | |
| Recourse 9% ↑ | 20 | √ | 33 | √ | 16 | √ | 25 | √ | Rcomp 3.5% ↓ | 0 | ^ | 0 | ^ | 0 | _ ^ | 0 | _ ^ | |
| R ₅₀₀₀₀ , 6.8% ↓ | 26 | √ | 16 | √ | 26 | √ | 35 | √ | Ccomp2 <u>%</u> ↑ | 0 | x | 15 | √ | 0 | х | | √ | |
| R _{comp1} 7.5%↑ | 0 | х | 1 | √ | 0 | х | 15 | √ | R1 2% ↓ | | х | | х | | х | 11 | х | |
| R _{compl} 5.6%↓ | 0 | х | 1 | √ | 0 | х | 35 | √ | Rcomp1 | 0 | х | 0 | x | 0 | x | 0 | √ | |
| Cht_8% ↑ | 18 | √ | 27 | √ | 3 | √ | 2 | √ | 1.5% ↓ | 0 | | 0 | | 0 | | 1 | | |
| Chf. 10 % ↓ | 0 | Х | 0 | Х | 0 | х | 35 | √ | Rcomp_2%↑ | _ | х | | х | | Х | | х | |
| <u>Ccorpp</u> ,6%↑ | 1 | √ | 2 | √ | 1 | √ | 7 | √ | | 0 | | 0 | | 0 | | 0 | | |
| Ç _{сопур,} 6%↓ | 0 | Х | 1 | √ | 0 | √ | 25 | √ | | PI COMPENSATOR OF A BUCK CONVERTER Various Detected Faults PI controller in the presence of MULTIPLE faults with deviations | | | | | | | | |
| R <u>1 43% ↑</u> | 1 | √ | 2 | √ | 1 | √ | 5 | √ | Injected | various Detec | | =500 | tne presence | e of MCLTIPLI | | | | |
| R1 28% ↓ | 1 | √ | 2 | √ | 1 | √ | 35 | √ | Faults | Optimal Or | | | odon 7th | Ontimal On | N _{MC} =1000 | | | |
| Rf2_35%_↑ | 22 | √ | 7 | √ | 18 | √ | 35 | √ | | Avg_dif = | =0.0162 | rder – 6 th , Optimal Order – 7 th , =0.0162 Avg_dif =0.0161 | | Optimal Order – 6 th , Avg_dif =0.0164 | | Optimal Order – 7th Avg_dif =0.0162 | | |
| Rf2_40% ↓ | 18 | √ | 1 | √ | 0 | Х | 36 | √ | | No. of Coefficients | Fault Detectio | No. of Coefficients | Fault Detectio | No. of Coefficients | Fault Detectio | No. of Coefficients | Fault Detectio | |
| Rf1 30% ↑ Rf1 25% ↓ | 26 | √ | 33 | √ | 26 | √ | 35 | √ √ | - | Out of Bound | n Status | Out of Bound | n Status | Out of Bound | n Status | Out of Bound | n Status | |
| Rcomp 42% | 27 0 | X | 33 24 | √ | 27 0 | X | 35 18 | √ | Comp 10%↓ Chf 20%↑ R1 12%↓ | | 1 | | √ | | , , | | 1 | |
| Rcomp 38% | 0 | х | 1 | √ | 0 | х | 35 | √ | R1 12%1 | 24 | | 24 | √ | 24 | -J | 36 | | |
| Rcomp1 37%↑ | 28 | √ | 35 | √ | 28 | √ | 15 | √ | Rcomp1 - 22%↑ Rcomp=13 %↓ | | | | | | | | - | |
| Rcomp1 40%↓ | 27 | √ | 36 | √ | 27 | √ | 13 | √ | Rcomp1 9%↓ Rcomp_ 13%↑ | 27 | √ | 35 | х | 27 | ✓ | 35 | √ | |
| Chf 28% ↑ | 0 | х | 1 | √ | 0 | х | 8 | √ | Rf1 18%↑ | | | | | | | | | |
| Chf 35% ↓ | 0 | х | 1 | √ | 0 | х | 35 | √ | Rf2 8% Ccomp 15% | 4 | √ | 0 | √ | 1 | -√ | 35 | -√ | |
| Ccomp 32% | 27 | √ | 35 | √ | 27 | √ | 35 | √ | Chf 11%↑ Rfl 9%↓ | | | | | | | | | |
| Ccomp45%↓ | 28 | √ | 36 | √ | 28 | √ | 34 | √ | Rf2 19%↑ R1 8%↑ | 3 | ✓ | 2 | -√ | 2 | -√ | 36 | √ | |
| COMMISSION | | | | | | | | | Rf1 12%↓ Rf2 14%↑ | 27 | | 35 | | 27 | | 34 | | |

