

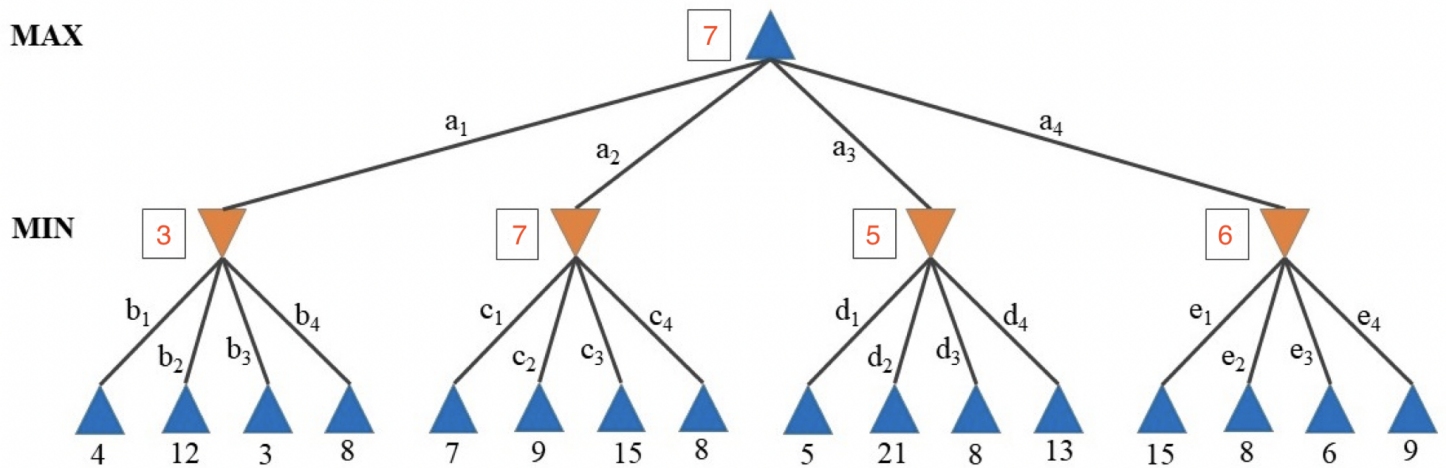
50.021 Artificial Intelligence

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Homework 6

1 Minimax Search

(a)



At the max level, the value is 7.

At the min level, the values are as follows:

- i) $a_1 = 3$
- ii) $a_2 = 7$
- iii) $a_3 = 5$
- iv) $a_4 = 6$

(b)

For all the MIN branches, the minimum possible value (**min**) is initially set to ∞ .

Exploring a_1 :

- When comparing between **min** and b_1 : $\text{MIN}(\infty, 4)$, b_1 is the smaller value. So the minimum changes from ∞ to 4.
- When comparing between **min** and b_2 : $\text{MIN}(4, 12)$, 4 is still the smaller value. So the minimum is unchanged.
- When comparing between **min** and b_3 : $\text{MIN}(4, 3)$, b_3 is the smaller value. So the minimum changes from 4 to 3.
- When comparing between **min** and b_4 : $\text{MIN}(3, 8)$, 3 is still the smaller value. So the minimum is unchanged.

3 is the minimum value of branch a_1 .

Exploring a_2 :

- When comparing between **min** and c_1 : $\text{MIN}(\infty, 7)$, c_1 is the smaller value. So the minimum changes from ∞ to 7.

- When comparing between **min** and c_2 : $\text{MIN}(7, 9)$, 7 is still the smaller value. So the minimum is unchanged.
- When comparing between **min** and c_3 : $\text{MIN}(7, 15)$, 7 is still the smaller value. So the minimum is unchanged.
- When comparing between **min** and c_4 : $\text{MIN}(7, 8)$, 7 is still the smaller value. So the minimum is unchanged.

7 is the minimum value of branch a_2 .

Exploring a_3 :

- When comparing between **min** and d_1 : $\text{MIN}(\infty, 5)$, d_1 is the smaller value. So the minimum changes from ∞ to 5.
- When comparing between **min** and d_2 : $\text{MIN}(5, 21)$, 5 is still the smaller value. So the minimum is unchanged.
- When comparing between **min** and d_3 : $\text{MIN}(5, 8)$, 5 is still the smaller value. So the minimum is unchanged.
- When comparing between **min** and d_4 : $\text{MIN}(5, 13)$, 5 is still the smaller value. So the minimum is unchanged.

