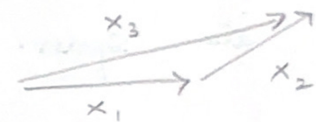
VECTORS & MATRICES

$$\boxed{\begin{array}{l} \bar{v} \cdot \bar{w} = |v||w| \cos\theta \\ \bar{v} \times \bar{w} = |v||w| \sin\theta \\ \\ \bar{v} \cdot \bar{w} \neq \bar{w} \cdot \bar{v} \\ \bar{v} \times \bar{w} = -\bar{w} \times \bar{v} \end{array}}$$

→ Vector ($\bar{v}$)

$$\begin{bmatrix} x \\ y \end{bmatrix} = x\hat{i} + y\hat{j}$$

$$\underset{X_1}{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}} + \underset{X_2}{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}} = \underset{X_3}{\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}}$$



$$S \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} S \cdot x \\ S \cdot y \end{bmatrix}$$

$$\underset{2\times 2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}}$$

① $M_1 \cdot M_2 \neq M_2 \cdot M_1$ but $M_1(M_2 \cdot M_3) = (M_1 M_2) \cdot M_3$

② for $A\vec{x} = \vec{v}$
$\Rightarrow \vec{x} = A^{-1}\vec{v}$] derived from $A^{-1} \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

③ $L(\vec{v} + \vec{w}) = L\vec{v} + L\vec{w}$ and $L(c\vec{v}) = cL\vec{v}$

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \cdot \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \cdot \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

→ Eigen Vector

These vectors which do not change their span on linear transformation

→ Eigen Value

It refers to the magnitude by which an eigen vector stretches /squishes about its span.

Mathematically, $A\vec{v} = \lambda \vec{v}$

$A\vec{v}$ → matrix - vector dot product

$\lambda \vec{v}$ → Scalar × vector

$\Rightarrow \quad A\vec{v} = (\lambda I)\vec{v}$

(λI) → becomes a matrix

$\Rightarrow \quad A\vec{v} - (\lambda I)\vec{v} = 0$

$\Rightarrow \quad (A - \lambda I) v = 0$

$\Rightarrow \quad \det (A - \lambda I) = 0$ ⟶ solving this gives us the value of λ. i.e. Eigen Value