MAT9004 MATHEMATICAL FOUNDATIONS FOR DATA SCIENCE

WEEK 1 – FUNCTIONS

Set – unordered collection, unique items

$x \in S$, $x \notin S$, $\{x \in S : P(x)\}$, $[a,b]$, (a,b)

cond^n closed open

interval

$$\sum_{x=a} f(x) = f(a) + f(a+1) + \dots f(b-1) + f(b)$$

$$\prod_{x=a}^{b} f(x) = f(a) \times f(a+1) \times \dots f(b-1) \times f(b)$$

$f : X \to Y$ $\{f(x) : x \in X\}$

domain co-domain image (set of outputs)

all possible outputs for given range of inputs

Zeroes of $f(x) = \{x \in X : f(x) = 0\}$

roots

Inverse : f^{-1} $\lceil f : X \to Y$

$\llcorner\ f^{-1} : Y \to X$

$f^{-1}(f(x)) = x \qquad f^{-1}(x) \neq \frac{1}{x}$

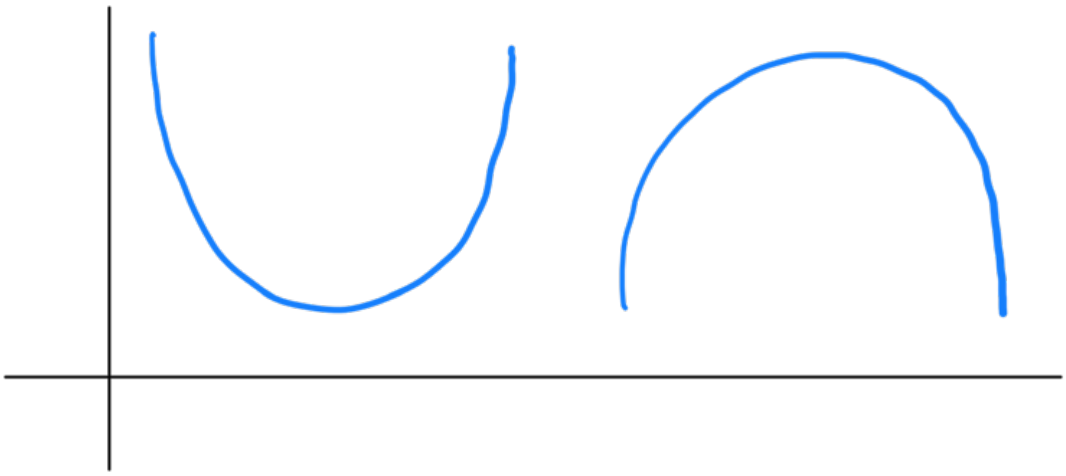
TYPES OF FUNCTION

(A) Injective — One to One (1)

Surjective — for every $f^{-1}(Y)$ there exists a $x \in X$ (2)

Bijective — BOTH (1) and (2)