NOISE ANALYSIS



- •Real-world circuits are affected by transient and steady-state noise.
- •Noise impacts behavior, performance, and fault detectability.
- •Simulating noise improves test robustness.



Noise Simulation Techniques

1. Random Noise Source

- •Add a voltage/current source with a defined DC value and noise amplitude.
- •Simulates real-time fluctuations at each time step.
- •Example: $1V \pm 0.2V$ generates a range from 0.9V to 1.1V.

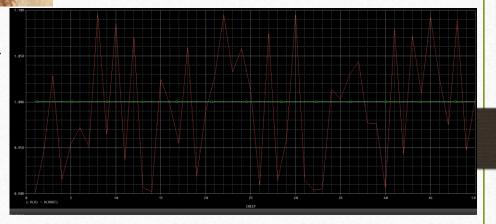
2. RNDR Function (Transient/DC Analysis)

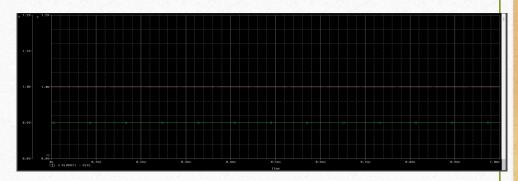
- •Injects a constant random value for the full simulation.
- •Example: (RNDR 0.5)/5 \rightarrow adds $\pm 0.1 \mathrm{V}$ random offset.

3. RNDC Function (Sweeps/Monte Carlo)

- •Generates different random values per sweep.
- •Useful for Monte Carlo, DC Sweep, Temperature Analysis.







COMPARISON

LEADLAG - in case of AC INPUT (SINE WAVE) with NOISE of 0.15mV

Various Detected Faults in LEADLAG in the presence of faults with deviations (5%-15%) & (20%-47%)

 N_{MC} =500, Avg_dif = 0.0420, Optimal Order – 18th

| Injected Faults | WITHOUT | NOISE | WITH NOISE | |
|--------------------|----------------------------------|------------------------|----------------------------------|------------------------|
| | No. of Coefficients Out of Bound | Fault Detection Status | No. of Coefficients Out of Bound | Fault Detection Status |
| R1 12%↑ | 113 | √ | 151 | √ |
| R1 8% ↓ | 1 | \checkmark | 73 | \checkmark |
| R2 9% ↑ | 0 | X | 63 | \checkmark |
| R2 6%↓ | 0 | X | 64 | √ |
| R3 10% ↑ | 71 | V | 61 | √ |
| R3 7%↓ | 0 | X | 100 | √ |
| C1 13%↑ | 0 | X | 66 | √ |
| C1 15% ↓ | 3 | √ | 63 | √ |
| C2 11% ↑ | 0 | X | 62 | √ |
| C2 14% ↓ | 10 | √ | 71 | √ |
| R1 20%↑ R1 30%↓ | 163 | √ , | 170 | √ |
| R2 32% | 183 | V | 179 | V |
| ↑ | 34 | √ | 73 | √ |
| R2 40% ↓ | 104 | V | 124 | √ |
| R3 25% ↑ | 174 | √ | 161 | √ |
| R3 45% ↓ | 185 | V | 182 | √ |
| C1 42% ↑ | 17 | $\sqrt{}$ | 77 | √ |
| C1 25% ↓ | 15 | V | 72 | √ |
| C2 35% ↑ | 38 | V | 72 | V |
| C2 47% ↓ | 140 | V | 149 | V |

