MHWI Blast Damage Model

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1 Introduction

The mathematical description of how status effects such as blast is well-described in *How Status Works: Definitive Status Guide MHW/Iceborne* (https://www.youtube.com/watch?v=iIPfkvvbGwY). However, I will attempt to summarize it here, in addition to clarifying some features of this game mechanic not mentioned in the video.

Status effects can be applied to monsters in a variety of ways, such as attacking while using a weapon with a poison/paralysis/sleep/blast stat.

When a status is applied to a monster, that status is "built-up". Only when this internal status application counter reaches certain thresholds does the status effect trigger (or *proc*).

1.1 Status Application

All monsters have unique tables regarding status effect application. Buildup usually involves four particular values:

- *Base Tolerance* (or simply *base*): The threshold before the first proc.
- *Tolerance Buildup* (or simply *buildup*): Every time we hit the threshold, we increase the new threshold by this value.
- *Tolerance Cap* (or simply *cap*): The maximum threshold.
- *Decay*: The internal status application counter decays at a constant rate over time.

To illustrate how these values work, let's consider Dodogama's sleep values:

- Base: 150
- Buildup: 100
- Cap: 550
- Decay: $-\frac{5}{10}$ per second

Additionally, let's specifically consider the following quest values:

- Base Multiplier: 1.25
- Buildup/Cap Multiplier: 1.90

Thus, our effective sleep values for that particular quest are:

- Base: $150 \times 1.25 = 187.5$
- Buildup: $100 \times 1.90 = 190$
- Cap: $550 \times 1.90 = 1045$

- Decay: $-\frac{5}{10}$ per second

Our first threshold is 188, so we must apply 188 sleep in order to get the first sleep proc.

After the first sleep proc, our new threshold is $188 + 190 = 378$, so we must apply 378 sleep for the second proc.

After the second sleep proc, our new threshold is $378 + 190 = 568$, so we must apply 568 sleep for the third proc.

This follows an arithmetic progression up until the cap, where it is clipped:

$$188 \longrightarrow 378 \longrightarrow 568 \longrightarrow 758 \longrightarrow 948 \longrightarrow \cancel{1138}\, 1045 \longrightarrow 1045 \longrightarrow 1045 \longrightarrow \cdots$$

IMPORTANT: This is a draft. I actually don't know how the rounding works, or other variables that status decay depends on, but my blast model is a continuous model, and blast doesn't decay, so it doesn't affect the remainder of this document.

1.2 Blast Buildup

Blast is a status effect that *does not decay* (i.e. decay is 0 blast per second).

Other than that, blast base/buildup/cap values work as expected.

1.3 Blast Procs

A blast proc deals fixed damage, regardless of monster (100 in Low Rank, 120 in High Rank, and 300 in Master Rank). The damage is dealt instantly, and the status can immediately continue being built-up.

2 The Continuous Blast Damage Model

2.1 Input Variables

Our model is based on the idea that a single specified attack always deals a predictable amount of average damage and blast application each time the attack is performed.

For example, if a greatsword draw attack dealt 100 raw damage and 5 status buildup on average, we can expect that using the attack n-times would deal $n \times 100$ raw damage and $n \times 5$ status buildup on average.

These variables shall be:

- ρ (average raw damage of an attack)
- σ (average status application of an attack)

Conceptually, we can try counting how many of the specified attack (such as the greatsword draw attack) we have to deal before we slay the monster, so we consider monster health to be another variable:

- H (monster total health)

If we ignore blast damage and assume all our damage comes from raw, it is trivial to see that we'd require $\frac{H}{\rho}$ uses of the specified attack in order to kill the monster. However, it gets tricky when we factor in blast.

The variables related to blast are:

- a_1 (blast base)
- d (blast buildup)
- c (blast cap)
- P (blast damage *per proc*)

Blast does not decay and blast procs do not cause status-application downtime (like how you cannot apply poison status if a monster is actively suffering from poison). This means that blast damage prediction can be completely independent of time, and dependent onnly on parameters relating to the sequence of attacks performed on the monster.