(B) Convex Concave



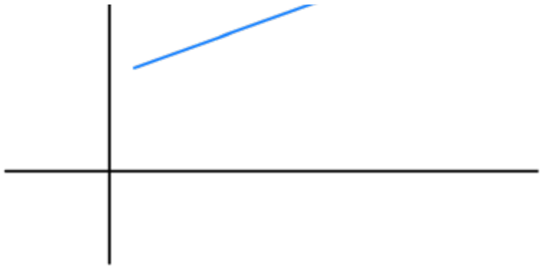
(C) Linear

$f(x) = mx + b$

Only 1 zero at $x = -b/m$

Bijective if $m \neq 0$

both convex and concave



Polynomial

$$f(x) = \sum_{i=0}^{n} a_i x_i$$

$$= a_0 + a_1 x + a_2 x^2 + \dots a_n x^n$$

$$\downarrow$$

$$x^0 = 1$$

Exponential

$$f(x) = b \cdot a^x$$

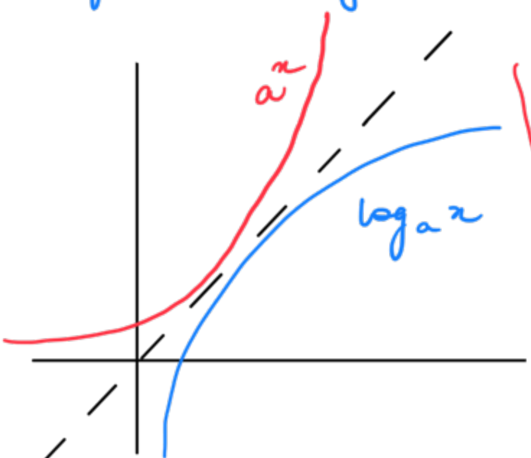
Logarithmic

$$f(x) = \log_a(x)$$

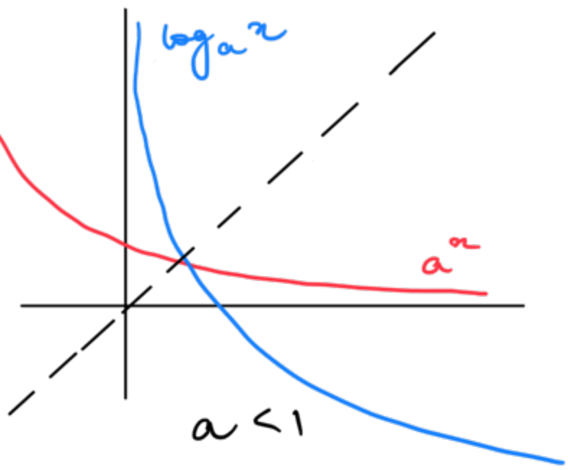
inverse of each other

if $f(x) = a^x$

$f^{-1}(x) = \log_a(x)$



$a > 1$



$a < 1$

$$\log_a(mn) = \log_a m + \log_a n$$

$$\log_a(m/n) = \log_a m - \log_a n$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$\log_a x^m = m \log_a x \longrightarrow \log_a(a^x) = x$

$a^{\log_a x} = x$

Power law

$f(x) = b \cdot x^{-a}$

exp $f(x)$ decay faster than power laws

$\downarrow$

$b \cdot a^x$ log $\longrightarrow$ power law $\longrightarrow$ st. line

WEEK 2 — DIFFERENTIATION

LINEAR TRANSFORMATION

* log $\log_a(x)$

* log - log power

data (x_1, y_1) --- (x_n, y_n)

plot $(\ln x_1, \ln y_1)$ --- $(\ln x_n, \ln y_n)$

$f(x) = b\, x^{-a}$ $[f(x) = y]$

$\ln(y) = \ln b + (-a) \ln x$

$\hat{y} = \ln b - a\ \hat{x}$

$m = (-a)$ and $b = \ln(b)$

* Semi - log

log lin   lin log

exp   log

log lin

$(x_1, \ln(y_1) - - - - x_n, \ln(y_n))$

$f(x) = b \cdot a^x$

$\ln(y) = \ln b + x \ln a$

$\hat{y} = (\ln a) x + \ln b$

$m = \ln a$ and $b = \ln(b)$

lin log

$(\ln(x_1), y_1 - - - - \ln(x_n), y_n)$

$f(x) = b \cdot \log_a x$

$= b \cdot \frac{\ln x}{\ln a}$

$y = \frac{b}{\ln a} (\ln x)$

$\hat{y} = \frac{b}{\ln a} \cdot \tilde{x}$ $m = b / \ln(a)$

DERIVATIVE

$f(x) \longrightarrow f'(x)$: slope of tangent

c	0
x^n	$n x^{n-1}$
a^x	$\ln a \cdot a^x$
$\log_a x$	$\frac{1}{\ln a \cdot x}$

$e^x \rightarrow e^x$

$\ln x \rightarrow \frac{1}{x}$

$f(x) + g(x)$ $f'(x) + g'(x)$

$f(x) - g(x)$ $\quad f'(x) - g'(x)$

$f(x)\, g(x)$ prod rule $\quad f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$f(g(x))$ chain rule $\quad g'(x) \cdot f'(g(x))$