5 is the minimum value of branch a_3 .

Exploring a_4 :

- When comparing between **min** and e_1 : $\text{MIN}(\infty, 15)$, e_1 is the smaller value. So the minimum changes from ∞ to 15.
- When comparing between **min** and e_2 : $\text{MIN}(15, 8)$, e_2 is the smaller value. So the minimum changes from 15 to 8.
- When comparing between **min** and e_3 : $\text{MIN}(8, 6)$, e_3 is the smaller value. So the minimum changes from 8 to 6.
- When comparing between **min** and e_4 : $\text{MIN}(6, 9)$, 6 is still the smaller value. So the minimum is unchanged.

6 is the minimum value of branch a_4 .

For the MAX branches, the MAXIMUM possible value (**max**) is initially set to $-\infty$.

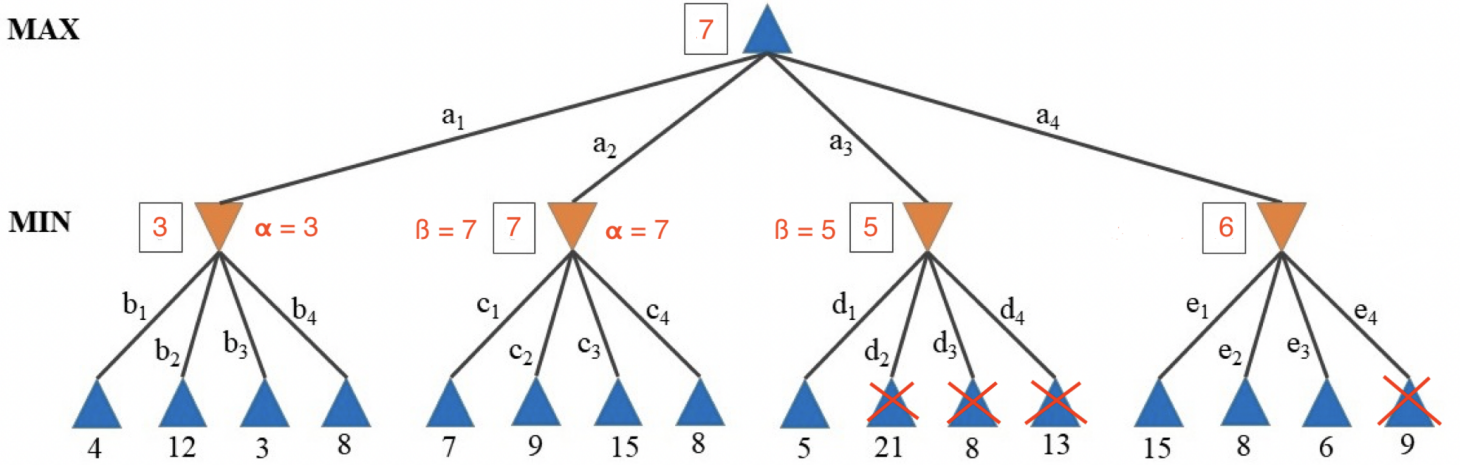
- When comparing between **max** and a_1 : $\text{MAX}(-\infty, 3)$, a_1 is the larger value. So the maximum changes from $-\infty$ to 3.
- When comparing between **max** and a_2 : $\text{MAX}(3, 7)$, a_2 is the larger value. So the maximum changes from 3 to 7.
- When comparing between **max** and a_3 : $\text{MAX}(7, 5)$, 7 is still the larger value. So the maximum is unchanged.
- When comparing between **max** and a_4 : $\text{MAX}(7, 6)$, 7 is still the larger value. So the maximum is unchanged.

7 (a_2) is the maximum value of the MAX branches.

(c) The chosen move is a_2 .

2 $\alpha - \beta$ Pruning

(a)



The following actions are pruned: d_2 , d_3 , d_4 and e_4 .

(b)

For all the MIN branches, β is initialised to ∞ and for all the MAX branches, α is initialised to $-\infty$.

