

# k-Nearest Neighbor (kNN) exercise

The kNN classifier consists of two stages:

- During training, the classifier takes the training data and simply remembers it
- During testing, kNN classifies every test image by comparing to all training images and transferring the labels of the k most similar training examples
- The value of k is cross-validated

In this exercise you will implement these steps and understand the basic Image Classification pipeline, cross-validation, and gain proficiency in writing efficient, vectorized code.

In [1]:

```
# Run some setup code for this notebook.

import random
import numpy as np
from cs175.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

# This is a bit of magic to make matplotlib figures appear inline in the notebook
k
# rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

In [2]:

```
# Load the raw CIFAR-10 data.
cifar10_dir = 'cs175/datasets/cifar-10-batches-py'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape: (50000, 32, 32, 3)
Training labels shape: (50000,)
Test data shape: (10000, 32, 32, 3)
Test labels shape: (10000,)
```

In [3]:

```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



In [4]:

```
# Subsample the data for more efficient code execution in this exercise
num_training = 5000
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]

num_test = 500
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]
```

In [5]:

```
# Reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
print(X_train.shape, X_test.shape)
```

```
(5000, 3072) (500, 3072)
```

In [6]:

```
from cs175.classifiers import KNearestNeighbor

# Create a kNN classifier instance.
# Remember that training a kNN classifier is a noop:
# the Classifier simply remembers the data and does no further processing
classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
```

We would now like to classify the test data with the kNN classifier. Recall that we can break down this process into two steps:

1. First we must compute the distances between all test examples and all train examples.
2. Given these distances, for each test example we find the  $k$  nearest examples and have them vote for the label

Lets begin with computing the distance matrix between all training and test examples. For example, if there are  $\mathbf{N_{tr}}$  training examples and  $\mathbf{N_{te}}$  test examples, this stage should result in a  $\mathbf{N_{te} \times N_{tr}}$  matrix where each element  $(i,j)$  is the distance between the  $i$ -th test and  $j$ -th train example.

First, open `cs175/classifiers/k_nearest_neighbor.py` and implement the function `compute_distances_two_loops` that uses a (very inefficient) double loop over all pairs of (test, train) examples and computes the distance matrix one element at a time.

In [7]:

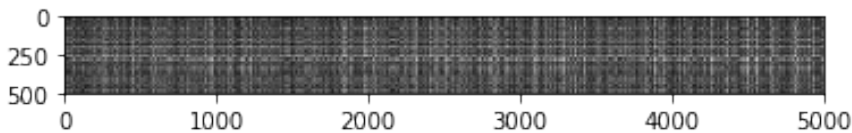
```
# Open cs175/classifiers/k_nearest_neighbor.py and implement
# compute_distances_two_loops.

# Test your implementation:
dists= classifier.compute_distances_two_loops(X_test)
print(dists.shape)
```

(500, 5000)

In [8]:

```
# We can visualize the distance matrix: each row is a single test example and
# its distances to training examples
plt.imshow(dists, interpolation='none')
plt.show()
```



**Inline Question #1:** Notice the structured patterns in the distance matrix, where some rows or columns are visible brighter. (Note that with the default color scheme black indicates low distances while white indicates high distances.)

- What in the data is the cause behind the distinctly bright rows?
- What causes the columns?

**Your Answer:** *Bright rows indicates high distances. High distances may be caused by class in test data which are not in training data or an images which have black background since the rgb value of black is (0,0,0) which increases the L2 distance between training and test data. Similarly, bright columns indicates that the images are different across the column i.e. there are no similar points between test data and training data.*

In [9]:

```
# Now implement the function predict_labels and run the code below:
# We use k = 1 (which is Nearest Neighbor).
y_test_pred = classifier.predict_labels(dists, k=1)

# Compute and print the fraction of correctly predicted examples
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

Got 137 / 500 correct => accuracy: 0.274000

You should expect to see approximately 27% accuracy. Now lets try out a larger k, say  $k = 5$ :

In [10]:

```
y_test_pred = classifier.predict_labels(dists, k=5)
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

Got 139 / 500 correct => accuracy: 0.278000

You should expect to see a slightly better performance than with  $k = 1$ .