$|x|$ is not differentiable

$f(x) = x^3 \quad f'(x) = 3x^2 \quad f''(x) = 6x \quad f'''(x) = 6$

WEEK 3 – OPTIMISING FUNCTION

INCREASING FUNCTION

$x \uparrow$ and $f(x) \uparrow$

$f'(x) > 0$

DECREASING FUNCTION

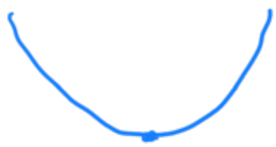
$x \downarrow$ and $f(x) \downarrow$

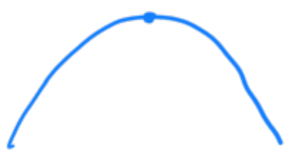
$f'(x) < 0$

x where $f'(x) = 0$: STATIONARY POINT

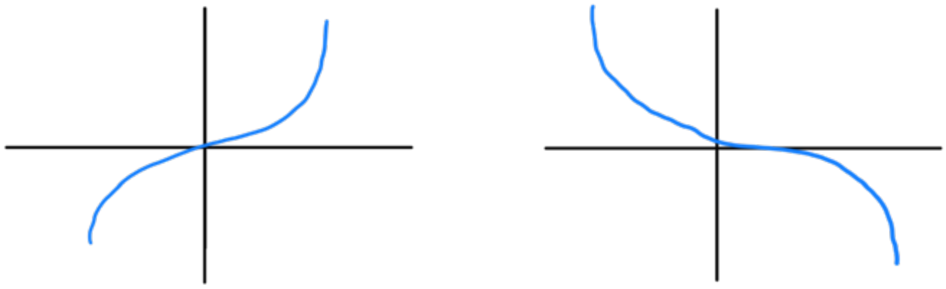
LOCAL MIN. LOCAL MAX

$f'(x)$ changes from $\quad$ −ve to +ve $\quad$ +ve to −ve

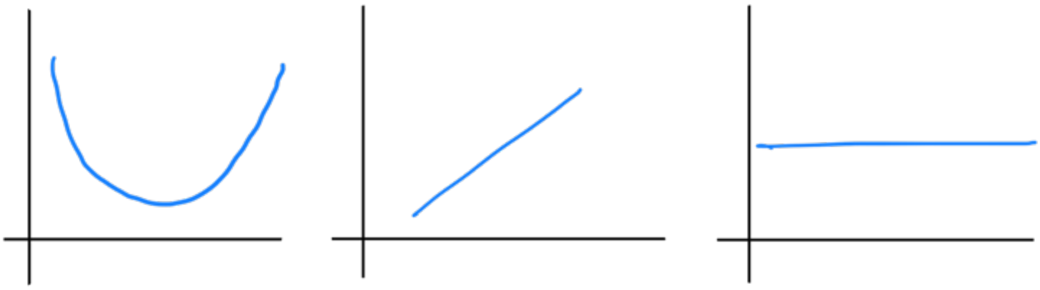




Inflection point – st. pt. but neither local max nor local min

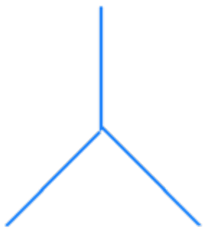


ex –



$f(x)$ $\qquad$ $f'(x)$ $\qquad$ $f''(x)$

GLOBAL EXTREMUM



MIN. $\qquad$ MAX

$f(a) \le f(x)$ $\qquad$ $f(a) \ge f(x)$ $\qquad$ $\forall x \in X$

for a $f(x)$ with $x \in [a,b]$, we may get global extrema at :

end pt. $\quad$ 1. a or b

st. pt. $\quad$ 2. c where $f'(c) = 0$

critical pt. $\quad$ 3. c where $f'(c)$ does not exist

i.e. $\quad$ $f''(x) > 0$ $\qquad$ Convexity

$f'(x) < 0$ concavity

$\because$ if a $f(x)$ is completely concave or completely convex, then its local extrema is also the global extrema.

