A Webcam-Based Random Quantum Generation: Photon Arrival Perspective

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Abstract

The fundamental quantum nature of light dictates that photons arrive at a detector randomly, following Poisson statistics, even from a perfectly stable source. This phenomenon, known as photon shot noise, constitutes an irreducible source of physical randomness. This article provides a theoretical foundation and quantifies how this intrinsic quantum flickering dominates the digital output of a standard consumer webcam. Through a detailed model of a common light source and its interaction with a typical sensor, we predict pixel noise due to photon shot noise. This prediction is then directly compared with empirical webcam measurements, demonstrating a precise correspondence between observed pixel value fluctuations and underlying quantum effects. We show that even in a brightly lit environment, pixel arrays are overwhelmingly dominated by quantum noise, highlighting how ubiquitous imaging devices can serve as accessible platforms for observing true quantum randomness, particularly by focusing on noise-dominant color channels.

1. Motivation: The Quest for True Randomness and Observable Quantum Effects

True Random Number Generators (TRNGs) are indispensable for modern cryptography, robust scientific simulations, and secure communication protocols. Unlike pseudo-random number generators (PRNGs), which produce sequences based on deterministic algorithms, TRNGs extract randomness from inherently unpredictable physical processes. While many TRNGs rely on specialized quantum hardware or complex physical phenomena, an accessible and ubiquitous source of true randomness exists in the quantum fluctuations of light itself.

Webcams, present in nearly all contemporary computing devices, are sophisticated arrays of photon detectors. Each pixel in a webcam's sensor acts as a tiny light-sensitive well, counting incident photons over a specific exposure period. The motivation for this work is to establish a clear theoretical understanding of how the intrinsic quantum randomness inherent in photon arrival directly influences the digital output of these consumer-grade sensors. By quantifying this effect, we aim to demonstrate how a standard webcam can serve as a foundation for observing fundamental quantum phenomena in an everyday setting.

2. Method Explanation: The Quantum Physics of Photon Arrival

2.1 Light's Granular Nature: From Planck to Poisson

The understanding that light is not a continuous wave but composed of discrete energy packets, or photons, revolutionized physics in the early 20th century. Max Planck's work on black-body radiation (1900) [1] introduced energy quantization, and Albert Einstein's explanation of the photoelectric effect (1905) [2] firmly established the photon as a fundamental particle.

Crucially, the emission and absorption of these individual photons are probabilistic quantum events. Even when a light source operates at a perfectly constant average intensity (e.g., an ideal LED lamp or ambient room light), the exact number of photons arriving at a specific detector (like a camera pixel) within a given time interval is not fixed. Instead, it fluctuates randomly around the average [4].

2.2 Photon Shot Noise: The Irreducible Randomness

The statistical nature of these fluctuations is precisely described by the **Poisson distribution**. For a light source like those found in an office, the number of photons detected (N) in a fixed time interval follows this distribution [4, 5].

A key property of the Poisson distribution is that its variance (σ^2_{photon}) is equal to its mean (μ or N):

$$\sigma^2_{\text{photon}} = \mu$$

Therefore, the standard deviation (σ_{photon}), which quantifies the typical size of the random fluctuation (in photons), is the square root of the mean number of photons:

$$\sigma_{\text{photon}} = \sqrt{\mu} \quad \text{or} \quad \sigma_{\text{photon}} = \sqrt{N}$$

This $\sqrt{N}$ fluctuation is known as **photon shot noise**. It is not caused by imperfections in the light source or the detector (e.g., as first analyzed by Walter Schottky in 1918 for electron emission [3]); it is an irreducible, fundamental consequence of the quantum mechanics of light. No matter how stable the light source or how perfect the sensor, this $\sqrt{N}$ uncertainty in the photon count will always be present, providing a true source of physical randomness.

2.2.1 Quantifying Quantum Flicker Domination

A pixel's digital output is mapped to a specific digital value (a "bin") based on its mean photon count. However, due to photon shot noise, the actual photon count for any given frame will fluctuate, potentially causing the pixel's value to fall into an adjacent "bin" (e.g., reading 99 or 101 when the average is 100). This phenomenon is referred to as quantum flickering, where the value "spreads on two bins" or causes visible "stripes" of color dither.