In [11]:

```
# Now lets speed up distance matrix computation by using partial vectorization
# with one loop. Implement the function compute_distances_one_loop and run the
# code below:
dists_one = classifier.compute_distances_one_loop(X_test)

# To ensure that our vectorized implementation is correct, we make sure that it
# agrees with the naive implementation. There are many ways to decide whether
# two matrices are similar; one of the simplest is the Frobenius norm. In case
# you haven't seen it before, the Frobenius norm of two matrices is the square
# root of the squared sum of differences of all elements; in other words, reshap
e
# the matrices into vectors and compute the Euclidean distance between them.
difference = np.linalg.norm(dists - dists_one, ord='fro')
print('Difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')
```

Difference was: 0.000000

Good! The distance matrices are the same

In [12]:

```
# Now implement the fully vectorized version inside compute_distances_no_loops
# and run the code
dists_two = classifier.compute_distances_no_loops(X_test)

# check that the distance matrix agrees with the one we computed before:
difference = np.linalg.norm(dists - dists_two, ord='fro')
print('Difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')
```

Difference was: 0.000000

Good! The distance matrices are the same

In [13]:

```
# Let's compare how fast the implementations are
def time_function(f, *args):
    """
    Call a function f with args and return the time (in seconds) that it took to
    execute.
    """
    import time
    tic = time.time()
    f(*args)
    toc = time.time()
    return toc - tic

two_loop_time = time_function(classifier.compute_distances_two_loops, X_test)
print('Two loop version took %f seconds' % two_loop_time)

one_loop_time = time_function(classifier.compute_distances_one_loop, X_test)
print('One loop version took %f seconds' % one_loop_time)

no_loop_time = time_function(classifier.compute_distances_no_loops, X_test)
print('No loop version took %f seconds' % no_loop_time)

# you should see significantly faster performance with the fully vectorized impl
ementation
```

```
Two loop version took 20.948889 seconds
One loop version took 42.650904 seconds
No loop version took 0.297069 seconds
```

## Cross-validation

We have implemented the k-Nearest Neighbor classifier but we set the value  $k = 5$  arbitrarily. We will now determine the best value of this hyperparameter with cross-validation.

In [14]:

```
num_folds = 5
k_choices = [1, 3, 5, 8, 10, 12, 15, 20, 50, 100]

X_train_folds = []
y_train_folds = []
#####
# TODO: #
# Split up the training data into folds. After splitting, X_train_folds and #
# y_train_folds should each be lists of length num_folds, where #
# y_train_folds[i] is the label vector for the points in X_train_folds[i]. #
# Hint: Look up the numpy array_split function. #
#####
X_train_folds=np.array_split(X_train,num_folds)
y_train_folds=np.array_split(y_train,num_folds)
#####
```

```

#                                END OF YOUR CODE                                #

#####

# A dictionary holding the accuracies for different values of k that we find
# when running cross-validation. After running cross-validation,
# k_to_accuracies[k] should be a list of length num_folds giving the different
# accuracy values that we found when using that value of k.
k_to_accuracies = {}

#####
# TODO:                                                                    #
# Perform k-fold cross validation to find the best value of k. For each    #
# possible value of k, run the k-nearest-neighbor algorithm num_folds times, #
# where in each case you use all but one of the folds as training data and the #
# last fold as a validation set. Store the accuracies for all fold and all    #
# values of k in the k_to_accuracies dictionary.                            #
#####
for k in k_choices:
    k_to_accuracies[k]=[]
    for i in range(num_folds):
        training_data=np.concatenate(X_train_folds[:i]+X_train_folds[i+1:])
        training_label=np.concatenate(y_train_folds[:i]+y_train_folds[i+1:])
        knn=KNearestNeighbor()
        knn.train(training_data,training_label)
        y_pred=knn.predict(X_train_folds[i],k,0)
        correct=np.sum(y_train_folds[i]==y_pred)
        k_to_accuracies[k].append(float(correct) / float(len(y_train_folds[i])))

#####
#                                END OF YOUR CODE                                #
#####

# Print out the computed accuracies
for k in sorted(k_to_accuracies):
    for accuracy in k_to_accuracies[k]:
        print('k = %d, accuracy = %f' % (k, accuracy))