$\hookrightarrow$ if NOT, break into sub domains

RESIDUAL SUM OF SQUARES

for data (x_1, y_1) ---- (x_n, y_n)

$$RSS = \sum_{i=1}^{n} (y_i - f(x_i))^2$$

Squaring to measure deviations ?

$\because$ it allows finding derivatives and penalise large deviations heavily

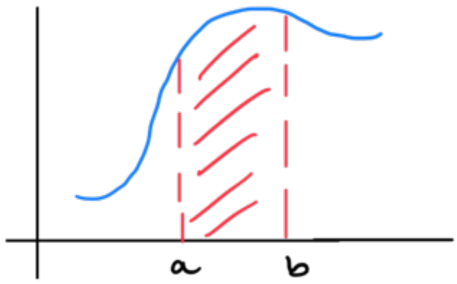
WEEK 4 — INTEGRATION

$$f(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x)$$

integrate

$\downarrow$

anti-derivate



- area under the curve

$\downarrow$

area below x-axis is -ve

$$\int_a^b f(x) \cdot dx = F(b) - F(a)$$

x^a $\qquad$ $\frac{1}{a+1} x^{a+1}$

$\frac{1}{x}$ $\quad\quad\quad$ $\ln(x)$

e^{ax} $\quad\quad\quad$ $\frac{1}{a}e^{ax}$

WEEK 5 — LINEAR ALGEBRA

VECTORS

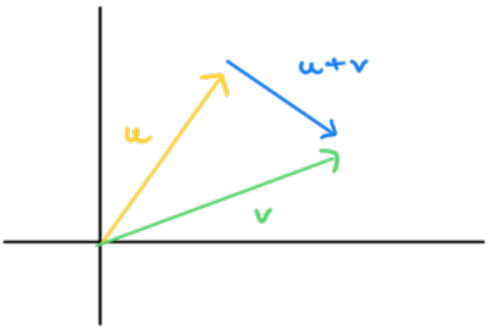
$$\underset{\text{set of d tuples}}{\mathbb{R}^d} = \left\{ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} : x_1, x_2, \ldots x_n \in \mathbb{R}^d \right.$$

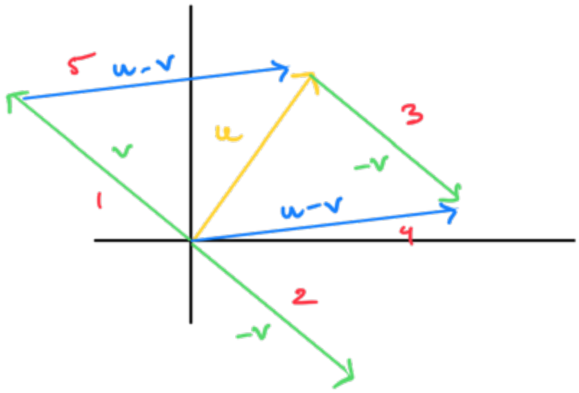
ex- $\mathbb{R}^3$ contains $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2.5 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} e \\ \pi \end{pmatrix}$