CONCLUSION

- ☐ Polynomial regression is effective for analog fault testing across both linear and non-linear circuits.
- ☐ Fault detection is accurate for:
 - Single faults: Detected reliably at deviations > ±5% (within expected design tolerances).
 - Multiple faults: Also successfully detected when two or more components deviate beyond tolerance.
- ☐ For deviations ≤ ±5%, faults are not flagged, as this lies within the allowable tolerance margin set during Monte Carlo construction.
- ☐ **Higher polynomial orders** improve detection capability but may introduce complexity or overfitting.
- ☐ Noise Robustness:
 - Verified using RNDR/RNDC noise sources in OrCAD PSpice.
 - Compared coefficients under noisy and clean conditions.

Future Work

Fault Diagnosis (Not Just Detection)

- Extend current approach to identify which component is faulty, not just detect that a fault exists.
- Use sensitivity ranking and component-wise coefficient deviation tracking.

Advanced Noise Modeling

- Simulate real-world noise sources including thermal drift, EMI, and fluctuating power supply noise.
- Analyze model stability under high-noise, low-SNR environments.

Integration with Machine Learning

 Combine polynomial features with ML classifiers (e.g. Decision Trees, SVMs) to detect and categorize multiple fault types

Toolchain Automation

Build a Python/MATLAB-based GUI or pipeline to automate tentire flow:
 Circuit → Simulation → Regression → Fault Report

References

- [1] S. Sindia, V. D. Agrawal and V. Singh, "Non-linear analog circuit test and diagnosis under process variation using V-Transform coefficients," 29th VLSI Test Symposium, Dana Point, CA, USA, 2011, pp. 64-69, doi: 10.1109/VTS.2011.5783756.
- [2] S. Sindia and V. D. Agrawal, "High sensitivity test signatures for unconventional analog circuit test paradigms," 2013 IEEE International Test Conference (ITC), Anaheim, CA, USA, 2013, pp. 1-10, doi: 10.1109/TEST.2013.6651884.
- [3] P. Bilski, "Analysis of the Ensemble of Regression Algorithms for the Analog Circuit Parametric Identification," Measurement, vol. 160, 2020, Art. no. 107829, doi: 10.1016/j.measurement.2020.107829.
- [4] Zhao, Dongsheng, and Yuzhu He. "A new test point selection method for analog circuit." Journal of Electronic Testing 31 (2015): 53-66.
- [5] A. A. Hatzopoulos, "Analog circuit testing," 2017 International Mixed Signals Testing Workshop (IMSTW), Thessaloniki, Greece, 2017, pp. 1-6, doi: 10.1109/IMS3TW.2017.7995206.
- [6] H. Kobayashi et al., "Analog/Mixed-Signal Circuit Testing Technologies in IoT Era," 2020 IEEE 15th International Conference on Solid-State & Integrated Circuit Technology (ICSICT), Kunming, China, 2020, pp. 1-4, doi: 10.1109/ICSICT49897.2020.9278194.
- [7] Sindia S, Agrawal VD, Singh V (2012) Parametric Fault Testing of Non-Linear Analog Circuits Based on Polynomial and V-Transform Coefficients. J Electron Test 28:757–771. https://doi.org/10.1007/s10836-012-5326-z
- [8] V. Riewruja and A. Rerkratn, "Four-quadrant analogue multiplier using operational amplifier," *International Journal of Electronics*, vol. 98, no. 4, pp. 459–474, 2011, doi: 10.1080/00207217.2010.520155.

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