```

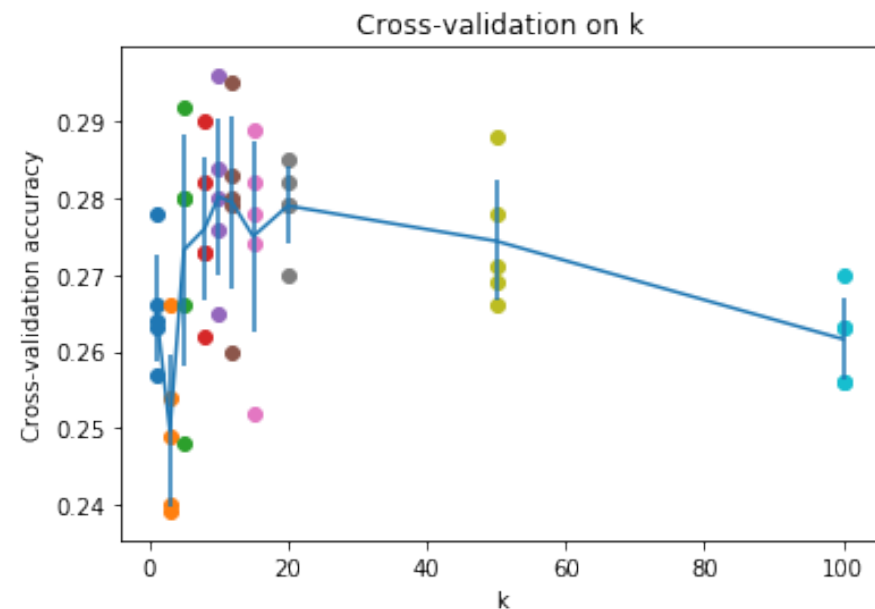
k = 1, accuracy = 0.263000  
k = 1, accuracy = 0.257000  
k = 1, accuracy = 0.264000  
k = 1, accuracy = 0.278000  
k = 1, accuracy = 0.266000  
k = 3, accuracy = 0.239000  
k = 3, accuracy = 0.249000  
k = 3, accuracy = 0.240000  
k = 3, accuracy = 0.266000  
k = 3, accuracy = 0.254000  
k = 5, accuracy = 0.248000  
k = 5, accuracy = 0.266000  
k = 5, accuracy = 0.280000  
k = 5, accuracy = 0.292000  
k = 5, accuracy = 0.280000  
k = 8, accuracy = 0.262000  
k = 8, accuracy = 0.282000  
k = 8, accuracy = 0.273000  
k = 8, accuracy = 0.290000  
k = 8, accuracy = 0.273000  
k = 10, accuracy = 0.265000  
k = 10, accuracy = 0.296000  
k = 10, accuracy = 0.276000  
k = 10, accuracy = 0.284000  
k = 10, accuracy = 0.280000  
k = 12, accuracy = 0.260000  
k = 12, accuracy = 0.295000  
k = 12, accuracy = 0.279000  
k = 12, accuracy = 0.283000  
k = 12, accuracy = 0.280000  
k = 15, accuracy = 0.252000  
k = 15, accuracy = 0.289000  
k = 15, accuracy = 0.278000  
k = 15, accuracy = 0.282000  
k = 15, accuracy = 0.274000  
k = 20, accuracy = 0.270000  
k = 20, accuracy = 0.279000  
k = 20, accuracy = 0.279000  
k = 20, accuracy = 0.282000  
k = 20, accuracy = 0.285000  
k = 50, accuracy = 0.271000  
k = 50, accuracy = 0.288000  
k = 50, accuracy = 0.278000  
k = 50, accuracy = 0.269000  
k = 50, accuracy = 0.266000  
k = 100, accuracy = 0.256000  
k = 100, accuracy = 0.270000  
k = 100, accuracy = 0.263000  
k = 100, accuracy = 0.256000  
k = 100, accuracy = 0.263000



In [15]:

```
# plot the raw observations
for k in k_choices:
    accuracies = k_to_accuracies[k]
    plt.scatter([k] * len(accuracies), accuracies)

# plot the trend line with error bars that correspond to standard deviation
accuracies_mean = np.array([np.mean(v) for k,v in sorted(k_to_accuracies.items())])
accuracies_std = np.array([np.std(v) for k,v in sorted(k_to_accuracies.items())])
plt.errorbar(k_choices, accuracies_mean, yerr=accuracies_std)
plt.title('Cross-validation on k')
plt.xlabel('k')
plt.ylabel('Cross-validation accuracy')
plt.show()
```



In [16]:

```
# Based on the cross-validation results above, choose the best value for k,
# retrain the classifier using all the training data, and test it on the test
# data. You should be able to get above 28% accuracy on the test data.
best_k = 1

classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
y_test_pred = classifier.predict(X_test, k=best_k)

# Compute and display the accuracy
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))

Got 137 / 500 correct => accuracy: 0.274000
```

# Multiclass Support Vector Machine exercise

In this exercise you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

In [1]:

```
# Run some setup code for this notebook.
```

```
import random
import numpy as np
from cs175.data_utils import load_CIFAR10
import matplotlib.pyplot as plt
```

```
from __future__ import print_function
```

```
# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
```

```
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'
```

```
# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

## CIFAR-10 Data Loading and Preprocessing

In [2]:

```
# Load the raw CIFAR-10 data.
cifar10_dir = 'cs175/datasets/cifar-10-batches-py'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape: (50000, 32, 32, 3)
Training labels shape: (50000,)
Test data shape: (10000, 32, 32, 3)
Test labels shape: (10000,)
```

In [3]:

```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



In [4]:

```
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape: (49000, 32, 32, 3)
Train labels shape: (49000,)
Validation data shape: (1000, 32, 32, 3)
Validation labels shape: (1000,)
Test data shape: (1000, 32, 32, 3)
Test labels shape: (1000,)
```

In [5]:

```
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

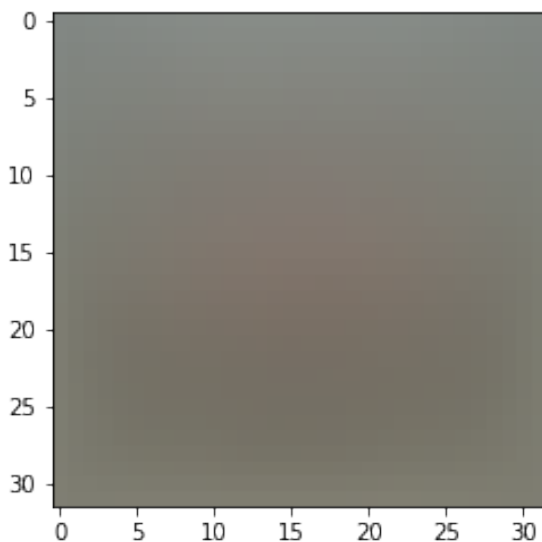
# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)
```

```
Training data shape: (49000, 3072)
Validation data shape: (1000, 3072)
Test data shape: (1000, 3072)
dev data shape: (500, 3072)
```

In [6]:

```
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean i
mage
plt.show()
```

```
[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]
```



In [7]:

```
# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image
```

In [8]:

```
# third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)

(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

## SVM Classifier

Your code for this section will all be written inside **cs175/classifiers/linear\_svm.py**.

As you can see, we have prefilled the function `compute_loss_naive` which uses for loops to evaluate the multiclass SVM loss function.

In [9]:

```
# Evaluate the naive implementation of the loss we provided for you:
from cs175.classifiers.linear_svm import svm_loss_naive
import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.000005)
print('loss: %f' % (loss, ))
```

loss: 8.975407

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

In [10]:

```
# Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions, and
# compare them with your analytically computed gradient. The numbers should matc
h
# almost exactly along all dimensions.
from cs175.gradient_check import grad_check_sparse
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = svm_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad)
```

numerical: 5.452293 analytic: 5.452293, relative error: 9.367385e-11  
numerical: -19.565726 analytic: -19.565726, relative error: 9.166351e-12  
numerical: -17.780532 analytic: -17.780532, relative error: 5.160872e-12  
numerical: -28.739971 analytic: -28.739971, relative error: 2.145735e-11  
numerical: -12.207504 analytic: -12.207504, relative error: 2.104780e-12  
numerical: -3.642083 analytic: -3.642083, relative error: 9.603972e-12  
numerical: -8.588827 analytic: -8.588827, relative error: 6.950765e-12  
numerical: -10.077323 analytic: -10.077323, relative error: 1.018918e-11  
numerical: -34.820867 analytic: -34.820867, relative error: 8.770658e-12  
numerical: 7.105618 analytic: 7.105618, relative error: 2.207735e-11  
numerical: -7.620453 analytic: -7.619434, relative error: 6.686054e-05  
numerical: -27.968442 analytic: -27.963746, relative error: 8.395620e-05  
numerical: -9.587827 analytic: -9.587417, relative error: 2.139413e-05  
numerical: -19.252627 analytic: -19.253781, relative error: 2.997435e-05  
numerical: 35.304743 analytic: 35.303370, relative error: 1.944763e-05  
numerical: 42.305025 analytic: 42.308084, relative error: 3.615000e-05  
numerical: -31.571840 analytic: -31.564771, relative error: 1.119598e-04  
numerical: -8.161366 analytic: -8.160050, relative error: 8.063040e-05  
numerical: -9.447753 analytic: -9.445924, relative error: 9.678954e-05  
numerical: 10.964797 analytic: 10.966333, relative error: 7.002237e-05

## Inline Question 1:

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? *Hint: the SVM loss function is not strictly speaking differentiable*

**Your Answer:** \*SVM loss function is not differentiable at hinge loss. Since gradient is taking the derivative, the hinge loss  $\max(0, 1-x)$  does not at certain points i.e when  $x=1$ . Similarly, the gradient will be different based on the direction. The discrepancy is not for reason of concern because it is caused by the differentiability of loss function.



In [11]:

```
# Next implement the function svm_loss_vectorized; for now only compute the loss
;
# we will implement the gradient in a moment.
tic = time.time()
loss_naive, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs175.classifiers.linear_svm import svm_loss_vectorized
tic = time.time()
loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# The losses should match but your vectorized implementation should be much faster.
print('difference: %f' % (loss_naive - loss_vectorized))
```

```
Naive loss: 8.975407e+00 computed in 0.065717s
Vectorized loss: 8.975407e+00 computed in 0.005323s
difference: -0.000000
```

In [12]:

```
# Complete the implementation of svm_loss_vectorized, and compute the gradient
# of the loss function in a vectorized way.

# The naive implementation and the vectorized implementation should match, but
# the vectorized version should still be much faster.
tic = time.time()
_, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss and gradient: computed in %fs' % (toc - tic))

tic = time.time()
_, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss and gradient: computed in %fs' % (toc - tic))

# The loss is a single number, so it is easy to compare the values computed
# by the two implementations. The gradient on the other hand is a matrix, so
# we use the Frobenius norm to compare them.
difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('difference: %f' % difference)
```

```
Naive loss and gradient: computed in 0.062188s
Vectorized loss and gradient: computed in 0.004839s
difference: 0.000000
```

## Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss.

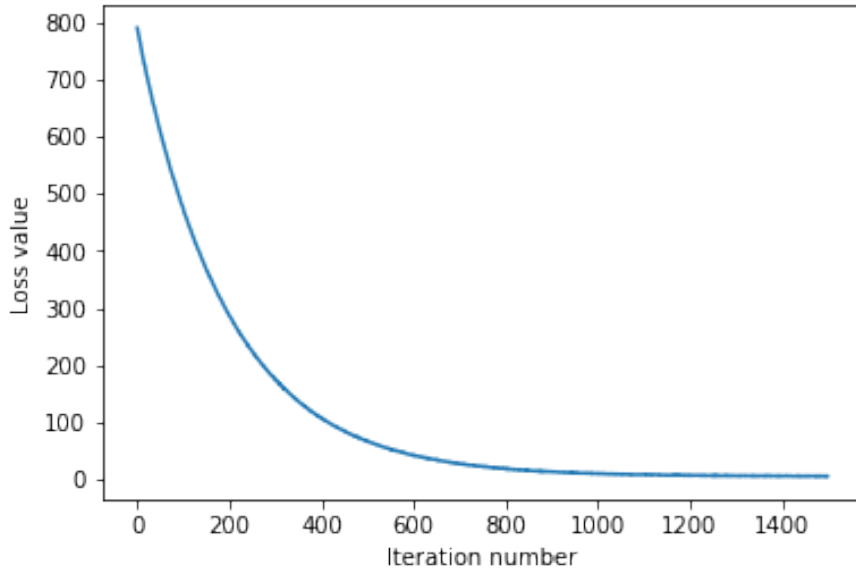
In [13]:

```
# In the file linear_classifier.py, implement SGD in the function  
# LinearClassifier.train() and then run it with the code below.  
from cs175.classifiers import LinearSVM  
svm = LinearSVM()  
tic = time.time()  
loss_hist = svm.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,  
                      num_iters=1500, verbose=True)  
toc = time.time()  
print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 789.448019  
iteration 100 / 1500: loss 473.747724  
iteration 200 / 1500: loss 285.618708  
iteration 300 / 1500: loss 174.405367  
iteration 400 / 1500: loss 107.181009  
iteration 500 / 1500: loss 67.094431  
iteration 600 / 1500: loss 41.567173  
iteration 700 / 1500: loss 27.532363  
iteration 800 / 1500: loss 18.736417  
iteration 900 / 1500: loss 13.567887  
iteration 1000 / 1500: loss 9.557464  
iteration 1100 / 1500: loss 8.876452  
iteration 1200 / 1500: loss 7.179011  
iteration 1300 / 1500: loss 6.162728  
iteration 1400 / 1500: loss 6.228678  
That took 4.377493s
```

In [14]:

```
# A useful debugging strategy is to plot the loss as a function of  
# iteration number:  
plt.plot(loss_hist)  
plt.xlabel('Iteration number')  
plt.ylabel('Loss value')  
plt.show()
```



In [15]:

```
# Write the LinearSVM.predict function and evaluate the performance on both the  
# training and validation set  
y_train_pred = svm.predict(X_train)  
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))  
y_val_pred = svm.predict(X_val)  
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

```
training accuracy: 0.380694  
validation accuracy: 0.370000
```

In [16]:

```
# Use the validation set to tune hyperparameters (regularization strength and  
# learning rate). You should experiment with different ranges for the learning  
# rates and regularization strengths; if you are careful you should be able to  
# get a classification accuracy of about 0.4 on the validation set.  
learning_rates = [1e-7, 5e-5]  
regularization_strengths = [2.5e4, 5e4]  
  
# results is dictionary mapping tuples of the form  
# (learning_rate, regularization_strength) to tuples of the form  
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction  
# of data points that are correctly classified.  
results = {}  
best_val = -1 # The highest validation accuracy that we have seen so far.  
best_svm = None # The LinearSVM object that achieved the highest validation rate
```

```

#####
# TODO: #
# Write code that chooses the best hyperparameters by tuning on the validation #
# set. For each combination of hyperparameters, train a linear SVM on the #
# training set, compute its accuracy on the training and validation sets, and #
# store these numbers in the results dictionary. In addition, store the best #
# validation accuracy in best_val and the LinearSVM object that achieves this #
# accuracy in best_svm. #
# #
# Hint: You should use a small value for num_iters as you develop your #
# validation code so that the SVMs don't take much time to train; once you are #
# confident that your validation code works, you should rerun the validation #
# code with a larger value for num_iters. #
#####

for lr in learning_rates:
    for reg in regularization_strengths:
        svm=LinearSVM()
        svm.train(X_train, y_train, learning_rate=lr, reg=reg, num_iters=800)

        y_train_pred = svm.predict(X_train)
        y_val_pred=svm.predict(X_val)

        accuracy_train=np.mean(y_train==y_train_pred)
        accuracy_val=np.mean(y_val==y_val_pred)

        results[(lr, reg)] = (accuracy_train, accuracy_val)

        if accuracy_val > best_val:
            best_val = accuracy_val
            best_svm = svm

#####
#                                     END OF YOUR CODE #
#####

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val
)

```

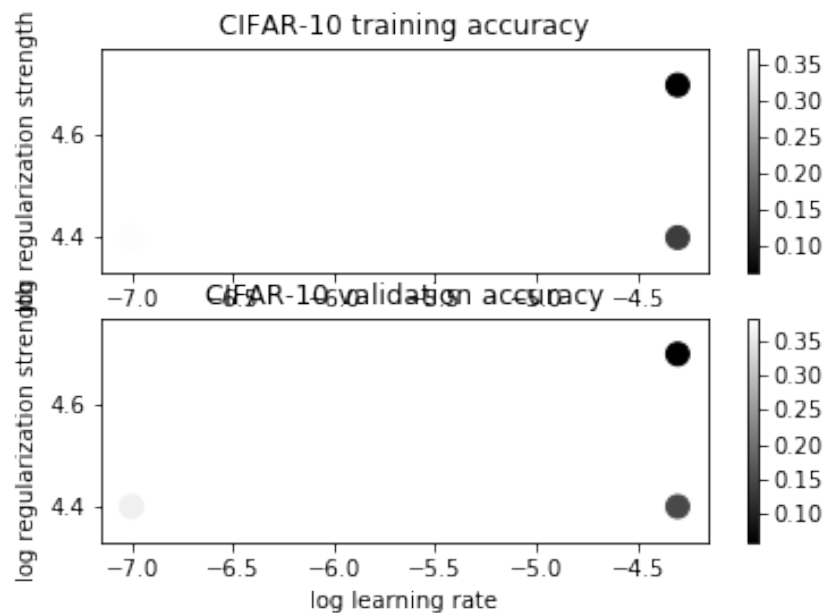
```
lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.366510 val accuracy: 0.362000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.369898 val accuracy: 0.382000
lr 5.000000e-05 reg 2.500000e+04 train accuracy: 0.134755 val accuracy: 0.152000
lr 5.000000e-05 reg 5.000000e+04 train accuracy: 0.060898 val accuracy: 0.059000
best validation accuracy achieved during cross-validation: 0.382000
```

In [17]:

```
# Visualize the cross-validation results
import math
x_scatter = [math.log10(x[0]) for x in results]
y_scatter = [math.log10(x[1]) for x in results]

# plot training accuracy
marker_size = 100
colors = [results[x][0] for x in results]
plt.subplot(2, 1, 1)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 training accuracy')

# plot validation accuracy
colors = [results[x][1] for x in results] # default size of markers is 20
plt.subplot(2, 1, 2)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 validation accuracy')
plt.show()
```