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ \vdots \\ x_n+y_n \end{pmatrix}$$

$$c\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} cx_1 \\ cx_2 \\ \vdots \\ cx_n \end{pmatrix}$$

a vector $(v, \vec{v}, \tilde{v})$ has length and dir^n but no pos^n





A line joining points u and v

contains v contains the points corresponding to $\alpha u + (1-\alpha)v$ where $\alpha \in [0,1]$

$$w = a_1 v_1 + a_2 v_2 - - - - + a_n v_n$$

linear comb^n of $v_1 - - - v_n$

$\rightarrow$ Linear dependence leads to redundancy

If not, then linearly Independent

ex - $\begin{pmatrix}1\\0\end{pmatrix}$ & $\begin{pmatrix}0\\1\end{pmatrix}$ are linearly independent

but $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\1\\0\end{pmatrix}$ are not.

as $$\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix}$$

if $v = \begin{pmatrix}v_1\\ \vdots\\ v_n\end{pmatrix}$ $\quad w = \begin{pmatrix}w_1\\ \vdots\\ w_n\end{pmatrix}$

$$v \cdot w = (v, w)$$
$$= v_1 w_1 + v_2 w_2 - - - - v_n w_n$$

if $v \cdot w = 0$ then v and w are orthogonal
i.e. perpendicular

EUCLIDEAN NORM

$$\|v\| = \sqrt{v_1^2 + v_2^2 + v_3^2 + - - - v_n^2}$$
$$= \sqrt{v \cdot v}$$

we can't use the term length in case of multiple dimensions therefore it is referred to as NORM

MATRICES

$(m \times n)$ matrix where m = row
n = column.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

$$c \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} & a_{11} b_{12} + a_{11} \cdot b_{22} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} \end{pmatrix}$$

$$M_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad M_2 = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\det M_1 = ad - bc$$

$$\det M_2 = a(ei - fh) - b(di - gf) + c(dh - ge)$$

Identity Matrix $(I) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Zero Matrix $= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

if $B \cdot A = I$ then $B = A^{-1}$ and $A = B^{-1} \rightarrow AB = I$

$A \cdot B \neq B \cdot A$

$\det(AB) = \det(A) \cdot \det(B)$

$\det(I) = 1$
$\det(A) \neq 0$ if A^{-1} exists

$\rightarrow$ let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

then $A^{-1} = \dfrac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$Ax = b \implies x = A^{-1}b.$

GAUSSIAN ELIMINATION

Steps –
1. Swap 2 rows
2. Mul. a row by non zero num.
3. Add a multiple of one row to another.

to achieve $\begin{pmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} * \\ * \\ * \end{pmatrix}$

for ex – $R_1 \longleftrightarrow R_2$

$R_2 \longrightarrow 5 * R_2$

$R_3 \longrightarrow 2R_3 - R_2$

Types of Solution :

1. Exactly ONE solution
2. NO solution

$$\begin{pmatrix} * & * \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} * \\ * \end{pmatrix}$$

$* \equiv$ non zero num. $\quad 0x + 0y = *$

$0 \neq *$

3. MULTIPLE Solutions

ex – $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$x + z = 1$
$y - 2z = 0$

$\Rightarrow$ $x = 1 - z$ and $y = 2z$
put $z = t$

Sol^n $= (1-t, 2t, t)$

$\rightarrow$ free variable : INFINITE sol^n(s)

WEEK 6 — EIGEN VALUES, VECTOR

$A x = \lambda x$ $\rightarrow$ eigen vector (can't be zero)

$\downarrow$

eigen value

$\Rightarrow (A - \lambda I) x = 0$ ①

$\det (A - \lambda I) = 0$ Characteristic eq^n ②

$n \times n$ matrix $\rightarrow n$ sol^n(s) $\rightarrow n$ eigen values

Eigen values aren't unique (multiples of λ)

Steps –

1. find λ from $\det(A - \lambda I) = 0$
2. use λ to find x from $(A - \lambda I) x = 0$

A is diagonalisable if $A = PDP^{-1}$ and

using $\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \end{pmatrix}^n = \begin{pmatrix} x^n & 0 & 0 \\ 0 & y^n & 0 \end{pmatrix}$ we get.