The underlying distribution of photon counts, for sufficiently large N, can be approximated by a Gaussian distribution. This Gaussian "smears" across the discrete digital bins that represent the pixel's output values. The Least Significant Bit (LSB) of a pixel value (which alternates between 0 for even bins and 1 for odd bins) is particularly sensitive to these fluctuations.

Let Δ_{LSB} be the number of photons corresponding to one digital gradation (one LSB). The probability that a pixel's measured photon count falls within its intended digital bin (i.e., its value remains "stable" and its LSB does not flip) is given by the error function:

$$P_{\text{stable}} = \text{erf}\left(\frac{\Delta_{\text{LSB}}/2}{\sqrt{2}\sigma_{\text{photon}}}\right)$$

where $\sigma_{\text{photon}} = \sqrt{N}$ is the photon shot noise for the mean photon count N associated with that pixel value. The term $\Delta_{\text{LSB}}/2$ represents the half-width of a digital bin in photon units.

The probability of quantum flicker (the value falling into another bin, causing the LSB to flip) is $P_{\text{flicker}} = 1 - P_{\text{stable}}$. For the quantum effect to strongly influence the pixel's output, leading to frequent LSB flips, the noise relative to the digital bin width must be significant. When the noise distribution is spread widely across many discrete LSB boundaries, the LSB of the pixel value effectively becomes a random coin flip, approaching a 0.5 probability for either state. This inherent quantum randomness can then be extracted.

3. Model and Case Study: Quantum Flicker from a 500 Lux Environment

3.1 Physical Model: Light Source and Sensor Parameters

For this case study, we consider a typical brightly lit indoor environment with an illuminance of **500 lux**. The webcam sensor is assumed to have pixels of size 1.3 µm x 1.3 µm, resulting in a pixel area (A_{px}) of $1.69 \times 10^{-12}\,\text{m}^2$ or $1.69\,\mu\text{m}^2$ [7]. The video frame rate is 30 frames per second, corresponding to an exposure time (t_{exp}) of 1/30 s.

3.1.1 Detailed Derivation of Photons per Unit Exposure per Unit Area

Following the methodology from [8], the mean number of photons (n_{ph}) incident on a sensor per unit exposure (lx-s) per unit area (m^2) can be estimated. This derivation assumes a Daylight illuminant (D53) and considers the photopic eye response function ($V(\lambda)$).

The mean energy of photons (E_m) in the visible range (taken as 395 nm to 718 nm, with a mean wavelength $\lambda_{avg} = 556.5\,\mathrm{nm}$) is calculated using Planck's constant ($h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$) and the speed of light ($c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$):

$$E_m = \frac{h \cdot c}{\lambda_{avg}} \text{ joules/photon}$$

$$E_m = \frac{(6.626 \times 10^{-34}\,\mathrm{J\,s}) \cdot (3 \times 10^8\,\mathrm{m\,s^{-1}})}{556.5 \times 10^{-9}\,\mathrm{m}} \approx 3.569 \times 10^{-19}\,\mathrm{J}$$

The integral of the photopic function $V(\lambda)$ from 380 nm to 780 nm is given as 113.042. Using these values, the number of photons per lx-s per m^2 (n_{ph}) is derived:

$$n_{ph} = \frac{\lambda_2 - \lambda_1}{E_m} \cdot \frac{1}{683 \cdot \int_{380_{nm}}^{780_{nm}} V(\lambda) d\lambda}, \frac{\text{photons}}{\mathrm{lx} \cdot \mathrm{s} \cdot \mathrm{m}^2}$$

Substituting the values ($\lambda_1 = 394.5\,\mathrm{nm}, \lambda_2 = 718.5\,\mathrm{nm}$):

$$n_{ph} = \frac{718.5 \times 10^{-9}\,\mathrm{m} - 394.5 \times 10^{-9}\,\mathrm{m}}{(3.569 \times 10^{-19}\,\mathrm{J}) \times 683.002 \times 113.042}$$

$$n_{ph} = \frac{324 \times 10^{-9}\,\mathrm{m}}{2.766 \times 10^{-14}\,\mathrm{J}} \approx 1.171 \times 10^{16}\,\text{photons/lx/s/m}^2$$