Our model will attempt to find the average blast damage per attack.

2.2 Pre-Cap Model Derivation

The following two equations are given for arithmetic progressions:

$$a_n = a_1 + (n-1)d \tag{1}$$
$$S_n = \frac{n}{2}(a_1 + a_n) \tag{2}$$

where a_1 is the first element of the sequence, a_n is the n^{th} element, $d = a_{i+1} - a_i$, and S_n is the sum of the first n elements.

Equivalently, a_n is the threshold for the n^{th} blast proc, and S_n is the total blast that must be applied to reach the n^{th} blast proc.

Eliminating a_n:

$$\begin{aligned} S_n &= \frac{n}{2}(a_1 + a_1 + (n-1)d) && (3) \\ &= \frac{n}{2}(2a_1 + (n-1)d) && (4) \end{aligned}$$

Solving for number of procs n:

$$\begin{aligned} 2S_n &= n(2a_1 + (n-1)d) && (5) \\ &= 2na_1 + n^2 d - nd && (6) \end{aligned}$$

$$\begin{aligned} 0 &= 2na_1 + n^2 d - nd - 2S_n && (7) \\ &= n^2 d + n(2a_1 - d) - 2S_n && (8) \end{aligned}$$

Since $n \geq 0$:

$$\begin{aligned} n &= \frac{-(2a_1 - d) + \sqrt{(2a_1 - d)^2 - 4d \times (-2S_n)}}{2d} && (9) \\ &= \frac{d - 2a_1 + \sqrt{(2a_1 - d)^2 + 8dS_n}}{2d} && (10) \end{aligned}$$

Let R_n and B_n denote the total raw and blast damage (respectively), given n blast procs.

Since P is blast damage per proc, total blast damage is simply:

$$B_n = Pn \tag{11}$$

Thus, we can calculate B as:

$$B_n = P\frac{d - 2a_1 + \sqrt{(2a_1 - d)^2 + 8dS_n}}{2d} \tag{12}$$

Since we know that we apply σ status for every ρ raw damage, it is trivial to calculate R_n using:

$$\frac{\rho}{\sigma} = \frac{R_n}{S_n} \tag{13}$$

Thus, we can introduce R_n into our equation. Additionally, we no longer care about counting blast procs, so we can simplify the notation of total raw and blast damage to R and B:

$$B = P\frac{d - 2a_1 + \sqrt{(2a_1 - d)^2 + 8Rd\frac{\sigma}{\rho}}}{2d} \tag{14}$$

In order to slay the monster, total raw and blast damage must add to equal the total health pool H:

$$H = R + B \tag{15}$$

Substituting into R:

$$B = P\frac{d - 2a_1 + \sqrt{(2a_1 - d)^2 + 8(H - B)d\frac{\sigma}{\rho}}}{2d} \tag{16}$$

$$\frac{2Bd}{P} = d - 2a_1 + \sqrt{(2a_1 - d)^2 + 8(H - B)d\frac{\sigma}{\rho}} \tag{17}$$

$$\frac{2Bd}{P} + 2a_1 - d = \sqrt{(2a_1 - d)^2 + 8(H - B)d\frac{\sigma}{\rho}} \tag{18}$$

$$\left(\frac{2Bd}{P} + 2a_1 - d\right)^2 = (2a_1 - d)^2 + 8(H - B)d\frac{\sigma}{\rho} \tag{19}$$

$$B^2\frac{4d^2}{P^2} + 2\frac{2Bd}{P}(2a_1 - d) + \cancel{(2a_1 - d)^2} = \cancel{(2a_1 - d)^2} + 8Hd\frac{\sigma}{\rho} - 8Bd\frac{\sigma}{\rho} \tag{20}$$

$$B^2\frac{4d^2}{P^2} + B\frac{4d}{P}(2a_1 - d) = 8Hd\frac{\sigma}{\rho} - 8Bd\frac{\sigma}{\rho} \tag{21}$$