$\begin{pmatrix} 0 & 0 & z \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & z^n \end{pmatrix}$

$$\boxed{A^n = P D^n P^{-1}}$$

$A^n \rightarrow$ $n \times n$ matrix

$D^n \rightarrow \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ upto λ_n

$P \rightarrow (v_1, v_2, \ldots v_n)$

WEEK 7 — BINARY RELATIONS

a binary rel^n relates 2 parameters

ex- points in xy plane $(1,2)$

every $f(x) \xrightarrow{\text{GIVES}}$ rel^n but NOT conversely

$\times$

eq^n of circle : $(x-u)^2 + (y-v)^2 = r^2$

ellipse : $\dfrac{(x-u)^2}{a^2} + \dfrac{(y-v)^2}{b^2} = r^2$

centre at (u,v)

MULTIVARIATE FUNCTION

$z = f(x,y)$

$f_x = \dfrac{\partial}{\partial x}(f) \qquad f_y = \dfrac{\partial}{\partial y}(f)$

$\Rightarrow$ PARTIAL DERIVATIVE

$\nabla f = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \rightarrow$ GRADIENT VECTOR

WEEK 8 — OPTIMISING MULT. FUNCTIONS

$$f(x+\Delta x) = f(x) + f'(x)\Delta x$$

$$f(x+\Delta x,\ y+\Delta y) \approx f(x,y) + f_x(x,y)\Delta x + f_y(x,y)\Delta y$$

STATIONARY POINT

$f(x,y)$ where $f_x = 0$ and $f_y = 0$

implying $\nabla f(x,y) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and is either local extrema or saddle pt.

HESSIAN MATRIX

$$H(x,y) = \begin{bmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{bmatrix}$$

second partial derivative

if $f_{xy} = f_{yx}$ the nice funcⁿ

$$D = \det(H(x,y))$$

$D < 0$ $\quad$ $D = 0$ $\quad$ $D > 0$

saddle pt. unconclusive local extremum

$f_{xx} > 0$ local min

$f_{xx} < 0$ local max.

GLOBAL EXTREMA

possible candidates – 1. stationary pt.s

f_x & f_y are undefined ← 2. singular pt.s

3. boundary

WEEK 9 — COMBINATORICS

Multiplication Rule $|S| = \prod_{j=1}^{n} k_j$ $k \times l$

Addition Rule $|S| = \sum_{j=1}^{n} S_j$ $k + l$

Complement Rule $S = S_g \cup S_b$, $S_g \cap S_b = 0$ $\Big\}$ $1-k$, $S_g = S - S_b$

TYPES OF SELECTION (4)

order matters: Y / N

rep^n allowed: Y / N

order matters → $AB \neq BA$

ordered sel^n w/o rep^n $\frac{n!}{(n-r)!}$

unordered sel^n w/o rep^n $\frac{n!}{r!(n-r)!}$

ordered sel^n with rep^n n^r

unordered sel^n with rep^n $\dfrac{(n+r-1)!}{r!\,(n-1)!}$

PASCAL'S TRIANGLE

$$\binom{n}{r} = \binom{n}{n-r}$$

PIGEON HOLE PRINCIPAL

1. n items placed in m containers
2. $n > m$
3. at least one container has $\left\lceil \frac{n}{m} \right\rceil$ items

rounding up.

WEEK 10 — PROBABILITY

Sample space : set of possible outcomes

$Pr : S \longrightarrow [0,1]$ prob. funct^n

sum of prob.(s) of outcomes $= 1$

uniform prob. space : each event has equal prob.