In [18]:

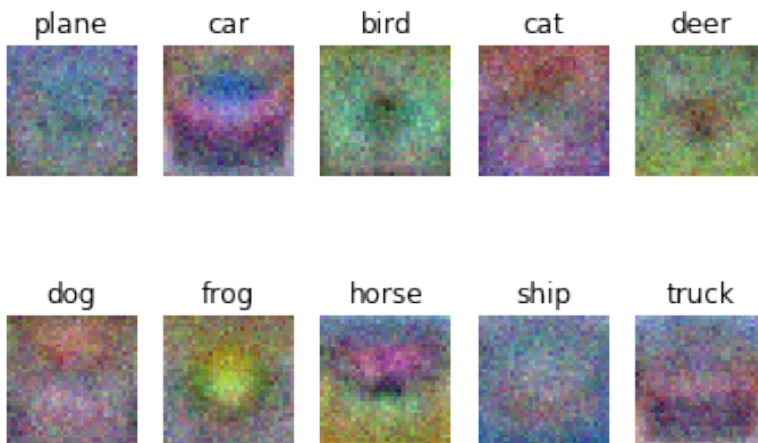
```
# Evaluate the best svm on test set
y_test_pred = best_svm.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('linear SVM on raw pixels final test set accuracy: %f' % test_accuracy)
```

linear SVM on raw pixels final test set accuracy: 0.355000

In [19]:

```
# Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, these may
# or may not be nice to look at.
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```



## Inline question 2:

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look the way that they do.

**Your answer:** Within a class there are different types of images for example in class car there are different types of car regarding make, model, color. Linear SVM generates the weight vector that takes the best generalizes all the image matrices within the class.

In [ ]:

# Softmax exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](http://vision.stanford.edu/teaching/cs175/assignments.html) (<http://vision.stanford.edu/teaching/cs175/assignments.html>) on the course website.

This exercise is analogous to the SVM exercise. You will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

In [1]:

```
import random
import numpy as np
from cs175.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

In [2]:

```
def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000, num_dev=500):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the linear classifier. These are the same steps as we used for the
    SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs175/datasets/cifar-10-batches-py'
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # subsample the data
```



```

# subsample the data
mask = list(range(num_training, num_training + num_validation))
X_val = X_train[mask]
y_val = y_train[mask]
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# Normalize the data: subtract the mean image
mean_image = np.mean(X_train, axis = 0)
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image

# add bias dimension and transform into columns
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev

# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIFAR10_data(
)
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)

```

```
Train data shape: (49000, 3073)
Train labels shape: (49000,)
Validation data shape: (1000, 3073)
Validation labels shape: (1000,)
Test data shape: (1000, 3073)
Test labels shape: (1000,)
dev data shape: (500, 3073)
dev labels shape: (500,)
```

## Softmax Classifier

Your code for this section will all be written inside **cs175/classifiers/softmax.py**.

In [3]:

```
# First implement the naive softmax loss function with nested loops.
# Open the file cs175/classifiers/softmax.py and implement the
# softmax_loss_naive function.

from cs175.classifiers.softmax import softmax_loss_naive
import time

# Generate a random softmax weight matrix and use it to compute the loss.
W = np.random.randn(3073, 10) * 0.0001
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As a rough sanity check, our loss should be something close to -log(0.1).
print('loss: %f' % loss)
print('sanity check: %f' % (-np.log(0.1)))
```

```
loss: 2.319922
sanity check: 2.302585
```

## Inline Question 1:

Why do we expect our loss to be close to  $-\log(0.1)$ ? Explain briefly.\*\*

**Your answer:** *We can interpret loss function as unnormalized log probabilities for each class. Weight vector was initialized with small values and since there are ten classes in our case, the softmax function will be closer to 0.1 assuming softmax function for each classes are somewhat similar.*