Exploring a_1 :

- We first explore b_1 . $\beta = \text{MIN}(\beta, b_1) = \min(\infty, 4) = 4$
- Similarly in b_2 , $\beta = \text{MIN}(\beta, b_2) = \text{MIN}(4, 12) = 4$
- In b_3 , $\beta = \text{MIN}(\beta, b_3) = \text{MIN}(4, 3) = 3$
- Finally, in b_4 , $\beta = \text{MIN}(\beta, b_4) = \text{MIN}(3, 8) = 3$

After exploring a_1 , $\alpha = 3$.

Exploring a_2 :

- We first explore c_1 . $\beta = \text{MIN}(\beta, b_1) = \min(\infty, 7) = 7$
- The same steps are repeated for b_2, b_3 and b_4 . The minimum (β) remains 7.
- As α is not greater or equal to β , so no pruning will happen.
- Finally, the value of α changes from 3 to 7 as $\max(\alpha, a_2)$ is 7.

After exploring a_2 , $\alpha = 7$.

β is reset to ∞ before we explore a_3 .

Exploring a_3 :

- We first explore d_1 . $\beta = \text{MIN}(\beta, d_1) = \min(\infty, 5) = 5$.
- The same steps are repeated for b_2, b_3 and b_4 . The minimum (β) remains 5.
- As α is greater or equal to β ($7 \geq 5$), pruning happens. All values which appear after β (5) is pruned.
- The d_2, d_3 and d_4 branches are pruned.

After exploring a_3 , α is still 7.

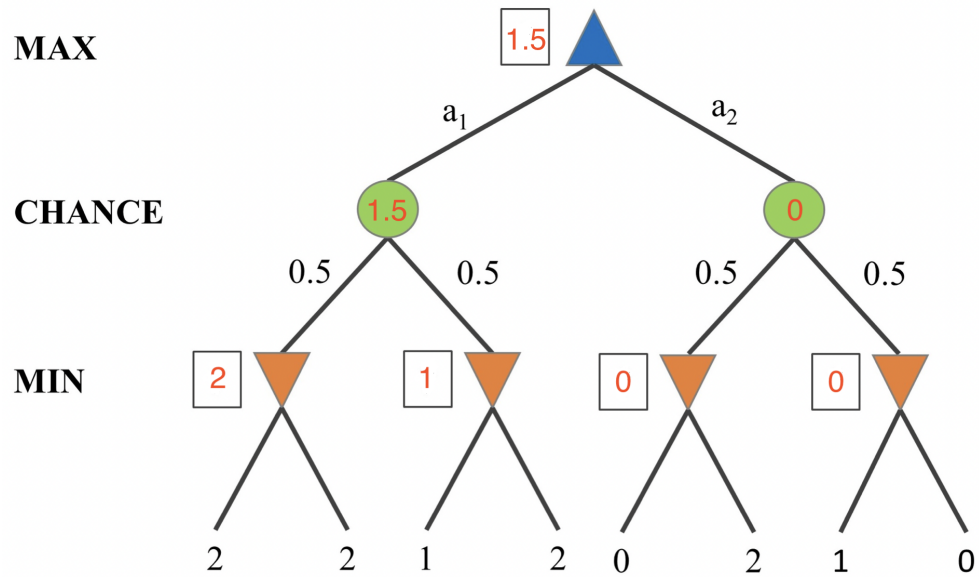
β is reset to ∞ before we explore a_4 .

Exploring a_4 :

- We first explore e_1 . $\beta = \text{MIN}(\beta, e_1) = \min(\infty, 15) = 15$.
- The same steps are repeated for b_2, b_3 and b_4 . The minimum (β) becomes 6 at e_3 .
- As α is greater or equal to β ($7 \geq 6$), pruning happens. All values which appear after β (6) is pruned.
- The e_4 branch is pruned.
- The value at the MAX branch remains 7.

3 ExpectiMinimax Search

(a)



At the max level, the value is 1.5.

At the chance level, the values are as follows:

- $a_1 = 1.5$
- $a_2 = 0$

At the min level, the values are as follows: 2, 1, 0, 0.

(b) The chosen move is a_1 .