$$B^2\frac{4d^2}{P^2} + B\left(\frac{4d}{P}(2a_1 - d) + 8d\frac{\sigma}{\rho}\right) = 8Hd\frac{\sigma}{\rho} \tag{22}$$

$$B^2\frac{4d^2}{P^2} + B4d\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) = \frac{8H\sigma d}{\rho} \tag{23}$$

$$B^2 + B\frac{P^2}{d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) = \frac{2HP^2\sigma}{\rho d} \tag{24}$$

$$B^2 + 2B\frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) = \frac{2HP^2\sigma}{\rho d} \tag{25}$$

$$B^2 + 2B\frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) + \left(\frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 = \frac{2HP^2\sigma}{\rho d} + \left(\frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 \tag{26}$$

$$\left(B + \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 = \frac{2HP^2\sigma}{\rho d} + \left(\frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 \tag{27}$$

$$B + \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) = P\sqrt{\frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2} \tag{28}$$

$$B = P\sqrt{\frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2} - \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) \tag{29}$$

Thus, we now have an expression for total blast damage, given the monster's total health pool.

Now, we want to find out the average amount of blast damage dealt per attack, given the attack's raw damage.

Let β_p denote the average blast damage per attack, *before hitting the blast cap.*

It is trivial to use the ratio between R and B:

$$\frac{\beta_p}{\rho} = \frac{B}{R} = \frac{B}{H - B} \tag{30}$$

Thus, we get an expression for β_p in terms of B, H, and ρ:

$$\beta_p = \rho\frac{B}{H - B} \tag{31}$$

Since we can use (29) to calculate B in terms of model input values, we now have an expression for calculating average blast damage per attack, given only our model input values.

For this model, it is useful to think of average blast damage per attack as a function of H, so we shall rewrite β_P as a function $[0, \infty) \to \mathbb{R}$ such that:

$$\beta_p(H) = \rho \frac{B(H)}{H - B(H)} \tag{32}$$

$\beta_p(H)$ is only valid up until H reaches the blast cap.

2.3 Blast Cap Derivation

The blast cap can be expressed as:

$$a_n \leq c \tag{33}$$

So to find where the pre-cap portion of the model stops, we must find the range of H in which (33) is true.

We start by substituting (1) into (2) via. n:

$$a_n - a_1 = (n-1)d \tag{34}$$

$$n = \frac{a_n - a_1}{d} + 1 \tag{35}$$

$$S_n = \left(\frac{a_n - a_1}{d} + 1\right)\frac{1}{2}(a_1 + a_n) \tag{36}$$

Substituting (13) into (36) via. S_n:

$$R_n \frac{\sigma}{\rho} = \left(\frac{a_n - a_1}{d} + 1\right)\frac{1}{2}(a_1 + a_n) \tag{37}$$

$$2R_n \frac{\sigma}{\rho} = \left(\frac{a_n - a_1}{d} + 1\right)(a_1 + a_n) \tag{38}$$

$$2R_n \frac{\sigma}{\rho} = \frac{a_n^2 - a_1^2}{d} + a_1 + a_n \tag{39}$$

$$0 = \frac{a_n^2}{d} - \frac{a_1^2}{d} + a_1 + a_n - 2R_n \frac{\sigma}{\rho} \tag{40}$$

$$0 = a_n^2 + a_n d + \left(a_1 d - a_1^2 - 2R_n \frac{\sigma}{\rho} d\right) \tag{41}$$

Since $a_n \geq 0$:

$$a_n = \frac{-d + \sqrt{d^2 - 4\left(a_1 d - a_1^2 - 2R_n \frac{\sigma}{\rho} d\right)}}{2} \tag{42}$$

$$a_n = \frac{-d + \sqrt{d^2 + 4\left(a_1^2 - a_1 d + 2R_n \frac{\sigma}{\rho} d\right)}}{2} \tag{43}$$