In [4]:

```
# Complete the implementation of softmax_loss_naive and implement a (naive)  
# version of the gradient that uses nested loops.  
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)  
  
# As we did for the SVM, use numeric gradient checking as a debugging tool.  
# The numeric gradient should be close to the analytic gradient.  
from cs175.gradient_check import grad_check_sparse  
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]  
grad_numerical = grad_check_sparse(f, W, grad, 10)  
  
# similar to SVM case, do another gradient check with regularization  
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 5e1)  
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 5e1)[0]  
grad_numerical = grad_check_sparse(f, W, grad, 10)
```

```
numerical: 1.355642 analytic: 1.355642, relative error: 1.589055e-08  
numerical: -3.840250 analytic: -3.840250, relative error: 6.119261e-09  
numerical: 1.354744 analytic: 1.354744, relative error: 3.931988e-08  
numerical: 1.624206 analytic: 1.624206, relative error: 3.678168e-09  
numerical: 1.104333 analytic: 1.104333, relative error: 3.453501e-09  
numerical: -0.159511 analytic: -0.159511, relative error: 1.599998e-07  
numerical: -1.923547 analytic: -1.923547, relative error: 3.107854e-09  
numerical: -0.528767 analytic: -0.528767, relative error: 3.132328e-08  
numerical: 1.760185 analytic: 1.760184, relative error: 4.535735e-08  
numerical: -2.439339 analytic: -2.439339, relative error: 2.625934e-08  
numerical: 1.041814 analytic: 1.047492, relative error: 2.717305e-03  
numerical: -1.555145 analytic: -1.557012, relative error: 5.998456e-04  
numerical: 0.421634 analytic: 0.415263, relative error: 7.612699e-03  
numerical: 0.388825 analytic: 0.381282, relative error: 9.794522e-03  
numerical: 1.616855 analytic: 1.622647, relative error: 1.787878e-03  
numerical: 3.097647 analytic: 3.095504, relative error: 3.460129e-04  
numerical: 0.077100 analytic: 0.078910, relative error: 1.160500e-02  
numerical: 2.392529 analytic: 2.394386, relative error: 3.880247e-04  
numerical: 0.393192 analytic: 0.396611, relative error: 4.329135e-03  
numerical: 1.001749 analytic: 0.996178, relative error: 2.787996e-03
```

In [5]:

```
# Now that we have a naive implementation of the softmax loss function and its g  
radient,  
# implement a vectorized version in softmax_loss_vectorized.  
# The two versions should compute the same results, but the vectorized version s  
hould be  
# much faster.  
tic = time.time()  
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)  
toc = time.time()  
print('naive loss: %e computed in %fs' % (loss_naive, toc - tic))  
  
from cs175.classifiers.softmax import softmax_loss_vectorized  
tic = time.time()  
loss_vectorized, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.00  
0005)  
toc = time.time()  
print('vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))  
  
# As we did for the SVM, we use the Frobenius norm to compare the two versions  
# of the gradient.  
grad_difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')  
print('Loss difference: %f' % np.abs(loss_naive - loss_vectorized))  
print('Gradient difference: %f' % grad_difference)
```

```
naive loss: 2.319922e+00 computed in 0.084736s  
vectorized loss: 2.319922e+00 computed in 0.005200s  
Loss difference: 0.000000  
Gradient difference: 0.000000
```

In [6]:

```
# Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of over 0.35 on the validation set.
from cs175.classifiers import Softmax
results = {}
best_val = -1
best_softmax = None
learning_rates = [1e-7, 5e-7]
regularization_strengths = [2.5e4, 5e4]

#####
# TODO:
# Use the validation set to set the learning rate and regularization strength.
# This should be identical to the validation that you did for the SVM; save
# the best trained softmax classifier in best_softmax.
#####
for lr in learning_rates:
    for reg in regularization_strengths:
        softmax=Softmax()
        softmax.train(X_train, y_train, learning_rate=lr, reg=reg, num_iters=800
        )

        y_train_pred = softmax.predict(X_train)
        y_val_pred=softmax.predict(X_val)

        accuracy_train=np.mean(y_train==y_train_pred)
        accuracy_val=np.mean(y_val==y_val_pred)

        results[(lr, reg)] = (accuracy_train, accuracy_val)

        if accuracy_val > best_val:
            best_val = accuracy_val
            best_softmax = softmax
#####
#                                     END OF YOUR CODE
#####

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val
)
```

```
lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.317061 val accuracy: 0.337000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.329755 val accuracy: 0.348000
lr 5.000000e-07 reg 2.500000e+04 train accuracy: 0.349184 val accuracy: 0.365000
lr 5.000000e-07 reg 5.000000e+04 train accuracy: 0.323857 val accuracy: 0.340000
best validation accuracy achieved during cross-validation: 0.365000
```

In [7]:

```
# evaluate on test set
# Evaluate the best softmax on test set
y_test_pred = best_softmax.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('softmax on raw pixels final test set accuracy: %f' % (test_accuracy, ))

softmax on raw pixels final test set accuracy: 0.352000
```

In [8]:

```
# Visualize the learned weights for each class
w = best_softmax.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)

w_min, w_max = np.min(w), np.max(w)

classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```