Event - subset of sample space.

ex - $Pr(A) = \dfrac{\text{no. of outcomes in } A}{\text{total no. of outcomes}}$

$= \dfrac{|A|}{|S|}$

$$Pr(\bar{A}) = 1 - Pr(A)$$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

if $Pr(A \cap B) = 0 \rightarrow$ mutually exclusive i.e. A and B cannot occur together

Independent Events : $Pr(A \cap B) = Pr(A) Pr(B)$

if 1 true then all true:

1. A and B are independent
2. $\bar{A}$ and $\bar{B}$ are independent
3. A and $\bar{B}$ are independent
4. $\bar{A}$ and B are independent

INDEPENDENT REPEATED TRIALS

Sample space $= S_1 \times S_2$

$$Pr(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B|A) \cdot P(A)}{P(B)}$$

if independent, $P(A|B) = P(A)$

BAYES THEOREM

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

$$= \frac{P(B|A) \cdot P(A)}{P(B)}$$

WEEK 11 — ADVANCED PROBABILITY

RANDOM VARIABLE

$P(X = x)$ a funcⁿ from sample space to $\mathbb{R}$

Random var.s are independent if $\forall$ x & y:

$P(X = x$ and $Y = y) = P(X = x) \cdot P(Y = y)$

Expected Value

$E[x] = p_1 x_1 + p_2 x_2 + \text{---} \; p_n x_n$

weights / probabilities

laws of large no.(s) -

$$\lim_{n \to \infty} \frac{1}{n} (x_1 + x_2 \text{---} x_n) = \mu$$

if $E[x] = \mu$

$Var[x] = E[(x-\mu)^2] = E[x^2] - \mu^2$

variance

$\sigma^2 = Var[x] \rightarrow$ std. dev^n

If X and Y are independent :

$E[X+Y] = E[X] + E[Y] \rightarrow$ linearity of Expec^n.

$E[X \cdot Y] = E[X] \cdot E[Y]$

$E[kX] = k\,E[X]$

$Var[kX] = k^2 \cdot Var[X]$

$Var[X+Y] = Var[X] + Var[Y]$

DISCRETE UNIFORM DISTRIBUTION

set of cons. integers (equally likely)

$P(X=k) = \dfrac{1}{b-a+1}$ for $k \in \{a, a+1 \ldots, b\}$

$E[X] = \dfrac{a+b}{2}$ $\quad Var[X] = \dfrac{(b-a+1)^2 - 1}{12}$

BERNOULLI DISTRIBUTION

prob. that a process succeeds or fails

$$P(X=k) = \begin{cases} p & k=1 \quad \text{success} \\ 1-p & k=0 \quad \text{fail} \end{cases} \qquad p \in [0,1]$$

$E[X] = p \qquad Var[X] = p(1-p)$

GEOMETRIC DISTRIBUTION

Seq of Bernolli trials — k failures before first success.

$P(X=k) = p(1-p)^k$ for $k \in N$

$E[X] = (1-p)/p \qquad Var[X] = (1-p)/p^2$

BINOMIAL DISTRIBUTION

Seq of n independent Bernolli Trials with k success.

$n \in Z^+$ and $p \in [0,1]$

$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k \in \{0,n\}$

POISSON DISTRIBUTION

with an avg of λ events per unit time it gives the prob. that k events occur in unit time.

$$P(X=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!} \qquad \text{for } k \in N$$

$E[X] = \lambda \qquad Var[X] = \lambda$

CONTINUOUS PROBABILITY

CONTINUOUS PROBABILITY

Probability Density Function

→ PDF can not be negative

→ integral of the PDF = 1 (over whole domain)

$P(a \le X \le b) = \int_a^b f \cdot dx$

$E[x] = \int_{-\infty}^{\infty} x \cdot f\,dx$ Var. stays same.

A continous uniform distⁿ has following PDF

$$f(x) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$$

$$E[x] = (a+b)/2 \qquad Var[x] = (b-a)^2/12$$

EXPONENTIAL DISTRIBUTION

Returns the time b/w events in a Poisson process.