We can now introduce B and H using (15), and substitute it into the blast cap condition (33):

$$c \geq \frac{-d + \sqrt{d^2 + 4\left(a_1^2 - a_1 d + 2(H-B)\frac{\sigma}{\rho}d\right)}}{2} \tag{44}$$

$$2c + d \geq \sqrt{d^2 + 4\left(a_1^2 - a_1 d + 2(H-B)\frac{\sigma}{\rho}d\right)} \tag{45}$$

$$(2c+d)^2 \geq d^2 + 4\left(a_1^2 - a_1 d + 2(H-B)\frac{\sigma}{\rho}d\right) \tag{46}$$

$$\frac{(2c+d)^2 - d^2}{4} \geq a_1^2 - a_1 d + 2(H-B)\frac{\sigma}{\rho}d \tag{47}$$

$$\frac{(2c+d)^2 - d^2}{4} - a_1^2 + a_1 d \geq 2(H-B)\frac{\sigma}{\rho}d \tag{48}$$

$$\frac{\rho}{2\sigma d}\left(\frac{(2c+d)^2 - d^2}{4} - a_1^2 + a_1 d\right) \geq H - B \tag{49}$$

$$\frac{\rho}{2\sigma d}\left(\frac{4c^2 + 4cd}{4} - a_1^2 + a_1 d\right) \geq H - B \tag{50}$$

$$\frac{\rho}{2\sigma d}\left(c^2 + cd + a_1 d - a_1^2\right) \geq H - B \tag{51}$$

Substituting (29) into (51) via. B:

$$\frac{\rho}{2\sigma d}\left(c^2 + cd + a_1 d - a_1^2\right) \geq H - \left(P\sqrt{\frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2} - \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right) \tag{52}$$

$$\frac{\rho}{2\sigma d}\left(c^2 + cd + a_1 d - a_1^2\right) - \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) \geq H - P\sqrt{\frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2} \tag{53}$$

Let X be the left side of (53):

$$X := \frac{\rho}{2\sigma d}\left(c^2 + cd + a_1 d - a_1^2\right) - \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) \tag{54}$$

Isolating H in (53):

$$X \geq H - P\sqrt{\frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2} \tag{55}$$

$$\frac{H - X}{P} \leq \sqrt{\frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2} \tag{56}$$

$$\frac{(H - X)^2}{P^2} \leq \frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 \tag{57}$$

$$H^2 - X^2 \leq H\frac{2P^2\sigma}{\rho d} + \left(\frac{P}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 \tag{58}$$

$$H^2 - H\frac{2P^2\sigma}{\rho d} \leq \left(\frac{P}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 + X^2 \tag{59}$$

$$H^2 - H\frac{2P^2\sigma}{\rho d} + \left(\frac{P^2\sigma}{\rho d}\right)^2 \leq \left(\frac{P}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 + X^2 + \left(\frac{P^2\sigma}{\rho d}\right)^2 \tag{60}$$

$$\left(H - \frac{P^2\sigma}{\rho d}\right)^2 \leq \left(\frac{P}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 + X^2 + \left(\frac{P^2\sigma}{\rho d}\right)^2 \tag{61}$$

$$H \leq \frac{P^2\sigma}{\rho d} + \sqrt{\left(\frac{P}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 + X^2 + \left(\frac{P^2\sigma}{\rho d}\right)^2} \tag{62}$$

Thus, we have our range for H in which the pre-cap part of the model (derived in subsection 2.2) is valid.

We shall define C to be the exact value of H where we hit the blast cap:

$$C \coloneqq \frac{P^2\sigma}{\rho d} + \sqrt{\left(\frac{P}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 + X^2 + \left(\frac{P^2\sigma}{\rho d}\right)^2} \tag{63}$$

2.4 Post-Cap Model Derivation

At this point, the status buildup threshold remains constant at c.

Since we have a constant threshold, then if we ignore pre-cap damage, any status buildup dealt will be linearly proportional to the damage it deals.

We will start by forming a relation between some number of hits k, average status per hit σ, and a blast threshold c:

$$k \coloneqq \frac{c}{\sigma} \tag{64}$$

Thus, k hits is required to cause a blast proc.