Converting to photons per lx-s per μm^2:

$$n_{ph} = 1.171 \times 10^{16}\,\text{photons/lx/s/m}^2 \times (1 \times 10^{-6}\,\mathrm{m\,\mu m^{-1}})^2 \approx 11\,710\,\text{photons/lx/s/}\mu\text{m}^2$$

The blog post notes that for a Daylight standard 'D' illuminant at 5300 K (D53), this value reduces to approximately 11 000 photons/lx/s/μm^2. We will use this more accurate estimate for our calculations.

3.1.2 Calculation of Total Photons per Pixel per Frame (N_{total})

The total number of photons incident on a single pixel per video frame (N_{total}) is calculated by multiplying the derived conversion factor ($n_{ph} = 11\,000$ photons/lx/s/μm^2) by the illuminance ($L = 500\,\mathrm{lx}$), exposure time ($t_{\text{exp}} = 1/30$ s), and pixel area ($A_{\text{px},} = 1.69\,\mu m^2$):

$$N_{\text{total}} = n_{ph} \times L \times t_{\text{exp}} \times A_{\text{px},}$$

$$N_{\text{total}} = 11000 \times 500 \text{ lx} \times (1/30 \text{ s}) \times 1.69$$

$$N_{\text{total}} \approx \mathbf{309833 \text{ photons}}$$

Thus, the total number of photons incident on a single pixel per video frame in a 500 lx environment is estimated to be approximately **309833 photons**. This value represents the total visible light photons incident on the pixel, which will then be filtered and detected by the individual color channels.

3.2 Photon Budget and Pixel Noise Prediction by Color (Model)

We distribute this total photon budget per pixel per frame (N_{total}) by color. While the 11 000 photons/lx/s/μm^2 factor is derived from the photopic (eye's) response, which peaks in the green region, it represents the overall visible light. For a typical white light source, the energy (and thus photon count) is distributed across the Red, Green, and Blue spectral bands. We use typical daylight energy distribution fractions for this model: Green $k_G = 0.45$, Red $k_R = 0.30$, and Blue $k_B = 0.25$.

The number of photons per channel is then:

$$N_G = k_G N_{\text{total}}, \qquad N_R = k_R N_{\text{total}}, \qquad N_B = k_B N_{\text{total}}.$$

- $N_{\text{Green}} = 0.45 \times 309833 \approx$ **139425 photons**

- $N_{\text{Red}} = 0.30 \times 309833 \approx$ **92950 photons**
- $N_{\text{Blue}} = 0.25 \times 309833 \approx$ **77458 photons**

The total photon budget per pixel per frame predicted by this physical model is approximately **309833 photons**.

Based on the instruction to use the formula $\sigma_{\text{pixel}} = \frac{100}{\sqrt{N}}$, where σ_{pixel} is the predicted pixel noise in Digital Numbers (DN) and N is the mean number of photons, the predicted pixel noise for each color channel is calculated. This model implies that the lower the photon count (N), the larger the sigma in pixel units (σ_{pixel}).

$$\sigma_{\text{pixel}} = \frac{100}{\sqrt{N}}.$$

- $\sigma_{\text{pixelGreen}} = \frac{100}{\sqrt{139425}} \approx \frac{100}{373.40} \approx$ **0.27 DN**
- $\sigma_{\text{pixelRed}} = \frac{100}{\sqrt{92950}} \approx \frac{100}{304.88} \approx$ **0.33 DN**
- $\sigma_{\text{pixelBlue}} = \frac{100}{\sqrt{77458}} \approx \frac{100}{278.31} \approx$ **0.36 DN**

3.3 Empirical Webcam Results and Comparison

** add here 0.33 per channel **

compute for white / black stripes of gasussian bias per sigma assuming it be 1 + a and 1 + b compute c for xor two pixels

make table

The full source code for `generator.py`, including setup instructions and usage details, can be found in the accompanying GitHub repository for this project.

References

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