→ randomly spread out with even density

PDF : $\lambda e^{-\lambda x}$ for $x \ge 0$

$E[X] = 1/\lambda$ $Var[x] = 1/\lambda^2$

NORMAL DISTRIBUTION

PDF : $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

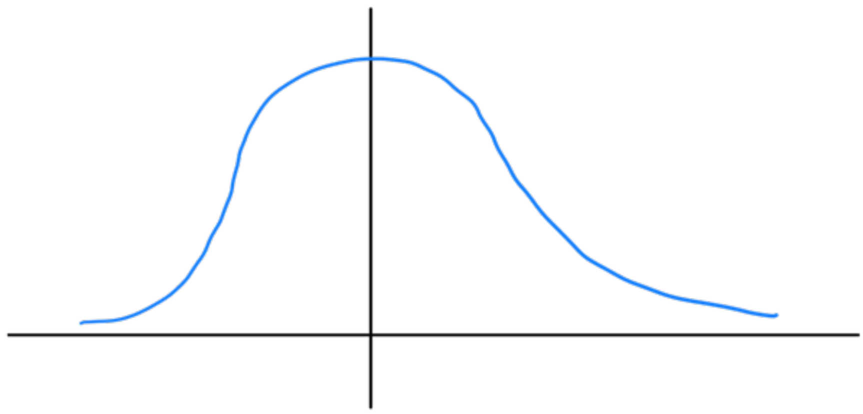
$E[x] = \mu$ $Var = \sigma^2$

many discrete distⁿ(s) tend towards a normal distⁿ

$Z = (x - \mu)/\sigma$

how many std. deviations above or below the mean x is.

$$Pr(a<x<b) = Pr\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$



WEEK 12 — GRAPH THEORY

Graph – vertices + edges

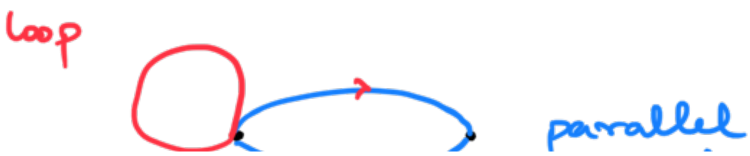
Vertices – may or may not be labelled

vertices sharing an edge are labelled as ADJACENT

Edge – connectⁿ b/w 2 vertices

loop – vertex related to itself

Parallel Edges – multiple routes b/w same vertices



edges.

Multi-graph : loops & parrallel allowed

Simple graph : NO loops / parrallel edges

Digraph – directed graph (dir^n on edges)

Walks – Sequence of vertices (adjacent)

Length of walk – no. of steps.

Path – walk with distinct vertices

Connected Graph Disconnected Graph

Cycle – start vertex is the end ver.

Degree (of a vertex) – no. of edges that include vertex

Regular Graph – If every vertex has same degree k.

k-regular graph.

HANDSHAKING LEMMA

In any graph,

Sum of degrees = 2 × (no. of edges)

⇒ Even no. of vertices of ODD degree.

TREE

Connected graph with no cycles

Prop(s) — If one true, all true:
(i) T is a tree
(ii) Any 2 vertices — linked by unique path
(iii) Delⁿ of an edge — DISCONNECTED
(iv) Adding an edge — CYCLE

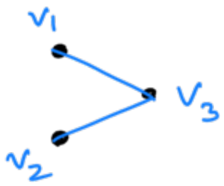
n vertices — (n-1) edges

SPANNING TREE

a tree contained in graph (includes all vertices)

Every connected graph contains a ST.

ADJACENCY MATRIX



$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$a_{ij} = \begin{cases} 0 & \text{adjacent} \\ 1 & \text{Not adjacent} \end{cases}$

EULER CIRCUIT

closed walk — uses every edge once

A connected graph is Eulerian iff every vertex has even degree

Euler Trail — start and end are diff

A connected grap has Euler Trail iff at most 2 vertices have odd degrees

HAMILTONIAN CYCLE

A cycle visiting each vertex exactly ONCE

if graph has hamiltonian cycle $\longrightarrow$ hamiltonian graph