Let β'_c be the effective average blast damage dealt per hit, but *only valid to damage dealt after C damage has been dealt to the monster*:

$$\beta'_c \coloneqq \frac{P}{k} = \frac{\sigma P}{c} \tag{65}$$

Note that β'_c does not depend on monster health.

We know that each hit after reaching blast cap deals an average of $\rho + \beta'_c$ damage, and we must distribute this damage in the remaining $H - C$ health of the monster.

This allows us to construct an expression involving the actual effective average blast damage dealt per hit β_c, which *includes damage dealt before C damage was dealt to the monster*. This expression is simply a weighted average:

$$\rho + \beta_c = \frac{C(\rho + \beta_p(C)) + (H - C)(\rho + \beta_c')}{H} \tag{66}$$

$$\beta_c = \frac{C}{H}(\rho + \beta_p(C)) + \frac{H - C}{H}\left(\rho + \frac{\sigma P}{c}\right) - \rho \tag{67}$$

Rewriting β_c as a function $[C, \infty) \to \mathbb{R}$ such that:

$$\beta_c(H) = \frac{C}{H}(\rho + \beta_p(C)) + \frac{H - C}{H}\left(\rho + \frac{\sigma P}{c}\right) - \rho \tag{68}$$

2.5 Summary

Our model takes the following input variables:

- ρ (average raw damage of an attack)
- σ (average status application of an attack)
- H (monster total health)
- a_1 (blast base)
- d (blast buildup)
- c (blast cap)
- P (blast damage *per proc*)

The *effective average blast damage per hit* can be calculated as a function of monster health $\beta : [0, \infty) \to \mathbb{R}$ such that:

$$\beta(H) = \begin{cases} \beta_p(H) & \text{if } H \leq C \\ \beta_c(H) & \text{if } H > C \end{cases} \tag{69}$$

Functions $\beta_p : [0, \infty) \to \mathbb{R}$ and $\beta_c : [C, \infty) \to \mathbb{R}$ are defined as:

$$\beta_p(H) = \rho \frac{B(H)}{H - B(H)} \tag{70}$$

$$\beta_c(H) = \frac{C}{H}(\rho + \beta_p(C)) + \frac{H - C}{H}\left(\rho + \frac{\sigma P}{c}\right) - \rho \tag{71}$$

C is the maximum amount of health before reaching the blast cap, and X is an anonymous helper variable:

$$C = \frac{P^2\sigma}{\rho d} + \sqrt{\left(\frac{P}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2 + X^2 + \left(\frac{P^2\sigma}{\rho d}\right)^2} \tag{72}$$

$$X = \frac{\rho}{2\sigma d}\left(c^2 + cd + a_1 d - a_1^2\right) - \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) \tag{73}$$

$B(H)$ is the total blast damage dealt when we've dealt a total of H damage to the monster:

$$B(H) = P\sqrt{\frac{2H\sigma}{\rho d} + \left(\frac{1}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right)\right)^2} - \frac{P^2}{2d}\left(\frac{2a_1 - d}{P} + \frac{2\sigma}{\rho}\right) \tag{74}$$

2.6 Model Extensions

I left out certain interpretations of the model in order to keep discussion as simple as possible. Now that the model has been introduced, we shall expand on these additional interpretations.

2.6.1 Combos

It turns out that if you can predict the average amount of raw damage and status application of a single attack, you can also do it for a given sequence of attacks.

Thus, ρ and σ can instead be interpreted as values for a given sequence attacks rather than just a single attack.

2.6.2 Target Health

H doesn't actually have to be the monster's total health pool if we don't intend on slaying it. For example, we can instead make H the damage required in order to capture the monster.

Multiplayer might also be considered here since multiple players share the full health pool of the monster. For example, if we assume four perfectly equal players, then we might want to divide the full 4-player health pool by 4.

2.7 Do we ever hit blast cap?

(This section will be written later!)

2.8 Limitations

(This section will be written later!)

3 The Discrete-Blasts Model

(This section will be written later!)